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LAB 2

Problem 1

The purpose of problem 1 is to create a finite-impulse response filter. In part 1.1, the filter was designed so that only the middle component will pass through the filter, and the output had a slight delay. We were also told to find the three unknown filter coefficients for a system of equations. This was done by using matrix division to find the values. In part 1.2, we then created a plot for the magnitude response and added the frequency points w_1 , w_2 , and w_3 to the graph. In part 1.3, we created a plot of the input and output, and we also plotted $s(n)$ which was similar to the output signal. We also created a table that compared n with $s(n)$, $s(n-2)$, $y(n)$, $v(n)$, and $y_v(n)$. Lastly, in part 1.4, we compared the FIR filter to the IIR filter. We measured for the noise ratio and saw that the IIR filter had a big decrease compared to the FIR filter.

Problem 1.1

```
w1 = 0.05*pi;
```

```
w2 = 0.1*pi;
```

```
w3 = 0.2*pi;
```

```
B = [2*cos(2*w1) 2*cos(w1) 1; 2*cos(2*w2) 2*cos(w2) 1; 2*cos(2*w3) 2*cos(w3) 1];
```

```
Y = [0; 1; 0];
```

```
b = B\Y;
```

```
disp(b);
```

```
%output
```

```
-48.0477
```

```
172.6550
```

```
-249.6665
```

Problem 1.2

```
B = @(w) 2*b(1)*cos(2*w)+2*b(2)*cos(w)+b(3);
```

```
H = @(w) exp(-2*1i*w).*B(w);
```

```
n = linspace(0,0.25*pi);
```

```
figure;
```

```
plot(n,abs(H(n)),'-b');
```

```
hold on;
```

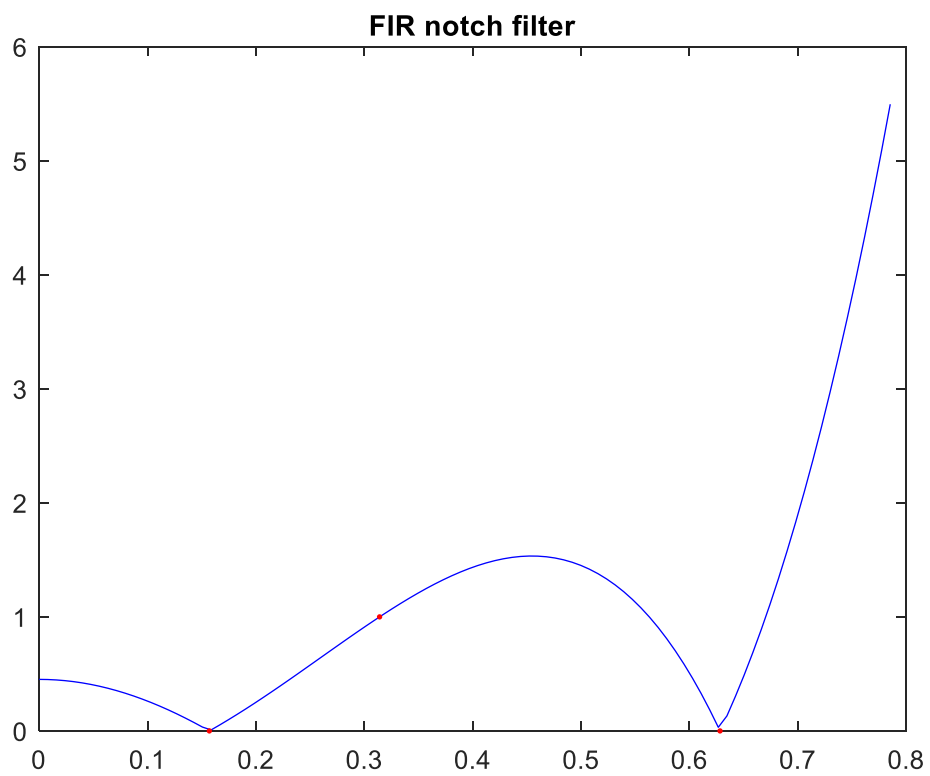
```
plot(w1,abs(H(w1)),'.r');
```

```
plot(w2,abs(H(w2)),'.r');
```

```
plot(w3,abs(H(w3)),'.r');
```

```
title("FIR notch filter");
```

```
hold off;
```



Problem 1.3

```
n = linspace(0,100);
```

```
H = [b(1), b(2), b(3), b(2), b(1)];
```

```
s = @(n) sin(w2.*n);
```

```
v = @(n) sin(w1.*n) + sin(w3.*n);
```

```
x = @(n) s(n) + v(n);
```

```
y = @(n) filter(H, 1, x(n));
```

```
yv = @(n) filter(H, 1, v(n));
```

```
s2 = @(n) sin(w2*(n-2)).*(n>=2);
```

```
figure;
```

```
plot(n, x(n));
```

```
hold on;
```

```
plot(n, s(n));
```

```
plot(n, y(n));
```

```
xlim([0, 100]);
```

```
ylim([-3, 3]);
```

```
title('input and output signals');
```

