Career concern model with voluntary information disclosure[⋆]

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Abstract

When an educational institution requires voluntary disclosure of test scores, what is the best way for a student to utilize their score? How does this strategic conduct influence the motivation to invest in ability? I study a career-concern model with voluntary disclosure and show that the optimal disclosure rule follows a threshold strategy in equilibrium. This strategic channel reduces incentives to invest in ability at the first stage.

Keywords: Career concern, Information disclosure

1. Introduction

Information plays an important role in a lot of circumstances. A student can submit a test score during his application; a seller can show a certificate of an object's quality to ask for a higher price; a candidate can provide a proof of prize during a competition to convince the interviewer to get a job. The underlying problem is simple but important: when the student (student/seller/candidate) has some private information, what is the optimal way to utilize it? In addition, when the disclosure of information is voluntary, how does it affect the incentive for the student to invest in his ability? To answer these questions, I propose a career concern model with strategic information disclosure which covers all the settings mentioned above, and throughout the paper, I will use 'student' and 'school' as two sides of market.

⋆For beneficial discussions and useful comments, I thank Atila Abdulkadiroglu, Attila Ambrus, Arjada Bardhi, Minjun Chang, Chang Liu, Adam Rosen, Huseyin Yildirim. All errors are my own.

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I consider an information model with one student and one school. The student first exerts effort e, which determines his (binary) type $\theta \in \{0, 1\}$. I assume that the student's true type is unknown to both the student and the school. The school then forms a prior belief e^* of the student's type. Then the student (exogenously) receives a signal about his ability (e.g. a standardized test score, prizes of competitions etc.) with a certain probability. A question then arises: should the student reveal his signal during the application? Further on, what is his optimal way to do that? It is intuitive to expect that the student will reveal the good signals and conceal the bad ones. Surprisingly, it is only possible for the student to conceal bad signals when the probability of signal arrival is strictly less than 1. Meanwhile, in equilibrium, the cutoff of revealing signals is independent of the choice of effort level.

When the signal arrives for sure, the school, although not knowing the content, perfectly knows the arrival of information. I prove that there is a unique class of equilibrium, in which the student strictly prefers to reveal if his signal is not the worst one and is indifferent otherwise, which echoes the results in Grossman (1981) and Milgrom (1981). The key intuition here is that everyone who conceals will be pooled together regardless of his signal quality. As a result, if the signal is not the worst one, he will reveal it to avoid being pooled with the ones whose signal quality is lower. The school can then perfectly infer the quality of concealed signals. When the arrival of information is not for sure, there is a positive probability where the student fails to get a private signal, i.e., not everyone has extra information. In this case, the unique equilibrium follows a thresholding feature. The student will prefer to reveal if his signal quality is above a certain threshold and to conceal otherwise. Here, those with bad signals lower than the threshold can hide because the ones fail to get a signal will have to be pooled with them. In expectation, the ones with extra information get higher payoff, at the expense of their unlucky colleagues.

A crucial assumption of my analysis and results is that the school knows the arrival of information. That is, although not knowing the content of information, the school has perfect knowledge of how much information the student possesses. This does not exclude the possibility that the student can exert effort to search for more information. However, it does not allow the case that the student secretly collects two pieces of information and only reveals the better one, where the school believes that this is the only information available to the student. The latter case will cause a problem similar to the winner's curse.

Besides the analysis on the disclosure game, I also show that voluntary disclosure will decrease the student's incentive to invest in his own ability, leading to a less socially efficient outcome.

1.1. Related Literature

To discuss the literature, it is helpful to start with the famous 'unraveling result.' in quality disclosure, derived by Grossman (1981) and Milgrom (1981), where sellers possesses better information about product quality than consumers do and there is no cost to verify and disclose it. The above two papers argue that sellers will always disclose. Starting from that, there is a gigantic body of literature studying seller's incentives to disclose quality information to uninformed buyers. For example, Jovanovic (1982) shows that when disclosure is costly, only seller with information better than a threshold will choose to disclose. Dye (1985) proves that when the investors are not sure if a manager possess private information, concealing bad information will be possible in equilibrium. Other than the frictions mentioned above, dynamic incentives can play a role in the disclosure game. Grubb (2011) shows that one may conceal information to avoid the pressure of disclosing in the future, when the quality may drop.[1]

Besides quality disclosure about products, there is a branch of literature regarding individuals' incentive to take different actions in a disclosure game. Holmström (1999) studies manager's (dynamic) incentive to exert effort when outcome of projects are publicly observable. Mukherjee (2008) shows that employers can strategically disclose information to influence incentives of employees and matching efficiency. Bar-Isaac and Lévy (2022) finds that employer may assign less informative tasks to avoid competition on workers.

