BDA - Project

Anonymous

Contents

Decision Analysis for Factory Data

This submission is completed in `python` with `pystan`.

```
import numpy as np
import matplotlib.pyplot as plt
# with out this, plots from matplotlib won't knit on windows
import matplotlib
matplotlib.use('TkAgg')
import pystan
import arviz as az
from pathlib import Path
from matplotlib.patches import Patch
from matplotlib.lines import Line2D

fig_size=(12,6)# parameters for figures throughout this assignment
num_bins = 20 #
font_size=15

import pystan
print("pystan version:", pystan.__version__)
```

```
## pystan version: 2.19.0.0
```

```
model_path = 'stan_models'
data_file = 'data/data.txt'
accident_data = np.loadtxt(data_file)
print(accident_data.shape)
```

```
## (6, 15)
```

```
mean_value = np.mean(accident_data) # mean value approximately 25 cases per 10,000 people
# it's very un likely to change 50% of the mean, so 2.57*sigma = mean_value/2
sigma = mean_value / (2*2.57)
sigma
```

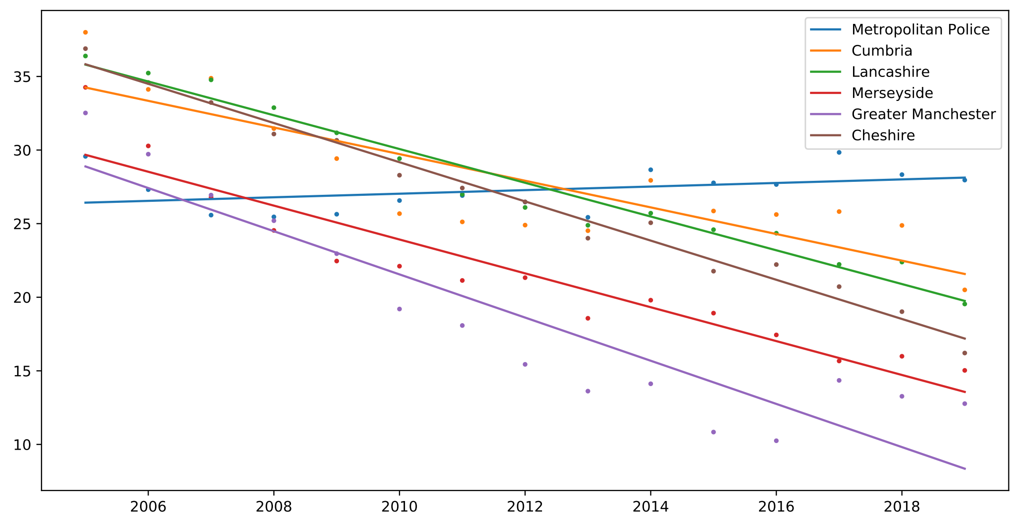
```
## 4.854734111543451
```

```
area_names = ["Metropolitan Police", 'Cumbria','Lancashire',
               'Merseyside','Greater Manchester','Cheshire']
```

```python
plt.figure(figsize=(12, 6));
years = np.arange(2005, 2020, 1).astype(np.int)
print(years.shape)
```

```
## (15,)
```

```python
for i in range(6):
    plt.scatter(years, accident_data[i, :], marker='.', s=20)
    fit = np.polyfit(years, accident_data[i, :], 1)
    fitted_values = np.polyval(fit, years)
    plt.plot(years, fitted_values, label=area_names[i])
plt.legend()
plt.show()
```



3.3 Hierarchical Model

```python
model_name = 'accident_hierarchical.stan'
stan_model = pystan.StanModel(file=model_path + '/' + model_name)
```

```
## INFO:pystan:COMPILING THE C++ CODE FOR MODEL anon_model_9fb09f0a30d029d43ce52bd2384506d8 NOW.
```

```python
print(stan_model.model_code)
```

```
## //
## // This Stan program defines a simple model, with a
## // vector of values 'y' modeled as normally distributed
## // with mean 'mu' and standard deviation 'sigma'.
## //
## // Learn more about model development with Stan at:
## //
## //    http://mc-stan.org/users/interfaces/rstan.html
## //    https://github.com/stan-dev/rstan/wiki/RStan-Getting-Started
## //
##
```

```
## data {
##   int<lower=0> N; // the number of police force
##   int<lower=0> Y; // the number of years has been studied, year 2005 corresponds to 1
##   matrix[N,Y] accidentData;//accident data
##   int prior_choice; // choose different setup for prior distribution
##   int xpred; // year of prediction (actual year)
## }
##
##
## parameters {
##   real mu_alpha;
##   real mu_beta;
##   real<lower=0> sigma_alpha;
##   real<lower=0> sigma_beta;
##   vector[N] alpha;
##   vector[N] beta;
##   vector<lower=0>[N] sigma;
## }
##
##
## transformed parameters{
##   matrix[N,Y]mu;
##   for(i in 1:N)
##     for(j in 1:Y)
##       mu[i,j]=alpha[i]+beta[i]*j;
## }
##
##
## model {
##   if (prior_choice==3){
##     // bigger variance
##     mu_alpha~normal(30,40);
##     mu_beta~normal(0,10);
##     sigma_alpha~normal(10,10);
##     sigma_beta~normal(3,6);
##   } else if (prior_choice==2){
##     // uniform prior
##   } else {
##     // default choice with moderate variance
##     mu_alpha~normal(30,20);
##     mu_beta~normal(0,4.85);
##     sigma_alpha~normal(10,5);
##     sigma_beta~normal(3,3);
##   }
##
##   //for each police force
##   for(i in 1:N){
##     alpha[i]~normal(mu_alpha,sigma_alpha);
##     beta[i]~normal(mu_beta,sigma_beta);
##   }
##
##   //for each police force
##   for(i in 1:N){
##     //for each observed year
```

```
##     for(j in 1:Y){
##      accidentData[i,j]~normal(mu[i,j],sigma[i]);
##     }
##   }
## }
##
##
## generated quantities{
##   //log likelihood
##   matrix[N,Y] log_lik;
##   matrix[N,Y] yrep;
##   //accident prediction in 2020 in different police force
##   vector[N] pred;
##
##   //for each police force
##   for(i in 1:N){
##     // 2005 -> 1, 2006 -> 2, ..., 2020 -> 16
##     pred[i]=normal_rng(alpha[i]+beta[i]*(xpred-2004),sigma[i]);
##   }
##
##   for(i in 1:N){
##     for(j in 1:Y){
##       // do posterior sampling and try to reproduce the original data
##       yrep[i,j]=normal_rng(mu[i,j],sigma[i]);
##       // prepare log likelihood for PSIS-LOO
##       log_lik[i,j]=normal_lpdf(accidentData[i,j]|mu[i,j],sigma[i]);
##     }
##   }
##
## }
```

```
data_for_stan = dict(
    N = accident_data.shape[0],
    Y = accident_data.shape[1],
    accidentData = accident_data,
    years = np.arange(1, accident_data.shape[1]+1), # stan index starts from 1
    xpred=2020,
    prior_choice=1
)
stan_results = stan_model.sampling(data=data_for_stan)
print(stan_results)
```

