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Ex-ante fairness under constrained school choice: An experimental approach☆

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ABSTRACT

In a college admission mechanism, students are often matched with colleges by using a noisy signal of their true abilities (e.g., their exam scores). The matching outcome thus may be imperfect in terms of ex-ante fairness, which suggests matching students with higher ability to better colleges. To achieve ex-ante fairness, we consider constraining student choice over colleges, by designing treatments with different constraint levels under the Boston and Serial Dictatorship mechanisms, with preference submission before or after the exam. Constraining student choice increases the probability of achieving ex-ante fairness under the Boston and Serial Dictatorship mechanisms with preference submission before the exam, compared with unconstrained mechanisms and mechanisms with preference submission after the exam. However, the probability of achieving highly unfair matching is also increased, resulting in a riskier matching outcome. Learning the game or providing students with recommended equilibrium strategies can decrease this risk and further increase ex-ante fairness.

1. Introduction

In college admission mechanisms, colleges often observe only one or several noisy signals of students' true ability (e.g., exam scores). One example is China's College Entrance Exam (CEE): a student's total score in this one-shot exam is used by colleges as the sole criterion for admission (Wu (2007)). Serial Dictatorship (or more broadly, Deferred Acceptance) mechanisms are widely used to achieve ex-post fairness, i.e., matching students with higher realized scores to better (or generally preferred) colleges. However, such an outcome is imperfect in terms of ex-ante fairness, which suggests matching students with higher ability to better colleges. Some researchers suggest that a Boston mechanism with preferences being submitted before the exam (i.e., a BOS-before mechanism) may help to achieve ex-ante fairness (Lien et al. (2016, 2017)). This solution, however, has only been partially justified either in theory or in the laboratory. One problem is that when a student can submit a *complete* preference list over all colleges, he has a minimum guarantee of his *non-first*

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choices, so he may list a college better than his qualified or "fair" college as his first choice, and expect to be the "lucky" one in the exam, i.e., to be admitted by his or her first choice. This incentive distortion damages ex-ante fairness. According to this reasoning, a mechanism with constrained school choice where a student can include some but not all colleges in his preference list (Haeringer and Klijn (2009)) may help to correct such distortion and achieve more ex-ante fairness (Lien et al. (2017)).

In this paper, we experimentally study the role of constrained school choice in achieving ex-ante fairness under different matching mechanisms. We implement a 2-by-2-by-3 experimental design. Following Lien et al. (2016, 2017), we first categorize matching mechanisms by two dimensions. One concerns the matching algorithm, i.e., Boston (BOS) vs. Serial Dictatorship (SD)[1] mechanism. The other dimension is the timing of preference submission: before the exam is taken versus after the scores are published. *Before* means that every student knows the distribution of the scores without knowing realized scores, and *after* stands for cases that each student knows everyone's realized score. The resulting four mechanisms are therefore called BOS-before, BOS-after, SD-before, and SD-after mechanisms. For each of these four mechanisms, we consider three levels of school choice constraint, i.e., unconstrained, moderately constrained, or fully constrained. Different levels of constraint are imposed simply by varying the number of colleges that students are allowed to choose. Note that under the fully constrained school choice where students can choose only one college, the BOS and SD mechanisms are equivalent.

We propose and test two hypotheses: i) the BOS/SD-before mechanisms outperforms the BOS/SD-after mechanisms under *constrained* school choice. ii) the BOS/SD-before under *constrained* school choice outperform the BOS/SD-before/after mechanisms under *unconstrained* school choice (Both in the sense of ex-ante fairness). Since *unconstrained* BOS/SD-before mechanisms are at least no worse off than *unconstrained* BOS/SD-after mechanisms, as shown in Lien et al. (2016, 2017), then we can reach the conclusion that the *constrained* BOS/SD-before mechanisms would outperform all other six alternative mechanisms, if our two hypotheses held. By comparing experimental results for those two hypotheses, we can gain a better understanding of the role of constraints in achieving ex-ante fairness.

The intuition for these two hypotheses is as follows: under the unconstrained BOS/SD-before mechanism, students have to submit their preference before the exam is taken. Therefore, students have to rely on their true ability, which is arguably the best estimation of their actual college entrance exam score when submitting their preference order. Ex-ante fairness is likely to be achieved if all students' incentives are aligned by listing their ex-ante fair college as their first choice.

However, one may have incentives to deviate. When a student is allowed to list many schools, he can put a college better than his ex-ante fair college as his first choice, and put the ex-ante fair college as his second choice. As long as he exceeds a student with higher ability in his *realized* scores, he will get that student's fair college, which is supposed to be better than his own fair college. Otherwise he will still get his fair college. Therefore, one way to recover ex-ante fairness is to reduce the number of colleges a student is allowed to list, so that the student faces fewer non-first-choice (or back-up) colleges. If he still chooses to "overstate" his first choice college but fails to be admitted by it, he will be admitted by a worse college than when he has an unconstrained choice set, or not be admitted at all. Constrained school choice then serves as a commitment device to encourage students to put their ex-ante fair colleges as their first choice, leading to ex-ante fair matching outcomes.

Based on our hypotheses, we conduct experiments examining how constraints affect ex-ante fairness in matching outcomes. Our results show that the level of ex-ante fairness does increase under *fully constrained BOS/SD-before* mechanisms. However, there is also a higher risk under such mechanisms, because students may reach highly unfair matching outcomes (such as not being admitted).

To reduce the risks that students face and possibly further improve ex-ante fairness, we then run a series of variants of those basic constrained mechanisms, including repeated games, games with recommended strategies, and dynamic games with a second-stage choice. We find that repeated games and recommended strategies can significantly improve the level of ex-ante fairness and reduce risks.[2] There is not enough evidence showing that dynamic games with a second-stage choice have similar effects.

Our research is related to two branches of the literature. One is the literature that reconsiders the Boston mechanism under some forms of uncertainty. In the earlier literature, the Boston mechanism was commonly regarded as inferior to the Serial Dictatorship mechanism in terms of efficiency and (ex-post) fairness; see Abdulkadiroğlu and Sönmez (2003) and Ergin and Sönmez (2006). Abdulkadiroğlu et al. (2011), however, argue that when students have homogeneous ordinal (but different cardinal) preferences for colleges and colleges have random priorities for students, BOS can outperform SD in terms of efficiency. Featherstone and Niederle (2014) argue that when students have private information on their preferences and schools have random priorities, the Boston mechanism can implement a truth-telling Bayesian Nash equilibrium and achieve more efficiency than the DA mechanism. Lien et al. (2016, 2017) argue that the Boston mechanism with preference submission before the exam can achieve greater ex-ante fairness than other mechanisms. Pan (2019) raised the point that overconfidence may harm the stability of matching outcomes under the PreExam-BOS (BOS-before) mechanism. Although this is different from out findings in this paper, her setting is also different from ours,

[1] Serial Dictatorship is a special case of the widely known Top Trading Cycles (TTC) mechanism (Abdulkadiroğlu and Sönmez, 2003), which implements the same matching as another widely known mechanism, Deferred Acceptance (DA), in their Nash equilibria, as long as college priorities are all the same, an extreme case of acyclic conditions (Haeringer and Klijn, 2009).

[2] Repeated games provide subjects with chances to learn. Recommended strategies are also a common way to avoid a "bad" equilibrium in the literature. For example, Van Huyck et al. (1992) study how recommendations affect the outcome in two-person coordination games. Brandts and MacLeod (1995) show that recommendation is an efficient method of equilibrium selection when there are multiple Nash equilibria. Seely et al. (2005) report that assignments (recommendation) can improve efficiency in a laboratory public goods game. Besides recommendation, Haruvy et al. (2017) showed that communication and visibility can be used to increase cooperation. In our experiment, we consider repeated game and recommendation for improving coordination. Note that repeated game can be seen as a way of communication.

in the sense that we inform the subjects their abilities (or types) perfectly. Our paper is a direct extension of these studies. In particular, we adapt our experimental design from Lien et al. (2016), but impose constrained school choice to reconsider the ex-ante fairness of some commonly-used mechanisms, as those in their paper.

Another branch of the literature focuses on constrained school choice. Haeringer and Klijn (2009) first raise this issue and study its welfare consequence on Boston, DA, and TTC mechanisms. The Boston mechanism is immune to such a quota restriction, in the sense that it can always implement stable matching outcomes under any quota, while other mechanisms may not. Calsamiglia et al. (2010) conduct an experimental study and conclude that such constraints reduce the efficiency and stability of all mechanisms. Furthermore, they find that BOS is not more stable than the DA and TTC mechanisms. Dur and Morrill (2020) showed that restricting students' choice in DA can lead to a Pareto-improvement of DA in equilibrium. To the best of our knowledge, the constrained school choice literature until now has not addressed the ex-ante fairness when student ability is only partially correlated to their scores, which we explore by introducing preference submission timing. It is therefore interesting to see how the combination of these two "imperfections" may potentially improve welfare.

The inspiration for our research comes from China's college admissions system, where a noisy signal of students' true ability, the college entrance exam score, is almost the only determinant of college priority. China's system is also moving from a BOS-before mechanism towards an SD-after mechanism (Chen and Kesten, 2017; Lien et al., 2016, 2017). Along the way, constrained school choice remains a prominent feature of this system: Although there are over 1000 colleges and vocational schools in China, students are allowed to fill in no more than 15 of them in their preference list. All of these features are reflected in our experimental design.

The remainder of the paper is organized as follows. We introduce our basic experiment design, theoretical predictions and hypotheses in Section 2. In Section 3, we present our experimental results on both matching outcomes and subject behavior. Section 4 discusses some variants of our *fully constrained BOS/SD-before* mechanisms and Section 5 concludes the paper.

2. Experimental design

Following Lien et al. (2016) and Lien et al. (2017), we first divide our treatments by two dimensions. One is by the matching mechanism (or more precisely, algorithm), i.e., the Boston (BOS) or the Serial Dictatorship (SD) mechanism. The other is the timing of preference submission: before the exam is taken or after. When preference submission occurs before the exam, each student is informed only the score distribution of all students, but no-one's realized score. When preference submission occurs after the exam, the exam scores are published so that each student knows the realized scores of all students. We then have four treatments (or mechanisms): BOS-before, BOS-after, SD-before, and SD-after.

Under each of these four mechanisms, we then introduce constraints on students' choice of colleges. In particular, there are three different level of constraints:(i)no constraint, where subjects can list all 3 colleges; (ii) partially constraint, where only 2 out of 3 can be listed; (iii)full constraint, where subjects can only submit 1 college. Except for the restriction on the number of colleges, students are free to list any colleges in any order. In our experiments, we have a total of three colleges for matching, so the fully constrained, partially constrained, and unconstrained school choice correspond to submitting at most 1, 2, or 3 colleges in preference list. This completes our 2*2*3 experimental design.

We measure the ex-ante fairness of matching outcomes by two methods. First, we consider whether a matching outcome is (fully) ex-ante fair, i.e., does not contain blocking pairs at all. Here a blocking pair is defined as a student–college pair in which both have incentives to alter their current matching and re-match with each other, according to student preferences and college priorities(Balinski and Sönmez (1999)). The other measure is the number of blocking pairs. The lower this number, the fairer a treatment is. To focus on the issue of ex-ante fairness, following the experiment design of Lien et al. (2016), we assume that students share exactly the same preferences for colleges, both ordinal and cardinal. However, students have different score distributions, and their realized scores are also always different from each other. Students' true abilities are measured by their mean scores, and we assume colleges always give higher priorities to students with higher abilities.

2.1. Environment

In our experiment, a matching (or college admissions) system contains three colleges A, B, C, and three students 1, 2, 3. Every college only has one vacant seat, and each student can only be admitted by one college. The students' score distribution and payoffs from being admitted by each college or not being admitted are listed in Table 1 and Table 2, and this information is common knowledge among all subjects.

We now describe all of the procedures used to match students and colleges in our experimental design. We first categorize them by matching mechanism (BOS or SD) and preference submission timing (before or after the exam). For each mechanism, we describe how

Table 1
Student score distributions.

Score type	High score	Normal score	Low score	Avg. score(true ability)
Probability	1/3	1/3	1/3	
Student 1	95	90	85	90
Student 2	91	86	81	86
Student 3	87	82	77	82

Table 2
Student payoffs.

Admission by college	A	B	C	Non-admission
Student 1's payoff	30	25	15	0
Student 2's payoff	30	25	15	0
Student 3's payoff	30	25	15	0

we vary the number of colleges the student is allowed to choose.

2.1.1. BOS-before mechanism

Step 1. All students simultaneously submit a preference list containing n colleges.
Step 2. Each student learns his and the other two students' realized scores, which are randomly and independently drawn according to everyone's score distribution.
Step 3. Each college considers the first choice of all of the students. Colleges admit the student with the highest realized score among those who list it as their first choice.

If $n = 1$, then the algorithm stops. Students not admitted in Step 3 are finally unadmitted.

Step 4. Colleges with vacancies then consider the second choice of all unadmitted students. They admit the student with the highest realized score among those who are as yet unadmitted and who list it as their second choice.

If $n = 2$, then the algorithm stops. Students not admitted in Steps 3–4 are finally unadmitted.

Step 5. Colleges with vacancies then consider the third choice of all unadmitted students. They admit the student with the highest realized score among those who are yet unadmitted and who list it as their third choice.

The algorithm stops.

When $n = 3$, the mechanism is an unconstrained BOS-before mechanism; when $n = 2$, it is a partially constrained BOS-before mechanism; when $n = 1$, it is a fully constrained BOS-before mechanism. The constraints on the other three mechanisms are similarly defined.