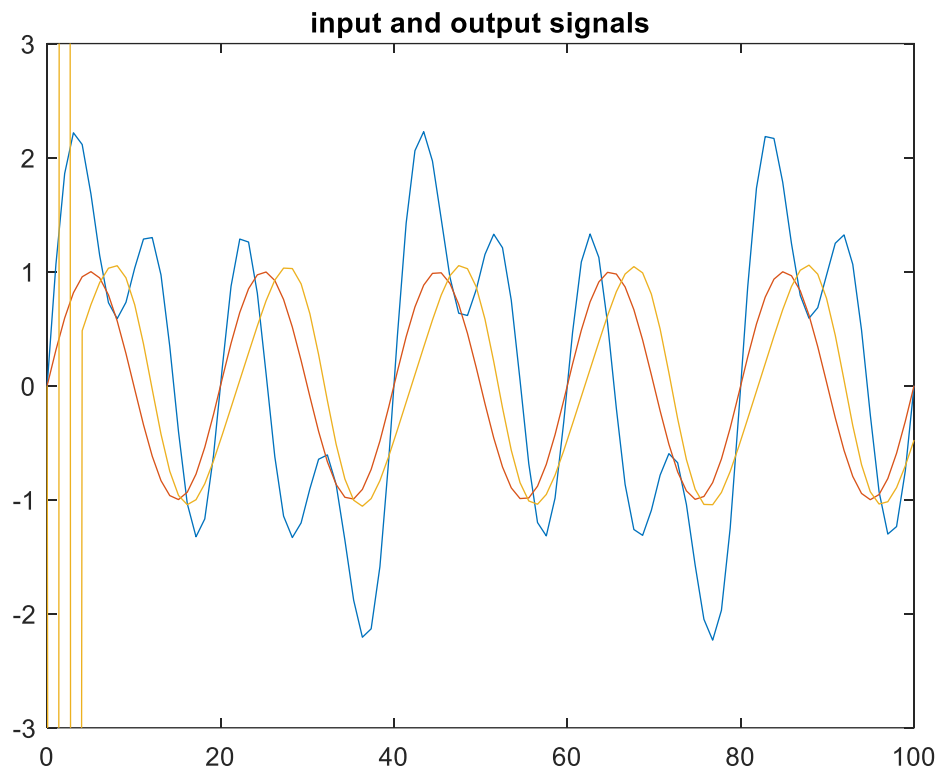
```
hold off;
```

```
n_table = 0:9;
```

```
fprintf('n    s(n)    s(n-2)    y(n)    v(n)    y_v(n)\n');
```

```
fprintf('-----\n');
```

```
fprintf('%d %9.4f %9.4f %9.4f %9.4f %9.4f\n',[n_table; s(0:9); s2(0:9); y(0:9); v(0:9); yv(0:9)]);
```



n	s(n)	s(n-2)	y(n)	v(n)	y_v(n)

0	0.0000	-0.0000	0.0000	0.0000	0.0000
1	0.3090	-0.0000	-50.6056	0.7442	-35.7580
2	0.5878	0.0000	93.0613	1.2601	67.9497
3	0.8090	0.3090	-50.2965	1.4050	-35.7580
4	0.9511	0.5878	0.5878	1.1756	-0.0000
5	1.0000	0.8090	0.8090	0.7071	-0.0000
6	0.9511	0.9511	0.9511	0.2212	-0.0000
7	0.8090	1.0000	1.0000	-0.0600	-0.0000
8	0.5878	0.9511	0.9511	-0.0000	-0.0000
9	0.3090	0.8090	0.8090	0.3999	0.0000

Problem 1.4

```
H = @(w) exp(-2*i*w).*B(w);
```

```
n = linspace(0, pi, 5000);
```

```
figure;
```

```
plot(n/pi, abs(H(n)));
```

```
hold on;
```

```
plot(w1/pi, abs(H(w1)), 'r');
```

```
plot(w2/pi, abs(H(w2)), 'r');
```

```
plot(w3/pi, abs(H(w3)), 'r');
```

```
xlim([0, 1]);
```

```
ylim([0, 800]);
```

```
title('FIR notch filter');
```

```
hold off;
```

```
for i=1:length(H)
```

```
    sum = sum - H(i).^2;
```

```
end
```

```
noise = sqrt(sum);
```

```
display(noise);
```

```
b = [.984011, -3.535954, 5.113142, -3.535954, 0.984011];
```

```
a = [1, -3.557832, 5.093644, -3.487380, 0.960788];
```

```
Hmag = abs(freqz(b, a, n));
```

```
figure;
```

```
plot(n/pi, Hmag);
```

```
hold on;
```

```

plot(w1/pi, abs(H(w1)), '.r');
plot(w2/pi, abs(H(w2)), '.r');
plot(w3/pi, abs(H(w3)), '.r');
xlim([0, 1]);
ylim([0, 2]);
title('cascade of IIR notch filters');
hold off;

```

```

n = 0:300;
H = impz(b, a, 301);
y = @(n) filter(H, 1, x(n));

```

```

figure;
plot(n, x(n));
hold on;
plot(n, s(n));
plot(n, y(n));
xlim([0, 300]);
ylim([-3, 3]);
title('input and output signals');
hold off;

```

```

n = 0:600;
H = impz(b, a, 601);
for i=1:length(H)
    sum = sum - H(i).^2;
end
noise = sqrt(sum);

```

```
display(noise);
```

```
yv = @(n) filter(H, 1, v(n));
```

```
figure;
```

```
plot(n, v(n));
```

```
hold on;
```

```
plot(n, yv(n));
```

```
xlim([0, 600]);
```

```
ylim([-3, 3]);
```

```
title('filtered interference');
```

```
hold off;
```

```
n40 = log(0.01)/log(max(abs(roots(a))));
```

```
display(n40);
```

```
%output
```

```
noise =
```