Another branch of literature closely related to this project is career concern models. Starting from the seminal paper, Holmström (1999), which gives a tractable framework to study career concerns, Dewatripont et al. (1999a) characterizes how to compare different information structure. Other development and extensions include Gibbons and Murphy (1992), Dewatripont et al. (1999b).

My work contributes to the literature in the following two aspects: firstly, I provide a clean characterization of the threshold on signal revealing with

[1] For more papers about quality disclosure, there is a nice review paper by Dranove and Jin (2010)

almost no assumptions; secondly, to the best of my knowledge, this is the first project combining career concerns with voluntary information disclosure.

The remainder of the paper is organized as follows: section 2 presents the model, section 3 discusses the main findings, and section 4 concludes the work.

2. Model

2.1. General Setup

I study a one-period disclosure game between a school and a student. The model consists of one school and one student. At the beginning of the game, the student exerts effort level $e \in [0, 1]$, with a cost of $c(e)$. The effort level will be the probability of being a high type, that is, $e = \mathbb{P}(\text{being high type})$. After the effort investment stage, the student's type $\theta \in \{0, 1\}$ is realized, which is unknown to both the school and himself. The school holds a prior belief $e^* \in (0, 1)$ which is the probability of the student being a high type. Then, the student receives a private signal s (e.g., a test score) with a fixed probability $p \in (0, 1]$, where p is a public knowledge. Conditional on receiving a signal, the signal s is randomly drawn from the distributions f_θ, F_θ, which depends on the true type of the student. I impose no restriction on the distributions except $f_0 \neq f_1$ and the two distributions have the same support. If the student gets a signal s, he decides between a binary action $\{R, C\}$, i.e., to reveal the signal or to conceal it. The school will form an updated belief μ after seeing the student's decision, and the student's payoff is then μ(e.g. admission offer with different level of scholarship). The timing of the game is as follows:

1. The student exerts effort level $e \in [0, 1]$, paying cost $c(e)$ (where $c(0) = 0, c'(0) = 0, c' > 0, c'' > 0$). His effort level e determines the probability of him being high type. (type is binary)
2. The school forms a belief e^*;
3. With probability p, the student privately observes a signal s, realized according to f_0, f_1;
4. The student chooses to reveal (R) the signal s or to conceal (C) it;
5. The school forms her updated belief $\mu = \mu_R(s)$ if the student reveals and $\mu = \mu_C$ otherwise, and then set the student's payoff $u = \mu$.

3. Analysis

3.1. The disclosure game

Before talking about the career concern game, let us first analyze the disclosure game given school's belief e^*.

3.1.1. $p = 1$

I first consider the case $p = 1$, i.e., the student for sure gets a signal. Let $r(s)$ denote a generic element of the student's strategy profile, which specifies the probability of revealing given the signal s. Adopting Bayes' rule, the school forms an updated belief $\mu = \mu_R(s)$ if seeing the student reveals as

$$\mu_R(s) = \frac{e^* f_1(s)}{(1-e^*)f_0(s) + e^* f_1(s)},$$

and forms an updated belief $\mu = \mu_C$ if seeing the student conceals as

$$\mu_C = \frac{\int e^*(1-r(x))f_1(x)dx}{\int (1-e^*)(1-r(x))f_0(x)dx + \int e^*(1-r(x))f_1(x)dx}.$$

For simplicity, I let $f(s) \equiv (1-e^*)f_0(s) + e^* f_1(s)$ denote the expected density of signal s according to the school's prior belief. In the following analysis, I employ the concept of perfect Bayesian equilibrium (PBE), where for a given signal realization s, the student chooses R if his payoff after revealing is higher, i.e., $\mu_R(s) > \mu_C$, chooses C if his payoff after revealing is lower, i.e., $\mu_R(s) < \mu_C$, and is indifferent otherwise.

I first show that in any equilibrium, the updated belief if seeing the student conceals is no larger than the prior. A formal result is as follows.

Lemma 1. *Given any equilibrium strategy profile $r^*(s)$, $\mu_C \le e^*$.*

Proof:

Suppose not, then we have $\mu_C > e^*$. There are two cases:

Case 1: there is an off-path strategy, that is, the student are always revealing or concealing. If always concealing, then $\mu_C = e^*$. If always revealing, then action C is off-path, PBE puts no constraint on μ_C. But in order to support such an equilibrium, I need $\mu_C \le \mu_R(s), \forall s$. However, Bayes plausibility requires that there is at least some signals s^*, such that $\mu_R(s^*) \leq e^*$, then $\mu_C \leq \mu_R(s^*) \leq e^*$, contradicting the assumption $\mu_C > e^*$.