2.1.2. BOS-after mechanism

Step 1. Each subject learns his and the other two students' realized scores, which are randomly and independently drawn according the score distribution.
Step 2. All of the subjects simultaneously submit a preference list containing n colleges.

All of the other steps are the same as listed in BOS-before.

2.1.3. SD-before mechanism

Steps 1 and 2 are the same as in BOS-before.
Step 3. The student with the highest realized score among the three is admitted by his highest ranked college in his preference list containing n colleges.
Step 4. The student with the second highest realized score among the three is admitted by his highest ranked college with vacancies.
Step 5. The student with the third highest realized score among the three is admitted by his highest ranked college with vacancies.

Students not admitted in Steps 4–5 are finally unadmitted.

2.1.4. SD-after mechanism

Steps 1 and 2 are the same as in BOS-after.
Steps 3 to 5 are the same as in SD-before.

2.2. Equilibrium

We now derive the (pure-strategy) Nash equilibrium for each treatment (mechanism) described above. Before that, in the general case, Lien et al. (2017) Corollary 3.2 indicates the following conclusion:

Lemma 1. *The fully constrained BOS-before/SD-before mechanism implements fully ex-ante fair matching outcomes in one of its pure-strategy*

Nash equilibria if the unconstrained BOS-before mechanism does so.

This result implies that imposing the full constraint on the BOS-before mechanism will always increase the likelihood of achieving ex-ante fairness.

2.2.1. Unconstrained mechanisms

Under unconstrained mechanisms, the equilibrium strategy and outcome have been identified by Lien et al. (2016). We only replicate their theoretical results here.[3]

Claim 1.1. *Under the unconstrained BOS-before mechanism: (1) The Nash equilibrium strategy profile for three students is of the form [(A,*,*), (B,*,*), (B,*,*)], where * denotes any college not yet listed. (2) The equilibrium outcome is [(1,A), (2,B), (3,C)] or [(1,A), (2,C), (3,B)], depending on whether student 2 gets a higher realized score than student 3.*

Claim 1.2. *Under the unconstrained BOS-after mechanism: (1) The Nash equilibrium is as follows: The student with the highest realized score submits (A,*,*), the student with the second highest realized score submits (B,*,*), the last one submits any list. (2) The equilibrium outcome is college A matching with the student with the highest score, college B matching with the student with the second highest score, college C matching with the remaining student.*

Claim 1.3. *Under the unconstrained SD-before mechanism: (1) The Nash equilibrium is that all students use a truth-telling strategy, i.e., (A,B,C). (2) The matching outcome is the same as for the BOS-after mechanism.*

Claim 1.4. *Under the unconstrained SD-after mechanism: (1) There are two Nash equilibria. One is as follows: The student with the highest realized score submits (A,*,*), the student with the second highest realized score submits (B,*,*), the remaining student submits any list. The other equilibrium is that all students use a truth-telling strategy, i.e., (A,B,C). (2) The equilibrium outcome is unique and the same as for the BOS-after mechanism.*

Under the unconstrained BOS-after and SD-before/after mechanisms, *ex-post* fairness is achieved. That is, students with higher scores are always matched to better or more preferred colleges.

2.2.2. Partially constrained mechanisms

When students are allowed to submit a preference ordering list containing two colleges, the equilibrium strategies differ somewhat from the corresponding unconstrained mechanisms due to the constraints. However, the equilibrium outcomes do not change. For BOS/SD-before mechanisms, we summarize our findings in Proposition 1 and Proposition 2, with proofs provided in Appendix A.

Proposition 1. *Under the partially constrained BOS-before mechanism*: (1) *The Nash equilibrium strategy profile for the three students is of the form* [(A,*), (B,C), (B,C)], *where * denotes any college not listed yet.* (2) *The equilibrium outcome is the same as under the unconstrained BOS-before mechanism.*

Proposition 2. *Under the partially constrained SD-before mechanism*: (1) *The Nash equilibrium is of the form* [(A,*), (B,C), (B,C)], *where * denotes any college not yet listed.* (2) *The equilibrium outcome is the same as under the unconstrained BOS-before mechanism.*

As for the BOS-after and SD-after mechanisms, the equilibrium outcomes are the same as for the corresponding unconstrained mechanisms. The formal descriptions are as follows:

***Claim* 2.1**. *Under the partially constrained BOS-after mechanism: (1) The Nash equilibrium is that the student with the highest ex-post score submits (A,*), the student with the second highest ex-post score submits (B,*), and the remaining student submits any list containing college C. (2) The equilibrium outcome is the same as for the unconstrained BOS-after mechanism.*

***Claim* 2.2**. *Under the partially constrained SD-after mechanism: (1) The Nash equilibrium is that the student with the highest ex-post score submits (A,*), the student with the second highest ex-post score submits (B,*) or (A, B), the remaining student submits any list containing college C. (2) The equilibrium outcome is the same as for the unconstrained BOS-after mechanism.*

2.2.3. Fully constrained mechanisms

When only one college may be listed, the BOS mechanism becomes the same as the SD mechanism. We summarize our results for BOS/SD-before mechanism in Proposition 3, and the proof is provided in Appendix A.

Proposition 3. *Under the fully constrained BOS/SD-before mechanism*: (1) *The Nash equilibrium strategy profile for the three students is either* [(A), (B), (C)] *or* [(B), (A), (C)]. (2) *The equilibrium outcome is* [(1,A), (2,B), (3,C)] *or* [(1,B), (2,A), (3,C)], *respectively*.

It can be verified that a sufficient condition for BOS/SD-before to implement ex-ante fairness, stated in Proposition 3.4 *in* Lien et al. (2017), *is satisfied in our experimental set-up. Proposition* 3, *together with Conclusions* 1.1 *and* 1.3, *is consistent with Corollary* 3.2 *in* Lien et al. (2017), *which claims that fully constrained BOS/SD-before mechanisms are more likely to implement ex-ante fairness than unconstrained mechanisms. However, it is true for only one of the two equilibria under the fully constrained case stated in Proposition* 3. *Proposition* 3 *also claims the existence of multiple equilibria, which is not discussed in* Lien et al. (2017). *Specifically, in our experiment design, given the specified score distributions, when* $3u(B) \geqslant u(A) \geqslant 1.5u(B)$, *and* $3u(C) \geqslant u(B) \geqslant 1.5u(C)$, *where* $u(i)>0$ *denotes the payoff of admission by college i, and non-admission payoffs are normalized to zero, there will be only one Nash equilibrium* [(1,A), (2,B), (3,C)]. *Our set-up does not satisfy those inequalities, so there are multiple equilibria.*

The equilibrium outcome under the BOS/SD-after mechanism is still not affected by the imposed constraints:

***Claim* 3.1**. *Under the fully constrained BOS/SD-after mechanism: (1) The student with the highest ex-post score submits (A), the student with*

[3] Claims 1.1–1.4 repeat the results in Lien et al. (2016). Propositions 1–4 and claims 2.1–2.2 and 3.1 are our own theoretical results.

Table 3
Theoretical predictions.

Unconstrained	BOS-before	BOS-after	SD-before	SD-after
Probability of achieving fairness	2/3	10/27	10/27	10/27
Number of blocking pairs	1/3	7/9	7/9	7/9
Partially constrained	BOS-before	BOS-after	SD-before	SD-after
Probability of achieving fairness	2/3	10/27	2/3	10/27
Number of blocking pairs	1/3	7/9	1/3	7/9
Fully constrained	BOS-before	BOS-after	SD-before	SD-after
Probability of achieving fairness	1 or 0	10/27	1 or 0	10/27
Number of blocking pairs	0 or 1	7/9	0 or 1	7/9

the second highest ex-post score submits (B), the remaining student submits (C). (2) The equilibrium outcome is the same as for the unconstrained BOS-after mechanism.

2.3. Theoretical predictions and hypotheses

From our equilibrium analysis in Section 2.2, we can derive the proportion of ex-ante fairness and the number of blocking pairs under different treatments/mechanisms. See Table 3.

For any given constraint level, BOS-before almost always (at least weakly) outperforms the other three mechanisms in ex-ante fairness. The only exception is in one of two equilibria under the fully constrained case, where BOS and SD-before actually perform worse. It should be interesting to see how these two equilibria manifest in the lab. Note also that the equilibrium outcomes under BOS-before and SD-before converge under the partially constrained case, although the mechanisms are not equivalent.

According to our theoretical predictions, in particular Table 3, we provide two hypotheses to be tested in our lab experiment.

Hypothesis 1. Under constrained school choice, BOS/SD-before outperforms the other two mechanisms in ex-ante fairness.

Hypothesis 2. Given BOS/SD-before mechanism, constrained school choice mechanisms outperforms unconstrained school choice mechanisms in ex-ante fairness.

Hypothesis 1 focuses on verifying if BOS/SD-before performs better than the other two mechanisms under constrained school choice. Hypothesis 2, with the focus moving to if constrained mechanisms with preference submission before the exam outperform all unconstrained mechanisms. We next investigate whether and how constrained school choice changes the relative performance of those four mechanisms by testing these two hypotheses.

2.4. Experimental implementation

We run different sessions corresponding to the BOS-before, BOS-after, SD-before and SD-after mechanisms. In each session, the subjects experience all three constraint levels, i.e., the unconstrained (listing three colleges), partially constrained (listing two colleges), and fully constrained (listing one college) environment sequentially, or in reverse order. For each constraint level, every subject plays each of the student types (student 1, 2, and 3) in turn. Subjects are randomly grouped in each period. All of the subjects know their own matched college (therefore payoff) after each period, but they are not told the matching outcomes of the other subjects. Each subject in total plays $3 * 3 = 9$ periods of the matching game. On finishing all 9 periods, they complete a risk attitude test (Tanaka et al., 2010) and a personal information survey.[4]

The payment, in Chinese yuan (RMB), is determined by the subject's total payoffs throughout the whole session, according to the exchange rate 1 ECU = 0.4 RMB. The average payoff to each subject was 100 RMB, including a participation fee of 20 RMB. All sessions were conducted on April 22, 2018, and March 20, 2021, with the full set of four sessions at each date. The number of subjects in each session was around 27 (min. 21 and max. 33).[5] The details are presented in Table 4.

All of the sessions were conducted in Tsinghua University's School of Economics and Management's Experimental Economics Laboratory (ESPEL) and all of the subjects played the matching game on computer terminals. As an example, the experimental instruction manual for BOS-before mechanism is shown in Appendix B.1.

[4] Personal information includes demographics (age, gender), CEE experience (exam year and province, which can be used to identify the matching mechanism), and college experience (college year, major). Risk attitude test and personal information survey were only conducted for sessions happened on April 22, 2018.

[5] To make it clear, let us take a session with 27 students as an example: in period 1, 27 subjects are divided into 9 groups, generating 9 results in our matching game. In period 2, with the same constraint level, the subjects change their roles as student 1, 2, and 3 and play the new matching game, with randomly assigned new group members. In period 3, their roles of student 1, 2, and 3 and group members change again so that every subject has played once as student 1, 2, and 3. As a result, there are 27 matching results for a specific constraint level within a session. Periods 4–6 repeat the process with a new constraint level and periods 7–9 with another level.

Table 4
Experimental sessions.

Session name	Level of constraint	Student role	# of subjects	# of matches
BOS-before	No/Partial/Full	student 1/2/3	33	33*3 = 99
BOS-before	Full/Partial/No	student 1/2/3	30	30*3 = 90
BOS-after	No/Partial/Full	student 1/2/3	27	27*3 = 81
BOS-after	Full/Partial/No	student 1/2/3	21	21*3 = 63
SD-before	No/Partial/Full	student 1/2/3	33	33*3 = 99
SD-before	Full/Partial/No	student 1/2/3	24	24*3 = 72
SD-after	No/Partial/Full	student 1/2/3	30	30*3 = 90
SD-after	Full/Partial/No	student 1/2/3	27	27*3 = 81

3. Results

In this section, we present our main experimental results. Our focus is investigating how each mechanism achieves ex-ante fairness in its matching outcomes. We also investigate individual behavior to explain how those matching outcomes are reached.

3.1. *Ex-ante fairness*

Following our theoretical prediction in Section 2.3, in particular Table 3, we consider two measures: the first is how likely it is that a mechanism can achieve ex-ante fairness, i.e., the proportion of ex-ante fairness within a mechanism (e.g., unconstrained BOS-before); the second is the number of blocking pairs averaged within such a mechanism.

3.1.1. *Proportion of ex-ante fair matching outcomes*

Figs. 1–3 and Panel A of Table 5 show the proportion of (fully) ex-ante fair matching outcomes under different mechanisms and different constraints. Under the unconstrained case, SD-after performs worse than the other three mechanisms while the proportions of ex-ante fair matching outcomes are not significantly different among the other three mechanisms. In particular, the observed ex-ante fairness proportion under BOS-before is far from the theoretical prediction (0.413 vs. 2/3), while the gap is smaller for the other three mechanisms (0.333–0.438 vs. 10/27). Our findings are consistent with Lien et al. (2016).

Under the partially constrained treatment, Hypothesis 1 is partially confirmed: the differences between BOS-before and BOS-after, SD-before and BOS-after are significant at the 99% level; differences between SD-before and SD-after are significant at the 95% level (see Panel B, Table 5); SD-before also outperforms BOS-before at the 90% level. Although there are no significant differences between BOS-before and SD-after, *before* dominates *after* as a whole (significant at the 99% level).

Under the fully constrained treatment, the proportions of ex-ante fair matching outcomes under all of the *before* mechanisms are higher than those under the *after* mechanisms, significantly at the 99% level. Between either the *before* or *after* mechanisms, the differences are small and insignificant. Hypothesis 1 is thus confirmed under the fully constrained treatment.