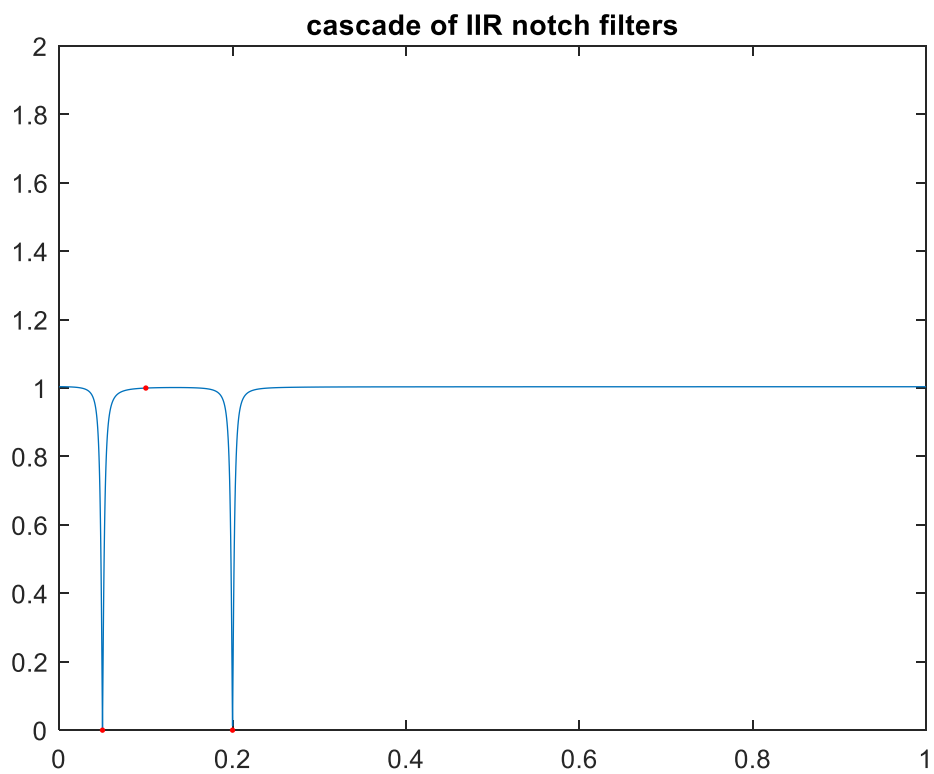
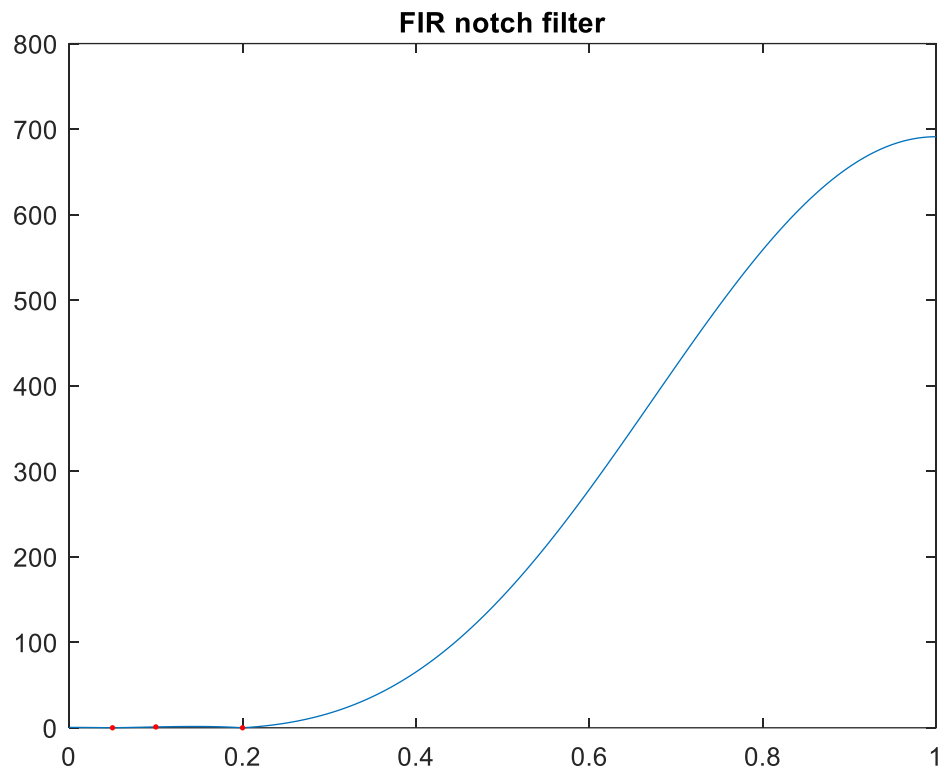
```
21.0093 - 9.6151i
```

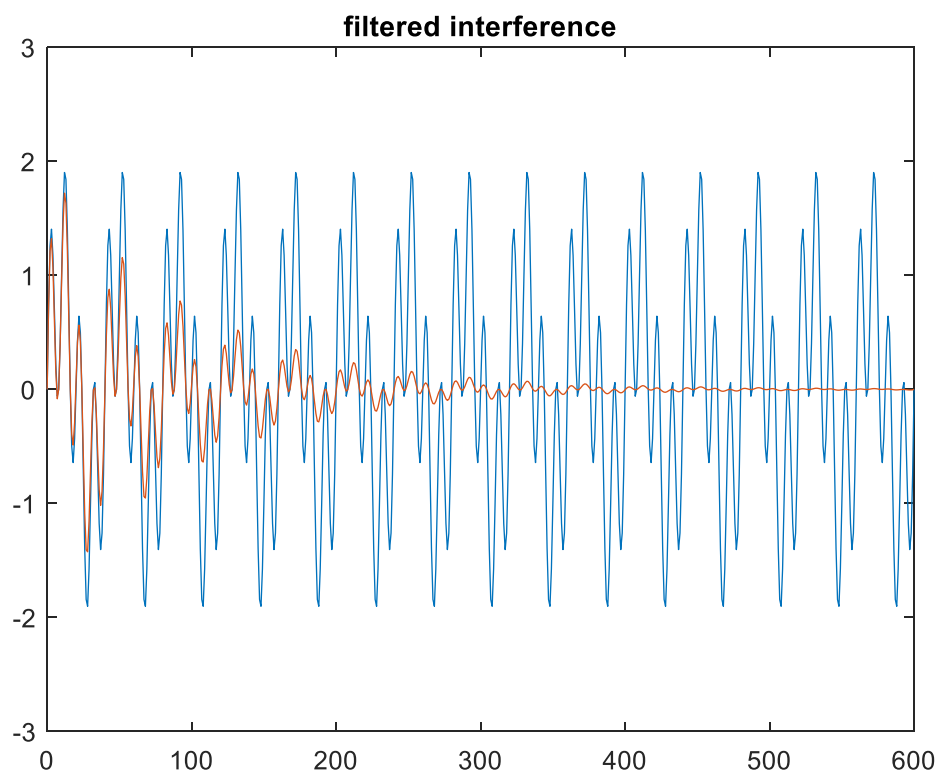
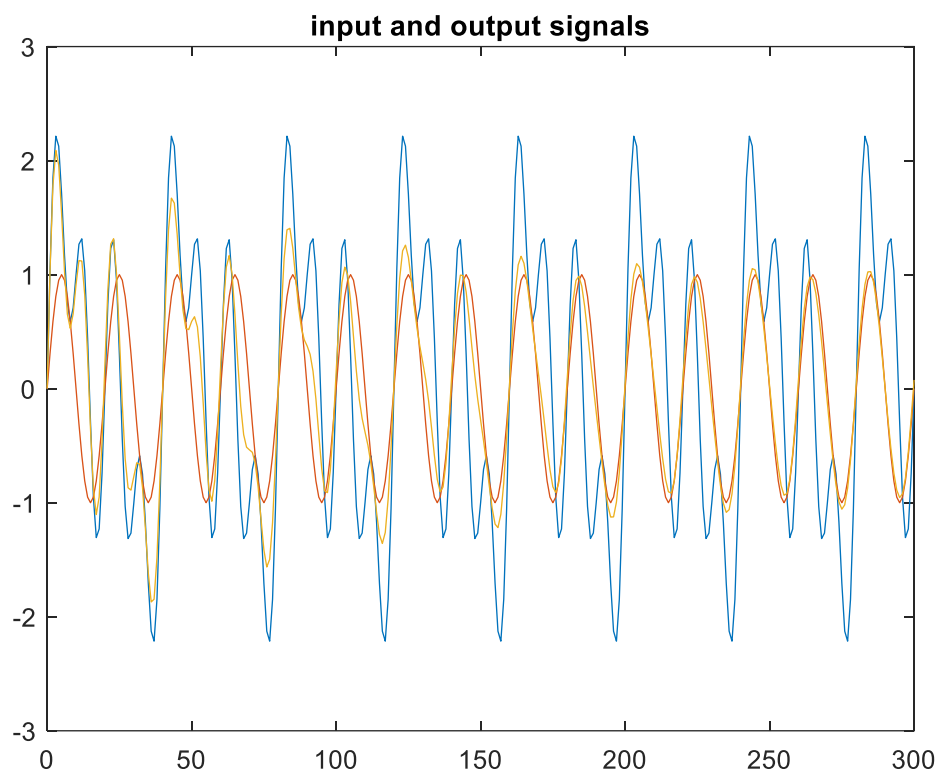
```
noise =
```