Case 2: there is no off-path strategy (R, C both have a positive probability). Then there exists an equilibrium strategy profile $r^*(\cdot)$ such that $\mu_C > e^*$. Bayes plausibility requires

$$e^* = \int f(x)r^*(x)\mu_R(x)dx + \int f(x)(1 - r^*(x))\mu_C dx.$$

If $\mu_C > e^*$, then there must exist some s^* with $r^*(s^*) > 0$ such that $\mu_R(s^*) < e^*$.By $\mu_C > e^*$, the student with signal s^* should strictly prefer concealing, contradicting $r^*(s^*) > 0$. □

Lemma 1 provides an intuitive result. The updated belief after seeing concealing is weakly lower than the school's prior belief, meaning that concealing is an indicator of bad signals. Given that, the student with any good enough signal will reveal it to distinguish himself. This logic naturally leads to the next corollary.

Corollary 1. *For any equilibrium strategy profile $r^*(\cdot)$, $r^*(s) = 1, \forall s \in \{s : \mu_R(s) > e^*\}$.*

The above results show that the student with good signals will reveal for sure in any equilibrium. Next, I show that there is one unique class of equilibrium, in which the student will reveal for sure if he gets any signal other than the worst one, and will be indifferent otherwise. This is equivalent to the case that the student always reveals since the school can update her belief perfectly 'as if' she observes the student's private signal.

Before formally stating the results, I introduce a likelihood ratio measurement $l(s) \equiv \frac{f_1(s)}{f_0(s)}$. Intuitively, a higher value of l indicates that the student is more likely to be a high type. The unique class of equilibrium is then formally stated as follows.

Proposition 1. *Assume f_θ share the same support. For any equilibrium strategy profile $r^*(\cdot)$,*

(i) $r^*(s) = 1$ *if* $l(s) > \inf_s l(s)$;

(ii) $r^*(s)$ *can take any value in* $[0, 1]$ *if* $l(s) = \inf_s l(s) = \min_s l(s)$.

Proof: see Appendix.

Proposition 1 echoes the results in Grossman (1981) and Milgrom (1981). It states that in equilibrium, the student strictly prefers revealing if his realized signal is not the worst one in the measurement of likelihood ratio. There are two key driving forces for the result. One is that revealing brings no cost. The student with the best signal will reveal for sure, forcing the second-best to distinguish themselves and so on, leading to the unique class of equilibrium.

These equilibriums can be equivalently interpreted as the case that the student always reveals his signal to the school. Because the concealing pool only includes the worst signal, seeing a concealing action is the same as observing a revealed worst signal. Therefore, in any equilibrium, the school can always update her belief perfectly as in the case where all signals are revealed.

In the next section, I generalize the result to the case $p \in (0, 1)$.

3.1.2. $p \in (0, 1)$

This section studies the general case $p \in (0, 1)$, i.e., the student gets a private signal with a probability strictly less than 1, which can be considered as some level of market friction. Such frictions in the real world are common. For example, students from poor families can study hard for the SAT tests, but they may lose the chance of taking the test because of financial restrictions. Another example can be the COVID-19, during the pandemic, students may lose the chances of joining international competitions (and they may have prepared years for that).

Adopting the same notations $r(\cdot)$ for the student's strategy profile and $l(\cdot)$ for the likelihood ratio measurement, the next result proves that there is again a unique class of equilibria. The existence of equilibrium with incomplete disclosure is showed in Dye (1985). In comparison, I show that the unique equilibrium follows a thresholding feature. That is, the student will prefer to reveal if the likelihood ratio of his signal is above a certain threshold; the student will prefer to conceal if the likelihood ratio of his signal is below the threshold; and the student will be indifferent if the likelihood ratio of his signal equals the threshold. This result does not require any commonly seen assumptions such as first order stochastic dominance or monotone likelihood ratio. A formal statement of the result is as follows.

Proposition 2. *Assume f_θ share the same support and are continuous. There exists a unique value of l^* such that for any equilibrium strategy profile $r^*(\cdot)$,*

(i) $r^*(s) = 1$ *if* $l(s) > l^*$;

(ii) $r^*(s) = 0$ *if* $l(s) < l^*$;

(iii) $r^*(s)$ *can take any value in* $[0, 1]$ *if* $l(s) = l^*$.

Proof: see Appendix.

Proposition 2 provides a similar result compared to Proposition 1. It states that in equilibrium, the student uses a threshold strategy. The key distinction here is that there is a possibility for the student to hide the content of information. The reason is that there is a positive probability where the student fails to get a private signal, i.e., not everyone has extra information. Similarly, those with good signals take the reveal action to distinguish themselves. However, those with bad signals can hide this time because the ones fail to get a signal will have to be pooled with them. In expectation, the ones with extra information get a higher payoff, in the cost of their unlucky colleagues.