Table 6 shows the same results from a different angle, i.e., by comparing proportions of fair matching outcomes among different constraint levels *within* each mechanism. Being fully constrained increases ex-ante fairness significantly for the *before* mechanisms, but not the *after* mechanisms. Partial constraint (vs. no constraint) raises the level of ex-ante fairness under SD-before and lowers it under BOS-after. Hypothesis 1 is still confirmed under the fully constrained case, but not so under the partially constrained case.

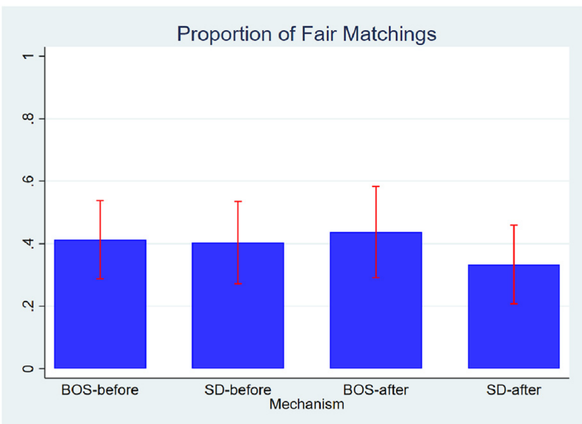

Fig. 1. Proportion of ex-ante fair matching outcomes under unconstrained mechanisms (the red line is the 95% confidence interval).

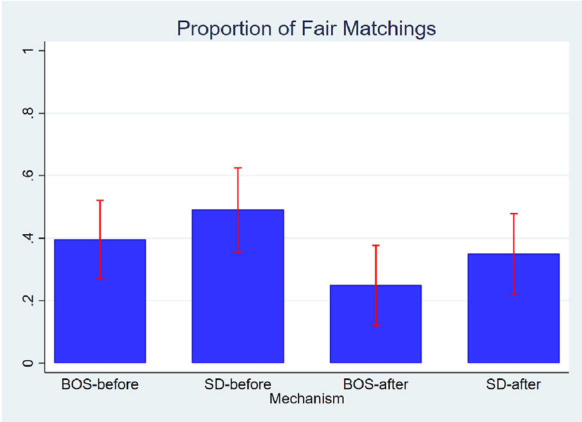

Fig. 2. Proportion of ex-ante fair matching outcomes under partially constrained mechanisms (the red line is the 95% confidence interval).

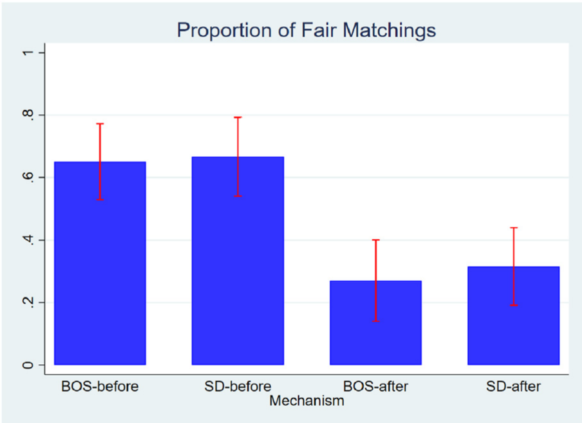

Fig. 3. Proportion of ex-ante fair matching outcomes under fully constrained mechanisms (the red line is the 95% confidence interval).

Table 5
Proportion of fair matching outcomes.

	Panel A: Mean						
	BOS-before	BOS-after		SD-before		SD-after	
Unconstrained	0.413	0.438		0.404		0.333	
Partially constrained	0.397	0.250		0.491		0.351	
Fully constrained	0.651	0.271		0.667		0.316	
	Panel B: Difference (Probit Regression Result)						
	Bb-Ba	Bb-Sa	Bb-Sb	Sb-Sa	Sb–Ba	Sa-Ba	before-after
Unconstrained	−0.025	0.079	0.009	0.070	−0.034	−0.104	0.027
P-value	(0.727)	(0.022)	(0.822)	(0.000)	(0.597)	(0.081)	(0.510)
Partially constrained	0.147	0.046	−0.094	0.140	0.241	0.101	0.136
P-value	(0.000)	(0.304)	(0.063)	(0.039)	(0.000)	(0.016)	(0.006)
Fully constrained	0.379	0.335	−0.016	0.351	0.396	0.045	0.363
P-value	(0.001)	(0.001)	(0.867)	(0.000)	(0.000)	(0.461)	(0.000)

Note: The results presented in the table are the mean values for all mechanisms and the differences between each pair; standard errors are clustered within each session of the experiment, with p-value shown in parentheses.

3.1.2. Number of blocking pairs

The results for the number of blocking pairs are shown in Figs. 4–6 and Table 7. There are still no statistically significant differences among the four mechanisms under the unconstrained case. When student choice is partially constrained, BOS-before works even worse (has more blocking pairs) than SD-before and SD-after at the 90% significance level, while all other differences among the four

Table 6
Proportion of fair matching outcomes by constraint levels.

	BOS-before	BOS-after	SD-before	SD-after	
Fully constrained	0.238	−0.154	0.261	−0.018	
P-value	(0.001)	(0.262)	(0.000)	(0.297)	
Partially constrained	−0.016	−0.174	0.088	0.017	
P-value	(0.734)	(0.021)	(0.020)	(0.741)	
	before	after	BOS	SD	Overall
Fully constrained	0.250	−0.083	0.062	0.123	0.093
P-value	(0.000)	(0.197)	(0.617)	(0.124)	(0.175)
Partially constrained	0.033	−0.073	−0.091	0.053	−0.018
P-value	(0.413)	(0.265)	(0.130)	(0.123)	(0.660)

Note: The results are generated from OLS regressions, compared to unconstrained groups; standard errors are clustered within each session of the experiment, with p-value shown in parentheses.

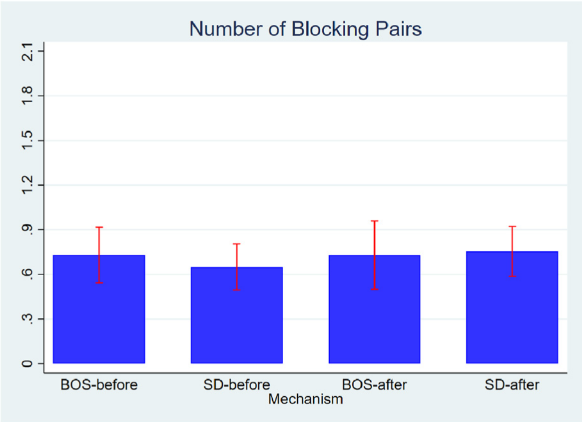

Fig. 4. Average number of blocking pairs under unconstrained mechanisms (the red line is the 95% confidence interval).

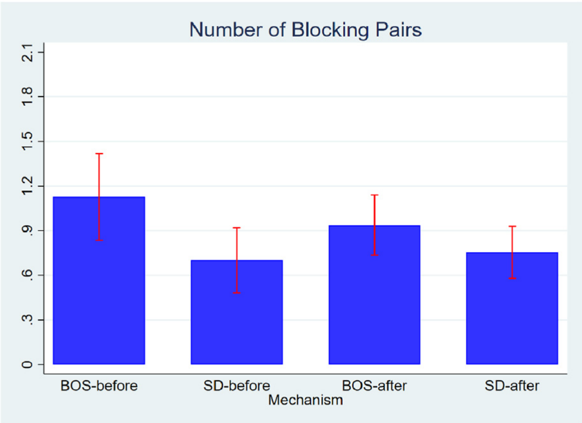

Fig. 5. Average number of blocking pairs under partially constrained mechanisms (the red line is the 95% confidence interval).

mechanisms are insignificant. Under the fully constrained mechanisms, all of the differences are insignificant. To summarize, there is little evidence supporting Hypothesis 1 when we use number of blocking pairs to measure fairness.

Table 8 then compares numbers of blocking pairs across different constraint levels within each fixed mechanism. Imposing school choice constraints increases the number of blocking pairs under all four mechanisms (although only one such increase is significant), and thus at least does not increase the level of ex-ante fairness.

Our first measure of ex-ante fairness, i.e., the proportion of ex-ante fair matching outcomes, gives strong support for Hypothesis 1 under the fully constrained mechanism: imposing constraints on student choice of colleges under the BOS/SD-before mechanisms does

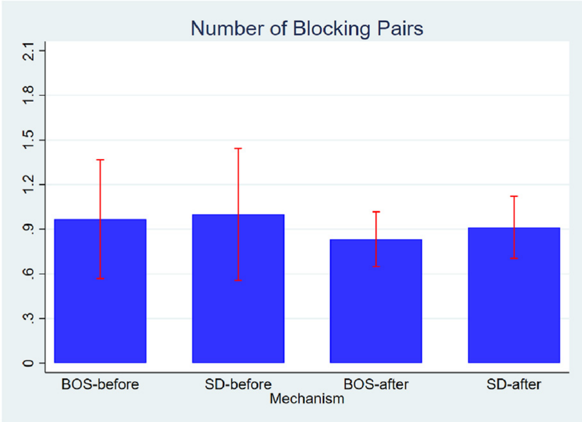

Fig. 6. Average number of blocking pairs under fully constrained mechanisms (the red line is the 95% confidence interval).

Table 7
Number of blocking pairs.

	Panel A: Mean of number of blocking pairs						
	BOS-before	BOS-after		SD-before		SD-after	
Unconstrained	0.730	0.729		0.649		0.754	
Partially constrained	1.127	0.938		0.702		0.754	
Fully constrained	0.968	0.833		1.000		0.912	
	Panel B: OLS Regression Result						
Difference of blocking pairs	Bb-Ba	Bb-Sa	Bb-Sb	Sb-Sa	Sb–Ba	Sa-Ba	before-after
Unconstrained	0.001	−0.024	0.081	−0.105	−0.080	0.025	−0.051
P-value	(0.992)	(0.678)	(0.296)	(0.100)	(0.435)	(0.776)	(0.357)
Partially constrained	0.189	0.373	0.425	−0.053	−0.236	−0.183	0.087
P-value	(0.330)	(0.077)	(0.065)	(0.526)	(0.124)	(0.164)	(0.583)
Fully constrained	0.135	0.056	−0.032	0.088	0.167	0.079	0.107
P-value	(0.681)	(0.857)	(0.927)	(0.586)	(0.393)	(0.427)	(0.520)

Note: The results presented in the table are the mean values for all of the mechanisms and differences between each pair; standard errors are clustered within each session of the experiment, with p-value shown in parentheses.

Table 8
Number of blocking pairs by constraint level.

	BOS-before	BOS-after	SD-before	SD-after	
Fully constrained	0.238	0.104	0.351	0.158	
P-value	(0.245)	(0.397)	(0.277)	(0.178)	
Partially constrained	0.397	0.208	0.053	0.000	
P-value	(0.022)	(0.242)	(0.577)	(1.000)	
	Before	After	BOS	SD	Overall
Fully constrained	0.292	0.133	0.180	0.254	0.218
P-value	(0.205)	(0.006)	(0.082)	(0.118)	(0.082)
Partially constrained	0.233	0.095	0.315	0.026	0.169
P-value	(0.032)	(0.365)	(0.034)	(0.799)	(0.075)

Note: The results are generated from OLS regressions compared to unconstrained groups; standard errors are clustered within each session of the experiment, with p-value shown in parentheses.

increase ex-ante fairness and causes these two mechanisms to dominate the other mechanisms. However, our second measure, the number of blocking pairs, does not support Hypothesis 1. The difference between the two measures needs further examination. Note that the first measure considers only the probability of achieving zero blocking pairs, while the second only takes the mean number of blocking pairs into account. For a more complete picture, we then look at the *distribution* of the number of blocking pairs.

3.1.3. Risk of mismatch

The histograms of the number of blocking pairs under various mechanisms and constraints are shown in Figs. 7–9. In addition, we use a two-sample Kolmogorov–Smirnov test to determine whether the distributions are different from each other; the results are shown in Table 9.

Under unconstrained cases, there are no significant differences in the distributions of the number of blocking pairs between any two treatments.

When partially constrained, the BOS-before mechanism is different from the BOS/SD-after mechanism (both at the 99% level) and the SD-before mechanism is different from the BOS-after mechanism (at the 90% level).

However, when we consider fully constrained mechanisms, all of the distributions between the *before* and *after* mechanisms are significantly different, with p-values all above 1%. From Fig. 9, we can see that the distributions become more dispersed under the *before* than under the *after* mechanisms. In particular, two *before* mechanisms have a higher possibility of reaching highly unfair matching outcomes with three or more blocking pairs, although they also have a higher possibility of reaching (fully) fair matching outcomes.

We further consider the probability of non-admission as an indicator of (un)fairness under different treatments. Non-admission must involve unfair matching outcomes in our experimental set-up. It can also be a serious problem in reality. In China, non-admitted students must wait and work hard for another year, then retake the CEE. If they do not, they drop out of the system forever.

The results are shown in Table 10. Under the unconstrained scenario, there is no chance of non-admission for all mechanisms. Under the partially constrained scenario, the chance of non-admission exists and is smaller under the *after* mechanisms than under the *before* mechanisms. The comparison becomes sharper when we consider the fully constrained scenario. Here, the chance of non-admission disappears under the *after* mechanisms, but remains under the *before* mechanisms, at around 10%. The outcomes confirm that the *before* mechanisms are indeed riskier than the *after* mechanisms for the subjects.