```
## Inference for Stan model: anon_model_9fb09f0a30d029d43ce52bd2384506d8.
## 4 chains, each with iter=2000; warmup=1000; thin=1;
## post-warmup draws per chain=1000, total post-warmup draws=4000.
##
##                 mean se_mean     sd   2.5%    25%    50%    75%  97.5%  n_eff   Rhat
## mu_alpha       32.68    0.06   2.79  27.02  31.13   32.7  34.31  38.18   2423    1.0
## mu_beta        -0.97  8.3e-3   0.37  -1.71  -1.17  -0.97  -0.77  -0.19   2013    1.0
## sigma_alpha     6.39    0.04   2.54   2.92   4.51   5.86   7.78  12.67   3234    1.0
## sigma_beta      0.82    0.01   0.42   0.38   0.56   0.71   0.95   1.88   1653    1.0
## alpha[1]       26.64    0.02   0.88  24.93  26.08  26.62  27.19  28.43   2631    1.0
## alpha[2]       34.92    0.03    1.5  31.92  33.98  34.95  35.86   37.8   2049    1.0
## alpha[3]       36.85    0.01   0.69  35.48  36.39  36.87   37.3  38.17   2411    1.0
## alpha[4]       30.87    0.03   1.16  28.64  30.14  30.86  31.58   33.2   2124    1.0
```

```
## alpha[5]       30.28     0.04    1.7  26.86  29.19   30.3  31.38   33.6   2219    1.0
## alpha[6]        37.1     0.01   0.51  36.08  36.78  37.11  37.44   38.1   2274    1.0
## beta[1]         0.08   2.0e-3    0.1  -0.12   0.02   0.08   0.15   0.27   2545    1.0
## beta[2]        -0.88   3.6e-3   0.17   -1.2  -0.99  -0.89  -0.78  -0.55   2154    1.0
## beta[3]        -1.14   1.5e-3   0.08  -1.28  -1.19  -1.14  -1.09  -0.99   2454    1.0
## beta[4]        -1.15   2.7e-3   0.13  -1.41  -1.23  -1.15  -1.07   -0.9   2240    1.0
## beta[5]        -1.45   4.0e-3   0.19  -1.81  -1.58  -1.45  -1.33  -1.07   2279    1.0
## beta[6]        -1.32   1.2e-3   0.06  -1.44  -1.36  -1.33  -1.29  -1.21   2123    1.0
## sigma[1]        1.56   7.1e-3   0.36   1.05   1.31    1.5   1.75   2.47   2675    1.0
## sigma[2]        2.86     0.01   0.65   1.92   2.39   2.75   3.21   4.44   3596    1.0
## sigma[3]        1.25   4.9e-3   0.28   0.84   1.05   1.21   1.39   1.91   3244    1.0
## sigma[4]        2.15   9.4e-3    0.5   1.44    1.8   2.06    2.4   3.37   2808    1.0
## sigma[5]        3.26     0.01   0.72   2.18   2.75   3.16   3.64   4.97   3223    1.0
## sigma[6]        0.93   4.0e-3   0.22   0.63   0.78    0.9   1.04   1.45   2855    1.0
## mu[1,1]        26.73     0.02   0.79  25.18  26.21   26.7  27.22  28.35   2674    1.0
## mu[2,1]        34.04     0.03   1.36  31.31  33.19  34.07  34.89  36.65   2080    1.0
## mu[3,1]        35.72     0.01   0.62  34.47   35.3  35.73  36.12  36.92   2436    1.0
## mu[4,1]        29.72     0.02   1.05  27.71  29.05  29.71  30.37  31.83   2150    1.0
## mu[5,1]        28.83     0.03   1.54  25.74  27.84  28.83  29.82  31.82   2249    1.0
## mu[6,1]        35.77   9.6e-3   0.46  34.84  35.49  35.78  36.08  36.68   2320    1.0
## mu[1,2]        26.81     0.01   0.71  25.44  26.35  26.79  27.25  28.26   2737    1.0
## mu[2,2]        33.15     0.03   1.22  30.65   32.4  33.17  33.93  35.54   2131    1.0
## mu[3,2]        34.58     0.01   0.56  33.46  34.21  34.59  34.95  35.67   2475    1.0
## mu[4,2]        28.56     0.02   0.94  26.79  27.97  28.56  29.14  30.45   2186    1.0
## mu[5,2]        27.38     0.03   1.38  24.63  26.48  27.38  28.27  30.07   2298    1.0
## mu[6,2]        34.45   8.6e-3   0.42   33.6  34.19  34.46  34.73  35.26   2388    1.0
## mu[1,3]        26.89     0.01   0.63  25.68  26.49  26.88  27.28  28.22   2832    1.0
## mu[2,3]        32.27     0.02    1.1  30.02   31.6  32.28  32.97  34.42   2215    1.0
## mu[3,3]        33.44   9.9e-3    0.5  32.44  33.11  33.45  33.77  34.41   2537    1.0
## mu[4,3]        27.41     0.02   0.84  25.84  26.88   27.4  27.94  29.09   2252    1.0
## mu[5,3]        25.93     0.03   1.24  23.48  25.13  25.93  26.72  28.36   2379    1.0
## mu[6,3]        33.13   7.5e-3   0.37  32.36  32.89  33.13  33.38  33.84   2490    1.0
## mu[1,4]        26.97     0.01   0.56  25.93  26.61  26.96  27.32  28.12   2975    1.0
## mu[2,4]        31.39     0.02   0.98  29.35  30.77   31.4  32.02  33.32   2354    1.0
## mu[3,4]        32.31   8.7e-3   0.45  31.43  32.01  32.31   32.6  33.19   2636    1.0
## mu[4,4]        26.26     0.02   0.75  