Small-Scale Helicopter Modeling and Adaptive Control

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In this report, the small-scale helicopter Maxi Joker 3 is explored via LQR, MRAC, and L1 adaptive control. The first step is to identify the dynamics and aerodynamics of the helicopter, and then build the complete nonlinear model in Simulink. The major objective of this independent study is to control the autonomous helicopter. LQR is used at first to stabilize the system and achieve path-following. Given that the moment of inertia is not accessible without measurement instruments or test beds, the feasibility of using adaptive control law is explored. MRAC is employed to control the helicopter with system uncertainties. Then results of L1 adaptive control and MRAC are compared with high gain values, time delay and in the presence of measurement noise and wind gusts.

I. Introduction

The use of autonomous unmanned aerial vehicles (UAV) have been proved its success in both military and civil applications. An autonomous helicopter has more maneuverability than that of an autonomous airplane, as a helicopter can land and take off in limited space, unprepared sites, have broad envelope of flight ranging from hovering to cruising, and fly at low altitude [1] [2]. Developing an autonomous helicopter is highly challenging, for the system is inherently unstable, high order, coupled and its dynamics is described in complex mathematical nonlinear model.

It's difficult for pilot to fly a helicopter and continuous attention is required for the whole flight [2]. There are four control inputs for pilot operate: longitudinal and lateral cyclic for horizontal motion maneuver, and collective input for vertical motion maneuver, and pedal input for yaw motion maneuver. Specifically, for the small-scale helicopter—Maxi Joker 3, control bar take the responsibility of horizontal motion control, pitch angle of main rotor blades decide the vertical performance, and the tail rotor introduces a yaw to the system as well as compensates the yawing rate created by the main rotor. The response of each control input could excite a secondary effect in addition to the primary response, and thus a corresponding compensation for the secondary response is needed to prevent a helicopter from out of order. Furthermore, helicopters are sensitive to atmospheric disturbances, and thus wind gusts may lead to sudden change of velocity and attitude. This has been tested in my research and good responses from adaptive controllers can be expected.

This report is organized as follows. Section Ⅱ discuss the fundamentals of helicopters. Dynamics modeling and simulation is in section Ⅲ. Section Ⅳgives theoretical proofs and examples of LQG, MRAC and L1 adaptive control. The last section is the conclusion.

II. RC helicopter overview

The helicopter is considered to be a rigid body with 6 degrees of freedom (6 DOF). For this study, I use body-fixed frame to derive equations of motion. And the center of gravity (COG) is set as the origin of this body-fixed frame. The translatory velocities are denoted as u,v and w, respectively. The rotary velocities p,q and r stand for roll, pitch and yaw motion. And the Euler angles are denoted as ϕ, θ and ψ.

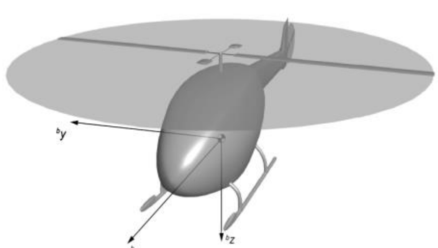

Fig. 1. *Body-fixed frame in XYZ direction* [1]

Actuator input	Rotary movement	Resulting translatory movement
Lateral	Roll (p)	Lateral (v)
Longitudinal	Pitch (q)	Longitudinal (u)
Collective	-	Heave (w)
Pedal	Yaw (r)	-

Table. 1. *Control inputs and resulting responses* [1]

The resulting rotary and translatory movement are caused by the main rotor, the control rotor and the tail rotor. The total pitch angle of the blades gives the corresponding thrust force perpendicular to the tip path plane (TPP), and the orientation of the TPP decides direction of the thrust vector. As a result, the propulsion and lift forces are the horizontal and vertical component of the thrust vector, respectively.