The riskier outcomes under the constrained mechanisms can be explained as follows. On the one hand, there are several different Nash equilibria under our settings of fully constrained mechanisms and our subjects do not have the chance to communicate with each other. Therefore, they may not be able to "cooperate", although one of these Nash equilibria may be more reasonable. On the other hand, the *before* mechanisms with a high level of constraints can cause non-admission problems, resulting in a relatively large number of blocking pairs, which also increase the possibility of mismatch.

In conclusion, although *fully constrained BOS/SD-before* mechanisms have a higher probability of achieving a (fully) ex-ante fair outcome, they are also more likely to cause highly unfair outcomes. They are more risky than other mechanisms.

As a final remark, we note that in our experimental design, every student has the same cardinal preferences for every school and non-admission. Therefore the proportion of non-admission is a "one-to-one mapping" on efficiency, i.e., the sum of payoffs for all students: lower proportion of non-admission implies higher efficiency. Therefore, unconstrained BOS-SD/before-after mechanism would deliver high efficiency than partially or fully constrained mechanisms, reflecting a trade-off between efficiency and fairness.

3.2. Analyzing subject behavior

Our results reveal a gap between theory and lab evidence over *equilibrium outcomes*, suggesting that subjects may not follow an *equilibrium strategy* in the lab. Who does not follow an equilibrium strategy? How do they affect matching outcomes? In this section, we discuss observed subject behaviors behind the aforementioned matching outcomes.

We first examine how each student role follows an equilibrium strategy under various treatments. Among multiple equilibria, we focus on those achieving the highest ex-ante fairness, i.e., the "good" equilibrium (or equilibria). In particular, under the BOS/SD-before mechanisms with full constraints, we focus on the equilibrium achieving (full) ex-ante fairness.

Table 11 shows the proportions of subjects following the equilibrium strategy under each mechanism. In general, the subjects follow the equilibrium strategy quite often. The proportion in most environments is over 80% or even 90%. However, there are quite a few cases where the proportion is low.

Under the unconstrained BOS-before mechanism, student 2 only follows the equilibrium strategy 49% of the time, and student 3 only does so 62% of the time. The results are consistent with Lien et al. (2016), and explain why the unconstrained BOS-before mechanism does not dominate the other mechanisms in terms of ex-ante fairness.

The discrepancy becomes even more pronounced under the partially constrained BOS-before mechanism, where student 2 only follows the equilibrium strategy 33% of the time, and student 3 only does so 60% of the time. Furthermore, about 45% of student 2

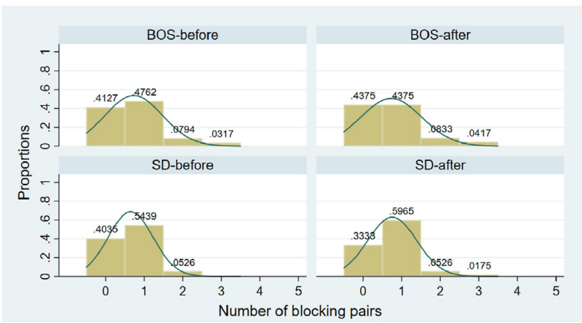

Fig. 7. Distribution of number of blocking pairs under unconstrained mechanisms.

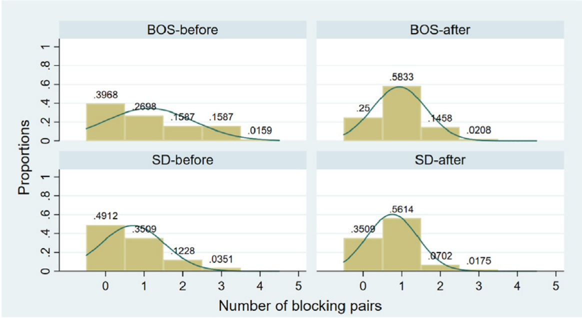

Fig. 8. Distribution of number of blocking pairs under partially constrained mechanisms.

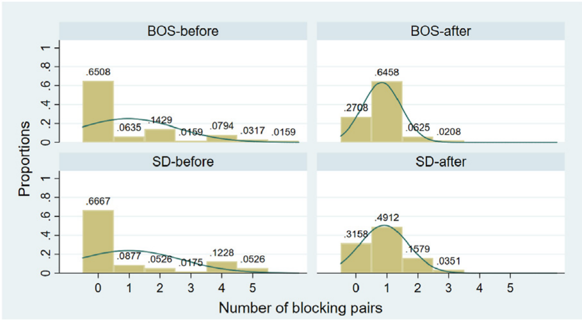

Fig. 9. Distribution of number of blocking pairs under fully constrained mechanisms.

Table 9
Difference in the distribution of the number of blocking pairs.

Unconstrained	Bb vs. Ba	Bb vs. Sa	Bb vs. Sb	Sb vs. Sa	Sb vs. Ba	Sa vs. Ba
χ^2	0.2	1.9	2.4	1.5	3.4	2.9
p-value	(0.976)	(0.598)	(0.493)	(0.678)	(0.333)	(0.407)
Partially constrained	Bb vs. Ba	Bb vs. Sa	Bb vs. Sb	Sb vs. Sa	Sb vs. Ba	Sa vs. Ba
χ^2	14.4	15.8	7.0	5.3	7.3	2.3
p-value	(0.006)	(0.003)	(0.136)	(0.154)	(0.062)	(0.507)
Fully constrained	Bb vs. Ba	Bb vs. Sa	Bb vs. Sb	Sb vs. Sa	Sb vs. Ba	Sa vs. Ba
χ^2	45.1	35.1	4.5	36.5	40.6	3.5
p-value	(0.000)	(0.000)	(0.613)	(0.000)	(0.000)	(0.315)

Note: The statistical values χ^2 and p-values are generated from Pearson's chi-squared test.

Table 10
Proportion of non-admitted students under different mechanisms.

Probability of non-admission	BOS-before	BOS-after	SD-before	SD-after
No constraint	0	0	0	0
Partial constraint	0.111	0.028	0.047	0.006
Full constraint	0.127	0	0.111	0

participants submit (A,*) instead of (B, C) (not reported in the table). If student 2 submits (A,*) while the other two students follow the equilibrium strategy, i.e., (A, *) and (B, C), ex-ante fairness can never be achieved under our settings. Therefore, a large-scale deviation by student 2 may be responsible for the gap between the lab results and theoretical predictions.

The student 2 participants also deviate by a large proportion under the partially constrained SD-before mechanism. Only 28% follow the equilibrium strategy (B,C), while the rest submit (A,*), and most of them choose (A,B) rather than (A,C) (not reported in the table). However, the SD-before mechanism performs surprisingly well under the partially constrained case: it actually has the highest level of ex-ante fairness and the lowest number of blocking pairs (see Tables 5 and 7). Why is this? Under the partially constrained BOS-before mechanism, the deviation of student 2 generates an unfair outcome for sure. However, under the SD-before mechanism, the probability of achieving ex-ante fairness is still more than 35% even for the non-equilibrium profile [(A,*)(A,B)(B,C)]. Therefore, the SD-before

Table 11
Proportion of students following a "good" equilibrium strategy.

No constraint	BOS-before	BOS-after	SD-before	SD-after
Student 1	0.952	0.896	0.982	1
Student 2	0.492	0.854	0.912	1
Student 3	0.619	0.979	0.719	1
Whole group	0.286	0.729	0.632	1
Partially constrained	**BOS-before**	**BOS-after**	**SD-before**	**SD-after**
Student 1	0.937	0.917	0.965	1
Student 2	0.333	1	0.281	0.982
Student 3	0.603	0.938	0.754	0.982
Whole group	0.190	0.854	0.228	0.965
Fully constrained	**BOS-before**	**BOS-after**	**SD-before**	**SD-after**
Student 1	0.921	1	0.842	1
Student 2	0.841	1	0.860	1
Student 3	0.762	0.979	0.860	1
Whole group	0.683	0.979	0.667	1

Table 12
Explanatory power of risk parameters for following the equilibrium strategy under partial constraints.

	Dependent variable: Whether following equilibrium strategy		
	Student 2 under B-b	Student 2 under S-b	Student 3 under B-b
σ	−0.308	−0.617	0.599
	(0.515)	(0.126)	(0.228)
λ	0.058	−0.060	0.009
	(0.307)	(0.233)	(0.867)
α	1.061	0.666	0.176
	(0.061)	(0.098)	(0.743)
Current profit	−0.001	−0.002	0.009
	(0.878)	(0.743)	(0.076)
Joint sig. of risk parameters (Prob>chi2)	0.217	0.381	0.432
Num of obs	33	33	33
Pseudo R^2	0.127	0.093	0.085

Note: p-values are shown in parentheses, generated from probit regressions. The dependent variable is an indicator of whether a subject plays equilibrium strategy; the independent variables are risk parameters α, λ, σ collected in the survey and subjects' current cumulative payoff. Coefficients report marginal effects at the mean level.

mechanism is not as sensitive to strategy deviations as the BOS-before mechanism in the partially constrained scenario.

In the fully constrained scenarios, the proportion of subjects following the equilibrium strategy is quite high. The proportion is 1 or close to 1 under the *after* mechanisms, and around 80%–90% under the *before* mechanisms. However, as we found in Section 3.1, under the *before* mechanisms, a significant proportion of matches turn out to be highly unfair. The observed discrepancy between behaviors and matching outcomes can only be explained by the fact that matching outcomes are highly sensitive to student behaviors: a small deviation in subjects' strategy may result in highly unfair matching outcomes (e.g., non-admission), simply because there are no back-up choices under such mechanisms.

Why do students deviate from the equilibrium strategy? In Table 12, we consider how the risk attitudes of subjects affect their choice. Lien et al. (2016) show that under unconstrained mechanisms, risk attitudes do not affect student behaviors. Here we focus on constrained mechanisms. In particular, we consider three cases where students deviate most, i.e., students 2 and 3 under BOS-before, and student 2 under SD-before, all under partially constrained mechanisms. Their risk attitudes are measured by three parameters (α, λ, σ) based on Tanaka et al. (2010). σ and λ measure a subject's value function and level of loss aversion, and α indicates the curvature of the gain and loss segment. As shown in the table, α significantly affects the behavior of student 2 under *partially constrained before* mechanisms. No other risk parameters are significant in the regressions, and they are jointly insignificant in all of the regressions. Our experimental design pays the subject for their gains in each round, so income effect may affect their behaviors.[6] We include subjects' current cumulative profit in the regression, showing that income effect only affects student 3's behavior under partially constrained BOS-before mechanism (significant at 10% level), but not the other two cases. We conclude that risk attitude and income effect play a very limited role in affecting students' behavior.[7]

[6] Since subjects' environment changes in each period (level of constraint and student role), their current cumulative profit should not introduce much income effect.

[7] We run the same regression without including current profit and results are stable. In addition, none of the personal characteristic variables collected in the survey show any significance when added into the regressions (not shown in the table).

An alternative reason for deviation might be that the subjects may not be able to figure out the equilibrium strategy. Under the BOS mechanisms and constrained SD mechanisms, students do not have a dominant strategy, e.g., truth-telling. When preference submission is set before the exam, they must take score distributions into account in calculating the expected payoffs for any strategy profile. This task is quite hard to complete in a limited time. This cognitive restriction argument can explain why only *before* mechanisms suffer from strategic deviations.

4. Some variants of constrained BOS/SD-before mechanisms

According to our results and analysis in Section 3, although *fully constrained BOS/SD-before* mechanisms can improve the likelihood of ex-ante fairness, they still have some disadvantages, such as a higher risk of mismatch. To find possible ways to further improve the ex-ante fairness of matching outcomes, we consider some variants of our aforementioned *fully constrained before* mechanisms. The variants of our basic models are motivated by the following reasoning: in our basic constrained settings, students may not reach the ex-ante fairness outcome because of multiple equilibria and failure to figure out the equilibrium strategies. Therefore, we use some common ways of addressing this coordination failure: repeated games (to let subjects learn), recommending strategies (to tell them the equilibrium strategy profile), and two-stage dynamic games (providing them with a supplementary option to avoid being unadmitted).[8] Table 13 summarizes our experimental sessions. The experimental instruction manual for BOS-dynamic is shown in Appendix C.2.

4.1. Repeated games

In the repeated games setting, all of the subjects play the original fully constrained matching game in the first three periods (as student 1, 2 and 3 in each period). After that, in periods 4–12, they repeatedly play the same matching game (playing as students 1, 2, and 3 in the same sequence for each set of 3 periods).

Our results are shown in Figure B.10–B.11 and Table B.15-B.16. We can see that ex-ante fairness is increasing and the number of blocking pairs is decreasing as subjects play the same matching game repeatedly. Under the BOS mechanism, the increase of proportion of fairness are significant at 99% level when we compare periods 1–3 and periods 10–12. The magnitude of such increase is more than 50% (0.3 out of 0.533). A similar but less significant increase is also observed under the SD mechanism. Results for the number of blocking pairs as the measure of ex-ante fairness are very similar.

We also examine whether there is evidence showing differences between our new design and our original or baseline results for the *fully constrained BOS/SD-before* mechanisms described in Section 3. When we compare our baseline result with periods 10–12 under the BOS mechanism, the increase in ex-ante fairness is significant at 90%, and all of the other comparison results are insignificant between repeated games and the baseline result.[9]

4.2. Recommended strategies

In the setting of recommended strategies, the subjects play the original fully constrained matching game in the first three periods. After that, in periods 4–12, they repeatedly play the same matching game except that they are told the recommended strategies for each student role this time. In our design, students 1, 2, 3 receive recommendations that they submit colleges A, B, C, correspondingly, and these recommended strategies are common knowledge to all of the subjects.