24.83   25.8  26.25  26.73  27.76   2377    1.0
## mu[5,4]        24.47     0.02   1.11  22.28  23.75  24.49  25.19  26.69   2511    1.0
## mu[6,4]         31.8   6.5e-3   0.33  31.11  31.59  31.81  32.02  32.44   2649    1.0
## mu[1,5]        27.06   8.9e-3    0.5  26.12  26.73  27.05  27.37  28.08   3157    1.0
## mu[2,5]         30.5     0.02   0.89  28.65  29.94  30.52  31.09  32.23   2586    1.0
## mu[3,5]        31.17   7.5e-3    0.4  30.38  30.91  31.17  31.42  31.97   2795    1.0
## mu[4,5]        25.11     0.01   0.67  23.82  24.69   25.1  25.53  26.43   2608    1.0
## mu[5,5]        23.02     0.02    1.0  21.04  22.36  23.03  23.67  25.03   2724    1.0
## mu[6,5]        30.48   5.5e-3    0.3  29.87  30.29  30.48  30.67  31.05   2883    1.0
## mu[1,6]        27.14   7.7e-3   0.45  26.29  26.85  27.13  27.42  28.05   3419    1.0
## mu[2,6]        29.62     0.01   0.81  27.92  29.12  29.63  30.15   31.2   2965    1.0
## mu[3,6]        30.03   6.5e-3   0.36  29.31   29.8  30.03  30.26  30.74   3041    1.0
## mu[4,6]        23.95     0.01    0.6  22.76  23.58  23.95  24.34  25.15   3029    1.0
## mu[5,6]        21.57     0.02   0.92  19.76  20.96  21.58  22.16  23.41   3045    1.0
## mu[6,6]        29.15   4.8e-3   0.27  28.59  28.98  29.15  29.32  29.68   3176    1.0
## mu[1,7]        27.22   6.7e-3   0.42  26.43  26.95  27.22  27.48  28.06   3833    1.0
## mu[2,7]        28.73     0.01   0.77  27.15  28.26  28.75  29.23  30.22   3532    1.0
## mu[3,7]        28.89   5.7e-3   0.33  28.24  28.68  28.89   29.1  29.56   3383    1.0
## mu[4,7]         22.8   9.1e-3   0.56   21.7  22.45   22.8  23.16  23.92   3766    1.0
```

```
## mu[5,7]        20.12    0.01   0.87  18.37  19.57  20.11  20.67  21.88   3581     1.0
## mu[6,7]        27.83  4.2e-3   0.25  27.32  27.67  27.83  27.99  28.32   3570     1.0
## mu[1,8]         27.3  6.3e-3   0.41  26.52  27.04   27.3  27.56  28.11   4122     1.0
## mu[2,8]        27.85    0.01   0.76   26.3  27.37  27.86  28.34  29.31   4046     1.0
## mu[3,8]        27.76  5.3e-3   0.32  27.12  27.55  27.75  27.96  28.41   3747     1.0
## mu[4,8]        21.65  8.0e-3   0.54  20.57   21.3  21.66   22.0  22.73   4594     1.0
## mu[5,8]        18.67    0.01   0.86  16.94  18.14  18.65  19.21  20.42   4278     1.0
## mu[6,8]         26.5  3.9e-3   0.24  26.03  26.35   26.5  26.66  26.98   3966     1.0
## mu[1,9]        27.38  6.6e-3   0.42  26.58  27.11  27.39  27.65  28.22   4074     1.0
## mu[2,9]        26.97    0.01   0.78  25.42  26.47  26.98  27.48  28.48   4344     1.0
## mu[3,9]        26.62  5.2e-3   0.33  25.96  26.41  26.61  26.83  27.29   3961     1.0
## mu[4,9]         20.5  7.9e-3   0.56  19.39  20.14   20.5  20.85  21.63   4977     1.0
## mu[5,9]        17.22    0.01   0.89  15.46  16.67   17.2  17.77  19.04   4543     1.0
## mu[6,9]        25.18  3.9e-3   0.25  24.68  25.02  25.18  25.34  25.68   4155     1.0
## mu[1,10]       27.47  7.4e-3   0.46  26.58  27.18  27.47  27.76  28.37   3855     1.0
## mu[2,10]       26.08    0.01   0.84  24.41  25.55  26.09  26.63  27.74   4303     1.0
## mu[3,10]       25.48  5.6e-3   0.35  24.78  25.26  25.48   25.7  26.19   3925     1.0
## mu[4,10]       19.34  8.7e-3    0.6  18.15  18.96  19.34  19.72  20.57   4770     1.0
## mu[5,10]       15.77    0.01   0.96  13.82  15.17  15.76  16.36   17.7   4343     1.0
## mu[6,10]       23.85  4.2e-3   0.27  23.32  23.69  23.85  24.03   24.4   4055     1.0
## mu[1,11]       27.55  8.5e-3   0.51  26.55  27.23  27.55  27.87  28.56   3628     1.0
## mu[2,11]        25.2    0.01   0.92  23.36  24.61  25.21  25.79  27.02   4047     1.0
## mu[3,11]       24.35  6.4e-3   0.39  23.59  24.09  24.34  24.59  25.13   3734     1.0
## mu[4,11]       18.19    0.01   0.67  16.84  17.76  18.18  18.61  19.54   4388     1.0
## mu[5,11]       14.32    0.02   1.06  12.19  13.66  14.31  14.99  16.43   4033     1.0
## mu[6,11]       22.53  4.9e-3    0.3  21.93  22.34  22.53  22.72  23.13   3632     1.0
## mu[1,12]       27.63  9.8e-3   0.57  26.48  27.27  27.63   28.0  28.76   3424     1.0
## mu[2,12]       24.32    0.02   1.03   22.2  23.67  24.32  24.97  26.39   3750     1.0
## mu[3,12]       