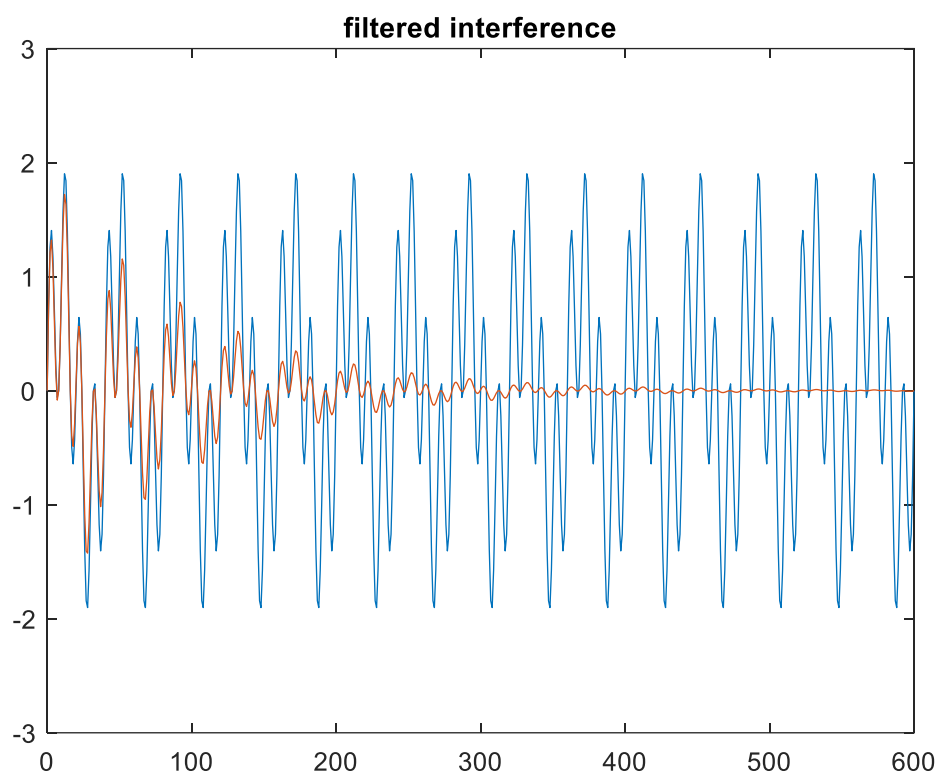
```
20.9899 - 9.6240i
```

```
n40 =
```

```
460.5267
```







Problem 2

In part 2.1, we had to plot the magnitude response of the filter with peak and side frequency points w_1 and w_2 . On the graph, we see a peak at $.2\pi$ with an output of 1 telling us that the frequency fully passes through the filter at that point. For this part, we also had to calculate the left and right 3-db frequencies and place them on the graph too. In part 2.2, we had to plot the phase delay of the filter. There is another peak at π which tells us that at that point, frequencies fully pass through the filter with no changes. In part 2.3, we see how the phase affects the input, there is a delay in the output signal compared to the input signal. In part 2.4, we changed the value of w_1 to $.3\pi$, and repeated parts 2.1-2.3 for this part. We saw a time delay and a shift due to this compared to before.

Problem 2.1

```
w0 = 0.2*pi;

B = 0.1;

w1 = 0.05*pi;

n = linspace(0, pi);

H = @(w) 1i*B.*sin(w)./(cos(w)-cos(w0)+1i*B.*sin(w));

left = acos((cos(w0)+B*sqrt(B^2+(sin(w0))^2))/(1+B^2));
right = acos((cos(w0)-B*sqrt(B^2+(sin(w0))^2))/(1+B^2));
w3dB = [left right];

figure;

plot(n/pi,abs(H(n)));

hold on;

plot(w0/pi,abs(H(w0)), 'ro');

plot(w1/pi,abs(H(w1)), 'rs');

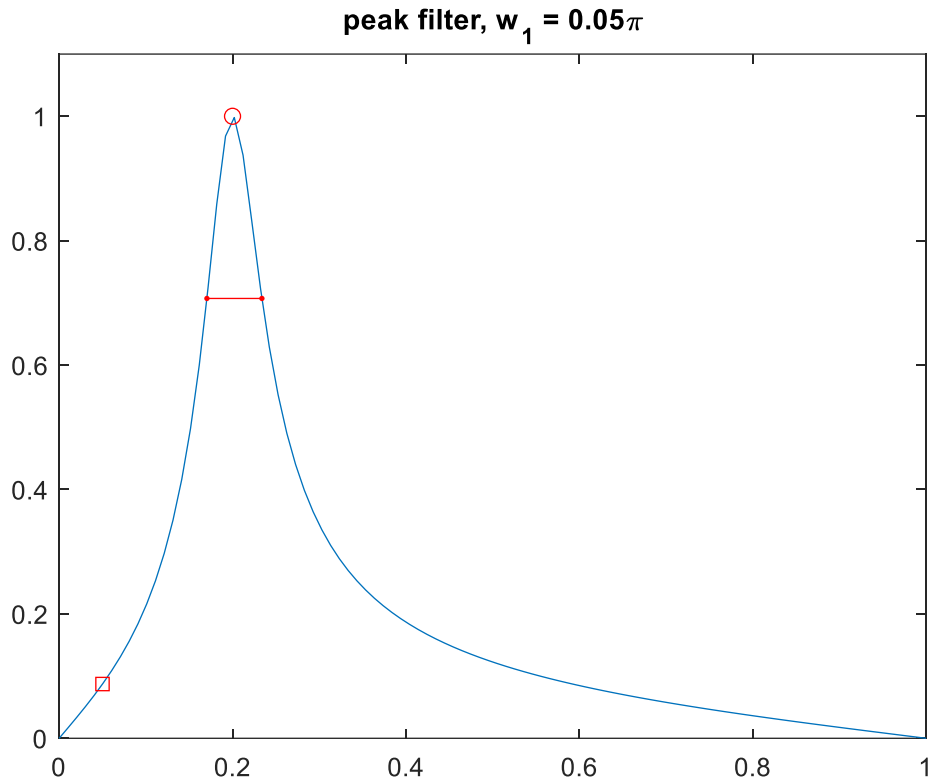
plot(w3dB/pi,abs(H(w3dB)), 'r.-');

xlim([0, 1]);

ylim([0, 1.1]);

title('peak filter, w_1 = 0.05\pi');
```

```
hold off;
```