The results are shown in Figures B.12–B.13 and Tables B.15–B.16. As a whole our result imply that the change arrives “faster” and leaves little room for further improvement. As for comparisons with the baseline results in Section 3, all of the differences are insignificant, but the results are still getting better in both measures.

4.3. Dynamic games

In the setting of dynamic games, the subjects play the original fully constrained games in the first three periods. Then, in periods 4–12, they play a matching game of two rounds. In round 1, all of the subjects play the same fully constrained matching game (i.e., submitting only one college). In round 2, if one student is admitted in round 1, then that college (and its corresponding payoff) will be his matching outcome. If one student is still available for matching (meaning that he failed to be admitted by the college he chose in round 1), then he will join the market again. All non-admitted students in the same group are informed which colleges are still available and the students' realized scores.[10] After that, all non-admitted students begin the second round of preference submission and are

[8] Gong and Liang (2020) studied dynamic matching mechanism used in Inner Mongolia, where students are given real-time allocation feedback based on their submitted lists and are allowed to revise their choices. However, in our setting, the matching result in the first round is definitive, only students unadmitted in the first round are provided with a supplementary round to be matched with schools with capacities.

[9] However, such results are cross-experiments because we held offline sessions on campus in 2018 and 2021, while due to COVID-19, we held smaller online sessions in 2020. Comparisons between the results from the online and offline experiments are not so informational because of the dramatic changes in our experimental environment. We only list them here for readers' reference.

[10] e.g., suppose that all three students submit college A in round 1, and the realized score is 95 for student 1, 91 for student 2, and 87 for student 3. Student 1 will be admitted by college 1 during the first round and the rest of the students will not be admitted. In round 2, both student 2 and 3 will know the realized scores of all students in this group; that is, 95 for student 1, 91 for student 2, and 87 for student 3. Meanwhile, they will also know that colleges B and C are still available.

Table 13
Experimental sessions for variant mechanisms.

Matching mechanism	Design	# of subjects	# of matches
BOS	Repeated games	15 + 15 = 30	30*4 = 120
BOS	Recommended strategies	15 + 15 = 30	30*4 = 120
BOS	Dynamic games	15 + 18 = 33	33*4 = 132
SD	Repeated games	15 + 18 = 33	33*4 = 132
SD	Recommended strategies	15 + 21 = 36	36*4 = 144
SD	Dynamic games	18 + 18 = 36	36*4 = 144

Note: Each mechanism, e.g., BOS-repeated, has two sessions.

restricted to choosing one of the available colleges on the market in round 2. Following the same matching rule, the subjects get their matching outcome after round 2.

In the setting of repeated games and recommended strategies, the Nash equilibrium strategies and outcomes are the same as those for the original *fully constrained BOS/SD-before* mechanisms. However, in the dynamic game, our previous results may not hold. We summarize our theoretical results in the following Propositions, proof of which is provided in Appendix A.

Proposition 4. *Under the dynamic mechanism: (1) The Nash equilibrium strategy profile in the first round for the three students takes the form [(A), (B), (B)]. One student will enter the second round based on his realized score, and his Nash equilibrium strategy in the second round of matching is [(C)]. (2) The equilibrium outcome is [(1,A), (2,B), (3,C)] or [(1,A), (2,C), (3,B)], depending on whether student 2 gets a higher realized score than student 3.*

Proposition 4 suggests that the expected proportion of ex-ante fairness will be 2/3, and the expected number of blocking pairs will be 1/3. These results are exactly the same as the theoretical predictions for partially constrained cases, and worse than our benchmark for *fully constrained BOS/SD-before* mechanisms.[11]

The results under dynamic games are shown in Figures B.14–B.15 and Tables B.15–B.16. We fail to find an increase in the proportion of ex-ante fairness or a decrease in the number of blocking pairs under dynamic games. On the contrary, there is even a statistically significant drop in the proportion of ex-ante fairness. A 32% drop is significant at the 99% level when we compare periods 4–12 and periods 1–3 under the BOS mechanism. In addition, in comparing periods 10–12 and periods 1–3, we find similar drops in the proportion of ex-ante fairness with a lower level of significance under both BOS and SD mechanisms.

4.4. *Risks of non-admission*

In Table B17, we present the non-admission rates under all variants. Similar to Table 10, these values indicate the efficiency of matching outcome in the sense of social welfare. We note that repeated games and recommended strategies reduce non-admission rates, as the subjects gather information and learn in later periods. Non-admission rates in the last three periods are also lower by around half than the baseline cases in Table 10. For the dynamic design, although it fails to increase the proportion of ex-ante fairness or decrease the number of blocking pairs according to the previous results, it does avoid the risk of non-admission. We also show the non-admission rate after the first round under the dynamic design, and generally 1/3 of students enter the second stage, which fits our theoretical predictions well.

To summarize section 4, we conclude that repeated games and strategy recommendations can improve the ex-ante fairness of matching outcomes and reduce the risk of mismatch, but not two-stage dynamic games.

5. Conclusion

Colleges want to select students with a high level of ability, but student ability is not observable, and colleges can only observe a noisy signal for it (e.g., exam scores). A carefully designed mechanism that properly incorporates these signals may induce students to self-select by their ability and achieve a socially desirable matching outcome. In this paper, we combine two design features, i.e., the timing of preference submission and constrained school choice, to explore potentiality in revealing true ability. Preference submission before the exam gives students opportunities to signal their true ability, while constrained school choice refines the signals, both helping us to achieve assortative matching between college qualities and student abilities.

We conduct experiments under three constraint levels (full, partial, or zero), two mechanisms (BOS or SD), and two preference submission timings (before-exam or after-exam). Our findings can be summarized in two conclusions, but with two caveats. The first conclusion is that the BOS and SD mechanisms under before-exam preference submission with full constraints achieve ex-ante fairness more frequently than other mechanisms, indicating that constrained school choice can be used as a signaling device to facilitate assortative matching. The second conclusion is that repeating the games or providing students with recommended strategies can increase ex-ante fairness and decrease risk under the BOS/SD-before mechanisms with full constraints, emphasizing the importance of coordination.

However, our first caveat is that *partial* constraints do not work as well for achieving ex-ante fairness. The second caveat is that the

[11] The equilibrium outcome of dynamic mechanism may not always equal to that of partially constrained BOS. In particular, under partially constrained BOS, students may have a chance of being unadmitted. But under the dynamic mechanism, students will never been unadmitted. Therefore, students may behave more aggressively in the first round under the dynamic mechanism than their first choice under the partially constrained BOS.

BOS/SD-before mechanisms with full constraints can be risky for players, because they may lead to highly unfair matching outcomes, albeit with a small probability. Here, the typical trade-off under asymmetric information, i.e., a trade-off between incentive and insurance, emerges: to discipline agents to behave well (i.e., to reveal their true types), they have to be less insured.

Our experiment design is "small-scale," as there are only three students and colleges in the matching system. How can those results be extended to a large-scale matching system (e.g., college admissions in China)? First, in the large-scale system, competition between students may not necessarily become more fierce. Although the number of students increases, those who are competing with each other (i.e., have overlapping realized scores) may still fall into a small group. Even if the competitive group becomes larger, a specific college often has as many slots as needed to accommodate competing students. Second, as the number of colleges increases, the number of equilibria may increase as the preference order becomes longer. However, an equilibrium in which each student lists his ex-ante fair school as a first choice may still be the focal point, under some proper mechanisms. Although, in large-scale matching, figuring out one's fair school can be challenging, such complexity may not be invincible. Note that in our small-scale matching in the lab, the subjects only had 10 min to understand the experimental set-up, including their score distributions, ordinal preferences, and the matching procedures. In reality, students spend plenty of time preparing for college admission, gathering information on their true abilities and eligible colleges. Under well-designed mechanisms, the system may approach ex-ante fairness.

Almost every province in China has reformed its admission rules by switching from the BOS-before mechanism to the SD-after mechanism. The SD-after mechanism generates an ex-post fair matching outcome, yet the "true" ability of students may be hidden under the noisy signal of exam scores. The issue appears to have become more severe in recent years. As the CEE difficulty decreases,[12] the CEE scores may reflect students' true ability as much as their fortunes. The current reform thus can be questioned. Our paper suggests that the "old" system, i.e., the BOS-before mechanism with a sufficiently high constraint level, may have its own advantages. Although it is still imperfect, some variants such as providing students with guidance on submitting preferences may be useful to balance risks. In addition, it would be also interesting to consider heterogeneous student college preferences, and to innovate other mechanisms to improve the results in future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proof of propositions

Proof of Proposition 1

With the score distribution presented in Table 1, the probability of a different score-rank is shown in Table A14.

Table A.14
Probability of a different score rank

Ranking of students by ex-post score	Probability
(1,2,3)	10/27
(1,3,2)	7/27
(2,1,3)	7/27
(2,3,1)	1/27
(3,1,2)	1/27
(3,2,1)	1/27

Claim A1.1 *There must be at least one student listing college A as his first choice in a Nash equilibrium.*

Proof. Under the Boston mechanism, if there is only one student listing college A as his first choice, he will be admitted by college A for sure. Therefore, if no one chooses A as his first choice, every student has the incentive to change his first choice to A to get 30, which is the largest possible payoff.

Claim A1.2 *No one lists college A as a second choice.*

Proof. By Claim 1.1, there must be at least one student listing A as his first choice; therefore, listing A as a second choice will lead to a 0 probability of being admitted by college A. Listing any other college as the second choice will be better than listing A.

Claim A1.3 *There is no equilibrium in which two students submit (C,B).*

Proof. By Claim 1.1, there must be at least one student listing A as his first choice. If the other two students submit (C,B), then each of them has an incentive to change to (B,C) to get 25 for sure.

Claim A1.4 *(C,B) is dominated by (B,C).*

Proof. By Claim 1.2, there are only four possible strategies left, that is (A,B) (A,C) (B,C) (C,B). By Claim 1.1 and 1.3, submitting (C,B) will get a 15 payoff for sure. If no other students submit (B,C), then he can change to (B,C) to get a 25 instead. If another student submits

[12] E.g., the minimum score for guaranteed admission into the first tier of colleges in Beijing increased significantly in 2013. In 2008–2012, the score required for science was between 477 and 502 (out of 750) and it rose to 532–550 in 2013–2018. In contrast, the score required for the arts was 495–532 in 2008–2012 and increased to 549–583 in 2013–2018. See also http://edu.people.com.cn/n/2013/0623/c1053-21939237.html for news from People's Daily Online (in Chinese) about the increase in admission scores in 2013.

(B,C), changing his choice to (B,C) will get a payoff of θ*25 + (1 − θ)*15, where $0 < \theta < 1$. Therefore, submitting (B,C) always dominates submitting (C,B).

Claim A1.5 *There is no equilibrium in which all students list A as first choice.*

Proof. By Claims 1.1–1.4, there are only three possible strategies left, that is (A,B) (A,C) (B,C). If all students list A as their first choice, there are two cases in total.

Case 1. All students submit (A,B). Under this case, we can easily calculate the payoff of student 3, which is $2/27 * 30 + 8/27 * 25 = 260/27 < 15$. Therefore, student 3 has an incentive to choose B or C as his first choice.

Case 2. Not all students submit (A,B), which means there is at least one student submitting (A,C). Consider the student who chooses (A,C): even if he is student 1, his payoff cannot be larger than $17/27 * 30 + 10/27 * 15 = 660/27 < 25$. Therefore, those submitting (A,C) have an incentive to submit (B,C) instead.

By Claim 1.5, with only three possible strategies (A,B) (A,C) (B,C) left, there are now five strategy combinations left.

Combination 1 (A,B) (A,B) (B,C)

Combination 2 (A,B) (A,C) (B,C)

Combination 3 (A,C) (A,C) (B,C)

Combination 4 (A,B) (B,C) (B,C)

Combination 5 (A,C) (B,C) (B,C)

Claim A1.6 *No one chooses (A,B) when only one student submits (B,C).*

Proof. When only one student submits (B,C), implying the other two list A as their first choice, he will get 25 for sure. If one student chooses (A,B), his second choice on the list is wasted and will certainly decrease his payoff, because he has a positive probability of being not admitted by A. Submitting (A,C) instead will increase his expected payoff for sure.

Claim A1.7 *No one chooses (B,C) when the other two students submit (A,C).*

Proof. When only one student submits (B,C) and the other two submit (A,C), the one who submits (B,C) will get 25 for sure. However, he will still be the only one who lists B in the preference ordering list if he submits (A,B) instead. Changing to (A,B), his expected payoff will be α*30 + (1 − α)*25, where $0 < \alpha < 1$. Therefore, (A,C) dominates (B,C) when the other two submit (A,C).

Claim A1.8 *Under combination 4 and 5, in any equilibrium, student 1 submits (A,*).*

proof. If (A,*) is submitted by student 2, this means that student 1 and student 3 are submitting (B,C) concurrently. Student 3 has an incentive to submit (A,C) instead. His expected payoff will be $1/3 * 30 + 2/3 * 15 = 20 > 1/9 * 25 + 8/9 * 15 = 145/9$. If (A,*) is submitted by student 3, this means that student 1 and student 2 are submitting (B,C) concurrently. Student 1 has an incentive to submit (A,C) instead. His expected payoff will be $8/9 * 30 + 1/9 * 15 = 85/3 > 2/3 * 25 + 1/3 * 15 = 65/3$.

By Claim 1.6 and Claim 1.7, combinations 1, 2, 3 can be dropped. By Claim 1.8, combinations 4 and 5 result in student 1 submitting (A,*). Therefore, the only possible equilibrium has the form [(A,*), (B,C), (B,C)]. It is straightforward to verify that such a strategy profile indeed forms a Nash equilibrium.