Another thing noteworthy about proposition 2 is that, this result does not depend on some commonly seen assumptions such as FOSD or MLR. The assumption of common support and continuity are also for cleanness of expression and proof. [2]

The above proposition shows the existence of cutoff (in the sense of likelihood), my next result further characterizes some nice properties of the cutoff

Proposition 3. *the (unique) cutoff value for likelihood ratio is specified below, independent of the school's belief e^*.*

$$\frac{(1-p) + p\int_{l(s)<l^*} f_1(s)\, ds}{(1-p) + p\int_{l(s)<l^*} f_0(s)\, ds} = l^* < 1.$$

[2] If we define the a positive number over 0 to be positive infinity and let our likelihood ratio function take values on extended real line, the thresholding feature still holds without these assumptions.

Proof: see Appendix.

Note that when we are only integrating signals with likelihood ratio smaller than a certain value, the left hand side is bounded by 1. Therefore, the value of l^* must be strictly smaller than 1.

Besides some mathematical details, there is an intuitive way to understand why this cutoff is independent of the school's belief. In the most standard Bayesian updating process, after seeing a signal s, one's belief will be

$$\frac{e^* f_1(s)}{(1-e^*)f_0(s) + e^* f_1(s)}$$

In the setting of $p < 1$, we can consider receiving 'no signal' and 'signal lower than cutoff' as receiving a signal called 'concealing'. Then one's belief after seeing 'conceal' will be

$$\frac{e^* \mathbb{P}(C|\text{high type})}{(1-e^*)\mathbb{P}(C|\text{low type}) + e^* \mathbb{P}(C|\text{high type})}$$

As a result, the only thing that matters in the updating process is the ratio between $\mathbb{P}(C|\text{low type}), \mathbb{P}(C|\text{high type})$, which are

$$\mathbb{P}(C|\text{high type}) = p + (1-p)\int_{s:l(s)<l^*} f_1(s)ds$$
$$\mathbb{P}(C|\text{low type}) = p + (1-p)\int_{s:l(s)<l^*} f_0(s)ds$$

which makes l^* independent of the school's belief e^*.

3.2. The career concern game

3.2.1. Socially optimal outcome

We first look at the benchmark case, where the school can perfectly observe the student's type. Since the type is directly observed, the student's (expected) payoff is $\mathbb{P}$(being high type), which is just effort level e. Therefore, he solves $\max_e e - c(e)$, which gives $c'(e) = 1$. By assumptions on the cost function ($c(0) = 0, c'(0) = 0, c' > 0, c'' > 0$), we have a unique socially optimal effort level, denoted as e^{opt}.

3.2.2. Equilibrium under voluntary disclosure

When the disclosure is voluntary, according to the strategy characterized in Proposition 2, the student's utility will be

$$\begin{aligned} u(e) = & -c(e) \\ & + (1-p)\mu_C \\ & + p[e \int_{s:l(s) \geq l^*} f_1(s)\mu_R(s)ds + (1-e) \int_{s:l(s) \geq l^*} f_0(s)\mu_R(s)ds] \\ & + p[e \int_{s:l(s) < l^*} f_1(s)\mu_C ds + (1-e) \int_{s:l(s) < l^*} f_0(s)\mu_C ds] \end{aligned}$$

where the first term is the cost of exerting effort; the second line stands for the utility when the student does not receive any signal; The third line and fourth line are utilities after signal arrivals.

Given the equation above, we can write down the new first order condition for the student:

$$\begin{aligned} c'(e) = p[\int_{s:l(s) \geq l^*} f_1(s)\mu_R(s)ds - \int_{s:l(s) \geq l^*} f_0(s)\mu_R(s)ds] \\ + p[\int_{s:l(s) < l^*} f_1(s)\mu_C ds - \int_{s:l(s) < l^*} f_0(s)\mu_C ds] \end{aligned}$$

the first line is the utility when he receives a signal good enough to reveal, and the second line is the case for concealing. We can further rewrite the equation as

$$\begin{aligned} c'(e) = p \int_s (f_1(s) - f_0(s))\mu_R(s)ds \\ + p \int_{s:l(s) < l^*} (f_1(s) - f_0(s))(\mu_C - \mu_R(s))ds \end{aligned}$$

This time, the first line will be utility under mandatory disclosure, and the second line is negative because $\mu_C \geq \mu_R(s)$ when $l(s) < l^*$, and $l^* < 1 \Rightarrow f_1(s) - f_0(s) < 0$.

This result suggests that voluntary disclosure reduces the incentive of exerting effort. An intuitive explanation is the following: when a student has the option to conceal signals, he must conceal bad signals. This means that voluntary disclosure benefits bad signals, which is realized more when one's type is low. Therefore, the student will have less incentive to exert effort at the first stage. The above results can be summarized in the following proposition.