Problem 2.2

```
w0 = 0.2*pi;
```

```
B = 0.1;
```

```
w1 = 0.05*pi;
```

```
n = linspace(0, pi);
```

```
T = @(w) -(1./w).*atan((cos(w)-cos(w0))./(B.*sin(w)));
```

```
figure;
```

```
plot(n/pi,T(n));
```

```
hold on;
```

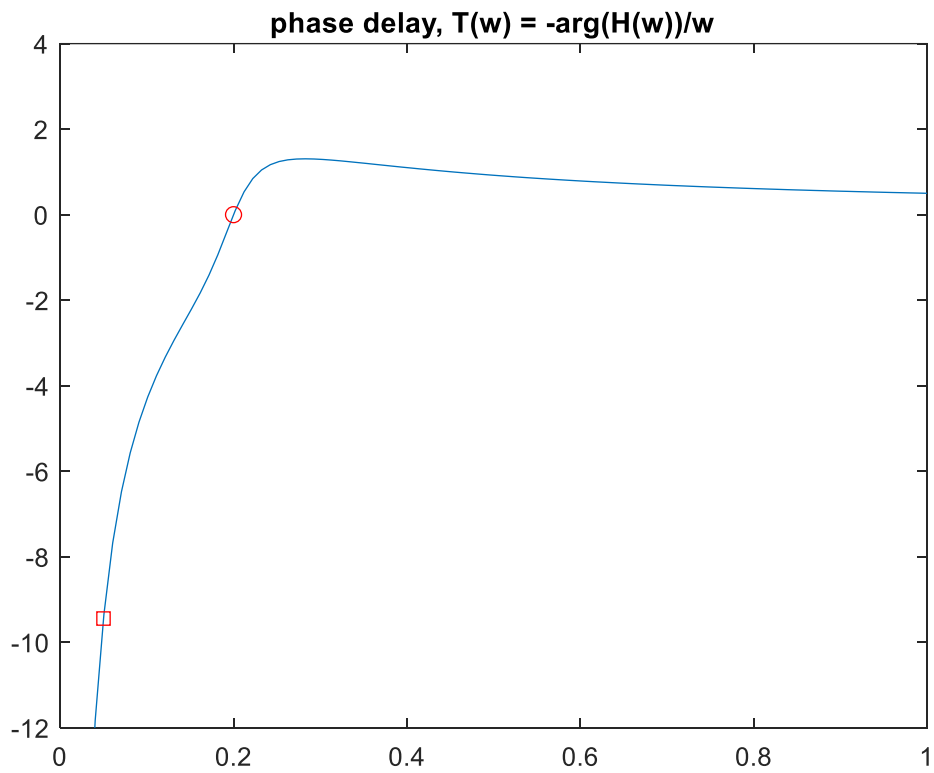
```
plot(w0/pi,T(w0), 'ro');
```

```
plot(w1/pi,T(w1), 'rs');
```

```

xlim([0, 1]);
ylim([-12, 4]);
title('phase delay,  $T(w) = -\arg(H(w))/w$ ');
hold off;

```



Problem 2.3

```

w0 = 0.2*pi;
B = 0.1;
w1 = 0.05*pi;
n = 0:100;

x = @(n) sin(w1.*n);
b = (B/(1+B)).*[1,0,-1];
a = [1,-2*cos(w0)/(1+B),(1-B)/(1+B)];
y = filter(b, a, x(n));

```

```

figure;
stem(n, x(n));
hold on;
stem(n, y);
xlim([0, 100]);
ylim([-1.2, 1.2]);
title('input and output signals');
hold off;

