1. (b)

$$g(z) = \left(K - S_0 e^{\sigma\sqrt{T} z + (r - \frac{1}{2}\sigma^2)T}\right)^+ \quad z \in (-\infty, +\infty)$$

$$K = S_0 e^{\sigma\sqrt{T} z^* + (r - \frac{1}{2}\sigma^2)T}$$

Then, $z^* = \left(\ln\frac{K}{S_0} - (r - \frac{1}{2}\sigma^2)T\right) / \sigma\sqrt{T}$

$\because \frac{\partial g(z)}{\partial z} = -S_0\sigma\sqrt{T} e^{\sigma T z} < 0 \quad z \in (-\infty, +\infty)$

① For $z \in (-\infty, z^*)$

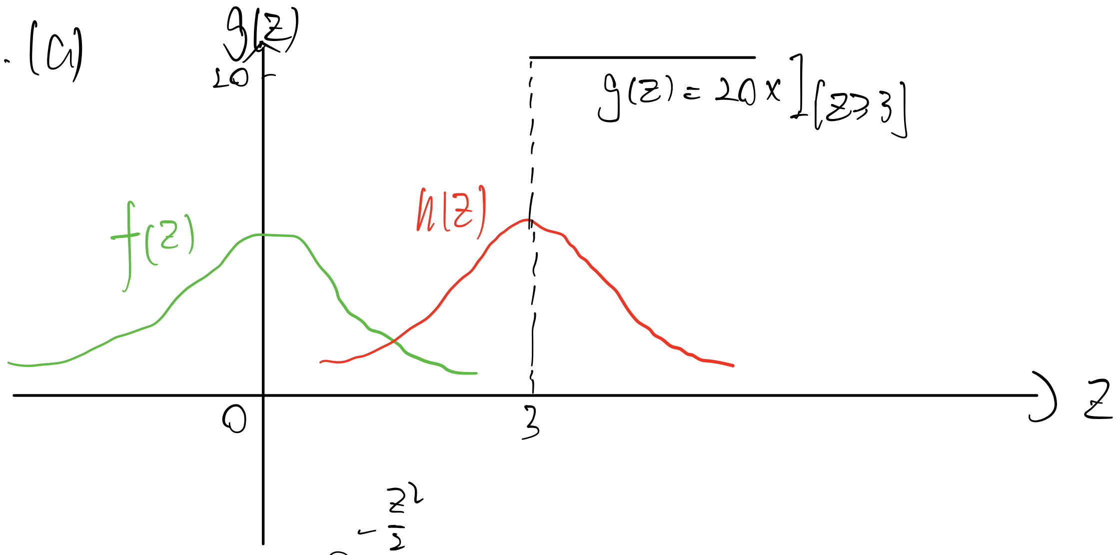
$$\frac{\partial g(z)}{\partial z} = -S_0\sigma\sqrt{T} e^{\sigma T z} < 0$$

② For $z \in (z^*, +\infty)$

$$g(z) = 0 \quad \frac{\partial g(z)}{\partial z} = 0$$

$\therefore \frac{\partial g(z)}{\partial z} \le 0$ with $z \in (-\infty, +\infty)$

2.(a)



$$f(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} \qquad z \in (-\infty, +\infty)$$

$$h(z) = \frac{e^{-\frac{(z-3)^2}{2}}}{\sqrt{2\pi}} \qquad z \in (-\infty, +\infty)$$

① via $f(z)$: $\theta = E^f[g(Z)] = \int g(z) f(z)\, dz$

$$= \int 20x\,\mathbb{1}_{\{z \geq 3\}} \cdot \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\, dz$$

② via $h(z)$: $\theta = E^f[g(Z)] = \int g(z) f(z)\, dz$

$$= \int g(z) \frac{f(z)}{h(z)} h(z)\, dz$$

$$\overset{K(z) = g(z)\frac{f(z)}{h(z)}}{=\!=} \int k(Z^h) h(Z^h)\, dz = E^h[K(Z^h)]$$

$$= E^h\left[20x\,\mathbb{1}_{\{Z^h \geq 3\}} \cdot e^{(-3Z^h + \frac{9}{2})}\right]$$