Proposition 4. *For any interior equilibrium e^* under the case $p = 1$. Consider any level of market friction $p < 1$*

1. *the corresponding equilibrium effort level (if exists) with mandatory disclosure drops.*

2. *the corresponding equilibrium effort level (if exists) with voluntary disclosure further drops.*

This result states that voluntary disclosure is actually harming the incentive of exerting effort, generating social inefficiency. The corresponding real world application can be the following: during the pandemic, a lot of educational institutions adopted test-optional policies, allowing students to strategically disclose their standardized test scores as they see fit; there are also empirical evidences from different areas (e.g. what to disclose about nutrition facts before nutrition labeling is mandatory, Mathios (2000), what rankings to disclose on MBA programs' website, Luca and Smith (2015)). Given the existence of strategic disclosure in a lot of contexts, my results suggest that mandatory disclosure can lead to a more socially efficient outcome.

4. Conclusion

The simple model proposed in this paper studies the role of voluntary disclosure in career concern game. I show that the information revealing has a thresholding feature and characterize a clean cutoff for such disclosure without strong assumptions. I further argue that voluntary disclosure harms the incentive of ability investment at the first stage, leading to socially inefficient outcome.

There are still many possible improvements to the current model. For example, the reason why schools do not require SAT scores can be that they hope students focus more on developing other abilities instead of concentrating on test scores. Allowing for multitasking can be another way to explain the adoption of policies. In addition, when there are two schools, one requiring mandatory disclosure, the other one asking for voluntary disclosure may also have interesting applications.

Proof of Proposition 1

To start the proof, we first prove that in equilibrium, everyone who has a strict positive probability of concealing will share the same likelihood ratio for their signals. A formal statement is as follows.

Lemma 2. *For any equilibrium strategy profile $r^*(\cdot)$, s_1, and $s_2 \neq s_1$, if $r^*(s_1) < 1$, $r^*(s_2) < 1$, then $l(s_1) = l(s_2)$.*

Proof of Lemma 2:

Suppose not, then there are two distinct signals $s_1 \neq s_2$ such that $r^*(s_1) < 1$, $r^*(s_2) < 1$, and $l(s_1) \neq l(s_2)$.

Let S_C denote the support of concealing, that is $S_C = \{s : r^*(s) < 1\}$. Let μ^* denote the average belief over S_C regardless of action, that is

$$\mu^* \equiv \frac{\int_{S_C} e^* f_1(x)dx}{\int_{S_C}(1-e^*)f_0(x)dx + \int_{S_C} e^* f_1(x)dx}.$$

Consider any signal $s \in S_C$, the school will obtain an updated belief if the student reveals s as

$$\mu_R(s) = \frac{e^* f_1(s)}{(1-e^*)f_0(s) + e^* f_1(s)}.$$

Substituting $l(s) = \frac{f_1(s)}{f_0(s)}$ and transforming the above equation, we get

$$\mu_R(s) = \frac{e^* l(s)}{(1-e^*) + e^* l(s)} = 1 - \frac{1-e^*}{(1-e^*) + e^* l(s)}.$$

The school will obtain an updated belief if the student conceals as

$$\begin{aligned}\mu_C &= \frac{\int e^*(1-r^*(x))f_1(x)dx}{\int (1-e^*)(1-r^*(x))f_0(x)dx + \int e^*(1-r^*(x))f_1(x)dx} \\ &= \frac{\int_{S_C} e^*(1-r^*(x))f_1(x)dx}{\int_{S_C}(1-e^*)(1-r^*(x))f_0(x)dx + \int_{S_C} e^*(1-r^*(x))f_1(x)dx},\end{aligned}$$

where the last equality is straightforward since $1 - r^*(s) = 0$ for all $s \notin S_C$. Then, Bayes' plausibility rule gives

$$\mu^* = \int_{S_C} [(1-e^*)f_0(x) + e^* f_1(x)] r^*(x) \mu_R(x) dx$$
$$+ \int_{S_C} [(1-e^*)f_0(x) + e^* f_1(x)](1 - r^*(x))\mu_C dx. \tag{.1}$$

Now consider a 'thought experiment' in which the student always reveal the signals in S_C. This may sound contradictory and may not support an equilibrium. But the imaginary condition still need to satisfy the Bayes' plausibility rule and will thus give us the following constraint

$$\mu^* = \int_{S_C} [(1-e^*)f_0(x) + e^* f_1(x)] \mu_R(x) dx. \tag{.2}$$

Equating (.1) and (.2), we get

$$\mu_C = \frac{\int_{S_C} [(1-e^*)f_0(x) + e^* f_1(x)](1 - r^*(x))\mu_R(x) dx}{\int_{S_C} [(1-e^*)f_0(x) + e^* f_1(x)](1 - r^*(x)) dx}, \tag{.3}$$

which implies that **the concealing belief μ_C is equal to a weighted average of the 'beliefs after revealing' of those who are concealing.**

Since $r^*(s_1) < 1$, $r^*(s_2) < 1$, we have $s_1, s_2 \in S_C$. By $l(s_1) \neq l(s_2)$, we have $\underline{l} \equiv \inf_{s \in S_C} l(s) < \sup_{s \in S_C} l(s) \equiv \bar{l}$.