According to the above result, student 1 will be admitted by college A for sure, and students 2 and 3 will be admitted by college B or C, depending on who gets a higher score.

Proof of Proposition 2

Claim A2.1 *In any equilibrium, the better the college, the higher its rank on any student's preference ordering list.*

Proof. Under the SD mechanism, a student will be admitted by his highest ranked college that remains unmatched. Therefore, for any two colleges that a student wants to include in his preference list, it is always weakly better to rank the preferred one higher. Because every student has a positive probability of being admitted by his first choice, it is then strictly better to rank the preferred one higher.

Claim 2.1 means that no one will lie (e.g., submit (B,A)).

Claim A2.2 *In any equilibrium, the preference ordering lists of the three students are not all the same.*

Proof. If all of the students submit the same preference, the expected payoff of student 3 is no more than $2/27 * 30 + 8/27 * 25 = 260/27 < 15$, but he can always get at least 15: if the other two do not include college A on their list, he can get 30 by listing (A,*); or if the other two do not include B on their list, he can get 25 by listing (B,C); otherwise he can get 15 by listing (A,C).

By Claim 2.1, there are only three possible strategies for a student, that is (A,B), (A,C), or (B,C).

Claim A2.3 *In any equilibrium, student 1 will not submit (B,C).*

Proof. Suppose that student 1 submits (A,B). When he ranks first (with prob. = 17/27), he gets 30. When he ranks second (with prob. = 8/27), he at least gets 25. His payoff by submitting (A,B) is thus at least $17/30 * 30 + 8/27 * 25 = 710/27 > 25$, but his payoff by submitting (B,C) cannot be larger than 25. Thus, he will never submit (B,C).

Claim A2.4 *If students 1 and 2 submit (A,*) but not the same list, student 3 will submit (B,C), instead of any form of (A,*).*

Proof. We only need to prove that submitting (B,C) is always better than submitting (A,*) for student 3. After some calculations, one can see that among submitting (A,B) and (A,C), submitting (A,B) is better for student 3, no matter who submits (A,B) (or (A,C)) among students 1 and 2. When student 3 submits (A,*), he will get A when he ranks first, and his second choice ("*") when he ranks second. The advantage of submitting (A,B) instead of (A,C) is largely due to the "second choice" advantage.

By comparing submitting (A,B) and (B,C), we find that submitting (B,C) is always better. By submitting (B,C), student 3 will always be admitted. However, if he submits (A,B), he will not be admitted with some probability. The advantage of submitting (B,C) instead of (A,B) is largely due to this "admission" advantage.

Claim A2.5 *The strategy profile [(A,C), (A,C), (A,B)] is not an equilibrium.*

Proof. It can be easily verified that under this strategy profile, student 1 has an incentive to deviate to (A,B).

Claim A2.6 *The strategy profile [(A,B), (A,B), (A,C)] is not an equilibrium.*

Proof. It can be easily verified that under this strategy profile, student 3 has an incentive to deviate to (B,C).

By Claims 2.4–2.6, the strategy profile of any form of [(A,*), (A,*), (A,*)] is not an equilibrium. Then, at least one of students 2 and 3 will submit (B,C).

Claim A2.7 *If student 1 submits (A,*), and student 2 submits (B,C), then student 3 will submit (B,C).*

Proof. This can be easily verified.

Claim A2.8 *If student 1 submits (A,*), and student 3 submits (B,C), then student 2 will submit (B,C).*

Proof. This can be easily verified.

Therefore, the strategy profile [(A,*), (B,C), (B,C)] is the only form of pure-strategy Nash equilibrium.

Proof of Proposition 3

Claim A3.1 *In any equilibrium, no students list the same colleges.*

Proof. If two or more students submit the same list, the one with the lowest ex-ante average score among them has a probability of no more than 1/3 of being admitted by that college. Therefore, his expected payoff cannot be more than $1/3 * 30 = 10 < 15$. Meanwhile, there must be at least one college which no student lists. He has an incentive to deviate and submit that unlisted college to get 15, 25, or 30, depending on which college is not listed by anyone.

Claim A3.2 *In any equilibrium, student 3 does not list college A.*

Proof. Suppose not. By Claim 3.1, every student will be admitted by his listed college. If student 3 lists A, then he will certainly get a payoff of 30. Consider the student admitted by C: he is now getting 15 and he has an incentive to challenge student 3. If he is student 1, his expected payoff will become $30 * 8/9 = 80/3 > 15$. If he is student 2, his expected payoff will become $30 * 2/3 = 20 > 15$.

By Claim 3.1 and Claim 3.2, it then can be easily verified that two Nash equilibria exist: [(A), (B), (C)] or [(B), (A), (C)].

Proof of Proposition 4

Before we analyze the whole matching game, we focus on the second stage of matching.

Claim A4.1 *There cannot be more than two students and two colleges entering the second round of matching.*

Proof. The student with the highest realized score will be admitted for sure in the first round, which finishes the proof.

Claim A4.2 *In the second round of matching, if there is only one student and one college in the market, then this student will list that college.*

Proof. Trivial.

Claim A4.3 *In the second round of matching, if there are two students and two colleges in the market, then the student with the higher realized score (second highest realized score among all three students) will list the college with higher payoff and vice versa.*

proof. In the second round, if there are two students and two colleges in the market, students will be informed of the realized scores of all three students and which colleges are still available. They can easily infer that the student with the highest score has been admitted by the college not in the market; thus every student in the second stage can infer whether the other student has a higher realized score than his. The student with the higher score can list the better college in the market because he will be admitted for sure, and then the one with the lower score can do nothing but list the college with a lower payoff.

So far we have covered all the cases in the second stage. We now begin the analysis of first stage.

Claim A4.4 *In any equilibrium, there at least one student lists college A in the first round.*

proof. Suppose not. Then there must be at least two students listing the same college in the first round. Given Table A14, each of them has a positive probability of being admitted by this college in the first round. Therefore, their current expected payoff must be both strictly smaller than 30 and both have an incentive to deviate to college A to get 30 with probability 1.

Claim A4.5 *In any equilibrium, there is at least one student listing college B in the first round.*

proof. Suppose not. Then there must be at least two students listing the same college in the first round. If they are competing for college C, given Claim 4.4, one student will list A and the other two will list C. Then each of those who list C in the first round has an incentive to deviate to college B, assuming the other sticks to C, to get a payoff of 25 with probability 1.

If they are competing for A, there are two cases: 1. Two students submit A and the other one submits C in the first round. 2. All three students submit A. In case 1, the one who submits C will get 15 for sure and has an incentive to deviate to B to get 25 instead. In case 2, given TableA14, Claim 4.3, the expected payoff of student 3 is $2/27*30 + 8/27*25 + 17/27*15 = 515/27 < 25$, so he has an incentive to deviate to B, which finishes the proof.

Claim A4.6 *In any equilibrium, there are no students listing college C in the first round.*

proof. Given Claims 4.4–4.5, if someone lists C in the first round, then he will get 15 for sure. Given Claim 4.2 and TableA14, he will get something strictly larger than 15 if he deviates to B or A.

Claim A4.7 *In any equilibrium, no two students list college A in the first round.*

proof. Suppose not. Given Claim 4.4 and 4.5, there are now two students submitting A and one student submitting B in the first round. There are three cases: 1. Student 1 submits B. 2. Student 2 submits B. 3. Student 3 submits B. In case 1, student 1 is getting 25 for sure. If he deviates to A (then all of them are submitting A in the first round), given TableA14 and Claim 4.3, he will get $17/27*30 + 8/27*25 + 2/27*15 = 740/27 > 25$, so he has an incentive to deviate. In case 2, given TableA14 and Claim 4.2, student 3 is getting $1/9*30 + 8/9*15 = 50/3$; if he deviates to B, he will get $1/3*25 + 2/3*15 = 55/3 > 50/3$. Therefore, student 3 has an incentive to deviate. In case 3, given TableA14 and Claim 4.2, student 2 is getting $1/3*30 + 2/3*15 = 20$. If he deviates to B, he will get $2/3*25 + 1/3*15 = 65/3 > 20$, so student 2 has an incentive to deviate, which finishes the proof.

Claim A4.8 *In any equilibrium, student 1 lists college A in the first round.*

proof. Suppose not. Given Claims 4.4–4.7, one student submits A and the other two submit B in the first round. There are two cases: 1. Student 2 submits A in the first round. 2. Student 3 submits A in the first round. In case 1: given TableA14, student 1 is getting 8/9*25 + 1/9*15 = 215/9, but he will get 2/3*30 + 1/3*15 = 25 > 215/9 if he deviates to A, so he will deviate. In case 2, student 1 is getting 2/3*25 + 1/3*15 = 65/3, but he will get 8/9*30 + 1/9*15 = 85/3 > 65/3 if he deviates to A, so he will deviate, which finishes the proof.

Claim A4.9 *Student 1 submitting A and students 2 and 3 submitting B is a Nash equilibrium.*

proof. We can easily see that student 1 does not deviate because he is getting 30 for sure, which is the highest possible payoff. As for students 2 and 3, they do not want to deviate to C by Claim 4.6. It remains to show they do not want to deviate to A. For student 2, he is getting 2/3*25 + 1/3*15 = 65/3; now he will get 1/3*30 + 2/3*15 = 20 < 65/3 instead, so he will not deviate. For student 3, he is getting 1/3*25 + 2/3*15 = 55/3; now he will get 1/9*30 + 8/9*15 = 50/3 < 55/3 instead, so he will not deviate either. Therefore, this is a Nash equilibrium.

Given Claims 4.4–4.8, [(1,A)(2,B)(3,B)] is the only possible candidate for a Nash equilibrium and Claim 4.9 confirms this, which finishes the proof of Proposition 4.

Appendix B. Tables and Figures

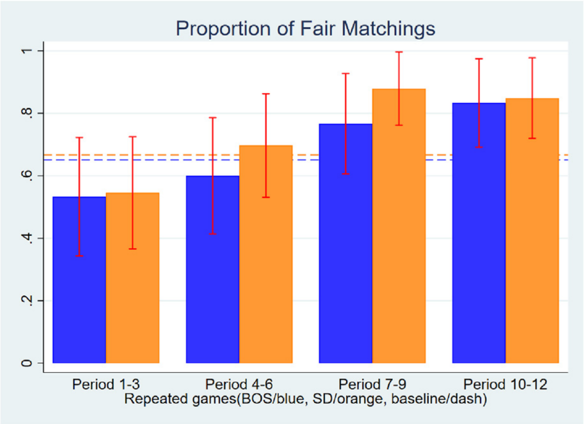

Fig. B.10. Proportion of ex-ante fair matching outcomes under repeated fully constrained mechanisms (the red line is the 95% confidence interval)

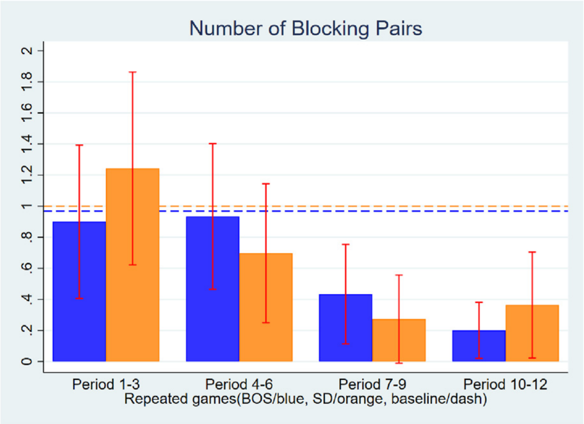

Fig. B.11. Average number of blocking pairs under repeated fully constrained mechanisms (the red line is the 95% confidence interval)

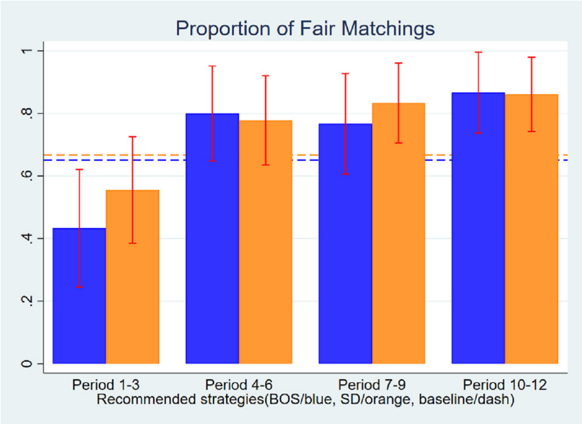

Fig. B.12. Proportion of ex-ante fair matching outcomes under fully constrained mechanisms with recommended strategies (the red line is the 95% confidence interval)

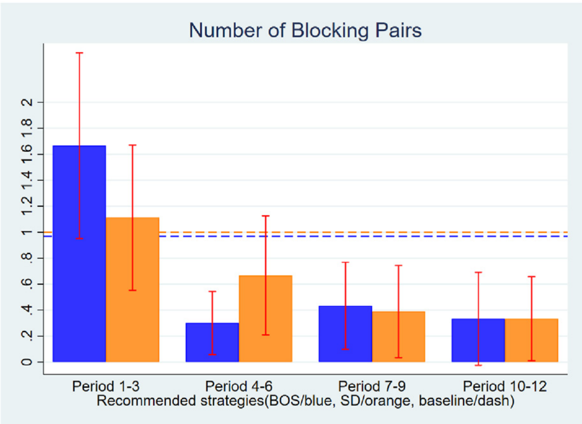

Fig. B.13. Average number of blocking pairs under fully constrained mechanisms with recommended strategies (the red line is the 95% confidence interval)