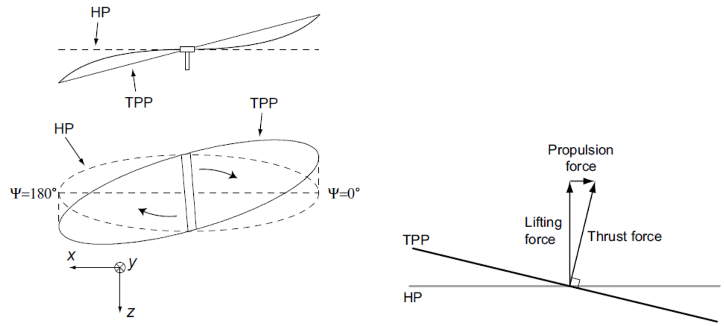

Fig. 2. *Illustration of thrust vector, TPP, and forces* [1]

With body-fixed frame, it is easy to explain the four control inputs mentioned in section Ⅰ. I use u_{lat} , u_{long} , u_{col} , and u_{ped} to describe the these inputs. The rotation about the *x* axis (roll) can be introduced by u_{lat} . The rotation about the *y* axis (pitch) is primarily caused by longitudinal input u_{long} . u_{col} controls the total pitch angle of blades, and by increasing the total pitch angle, the helicopter gets more thrust. Therefore the input vector is defined as

$$u = [u_{lat}, u_{long}, u_{col}, u_{ped}]^T \qquad (1)$$

III. Dynamics Modeling and Simulation

In this section, I am going to implement the mathematical model of Maxi Joker 3 in Simulink and test the response of nonlinear system at the hovering operation point. Then a linear helicopter model is deduced using small-angle approximation and operating point linearization.

The modeling of helicopter consists of three boxes: flapping and thrust equations, force and torque equations and rigid body equations. And it is presented as following:

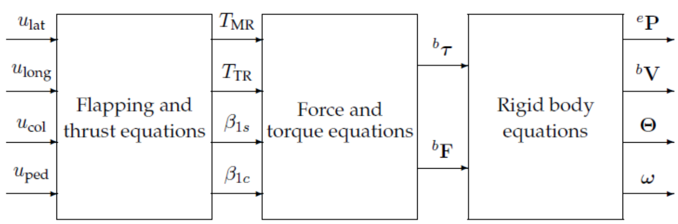

Fig. 3. *Modelling boxes* [1]

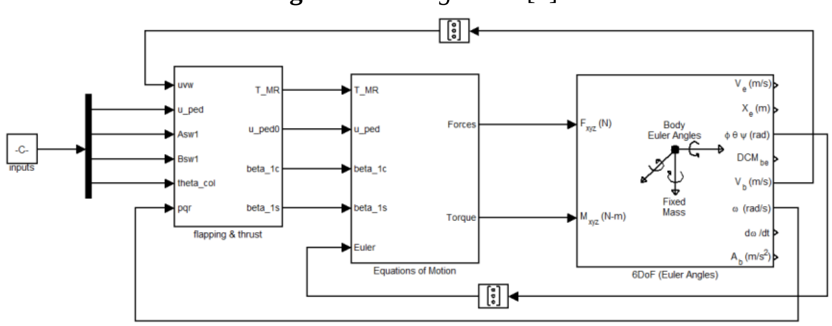

Fig. 4. *Modelling Toolbox in Simulink*

The nonlinear model of Maxi Joker 3 can be described by the following equations. The translatory accelerations are

$$^b\dot{\mathbf{V}} = \begin{bmatrix} ^b\dot{u} \\ ^b\dot{v} \\ ^b\dot{w} \end{bmatrix} = \begin{bmatrix} \frac{^bf_x}{m} + {^bv} \cdot r - {^bw} \cdot q \\ \frac{^bf_y}{m} - {^bu} \cdot r + {^bw} \cdot p \\ \frac{^bf_z}{m} + {^bu} \cdot q - {^bv} \cdot p \end{bmatrix} \quad (2)$$