Notice that, for any $s \in S_C$,

$$\mu_R(s) = 1 - \frac{1-e^*}{(1-e^*) + e^* l(s)} \leq 1 - \frac{1-e^*}{(1-e^*) + e^* \bar{l}}.$$

By $l(s_1) \neq l(s_2)$ and (.3), we have the following strict inequality

$$\mu_C < 1 - \frac{1-e^*}{(1-e^*) + e^* \bar{l}}.$$

This implies that we can find some value $\mu' \in (\mu_C, 1 - \frac{1-e^*}{(1-e^*)+e^*\bar{l}})$. By definition of sup and $\mu_R(s)$ being a continuous function of $l(s)$, there exists $s^* \in S_C$ such that $\mu_R(s^*) \in (\mu', 1 - \frac{1-e^*}{(1-e^*)+e^*\bar{l}})$. Then we have $\mu_R(s^*) > \mu' > \mu_C$. The student will strictly prefer to reveal signal s^*, meaning that $r^*(s^*) = 1$, which contradicts $s^* \in S_C$. This completes the proof for Lemma

2. $\square$

By Lemma 2, we can easily conclude that $l(s) = l(s'), \forall s, s' \in S_C$. We denote this value as $c \equiv l(s), s \in S_C$. Then, for any $s \in S_C$, we have

$$\mu_R(s) = 1 - \frac{1-e^*}{(1-e^*) + e^* l^*(s)} = 1 - \frac{1-e^*}{(1-e^*) + e^* c} = \mu_C. \tag{.4}$$

Substituting into (.3), we have for any $s \in S_C$,

$$\begin{aligned}
\mu_C &= \frac{\int_{S_C} [(1-e^*) f_0(x) + e^* f_1(x)](1 - r^*(x))\mu_R(x) dx}{\int_{S_C} [(1-e^*) f_0(x) + e^* f_1(x)](1 - r^*(x)) dx} \\
&= \frac{\int_{S_C} [(1-e^*) f_0(x) + e^* f_1(x)](1 - r^*(x))[1 - \frac{1-e^*}{(1-e^*)+e^* c}] dx}{\int_{S_C} [(1-e^*) f_0(x) + e^* f_1(x)](1 - r^*(x)) dx} \\
&= 1 - \frac{1-e^*}{(1-e^*) + e^* c}.
\end{aligned}$$

Since S_C is the support of concealing, it remains to show that $c = \inf_s l(s)$. We again prove by contradiction. Suppose not, i.e., $c > \inf_s l(s)$. Then, there exists s^* such that $r^*(s^*) = 1$, $l(s^*) < c$. However,

$$\mu_R(s^*) = 1 - \frac{1-e^*}{(1-e^*) + e^* l^*(s^*)} < 1 - \frac{1-e^*}{(1-e^*) + e^* c} = \mu_C.$$

That is, the student with the signal s^* will have a strict incentive to deviate. Contradicting $r^*(\cdot)$ being an equilibrium. Thus, $c = \inf_s l(s)$. This finishes the proof for the first part of Proposition 1.

The second part of Proposition 1 follows directly from (.4), which is the indifference condition between revealing and concealing. This completes the proof. $\square$

Proof of Proposition 2

We prove Proposition 2 with two lemmas.

Lemma 3. *For any equilibrium strategy profile $r^*(\cdot)$, there exists a constant c (depending on $r^*(\cdot)$) such that*

(i) $r^*(s) = 1$ *if* $l(s) > c$,

(ii) $r^*(s) = 0$ *if* $l(s) < c$*,*

(iii) $r^*(s)$ *can take any value in* $[0, 1]$ *if* $l(s) = c$*.*

Proof of Lemma 3

Consider any equilibrium strategy $r^*(\cdot)$. Since $1 - p > 0$, i.e., there is a positive probability of not receiving a signal, the updated belief of concealing always exists. Using $f(x) = (1 - e^*)f_0(x) + e^* f_1(x)$, we have

$$\mu_C = \frac{pe^* + (1-p)\int f(x)(1-r^*(x))dx \frac{\int e^*(1-r^*(x))f_1(x)dx}{\int (1-e^*)(1-r^*(x))f_0(x)dx + \int e^*(1-r^*(x))f_1(x)dx}}{p + (1-p)\int f(x)(1-r^*(x))dx} \tag{.5}$$

$$= \frac{pe^* + (1-p)\int f(x)(1-r^*(x))\mu_R(x)dx}{p + (1-p)\int f(x)(1-r^*(x))dx}, \tag{.6}$$

where the first equality represents a weighted average among those not receiving a signal and those who conceal, and the second equality follows by Bayes plausibility. **That is, the average belief of those who conceal should be the same as the average belief of the same set of student who reveals.**