23.21  7.4e-3   0.44  22.37  22.92  23.19  23.49  24.08   3513     1.0
## mu[4,12]       17.04    0.01   0.75  15.54  16.57  17.01  17.52  18.53   3995     1.0
## mu[5,12]       12.87    0.02   1.18  10.49  12.13  12.86  13.61  15.25   3715     1.0
## mu[6,12]       21.21  5.9e-3   0.33  20.53   21.0   21.2  21.42  21.87   3175     1.0
## mu[1,13]       27.71    0.01   0.65  26.39  27.31  27.72  28.13  28.98   3255     1.0
## mu[2,13]       23.43    0.02   1.15  21.08   22.7  23.42  24.17  25.74   3491     1.0
## mu[3,13]       22.07  8.5e-3   0.49  21.13  21.74  22.06  22.39  23.04   3322     1.0
## mu[4,13]       15.89    0.01   0.84  14.24  15.35  15.86  16.42  17.56   3619     1.0
## mu[5,13]       11.42    0.02   1.32    8.8  10.59  11.41  12.26  14.07   3450     1.0
## mu[6,13]       19.88  6.9e-3   0.37  19.13  19.65  19.88  20.13  20.62   2889     1.0
## mu[1,14]        27.8    0.01   0.73  26.33  27.34   27.8  28.26   29.2   3114     1.0
## mu[2,14]       22.55    0.02   1.28  19.94  21.75  22.53  23.36  25.15   3286     1.0
## mu[3,14]       20.93  9.8e-3   0.55  19.88  20.57  20.93  21.29  22.03   3171     1.0
## mu[4,14]       14.73    0.02   0.95  12.89  14.13  14.72  15.33  16.63   3347     1.0
## mu[5,14]        9.97    0.03   1.47   7.07   9.02   9.98  10.91  12.89   3244     1.0
## mu[6,14]       18.56  8.0e-3   0.42  17.72  18.29  18.54  18.83   19.4   2723     1.0
## mu[1,15]       27.88    0.01   0.81  26.23  27.37  27.89   28.4  29.46   3011     1.0
## mu[2,15]       21.66    0.03   1.41   18.8  20.77  21.64  22.56  24.52   3128     1.0
## mu[3,15]        19.8    0.01   0.61   18.6  19.39  19.79  20.19  21.03   3055     1.0
## mu[4,15]       13.58    0.02   1.05  11.55  12.91  13.57  14.24  15.67   3150     1.0
## mu[5,15]        8.51    0.03   1.62   5.35   7.46   8.52   9.56  11.72   3087     1.0
## mu[6,15]       17.23  9.1e-3   0.46  16.31  16.93  17.22  17.53  18.18   2605     1.0
## log_lik[1,1]   -3.43    0.02   1.26  -6.57  -4.09  -3.18  -2.49  -1.77   3315     1.0
## log_lik[2,1]   -3.12    0.01   0.71  -4.84  -3.52   -3.0  -2.61  -2.11   2827     1.0
## log_lik[3,1]   -1.39  7.1e-3   0.35  -2.24  -1.58  -1.34  -1.15  -0.86   2475     1.0
## log_lik[4,1]   -4.34    0.02   1.39  -7.65  -5.12  -4.11   -3.3  -2.35   3756     1.0
```

```
## log_lik[5,1]   -2.91    0.01    0.6  -4.41  -3.22  -2.79  -2.49  -2.08   2779    1.0
## log_lik[6,1]   -1.75    0.01   0.66  -3.37  -2.09  -1.61  -1.27  -0.85   3142    1.0
## log_lik[1,2]   -1.51  5.4e-3   0.28  -2.15  -1.66  -1.48  -1.32  -1.06   2643    1.0
## log_lik[2,2]   -2.09  5.5e-3   0.27  -2.71  -2.25  -2.07   -1.9  -1.64   2474    1.0
## log_lik[3,2]   -1.36  6.2e-3   0.31  -2.09  -1.54  -1.32  -1.14  -0.86   2540    1.0
## log_lik[4,2]   -2.12  7.2e-3   0.39  -3.06  -2.32  -2.05  -1.84  -1.54   2888    1.0
## log_lik[5,2]   -2.46  7.3e-3   0.36  -3.35  -2.64   -2.4  -2.21  -1.91   2508    1.0
## log_lik[6,2]   -0.93  5.8e-3   0.26  -1.52  -1.08   -0.9  -0.75  -0.51   2004    1.0
## log_lik[1,3]   -1.81  7.2e-3   0.39  -2.72  -2.01  -1.75  -1.53  -1.21   2885    1.0
## log_lik[2,3]   -2.48  7.4e-3   0.38  -3.38  -2.69  -2.43  -2.21  -1.89   2544    1.0
## log_lik[3,3]   -1.83  8.1e-3   0.45  -2.85  -2.09  -1.75   -1.5  -1.16   3104    1.0
## log_lik[4,3]   -1.78  5.2e-3   0.26  -2.35  -1.93  -1.75   -1.6  -1.35   2476    1.0
## log_lik[5,3]    -2.2  5.0e-3   0.25  -2.74  -2.34  -2.18  -2.03  -1.78   2450    1.0
## log_lik[6,3]   -0.91  5.3e-3   0.24  -1.46  -1.06  -0.88  -0.74   -0.5   2153    1.0
## log_lik[1,4]   -1.92  6.7e-3   0.38  -2.82  -2.13  -1.87  -1.66  -1.33   3251    1.0
## log_lik[2,4]    -2.0  4.2e-3   0.23  -2.49  -2.15  -1.99  -1.84   -1.6   3052    1.0
## log_lik[3,4]   -1.29  4.8e-3   0.25  -1.85  -1.45  -1.28  -1.12  -0.86   2734    1.0
## log_lik[4,4]   -2.08  5.8e-3   0.32  -2.81  -2.26  -2.05  -1.86  -1.59   3020    1.0
## log_lik[5,4]   -2.16  4.4e-3   0.23  -2.65   -2.3  -2.15   -2.0  -1.75   2635    1.0
## log_lik[6,4]   -1.23  5.5e-3   0.31  -1.95  -1.41  -1.19  -1.01  -0.72   3303    1.0
## log_lik[1,5]   -1.85  5.7e-3   0.32   -2.6  -2.03  -1.81  -1.62  -1.32   3173    1.0