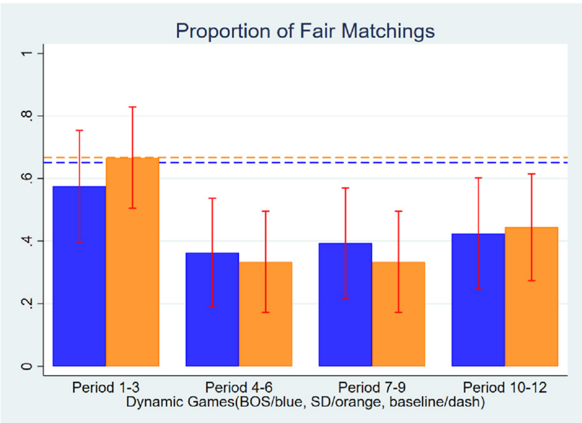

Fig. B.14. Proportion of ex-ante fair matching outcomes under dynamic fully constrained mechanisms (the red line is the 95% confidence interval)

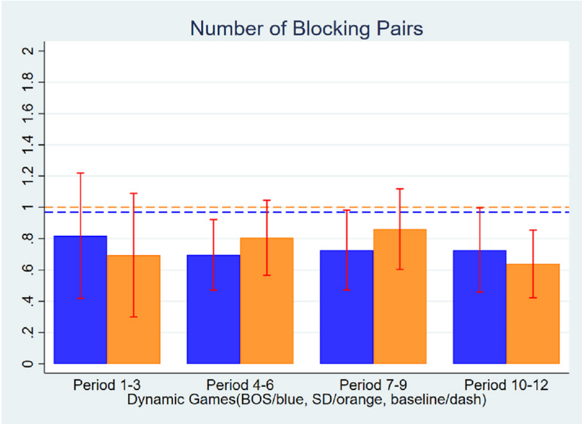

Fig. B.15. Average number of blocking pairs under dynamic fully constrained mechanisms (the red line is the 95% confidence interval)

Table B.15
Proportion of fair matching outcomes under variants

		Panel A: Mean				
		Periods 1-3	Periods 4-6	Periods 7-9	Periods 10-12	
BOS	Repeated games	0.533	0.6	0.767	0.833	
	Recommended strategies	0.433	0.8	0.767	0.867	
	Dynamic games	0.575	0.364	0.394	0.424	
SD	Repeated games	0.545	0.697	0.879	0.848	
	Recommended strategies	0.556	0.778	0.833	0.861	
	Dynamic games	0.667	0.333	0.333	0.444	
		Panel B: Probit Regression Results				
		Periods 4–12 - 1-3	Periods 10–12 - 1-3	Periods 10–12 - 4-6	Periods 4–12 - Baseline	Periods 10–12 - Baseline
BOS	Repeated games	0.2	0.3	0.233	0.083	0.183
	(p-value)	(0.075)	(0.003)	(0.000)	(0.384)	(0.032)
	Recommended strategies	0.378	0.433	0.067	0.160	0.215
	(p-value)	(0.050)	(0.069)	(0.480)	(0.265)	(0.199)
	Dynamic games	−0.182	−0.152	0.061	−0.257	−0.227
	(p-value)	(0.000)	(0.153)	(0.252)	(0.008)	(0.055)
SD	Repeated games	0.263	0.303	0.152	0.141	0.182
	(p-value)	(0.256)	(0.136)	(0.011)	(0.404)	(0.217)
	Recommended strategies	0.269	0.306	0.083	0.157	0.194
	(p-value)	(0.013)	(0.082)	(0.446)	(0.235)	(0.278)
	Dynamic games	−0.296	−0.222	0.111	−0.296	−0.222
	(p-value)	(0.287)	(0.360)	(0.000)	(0.000)	(0.000)

Note: The results presented in panel A are the mean values for all mechanisms. Panel B shows the differences between each pair of period intervals; standard errors are clustered within each session.

Table B.16
Number of blocking pairs under variants.

		Panel A: Mean			
		Periods 1-3	Periods 4-6	Periods 7-9	Periods 10-12
BOS	Repeated games	0.9	0.933	0.433	0.2
	Recommended strategies	1.667	0.3	0.433	0.333
	Dynamic games	0.818	0.697	0.727	0.727
SD	Repeated games	1.242	0.697	0.273	0.364
	Recommended strategies	1.111	0.667	0.389	0.333
	Dynamic games	0.694	0.806	0.861	0.639

(*continued on next page*)

Table B.16 (*continued*)

		Panel A: Mean				
		Periods 1-3	Periods 4-6	Periods 7-9	Periods 10-12	
		Panel B: OLS Regression Results				
		Periods 4–12 - 1-3	Periods 10–12 - 1-3	Periods 10–12 - 4-6	Periods 4–12 - Baseline	Periods 10–12 - Baseline
BOS	Repeated games	−0.378	−0.7	−0.733	−0.446	−0.768
	(p-value)	(0.281)	(0.091)	(0.171)	(0.224)	(0.074)
	Recommended strategies	−1.311	−1.333	0.033	−0.613	−0.635
	(p-value)	(0.032)	(0.064)	(0.910)	(0.186)	(0.207)
	Dynamic games	−0.101	−0.091	0.030	−0.251	−0.241
	(p-value)	(0.291)	(0.642)	(0.911)	(0.443)	(0.511)
SD	Repeated games	−0.798	−0.879	−0.333	−0.555	−0.636
	(p-value)	(0.445)	(0.381)	(0.472)	(0.221)	(0.130)
	Recommended strategies	−0.648	−0.778	−0.333	−0.537	−0.667
	(p-value)	(0.111)	(0.206)	(0.376)	(0.117)	(0.153)
	Dynamic games	0.074	−0.056	−0.167	−0.231	−0.361
	(p-value)	(0.913)	(0.930)	(0.000)	(0.217)	(0.108)

Note: The results presented in panel A are the mean values for all mechanisms. Panel B shows the differences between each pair of period intervals; standard errors are clustered within each session.

Table B.17
Probability of non-admission under variant mechanisms

Probability of non-admission	Periods 1-3	Periods 4-6	Periods 7-9	Periods 10-12
BOS-repeated games	0.156	0.133	0.067	0.056
BOS-recommended strategies	0.211	0.067	0.078	0.044
BOS-dynamic (after the first stage)	NA	0.354	0.343	0.343
BOS-dynamic (after the second stage)	0.141	0	0	0
SD-repeated games	0.141	0.081	0.040	0.051
SD-recommended strategies	0.176	0.065	0.065	0.047
SD-dynamic (after the first stage)	NA	0.371	0.343	0.352
SD-dynamic (after the second stage)	0.111	0	0	0

Appendix C. Experimental Instruction Manual

We only provide instruction manual for BOS-before and BOS-dynamic since instruction manual for other mechanisms are highly analogous to these two.

Appendix C.1. Instruction Manual for BOS-before

Thank you for participating in this experiment on decision making. From now until the end of the session any communication with other participants is forbidden. If you have any question, feel free to ask at any point of the experiment. Please do so by raising your hand and one of us will come to your desk to answer your question.

In this experiment we simulate 3 procedures to allocate students to schools. For each procedure, there are 3 independent rounds of games. So the whole experiment will have totally 9 rounds. In each round, we will form groups of three participants, so that you will be randomly grouped with 2 other participants, whose identity you will not know. You will play one of three roles of students, namely student 1, 2 or 3, and the other 2 players will play the remained roles respectively. You will play all the three roles of student 1, 2 and 3 one by one in the 3 consecutive rounds for each procedure. The sequence is assigned randomly. Note that groups will be reformed after each round.

In each round, all the participants have to indicate a preference ordering over schools. There are three schools (A, B, and C) and every school has one slot available. The assignment of each slot is based on the preference ordering submitted by the 3 participants of the group, and also a score ranking assigned to each of the 3 participants. Schools differ in quality, and the desirability of schools in terms of quality is summarized by the amounts shown in the payoff table (see Decision Sheets), which contains the payoff amounts in experimental currency units (ECU) corresponding to each participant and school slot. This table is known by all the participants. Meanwhile, in each round of the matching game, it is possible that some participants are unadmitted by any school, who will earn a payoff of 0 ECU.

Submitted school ranking. In each round during the experiment, you will be asked to complete the Decision Sheet by indicating the preference ordering over schools you wish to submit. In procedure 1, you have to rank only 1 school. In procedure 2, you have to rank only 2 schools and in procedure 3, you have to rank all of 3 schools.

Score Assignment and ranking. Schools build a priority ordering when offering slots where all candidates are ranked. The rankings are solely determined by score rankings of all candidates. All the three schools give the student with the highest score the highest priority, the second highest score the second highest priority, and the third highest (or the lowest) score the third highest (or lowest) priority. Score rankings are determined by score numbers all the participants have. The rules of score assignment and ranking are described below:

Each student will have a score number. Score numbers of all the participants will determine score rankings. Students who have the highest score will be ranked no. 1, the second highest no. 2, and the third highest (or the lowest) no. 3.

Each student will have an equal probability of getting three types of scores (namely high, normal, and low), where 100 represents full marks. However, those three scores are different for each of the three participants. The following table contains the score distribution of each student (this is known by all participants):

Score type	High score	Normal score	Low score	Avg. score
Probability	1/3	1/3	1/3	
Student 1	95	90	85	90
Student 2	91	86	81	86
Student 3	87	82	77	82

It can be seen from the table above that student 1 has an average score higher than 2, and 2 higher than 3. However, when student 1 has a normal score and 2 has a high score, 2 will have a higher score than 1 thus rank ahead of 1. The similar event happens between student 2 and 3. Furthermore, if student 1 has a low score 33 and student 3 has a high score, even student 3 will surpass student 1.

Payoffs. You can earn money during the session. You will receive 20 RMB for your participation, in addition to the amount you earn in the experiment. The payoff for each student in each round is displayed in the payoff table, corresponding to the slot you hold at the end of each round. Note that the slot you hold at the end of each round depends on your submitted ordering and the submitted ordering of the other participants of your group (which you will not know at the moment of submitting your preference ordering).

The total payoff you earn is the sum of payoffs you earn in each of the 9 rounds, plus the 20 RMB participation fee. Once the whole experiment has finished and all the 9 allocations (corresponding to 9 rounds of games) of the participants are determined, each participant will get paid her total payoff in RMB. 1 ECU is equal to 0.4 RMB.

Allocation Procedures. In each round, each participant is assigned a slot at the best possible school reported in her Decision Sheet that is consistent with the priority ordering of schools, the ordering being solely determined by score rankings among all the participants. The only difference between procedures are the number of schools she needs to rank (from 1 to 3). The detailed process of each procedure is the following:

Step 1. All students simultaneously submit a preference list containing n colleges.
Step 2. Each student learns his and the other two students' realized scores, which are randomly and independently drawn according to everyone's score distribution.
Step 3. Each college considers the first choice of all of the students. Colleges admit the student with the highest realized score among those who list it as their first choice.

If $n = 1$, then the algorithm stops. Students not admitted in Step 3 are finally unadmitted.

Step 4. Colleges with vacancies then consider the second choice of all unadmitted students. They admit the student with the highest realized score among those who are as yet unadmitted and who list it as their second choice.

If $n = 2$, then the algorithm stops. Students not admitted in Steps 3–4 are finally unadmitted.

Step 5. Colleges with vacancies then consider the third choice of all unadmitted students. They admit the student with the highest realized score among those who are yet unadmitted and who list it as their third choice.

The game stops at this point.
Examples. We will go through two simple examples to see how this allocation procedure works.

Example 1. Participants have to rank all 3 schools.

Step 1. Submitted school ranking: Suppose the submitted school rankings of each participant are the following:

	Student 1	Student 2	Student 3
1st choice	A	A	A
2nd choice	B	B	B
3rd choice	C	C	C

Step 2. Score assignment and ranking: Suppose after three lotteries being drawn randomly and independently for each participant, students have scores and therefore ranks as:

Student/Applicant	1	2	3
Score	95	91	87
Rank	1	2	3

Step 3–6. Allocation. The allocation procedure consists of the following steps:

Step 3: Each applicant applies to her first choice:
—Applicant 1, 2, 3 apply all to school A.
Step 4: Each school accepts the applicant with the highest score ranking and rejects others:
—School A retains applicant 1 and reject applicant 2 and 3.
—Applicant 1 and school A are removed from the subsequent process.
Step 5: Each applicant who is rejected in round 1 applies to her second choice:
—Applicants 2 and 3 apply to school B.
—School B accepts applicant 2 and rejects applicant 3.
—Applicant 2 and school B are removed from the subsequent process.
Step 6: Each remaining participant is assigned her last choice.
—Applicant 3 gets the remaining slot in school C.

Here the process finishes; and the final allocations are the following:

Student/Applicant	1	2	3
School	A	B	C

Example 2. Participants have to rank only 2 schools.

Step 1. Submitted school ranking: Suppose the submitted school rankings of each participant are the following:

	Student 1	Student 2	Student 3
1st choice	A	A	A
2nd choice	B	B	B

Step 2. Score assignment and ranking: Suppose after three lotteries being drawn randomly and independently for each participant, students have scores and therefore ranks as:

Student/Applicant	1	2	3
Score	95	91	87
Rank	1	2	3

Step 3–5. Allocation. The allocation procedure consists of the following steps:

Step 3: Each applicant applies to her first choice:
—Applicant 1, 2, 3 apply all to school A.
Step 4: Each school accepts the applicant with the highest score ranking and rejects others:
—School A retains applicant 1 and reject applicant 2 and 3.
—Applicant 1 and school A are removed from the subsequent process.
Step 5: Each applicant who is rejected in round 1 applies to her second choice:
—Applicants 2 and 3 apply to school B.
—School B accepts applicant 2 and rejects applicant 3.