For a fixed equilibrium strategy profile $r^*(\cdot)$, μ_C is a constant. Recall that the school will obtain an updated belief if the student reveals a signal s as

$$\mu_R(s) = \frac{e^* f_1(s)}{(1-e^*)f_0(s) + e^* f_1(s)} = 1 - \frac{1-e^*}{(1-e^*) + e^* l(s)}.$$

Then, for a constant μ_C, the student will strictly prefer revealing if and only if

$$\mu_C < \mu_R(s) \iff \mu_C < 1 - \frac{1-e^*}{(1-e^*) + e^* l(s)}$$
$$\iff l(s) > \frac{\mu_C(1-e^*)}{e^*(1-\mu_C)}.$$

The student will strictly prefer concealing if and only if

$$\mu_C > \mu_R(s) \iff l(s) < \frac{\mu_C(1-e^*)}{e^*(1-\mu_C)}.$$

And the student will be indifferent if and only if

$$\mu_C = \mu_R(s) \iff l(s) = \frac{\mu_C(1-e^*)}{e^*(1-\mu_C)}.$$

Therefore, $c = \frac{\mu_C(1-e^*)}{e^*(1-\mu_C)}$ is a desired constant as described in Lemma 3. This finishes the proof. $\square$

With the existence of a threshold established, we next prove its uniqueness.

Lemma 4. *The constant c in Lemma 3 is unique.*

Proof of Lemma 4

We prove the result by contradiction. Suppose not. That is, there exist two distinct values $c_1 \neq c_2$ that correspond to two equilibrium profiles r_1^*, r_2^*. WLOG, let $c_1 < c_2$. We then have $\frac{\mu_{C_1}(1-e^*)}{e^*(1-\mu_{C_1})} < \frac{\mu_{C_2}(1-e^*)}{e^*(1-\mu_{C_2})}$, which further gives $\mu_{C_1} < \mu_{C_2}$ since $\mu_{C_1}, \mu_{C_2} \in (0,1)$.

By Lemma 3, we have

$$\begin{aligned}
r_1^*(s) &= 1, \ \forall s \in \{s : l(s) > c_1\}, \\
r_1^*(s) &= 0, \ \forall s \in \{s : l(s) < c_1\}, \\
r_2^*(s) &= 1, \ \forall s \in \{s : l(s) > c_2\}, \\
r_2^*(s) &= 0, \ \forall s \in \{s : l(s) < c_2\}.
\end{aligned}$$

Since μ_{C_1}, μ_{C_2} are the updated belief of concealing, by definition, we have

$$\begin{aligned}
\mu_{C_1} &= \frac{pe^* + (1-p)\int f(x)(1-r_1^*(x))\mu_R(x)dx}{p + (1-p)\int f(x)(1-r_1^*(x))dx} \\
&= \frac{pe^* + (1-p)\int_{s:l(s)\leq c_1} f(x)(1-r_1^*(x))\mu_R(x)dx}{p + (1-p)\int_{s:l(s)\leq c_1} f(x)(1-r_1^*(x))dx} \\
&= \frac{pe^* + (1-p)\int_{s:l(s)< c_1} f(x)(1-r_1^*(x))\mu_R(x)dx}{p + (1-p)\int_{s:l(s)< c_1} f(x)(1-r_1^*(x))dx} \\
&= \frac{pe^* + (1-p)\int_{s:l(s)< c_1} f(x)\mu_R(x)dx}{p + (1-p)\int_{s:l(s)< c_1} f(x)dx},
\end{aligned} \tag{.7}$$

where the first line follows by (.6), the second line follows by the first half of Lemma 3, the third line follows by $\mu_R(s) = \mu_{C_1}$ when $l(s) = c_1$, and the last line follows by the second half of Lemma 3.