## log_lik[2,5]   -2.08  4.0e-3   0.22  -2.56  -2.22  -2.06  -1.92  -1.68   3126    1.0
## log_lik[3,5]   -1.16  4.0e-3   0.22  -1.65   -1.3  -1.15  -1.01  -0.77   3049    1.0
## log_lik[4,5]   -2.57  7.1e-3   0.44  -3.59   -2.8   -2.5  -2.27  -1.93   3825    1.0
## log_lik[5,5]   -2.12  4.1e-3   0.22  -2.58  -2.26  -2.12  -1.97  -1.73   2767    1.0
## log_lik[6,5]   -0.89  4.7e-3   0.23   -1.4  -1.04  -0.88  -0.73   -0.5   2430    1.0
## log_lik[1,6]   -1.45  4.6e-3   0.23  -1.96  -1.59  -1.43  -1.29  -1.05   2576    1.0
## log_lik[2,6]   -3.08  8.2e-3    0.5   -4.3  -3.35   -3.0  -2.72  -2.37   3708    1.0
## log_lik[3,6]   -1.29  4.0e-3   0.22  -1.78  -1.43  -1.28  -1.14  -0.91   3003    1.0
## log_lik[4,6]   -2.12  4.6e-3   0.27  -2.71  -2.28  -2.09  -1.93  -1.68   3453    1.0
## log_lik[5,6]   -2.42  4.4e-3   0.24  -2.96  -2.56   -2.4  -2.25  -2.01   3018    1.0
## log_lik[6,6]   -1.36  5.0e-3    0.3  -2.03  -1.53  -1.33  -1.15  -0.88   3510    1.0
## log_lik[1,7]    -1.4  4.3e-3   0.22  -1.89  -1.53  -1.38  -1.24  -1.01   2681    1.0
## log_lik[2,7]    -2.9  6.7e-3   0.41  -3.89  -3.12  -2.84  -2.61   -2.3   3801    1.0
## log_lik[3,7]   -2.48  8.9e-3   0.56  -3.83  -2.78  -2.38  -2.08  -1.68   3950    1.0
## log_lik[4,7]   -2.03  4.0e-3   0.24  -2.54  -2.18  -2.02  -1.87  -1.64   3506    1.0
## log_lik[5,7]   -2.34  4.0e-3   0.22  -2.82  -2.48  -2.32  -2.18  -1.96   3071    1.0
## log_lik[6,7]   -0.97  3.9e-3   0.21  -1.42   -1.1  -0.96  -0.82  -0.59   2993    1.0
## log_lik[1,8]   -1.53  4.2e-3   0.22  -2.01  -1.67  -1.52  -1.38  -1.14   2773    1.0
## log_lik[2,8]   -2.59  5.0e-3    0.3  -3.29  -2.76  -2.55  -2.38  -2.11   3691    1.0
## log_lik[3,8]   -2.15  6.8e-3   0.43  -3.18  -2.38  -2.08  -1.85  -1.52   4004    1.0
## log_lik[4,8]    -1.7  4.1e-3   0.22  -2.17  -1.84  -1.69  -1.55  -1.32   2939    1.0
## log_lik[5,8]   -2.67  4.6e-3   0.29  -3.33  -2.83  -2.63  -2.47  -2.21   3868    1.0
## log_lik[6,8]   -0.86  4.3e-3   0.22  -1.33  -0.99  -0.84   -0.7  -0.48   2702    1.0
## log_lik[1,9]   -2.28  6.8e-3   0.41  -3.24  -2.49  -2.21  -1.99  -1.68   3542    1.0
## log_lik[2,9]    -2.4  4.4e-3   0.26  -2.97  -2.56  -2.38  -2.22  -1.97   3480    1.0
## log_lik[3,9]   -2.24  7.2e-3   0.46  -3.36  -2.49  -2.17  -1.91  -1.56   4201    1.0
## log_lik[4,9]   -2.15  4.1e-3   0.26  -2.74   -2.3  -2.13  -1.97  -1.73   4105    1.0
## log_lik[5,9]   -2.81  5.3e-3   0.34  -3.59  -2.98  -2.75  -2.58  -2.28   4051    1.0
## log_lik[6,9]   -1.76  6.5e-3    0.4  -2.75  -1.98   -1.7  -1.48  -1.16   3816    1.0
## log_lik[1,10]  -1.71  4.7e-3   0.26  -2.26  -1.87  -1.69  -1.53  -1.26   3050    1.0
## log_lik[2,10]  -2.23  4.3e-3   0.24  -2.75  -2.38   -2.2  -2.06  -1.81   3243    1.0
## log_lik[3,10]  -1.17  3.8e-3   0.21  -1.64  -1.31  -1.17  -1.02  -0.79   3262    1.0
## log_lik[4,10]  -1.72  4.2e-3   0.22  -2.19  -1.86  -1.71  -1.57  -1.34   2752    1.0
```

```
## log_lik[5,10]  -2.26  4.1e-3  0.23  -2.78   -2.4  -2.25   -2.1  -1.87   3045    1.0
## log_lik[6,10]  -1.82  7.0e-3  0.44  -2.87  -2.06  -1.75  -1.51  -1.16   4011    1.0
## log_lik[1,11]   -1.4  4.7e-3  0.23  -1.91  -1.55  -1.38  -1.23   -1.0   2422    1.0
## log_lik[2,11]  -2.03  4.2e-3  0.23  -2.52  -2.17  -2.01  -1.86  -1.63   2996    1.0
## log_lik[3,11]  -1.19  3.9e-3  0.22  -1.65  -1.33  -1.18  -1.03   -0.8   3212    1.0
## log_lik[4,11]  -1.77  4.4e-3  0.23  -2.27  -1.92  -1.76  -1.61  -1.37   2699    1.0
## log_lik[5,11]  -2.77  6.2e-3  0.38  -3.67  -2.98  -2.71  -2.51  -2.19   3716    1.0
## log_lik[6,11]  -1.25  5.4e-3  0.29  -1.91  -1.42  -1.22  -1.05  -0.77   2854    1.0
## log_lik[1,12]   -1.4  4.8e-3  0.24  -1.92  -1.55  -1.38  -1.23  -0.99   2458    1.0
## log_lik[2,12]  -2.13  4.6e-3  0.26  -2.72  -2.28   -2.1  -1.95  -1.69   3032    1.0
## log_lik[3,12]  -1.66  5.8e-3  0.35  -2.49  -1.85  -1.61  -1.41  -1.11   3662    1.0
## log_lik[4,12]  -1.74  4.7e-3  0.23  -2.25  -1.88  -1.72  -1.57  -1.33   2524    1.0
## log_lik[5,12]   -2.5  5.8e-3  0.33  -3.27  -2.68  -2.46  -2.28  -1.99   3194    1.0
## log_lik[6,12]  -1.57  7.1e-3  0.43  -2.59   -1.8   -1.5  -1.26  -0.95   3661    1.0
## log_lik[1,13]  -2.46  9.7e-3  