```

```

figure;
plot(n,x(n));
hold on;
plot(n, y);
xlim([0, 100]);
ylim([-1.2, 1.2]);
title('input and output signals');
hold off;

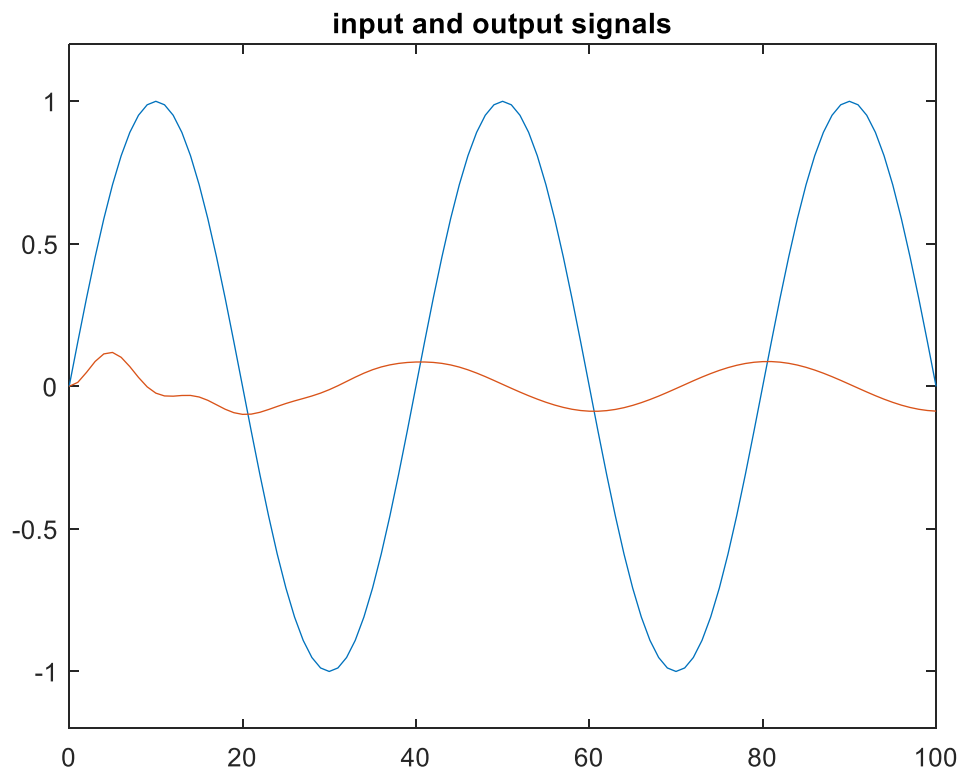
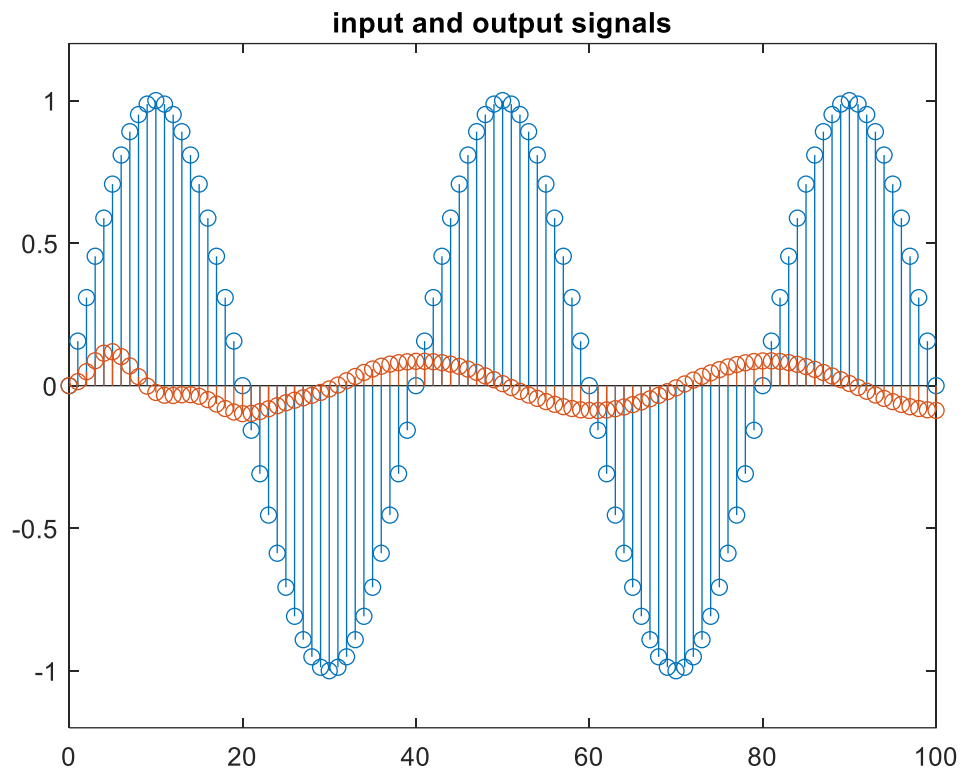
```

```

display(T(w1));
display(abs(T(w1)));
%output
-9.4440
9.4440

```

This seems correct as we see on the graph that it seems there is around a 10 unit shift.



Problem 2.4

```
w0 = 0.2*pi;
```

```
B = 0.1;
```

```
w1 = 0.3*pi;
```

```
n = linspace(0, pi);
```

```
H = @(w) 1i*B.*sin(w)./(cos(w)-cos(w0)+1i*B.*sin(w));
```

```
left = acos((cos(w0)+B*sqrt(B^2+(sin(w0))^2))/(1+B^2));
```

```
right = acos((cos(w0)-B*sqrt(B^2+(sin(w0))^2))/(1+B^2));
```

```
w3dB = [left right];
```

```
figure;
```

```
plot(n/pi, abs(H(n)));
```

```
hold on;
```

```
plot(w0/pi, abs(H(w0)), 'ro');
```

```
plot(w1/pi, abs(H(w1)), 'rs');
```

```
plot(w3dB/pi, abs(H(w3dB)), 'r.-');
```

```
xlim([0, 1]);
```

```
ylim([0, 1.1]);
```

```
title('peak filter, w_1 = 0.30\pi');
```

```
hold off;
```

```
T = @(w) -(1./w).*atan((cos(w)-cos(w0))./(B.*sin(w)));
```

```
figure;
```

```
plot(n/pi, T(n));
```

```
hold on;
```



```

plot(w0/pi, T(w0), 'ro');
plot(w1/pi, T(w1), 'rs');
xlim([0, 1]);
ylim([-12, 4]);
title('phase delay,  $T(w) = -\arg(H(w))/w$ ');
hold off;

```

```

n = 0:100;

```

```

x = @(n) sin(w1.*n);
b = (B/(1+B)).*[1,0,-1];
a = [1,-2*cos(w0)/(1+B),(1-B)/(1+B)];
y = filter(b,a,x(n));

```

```

figure;
stem(n, x(n));
hold on;
stem(n, y);
xlim([0, 100]);
ylim([-1.2, 1.2]);
title('input and output signals');
hold off;

```

```

figure;
plot(n, x(n));
hold on;
plot(n, y);
xlim([0, 100]);