Now consider μ_{C_2}. Similarly, we have

$$\begin{aligned}
\mu_{C_2} &= \frac{pe^* + (1-p)\int f(x)(1-r_2^*(x))\mu_R(x)dx}{p + (1-p)\int f(x)(1-r_2^*(x))dx} \\
&= \frac{pe^* + (1-p)\int_{s:l(s)\leq c_2} f(x)(1-r_2^*(x))\mu_R(x)dx}{p + (1-p)\int_{s:l(s)\leq c_2} f(x)(1-r_2^*(x))dx} \\
&= \frac{pe^* + (1-p)\int_{s:l(s)<c_1} f(x)\mu_R(x)dx + (1-p)\int_{s:l(s)\in[c_1,c_2]} f(x)(1-r_2^*(x))\mu_R(x)dx}{p + (1-p)\int_{s:l(s)<c_1} f(x)dx + (1-p)\int_{s:l(s)\in[c_1,c_2]} f(x)(1-r_2^*(x))dx} \\
&= \frac{[p + (1-p)\int_{s:l(s)<c_1} f(x)dx]\mu_{C_1} + (1-p)\int_{s:l(s)\in[c_1,c_2]} f(x)(1-r_2^*(x))\mu_R(x)dx}{p + (1-p)\int_{s:l(s)<c_1} f(x)dx + (1-p)\int_{s:l(s)\in[c_1,c_2]} f(x)(1-r_2^*(x))dx} \\
&\leq \frac{[p + (1-p)\int_{s:l(s)<c_1} f(x)dx]\mu_{C_1} + [(1-p)\int_{s:l(s)\in[c_1,c_2]} f(x)(1-r_2^*(x))dx]\mu_{C_2}}{p + (1-p)\int_{s:l(s)<c_1} f(x)dx + (1-p)\int_{s:l(s)\in[c_1,c_2]} f(x)(1-r_2^*(x))dx} \\
&< \mu_{C_2},
\end{aligned}$$

where the first line follows by (.6), the second line follows by the first half of Lemma 3, the third line follows by splitting $\{s : l(s) \leq c_2\}$ into $\{s : l(s) < c_1\}$ and $\{s : l(s) \in [c_1, c_2]\}$ and applying the second half of Lemma 3, the fourth line follows by using (.7), the fifth line follows by $\mu_R(s) \leq \mu_{C_2}$ when $l(s) \leq c_2$, and the last line follows by $\mu_{C_1} < \mu_{C_2}$, and the fact that a weighted average of μ_{C_1}, μ_{C_2} is strictly smaller than the larger one, which is μ_{C_2} in this case. This gives a contradiction, which finishes the proof. $\square$

Proof of Proposition 2

Combining Lemma 3 and Lemma 4, we have shown that if there exists equilibria, they share the same unique threshold of likelihood ratio l^*, such that the student follows a cutoff strategy on his realized $l(s)$. It remains to show the existence of such an equilibrium.

To show existence, consider the two following functions:

$$\begin{aligned}
h_1(l) &= 1 - \frac{1-e^*}{(1-e^*) + e^* l}, \\
h_2(l) &= \frac{pe^* + (1-p)\int_{s:l(s)<l} f(x)[1 - \frac{1-e^*}{(1-e^*)+e^* l}]dx}{p + (1-p)\int_{s:l(s)<l} f(x)dx}.
\end{aligned}$$

where $h_2(\underline{l}) = e^* > h_1(\underline{l})$, $h_2(\bar{l}) \leq e^* \leq h_1(\bar{l})$. Since both h_1, h_2 are continuous functions w.r.t. l, intermediate value theorem gives the existence of l^*, which is the desired threshold in Proposition 2, Lemma 4 gives the uniqueness, completing the proof. □

Proof of Proposition 3

To find the value of l^*, we require the following indifference condition

$$\begin{aligned}\mu_C &= \frac{(1-p)e^* + p\int_{s:l(s)<l^*} e^* f_1(s)ds}{(1-p) + p\int_{s:l(s)<l^*} e^* f_1(s) + (1-e^*)f_0(s)ds} \\ &= \frac{e^* l^*}{(1-e^*) + e^* l^*}\end{aligned}$$

where the first line is the school's belief over who conceal signals, the second line is the updated belief if a student reveal a signal with likelihood l^*. The existence of such signal is guaranteed by the continuity of f_θ.

On both sides of the equation, we take away the numerator from the denominator, which gives

$$\begin{aligned}\frac{(1-p)e^* + p\int_{s:l(s)<l^*} e^* f_1(s)ds}{(1-p)(1-e^*) + p\int_{s:l(s)<l^*}(1-e^*)f_0(s)ds} &= \frac{e^* l^*}{(1-e^*)} \\ \Rightarrow \frac{(1-p) + p\int_{s:l(s)<l^*} f_1(s)ds}{(1-p) + p\int_{s:l(s)<l^*} f_0(s)ds} &= l^*\end{aligned}$$

where the second line is cancelling out $\frac{e^*}{1-e^*}$, which finishes the proof. □

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