0.61   -3.9  -2.82  -2.37  -2.01  -1.55   3968    1.0
## log_lik[2,13]  -2.43  6.5e-3  0.38  -3.35  -2.62  -2.37  -2.17  -1.86   3384    1.0
## log_lik[3,13]   -1.2  4.3e-3  0.23  -1.72  -1.34  -1.19  -1.04  -0.79   2997    1.0
## log_lik[4,13]  -1.74  5.0e-3  0.24  -2.27  -1.89  -1.71  -1.57  -1.33   2370    1.0
## log_lik[5,13]  -2.62  7.1e-3  0.41  -3.65  -2.82  -2.55  -2.34  -2.03   3272    1.0
## log_lik[6,13]  -1.37  6.9e-3   0.4  -2.34  -1.57  -1.31  -1.09  -0.79   3346    1.0
## log_lik[1,14]   -1.5  6.2e-3   0.3  -2.19  -1.67  -1.47  -1.29  -1.03   2264    1.0
## log_lik[2,14]  -2.43  7.2e-3  0.42  -3.47  -2.63  -2.36  -2.14  -1.83   3297    1.0
## log_lik[3,14]   -2.0  9.5e-3  0.58  -3.43  -2.31  -1.89  -1.58  -1.18   3711    1.0
## log_lik[4,14]  -1.95  6.3e-3  0.33  -2.72  -2.13  -1.91  -1.72  -1.43   2719    1.0
## log_lik[5,14]  -2.77  8.8e-3  0.51  -4.08  -3.02  -2.66  -2.41  -2.07   3355    1.0
## log_lik[6,14]  -1.06  6.0e-3  0.31   -1.8  -1.23  -1.02  -0.85  -0.58   2697    1.0
## log_lik[1,15]  -1.47  6.0e-3  0.29  -2.12  -1.62  -1.44  -1.27  -1.01   2224    1.0
## log_lik[2,15]  -2.15  6.2e-3  0.32  -2.94  -2.33  -2.11  -1.93  -1.66   2666    1.0
## log_lik[3,15]  -1.25  5.7e-3  0.28   -1.9  -1.41  -1.23  -1.06   -0.8   2477    1.0
## log_lik[4,15]  -2.04  7.5e-3   0.4  -3.02  -2.23  -1.97  -1.77  -1.45   2790    1.0
## log_lik[5,15]  -3.18    0.01  0.74  -5.05  -3.56  -3.02  -2.64   -2.2   3528    1.0
## log_lik[6,15]  -1.62    0.01  0.59  -3.08  -1.96   -1.5   -1.2  -0.81   3131    1.0
## yrep[1,1]       26.7    0.03  1.77   23.2  25.59  26.67  27.82  30.36   3725    1.0
## yrep[2,1]       34.1    0.05  3.25  27.72  32.07  34.07  36.27  40.52   3638    1.0
## yrep[3,1]       35.7    0.02  1.44  32.84  34.78  35.68   36.6  38.59   3536    1.0
## yrep[4,1]      29.78    0.04  2.46  24.79  28.27  29.76  31.33  34.84   3398    1.0
## yrep[5,1]      28.89    0.07  3.62  21.91   26.6  28.85  31.13  36.28   3057    1.0
## yrep[6,1]      35.76    0.02  1.05  33.61  35.11  35.77  36.43  37.85   3134    1.0
## yrep[1,2]      26.83    0.03   1.7  23.48  25.73   26.8  27.88   30.3   3735    1.0
## yrep[2,2]      33.13    0.05   3.2  26.76  31.11  33.13  35.18  39.52   3532    1.0
## yrep[3,2]      34.57    0.02   1.4  31.68  33.67  34.61  35.49  37.25   3779    1.0
## yrep[4,2]      28.66    0.04   2.4  23.86  27.12  28.66   30.2   33.5   3572    1.0
## yrep[5,2]       27.4    0.06  3.55  20.28  25.11  27.43  29.64  34.51   3642    1.0
## yrep[6,2]      34.48    0.02  1.04  32.34  33.84  34.49  35.13  36.58   3598    1.0
## yrep[1,3]      26.91    0.03  1.74  23.59  25.76  26.89  27.99  30.43   3455    1.0
## yrep[2,3]      32.28    0.05  3.07  26.12  30.38  32.33  34.27  38.26   3658    1.0
## yrep[3,3]      33.43    0.02  1.39  30.73  32.53  33.42   34.3  36.18   3860    1.0
## yrep[4,3]       27.4    0.04  2.36  22.59  25.98  27.39  28.85  32.11   3744    1.0
## yrep[5,3]      25.91    0.06  3.58  18.72  23.66  25.93  28.22  33.01   3806    1.0
## yrep[6,3]      33.12    0.02  1.03  30.98  32.46  33.13  33.76   35.1   3468    1.0
## yrep[1,4]      26.97    0.03   1.7  23.54  25.91  26.99  28.03   30.4   4186    1.0
## yrep[2,4]       31.4    0.05  3.11  24.94  29.46  31.41   33.4  37.41   3542    1.0
## yrep[3,4]      32.31    0.02  1.34  29.71  31.47  32.33  33.17  34.92   3818    1.0
## yrep[4,4]      26.21    0.04  2.32   21.6  24.78  26.24  27.67  30.76   3532    1.0
```

```
## yrep[5,4]      24.48    0.06   3.56   17.4  22.25  24.44  26.73  31.48    3462     1.0
## yrep[6,4]      31.77    0.02    1.0  29.74  31.13  31.79  32.41  33.74    3893     1.0
## yrep[1,5]      27.05    0.03   1.69  23.71  25.99  27.04   28.1  30.43    4040     1.0
## yrep[2,5]       30.5    0.05   3.09  24.38  28.49  30.53  32.54  36.47    3981     1.0
## yrep[3,5]      31.21    0.02   1.35  28.51  30.39  31.21  32.08  33.86    3793     1.0
## yrep[4,5]      25.11    0.04   2.33  20.68  23.59  25.08  26.63  29.64    3954     1.0
## yrep[5,5]       23.1    0.05   3.43  16.42  20.92  23.14  25.22  30.06    4131     1.0
## yrep[6,5]      30.47    0.02   1.03  28.46  29.84  30.45  31.11  32.53    