Here the process finishes; and the final allocations are the following:

Student/Applicant	1	2	3
School	A	B	Unadmitted

Now you can to go over the instructions at your place, and make your decisions. You will make decisions under 3 procedures mentioned above separately. Within each procedure, you will be asked to play the role of student 1, 2, 3 alternately. So you will have 9 total decisions to make, i.e., 9 preference orderings to submit. The final allocation and your payoff for each role you play in each procedure will be shown you at the end of the entire experimental session.

Are there any questions?

Decision Sheet BOS-before

Recall: Every student will be assigned a score number which is independently and randomly draw by the experimenter from a distribution shown in the table below:

Score type	High score	Normal score	Low score	Avg. score
Probability	1/3	1/3	1/3	
Student 1	95	90	85	90
Student 2	91	86	81	86
Student 3	87	82	77	82

Your payoff amount for each role you play in each procedure depends on the school slot you hold at the end of it. Your possible payoff amounts in each round are shown in the following table:

Admission by college	A	B	C	Non-admission
Student 1's payoff	30	25	15	0
Student 2's payoff	30	25	15	0
Student 3's payoff	30	25	15	0

This means, that if at the end of one game you:

—hold a slot at school A, you will be paid 30 ECU for this round;
—hold a slot at school B, you will be paid 25 ECU for this round;
—hold a slot at school C, you will be paid 15 ECU for this round;
—are unadmitted by any school, you will be paid 0 ECU for this round.

Procedure 1

Recall: You will make decisions before you know your realized score. All the information you (and other participants in your group) have is the score distribution of each student.

Round 1.

You are playing the role of student _(1, 2, or 3 - will be shown on your screen) in this game. Please submit your ranking of the schools. You need to rank only 1 school this round.

(For sample screeshot, please see Figure C16)

1st choice

Round 2
......
Round 3
......

Procedure 2

Recall: You will make decisions before you know your realized score. All the information you (and other participants in your group) have is the score distribution of each student.

Round 4.

You are playing the role of student _(1, 2, or 3 - will be shown on your screen) in this game. Please submit your ranking of the schools. You need to rank only 2 schools this round.

1st choice	2nd choice

Round 5
......
Round 6
......

Procedure 3

Recall: You will make decisions before you know your realized score. All the information you (and other participants in your group) have is the score distribution of each student.

Round 7.

You are playing the role of student _(1, 2, or 3 - will be shown on your screen) in this game. Please submit your ranking of the schools. You need to submit all 3 schools this round.

1st choice	2nd choice	3rd choice

Round 8
......
Round 9
......

This is the end of the experiment. Please go on to fill out the following short form concerning your personal background and risk attitudes. The form is important for analyzing the experimental results. During this period, our experimental system will calculate your payoff. After you finish the form, please wait until everybody finishes and you will be informed of the result.

Appendix C.2. Instruction Manual for BOS-dynamic

Thank you for participating in this experiment on decision making. From now until the end of the session any communication with other participants is forbidden. If you have any question, feel free to ask at any point of the experiment. Please do so by raising your hand and one of us will come to your desk to answer your question. There are two phases in this experimental session and we will explain phase 1 now.

Phase 1

In this experiment there are 3 independent rounds of games. In each round, we will form groups of three participants, so that you will be randomly grouped with 2 other participants, whose identity you will not know. You will play one of three roles of students, namely student 1, 2 or 3, and the other 2 players will play the remained roles respectively. You will play all the three roles of student 1, 2 and 3 one by one in the 3 consecutive rounds for each procedure. The sequence is assigned randomly. Note that groups will be reformed after each round … …

Submitted school ranking. In each round during the experiment, you will be asked to complete the Decision Sheet by indicating the preference ordering over schools you wish to submit. In each round, you have to rank only 1 school … …

Payoffs. You can earn money during the session. You will receive 10 RMB for your participation, in addition to the amount you earn in the experiment. The payoff for each student in each round is displayed in the payoff table, corresponding to the slot you hold at the end of each round. Note that the slot you hold at the end of each round depends on your submitted ordering and the submitted ordering of the other participants of your group (which you will not know at the moment of submitting your preference ordering).

The total payoff you earn is the sum of payoffs you earn in each of the 9 rounds, plus the 20 RMB participation fee. Once the whole experiment has finished and all the 9 allocations (corresponding to 9 rounds of games) of the participants are determined, each participant will get paid her total payoff in RMB. 1 ECU is equal to 0.15 RMB … …

Phase 2

Now, there are two stages of preference submitting in each round of the matching game. Details are as follows:

At stage 1, settings are exactly the same as phase 1 (note there is no second chance in phase 1).

If someone is admitted at stage 1, then his game ends in this round. All **unadmitted** students will enter stage 2. If you enter stage 2, you will be informed of all realized scores of members in your group and schools with vacancies after stage 1. You need to submit one school among those with vacancies and join the matching procedure (same as phase 1) with other unadmitted students in your group. The payoff of being admitted at stage 2 are the same as stage 1 (and phase 1).

Examples. We will go through one simple example to see how this allocation procedure works.

Step 1. Submitted school ranking: Suppose the submitted school rankings of each participant are the following:

	Student 1	Student 2	Student 3
1st choice	A	A	A

Step 2. Score assignment and ranking: Suppose after three lotteries being drawn randomly and independently for each participant, students have scores and therefore ranks as:

Student/Applicant	1	2	3
Score	95	91	87
Rank	1	2	3

Step 3–4. Allocation. The allocation procedure consists of the following steps:

Step 3: Each applicant applies to her first choice:

—Applicant 1, 2, 3 apply all to school A.

Step 4: Each school accepts the applicant with the highest score ranking and rejects others:

—School A retains applicant 1 and rejects applicant 2 and 3.
—Applicant 1 and school A are removed from the subsequent process.
Stage 1 ends. Student 2 and 3 are unadmitted so they enter stage 2.
They are informed that school B and C are still available and they know realized scores of all students in this group now.

Step 1. Submitted school ranking: Suppose the submitted school rankings of each participant are the following:

	Student 2	Student 3
1st choice	B	B

Step 2. Scores remain the same and therefore:

Student/Applicant	2	3
Score	91	87
Rank	1	2

Step 3–4. Allocation. The allocation procedure consists of the following steps:

Step 3: Each applicant applies to her first choice:

—Applicant 2, 3 apply all to school B.

Step 4: Each school accepts the applicant with the highest score ranking and rejects others:

—School B retains applicant 2 and rejects applicant 3.
Here the process finishes; and the final allocations are the following:

Student/Applicant	1	2	3
School	A	B	Unadmitted

Now you can to go over the instructions at your place, and make your decisions. Y you will have 12 total decisions to make. The final allocation and your payoff for each role you play in each procedure will be shown you at the end of the entire experimental session.
Are there any questions?

Decision sheet BOS-dynamic
Phase 1.
… …
Round 1.
You are playing the role of student _(1, 2, or 3 - will be shown on your screen) in this game. Please submit your ranking of the schools. You need to rank only 1 school this round.
(For sample screeshot, please see Figure C16)

1st choice

Round 2
……
Round 3
……

Phase 2.
Round 4.
You are playing the role of student _(1, 2, or 3 - will be shown on your screen) in this game. Please submit your ranking of the schools. You need to rank only 1 school this round.
(For sample screeshot, please see Figure C16)

1st choice

If you are unadmitted:
You are playing the role of student _(1, 2, or 3 - will be shown on your screen) in this game. Student 1's realized score is _(will be shown on your screen), student 2's realized score is _(will be shown on your screen), student 3's realized score is _(will be shown on your screen). School _(will be shown on your screen) are still available. Please submit your ranking of the schools. You need to rank only 1 school this round.
(For sample screeshot, please see Figure C17)

1st choice

Round 5–12 are the same as round 4.

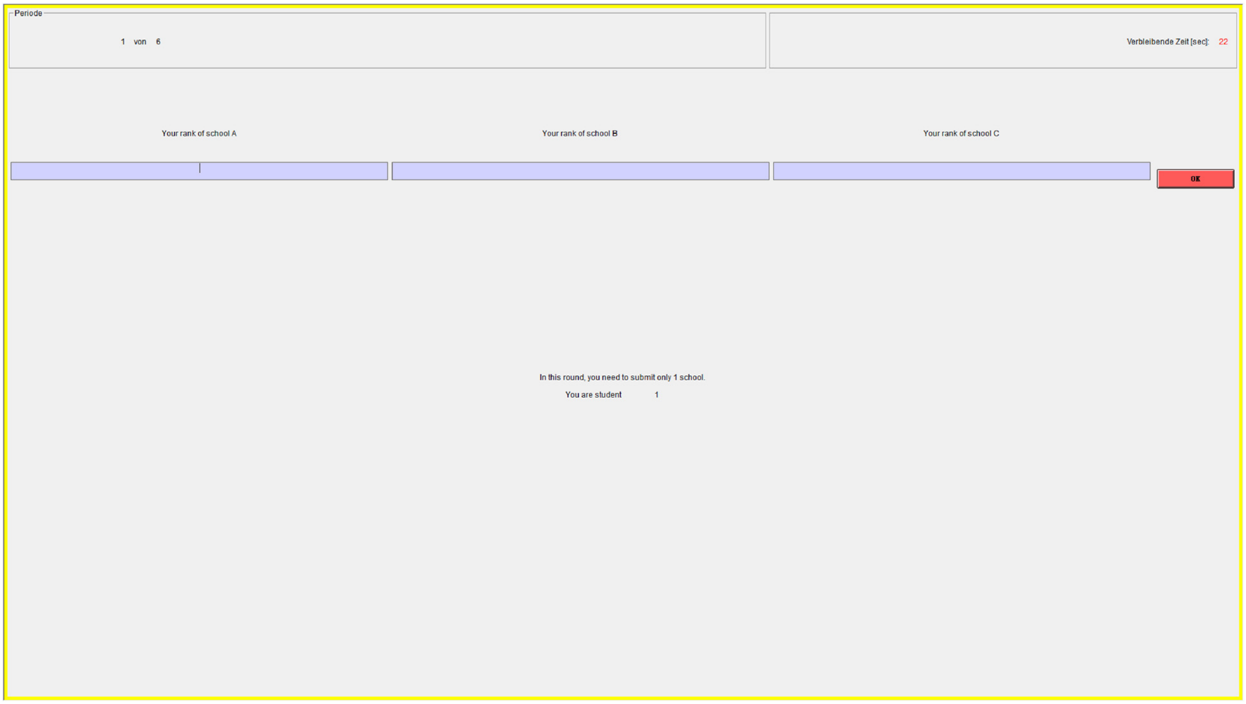

Fig. C.16. Sample screenshot in BOS-before sessions (full constraint)

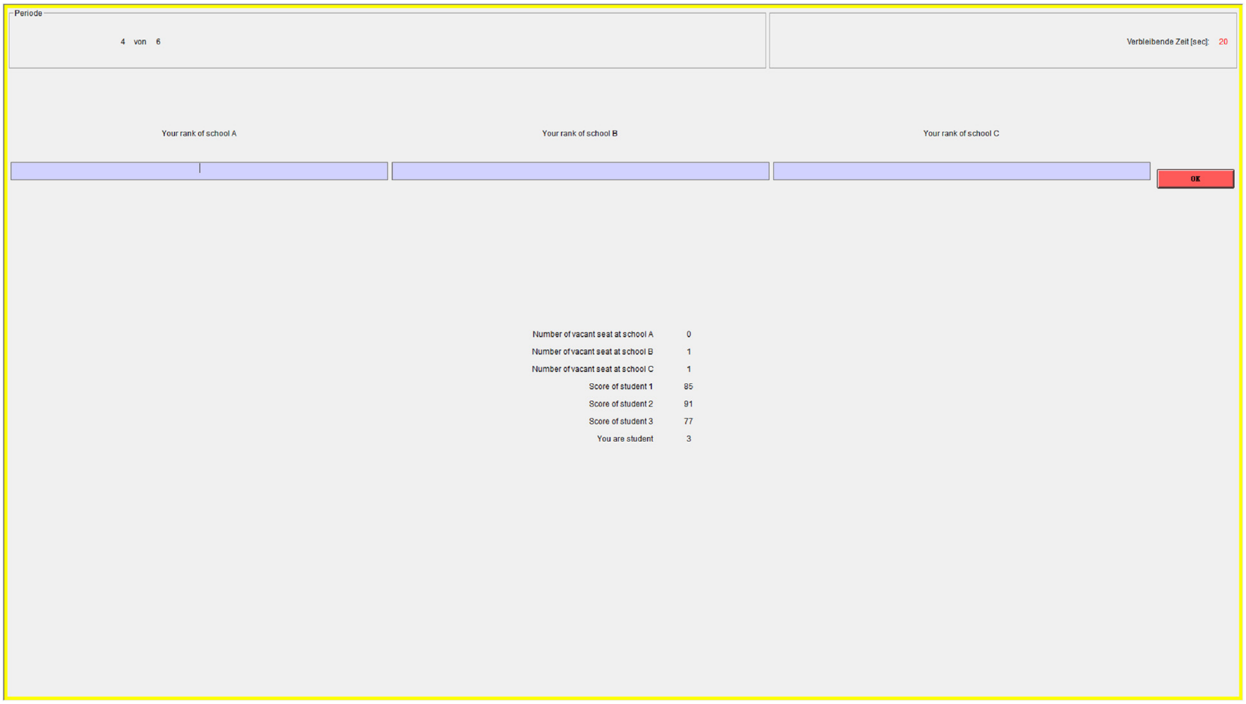

Fig. C.17. Sample screenshot in BOS-dynamic sessions (second round)

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