```

```
ylim([-1.2, 1.2]);  
title('input and output signals');  
hold off;
```

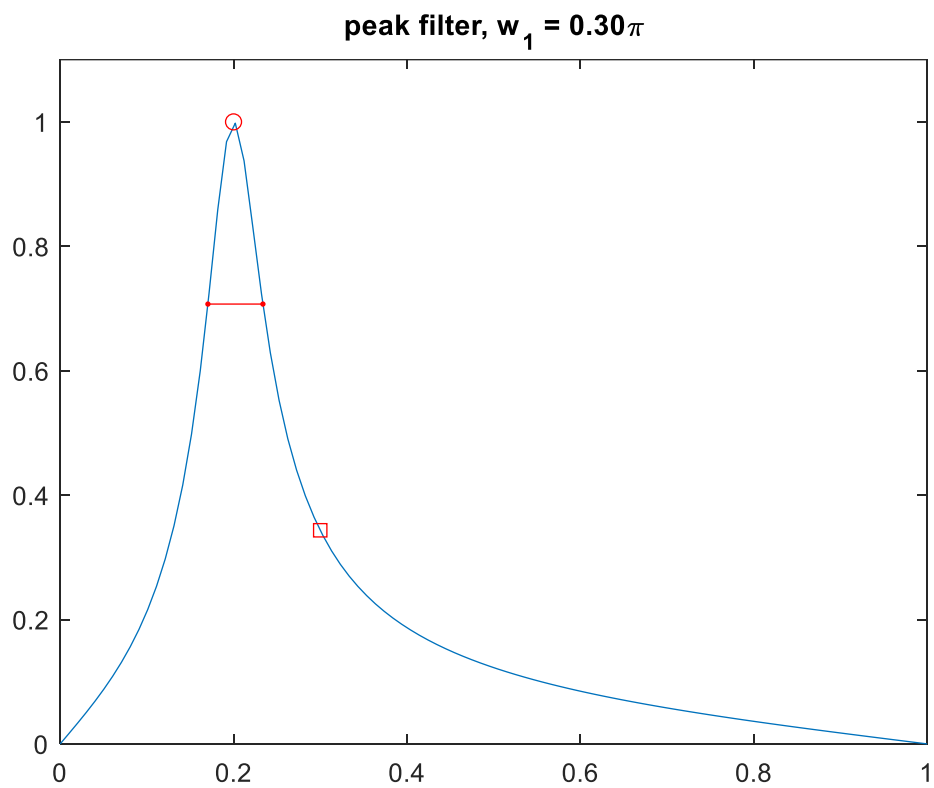
```
display(T(w1));  
display(abs(T(w1)));
```

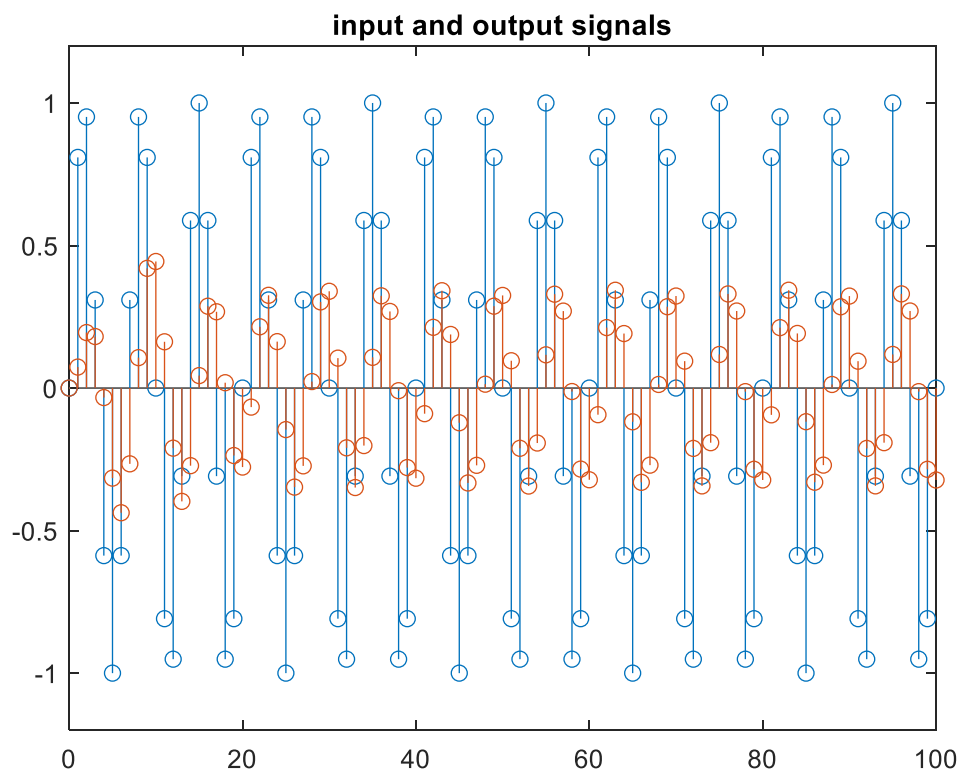
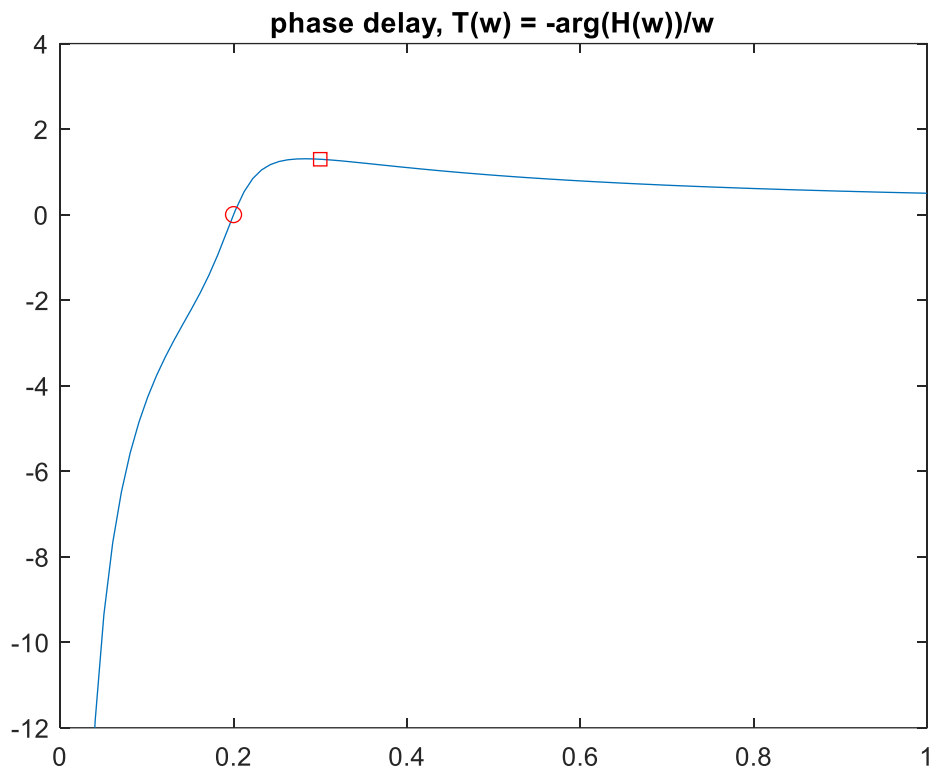
```
%output
```