3647     1.0
## yrep[1,6]      27.11    0.03   1.63  23.87  26.07  27.09  28.14  30.34    3968     1.0
## yrep[2,6]      29.61    0.05   3.04  23.58  27.67  29.66  31.53   35.8    4107     1.0
## yrep[3,6]      30.04    0.02   1.34  27.35  29.21  30.05   30.9  32.67    3709     1.0
## yrep[4,6]      23.94    0.04   2.29  19.53  22.47  23.95  25.42  28.56    3965     1.0
## yrep[5,6]      21.65    0.05   3.47   15.0  19.45  21.67  23.81   28.5    4094     1.0
## yrep[6,6]      29.15    0.02   0.99  27.24  28.51  29.15  29.78  31.16    3809     1.0
## yrep[1,7]      27.22    0.03   1.67  23.86  26.17  27.24  28.28   30.5    4040     1.0
## yrep[2,7]      28.68    0.05   3.01  22.67  26.75  28.75  30.59  34.47    3814     1.0
## yrep[3,7]      28.93    0.02   1.33  26.28  28.09  28.92  29.75  31.66    3922     1.0
## yrep[4,7]       22.8    0.04   2.28  18.16  21.35   22.8  24.29  27.25    3565     1.0
## yrep[5,7]      20.12    0.05   3.45  13.24  17.93  20.13  22.31   27.0    4012     1.0
## yrep[6,7]      27.82    0.02    1.0  25.88  27.16  27.82  28.46  29.82    4096     1.0
## yrep[1,8]      27.29    0.03   1.62  24.07  26.26  27.29  28.32  30.57    3993     1.0
## yrep[2,8]      27.81    0.05   3.01   21.7  26.01  27.83   29.7  33.67    4131     1.0
## yrep[3,8]      27.76    0.02   1.32  25.05  26.93  27.76   28.6  30.33    3935     1.0
## yrep[4,8]      21.62    0.03   2.23  17.15   20.2  21.62  23.01  26.02    4162     1.0
## yrep[5,8]      18.71    0.05   3.41  11.94  16.52  18.68  20.92  25.47    3930     1.0
## yrep[6,8]      26.49    0.01   0.99  24.46  25.88  26.49  27.11  28.45    4356     1.0
## yrep[1,9]      27.43    0.03   1.65  24.13  26.39  27.42  28.48  30.78    3860     1.0
## yrep[2,9]      27.03    0.05    3.0  21.07  25.17  27.02  28.89  33.03    3760     1.0
## yrep[3,9]      26.63    0.02   1.32  24.06  25.78  26.62  27.44  29.32    3660     1.0
## yrep[4,9]      20.44    0.04   2.27  16.02  18.99  20.42  21.89  24.93    4086     1.0
## yrep[5,9]      17.19    0.06   3.53   9.89  14.92   17.2  19.41  24.19    3754     1.0
## yrep[6,9]      25.15    0.02   1.01  23.16  24.54  25.15  25.77  27.13    4001     1.0
## yrep[1,10]     27.48    0.03   1.69  24.04  26.44  27.49  28.56  30.81    4108     1.0
## yrep[2,10]     26.18    0.05   3.03   19.9  24.25  26.21  28.09   32.2    4122     1.0
## yrep[3,10]     25.46    0.02   1.34  22.75  24.63  25.45  26.34  28.09    3667     1.0
## yrep[4,10]     19.29    0.04   2.27  14.64  17.89  19.29  20.71  23.79    4049     1.0
## yrep[5,10]     15.75    0.06   3.49   8.82  13.48  15.82  17.97  22.62    3971     1.0
## yrep[6,10]     23.86    0.02   0.99  21.87  23.23  23.85  24.47  25.88    3902     1.0
## yrep[1,11]     27.54    0.03   1.73  24.04  26.48   27.5  28.65  30.89    4106     1.0
## yrep[2,11]     25.26    0.05   3.03  19.21  23.38  25.26  27.17   31.2    3652     1.0
## yrep[3,11]     24.35    0.02   1.32  21.81   23.5   24.3  25.17  27.06    3638     1.0
## yrep[4,11]     18.19    0.04   2.26  13.65  16.76  18.17   19.6  22.78    3930     1.0
## yrep[5,11]     14.28    0.06   3.46    7.3  12.06  14.26  16.46  21.19    3832     1.0
## yrep[6,11]      22.5    0.02   0.99  20.53  21.87   22.5  23.11  24.51    3868     1.0
## yrep[1,12]     27.62    0.03   1.71  24.22  26.55  27.64  28.66  31.06    3861     1.0
## yrep[2,12]     24.27    0.05   3.09  18.01  22.26  24.31  26.29  30.21    3691     1.0
## yrep[3,12]     23.22    0.02   1.33  20.58   22.4   23.2  24.03  25.93    3855     1.0
## yrep[4,12]     17.02    0.04   2.35   12.4  15.52  17.01   18.5  21.95    4039     1.0
## yrep[5,12]     12.91    0.06   3.58   6.07  10.55  12.88  15.25   20.1    4175     1.0
## yrep[6,12]     21.22    0.02   1.01  19.27  20.57   21.2  21.86   23.2    3850     1.0
## yrep[1,13]     27.71    0.03   1.73  24.24  26.61  27.71   28.8  31.14    3951     1.0
## yrep[2,13]     23.48    0.05    3.1  17.41  21.54  23.48   25.4  29.62    3604     1.0
## yrep[3,13]     22.07    0.02   1.34  19.46  21.17  22.07  22.95  24.78    3887     1.0
## yrep[4,13]     15.85    0.04   2.35  11.04  14.39  15.85  17.32  20.49    4072     1.0
```

```
## yrep[5,13]     11.43    0.06   3.57   4.33   9.19  11.41  13.71  18.65   3992    1.0
## yrep[6,13]     19.88    0.02   1.01  17.88  19.21  19.89  20.55  21.91   4081    1.0
## yrep[1,14]     27.75    0.03   1.79  24.08  26.63  27.78  28.93  31.17   3820    1.0
## yrep[2,14]     22.46    0.05   3.22  16.18  20.39  22.47  24.54  28.67   3915    1.0
## yrep[3,14]     20.92    0.02   1.38  18.12  20.05   20.9  21.78  23.73   3932    1.0
## yrep[4,14]      14.7    0.04   2.42   9.72   13.2  14.75  16.16   19.4   3646    1.0
## yrep[5,14]      9.93    0.06   3.73   2.46   7.52   9.93  12.27  17.55   3827    1.0
## yrep[6,14]     18.56    0.02   1.05  16.45  17.89  18.55  19.23  20.67   3857    1.0
## yrep[1,15]      27.9    0.03   1.78  24.28  26.81  27.94  29.05  31.45   3816    1.0
## yrep[2,15]     21.73    0.05   3.34  15.16  19.64  21.69  23.88  28.27   3795    1.0
## yrep[3,15]      19.8    0.02   1.42  16.92  18.92  19.81  20.67  22.66   3712    1.0
## yrep[4,15]     13.67    0.04   2.44   8.81  12.14  13.69  15.24  18.47   3909    1.0
## yrep[5,15]       8.6    0.06   3.73   1.26   6.28   8.52   10.9  16.01   3901    1.0
## yrep[6,15]     17.24    0.02   1.06  15.11  16.56  17.22  17.91  19.41   3555    1.0
## pred[1]        27.96    0.03   1.86  24.27   26.8  27.97  29.11  31.68   3786    1.0
## pred[2]        20.73    0.05   3.27   14.2  18.59   20.7  22.84  27.23   3687    1.0
## pred[3]        18.67    0.02   1.44  15.92  17.75  18.65  19.55  21.69   3534    1.0
## pred[4]        12.45    0.04   2.49    7.7  10.82  12.45  13.99  17.49   3507    1.0
## pred[5]         7.08    0.07   3.82  -0.53   4.66   7.09   9.49  14.73   3325    1.0
## pred[6]        15.91    0.02   1.06   13.8  15.26  15.89  16.57  18.05   3037    1.0
## lp__          -102.9     0.1   3.76 -111.3 -105.1 -102.5 -100.2 -96.68   1450    1.0
##
## Samples were drawn using NUTS at Sun 29 Nov 2020 06:48:27 PM .
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```python
def get_psis_loo_result(stan_results):
    idata = az.from_pystan(stan_results, log_likelihood="log_lik")
    loo_results = az.loo(idata, pointwise=True)
    print(loo_results)
    khats = loo_results.pareto_k
    az.plot_khat(khats, xlabels=True, annotate=True)

get_psis_loo_result(stan_results)
```

```
## Computed from 4000 by 90 log-likelihood matrix
##
##          Estimate       SE
## elpd_loo  -186.24     7.53
## p_loo       15.90        -
##
## There has been a warning during the calculation. Please check the results.
## ------
##
## Pareto k diagnostic values:
##                          Count   Pct.
## (-Inf, 0.5]   (good)       86   95.6%
##  (0.5, 0.7]   (ok)          2    2.2%
##    (0.7, 1]   (bad)         2    2.2%
##    (1, Inf)   (very bad)    0    0.0%
##
##
## /home/weijiang/anaconda3/envs/bda/lib/python3.7/site-packages/arviz/stats/stats.py:684: UserWarning:
##   "Estimated shape parameter of Pareto distribution is greater than 0.7 for "
```

```
_ = az.plot_ess(
    stan_results, var_names=["alpha", "beta", "sigma_alpha", "sigma_beta"],
    kind="local", marker="_", ms=20, mew=2, figsize=(20, 20)
)
```

```
_ = az.plot_trace(stan_results, var_names = ["alpha", "beta", "sigma_alpha", "sigma_beta"], figsize=(20
```

```
h = stan_results.to_dataframe(diagnostics=True)
print('max treedepth for draws: ', h['treedepth__'].max())
```

```
## max treedepth for draws:  6
```

```
print('min treedepth for draws: ', h['treedepth__'].min())
```

```
## min treedepth for draws:  3
```

```
print('mean treedepth for draws: ', h['treedepth__'].mean())
```

```
## mean treedepth for draws:  4.05475
```

```
print('divergent transitions: ', any(h['divergent__']))
```

```
## divergent transitions:  False
```

```
plt.figure(figsize=(15,10))
```

```
## <Figure size 1500x1000 with 0 Axes>
```

```
year_idx = np.arange(accident_data.shape[1])+1
actual_years = year_idx + 2004

colors = ['red', 'pink', 'orange', 'gray', 'green', 'purple']
for x in range(1, 7):
    print()
    for i in range(100):
        y = stan_results["beta"][:, x-1][i] * year_idx + stan_results["alpha"][:, x-1][i]
        _ = plt.plot(actual_years, y, color=colors[x-1], alpha=0.05)

for x in range(1, 7):
    for j in reversed(range(1, 16)):
        yrep = stan_results['yrep[{},{}]'.format(x, j)]
        _ = plt.errorbar(
            x = actual_years[j-1],
            y = np.mean(yrep),
            yerr=np.std(yrep),
            fmt='--o', zorder=i+j,
            ecolor='black', capthick=2,
            color='black',
            alpha=0.5
        )

for k in range(1, 7):
    ypred = stan_results['pred[{}]'.format(k)]
    _ = plt.errorbar(
        x = 2020,
        y = np.mean(ypred),
        yerr=np.std(ypred),
        fmt='--o', zorder=i+j+100,
```

```
        ecolor='red', capthick=2,
        color='red',
    )


_ = plt.scatter(np.tile(years, 6), accident_data.flatten(), zorder=j+i+100, edgecolors='black')
# _ = plt.scatter(data_for_stan["years"], data_for_stan["accidentData"], zorder=j+i+100, edgecolors='bl
_ = plt.title("Posterior predictive check")
_ = plt.legend(bbox_to_anchor=(1.05, 1), loc='lower left', borderaxespad=0.)


# area_names = ["Metropolitan Police", 'Cumbria','Lancashire',
#                 'Merseyside','Greater Manchester','Cheshire']

custom_lines = [
    Line2D([0], [0], color='red', lw=4, label='Metropolitan Police'),
    Line2D([0], [0], color='yellow', lw=4, label='Cumbria'),
    Line2D([0], [0], color='orange', lw=4, label='Lancashire'),
    Line2D([0], [0], color='gray', lw=4, label='Merseyside'),
    Line2D([0], [0], color='green', lw=4, label='Greater Manchester'),
    Line2D([0], [0], color='purple', lw=4, label='Cheshire'),
    Line2D([0], [0], marker='o', color='black', label='Original datapoint', markerfacecolor='b', markers
    Line2D([0], [0], marker='o', color='red', label='Predictions 2020', markersize=15),
    Line2D([0], [0], marker='o', color='black', label='Posterior samples', markersize=15),
]

_ = plt.legend(handles=custom_lines, bbox_to_anchor=(1, 1))
_ = plt.xticks(np.arange(2005, 2021), fontsize=13)
_ = plt.yticks(fontsize=14)
```

```
data_dict = dict()
names = ["default_prior", "uniform_prior", "bigger_variance"]
for i in range(3):
    current_stan_data = dict(
        N = accident_data.shape[0],
        Y = accident_data.shape[1],
        accidentData = accident_data,
        years = np.arange(1, accident_data.shape[1]+1), # stan index starts from 1
        xpred=2020,
        prior_choice= i+1
    )
    data_dict[names[i]] = current_stan_data
```

```
result_dict = dict()
for key, stan_data in data_dict.items():
    print("Generating results with prior:{} {}".format(stan_data["prior_choice"], key))
    sampling_result = stan_model.sampling(data=stan_data)
    #print(sampling_result)
    result_dict[key] = sampling_result
```

```
## Generating results with prior:1 default_prior
## Generating results with prior:2 uniform_prior
## Generating results with prior:3 bigger_variance
```

```
_ = az.plot_forest(
    list(result_dict.values()),
    model_names=list(result_dict.keys()), var_names=["beta"], markersize=10,
    kind='ridgeplot', ridgeplot_overlap=3, ridgeplot_alpha=0.3, r_hat=True, ess=True, figsize=(20, 20),
)
plt.rcParams['xtick.labelsize'] = 20
plt.rcParams['ytick.labelsize'] = 20
plt.show()
```