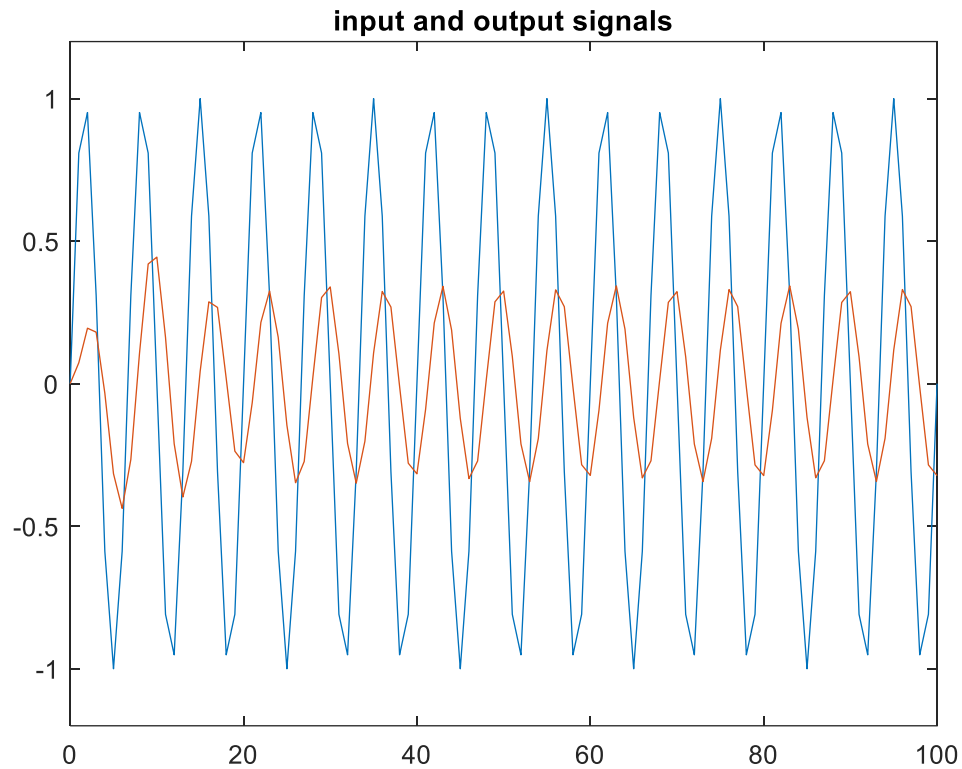
```
1.2947
```

```
1.2947
```

This seems correct as we see on the graph that it seems there is around a 1 unit shift.







Problem 3

In problem3, we demonstrated the effects of aliasing from improper sampling. We did this with 2 different frequencies of 5KHz and 10KHz. We were able to see that the 10KHz had less aliasing than the 5KHz sampled signal.

Problem 3.1

```
T = 0:.2:2;
```

```
n = linspace(0, 2, 1000);
```

```
x = @(t) cos(2*pi*t) + cos(8*pi*t) + cos(12*pi*t);
```

```
xa = @(t) 3 * cos(2*pi*t);
```

```
figure;
```

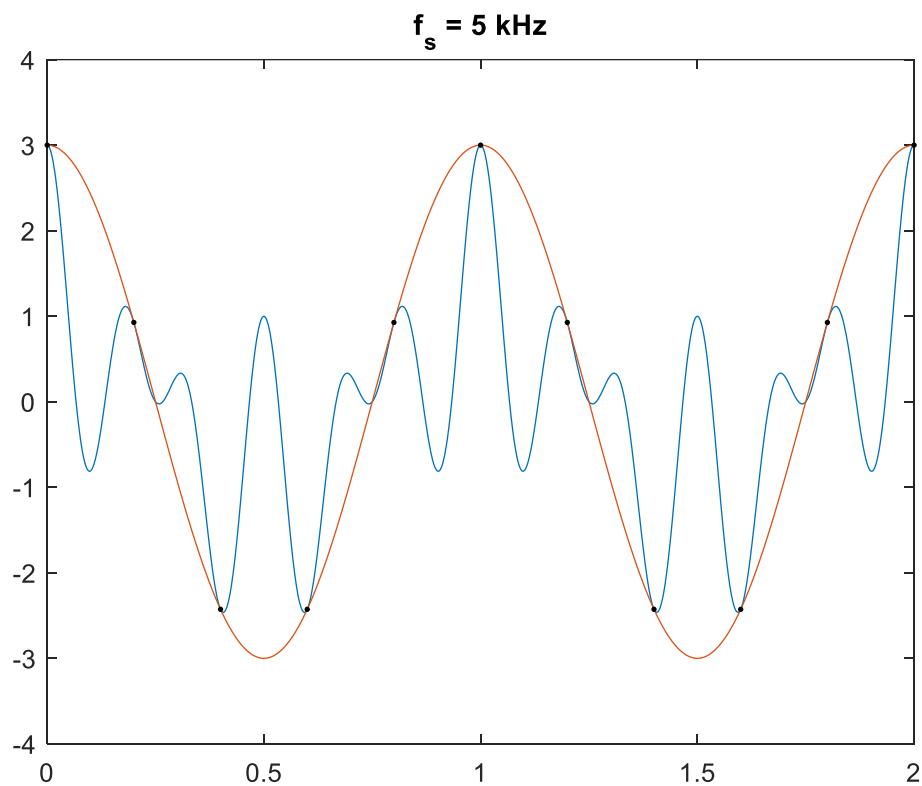
```
plot(n, x(n));
```

```
hold on;
```

```

plot(n, xa(n));
plot(T,x(T),'k. ');
xlim([0, 2]);
ylim([-4, 4]);
title('f_s = 5 kHz')
hold off;

```



Problem 3.2

```

T = 0:1:2;
n = linspace(0, 2, 1000);

x = @(t) cos(2*pi*t) + cos(8*pi*t) + cos(12*pi*t);
x_a = @(t) cos(2*pi*t) + 2 * cos(8*pi*t);

```

```
figure;
```

```
plot(n,x(n));  
hold on;  
plot(n,x_a(n));  
plot(T,x(T),'k.');
```

`xlim([0, 2]);`
`ylim([-4, 4]);`
`title('fs = 10 kHz')`