The Euler rates are expressed as

$$\dot{\mathbf{\Theta}} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p + \sin(\phi) \cdot \tan(\theta) \cdot q + \cos(\phi) \cdot \tan(\theta) \cdot r \\ \cos(\phi) \cdot q - \sin(\phi) \cdot r \\ \frac{\sin(\phi)}{\cos(\theta)} \cdot q + \frac{\cos(\phi)}{\cos(\theta)} \cdot r \end{bmatrix} \quad (3)$$

The angular acceleration of the body frame is presented as

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{(I_{yy}-I_{zz})\cdot q \cdot r + L}{I_{xx}} \\ -\frac{(I_{xx}-I_{zz})\cdot p \cdot r - M}{I_{yy}} \\ \frac{(I_{xx}-I_{yy})\cdot p \cdot q + N}{I_{zz}} \end{bmatrix} \quad (4)$$

The moment of inertia is expressed as

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (5)$$

The expressions for forces and torques are given below

$$^b\mathbf{F} = \begin{bmatrix} ^bf_x \\ ^bf_y \\ ^bf_z \end{bmatrix} = \begin{bmatrix} -T_{\mathrm{MR}} \cdot \sin(\beta_{1c}) - \sin(\theta) \cdot m \cdot g \\ T_{\mathrm{MR}} \cdot \sin(\beta_{1s}) + T_{\mathrm{TR}} + \sin(\phi) \cdot \cos(\theta) \cdot m \cdot g \\ -T_{\mathrm{MR}} \cdot \cos(\beta_{1s}) \cdot \cos(\beta_{1c}) + \cos(\phi) \cdot \cos(\theta) \cdot m \cdot g \end{bmatrix} \quad (6)$$

$$^b\boldsymbol{\tau} = \begin{bmatrix} ^bL \\ ^bM \\ ^bN \end{bmatrix} = \begin{bmatrix} ^bf_{y,\mathrm{MR}} \cdot h_m - {^bf_z} \cdot y_m + {^bf_{y,\mathrm{TR}}} \cdot h_t + Q_{\mathrm{MR}} \cdot \sin(\beta_{1c}) \\ -{^bf_x} \cdot h_m - {^bf_z} \cdot l_m - Q_{\mathrm{MR}} \cdot \sin(\beta_{1s}) \\ ^bf_x \cdot y_m + {^bf_{y,\mathrm{MR}}} \cdot l_m - {^bf_{y,\mathrm{TR}}} \cdot l_t + Q_{\mathrm{MR}} \cdot \cos(\beta_{1c}) \cdot \cos(\beta_{1s}) \end{bmatrix} \quad (7)$$

And

$$Q_{MR} = -\left(A_{Q,MR} \cdot T_{MR}^{1.5} + B_{Q,MR}\right) \quad (8)$$

$$T_{MR} = -(w_b - v_i) \cdot \frac{\rho \cdot \Omega \cdot R^2 \cdot a \cdot B \cdot c}{4} \quad (9)$$

$$T_{TR} = \frac{^bf_{y,MR} \cdot l_m + {^bf_{x,MR}} \cdot y_m - Q_{MR} \cdot \cos(\beta_{1c}) \cdot \cos(\beta_{1s})}{l_t} + u_{ped} \quad (10)$$

Given default inputs $u = [u_{lat}, u_{long}, u_{col}, u_{ped}]^T = [0, 0, 0.1, 3.0713]^T$, where u_{col} is used to balance gravity and u_{ped} to compensate the torque of main rotor, its translatory velocities of hovering point are in Fig. 5. It is clear that the nonlinear system is inherently unstable and thus the translaory velocities cannot stay at equilibrium point.

To stabilize the helicopter system and control it to follow certain commands, I am going to linearize the system. However it is not reasonable to use the linearization at the hovering point to test all flight conditions. As a matter of fact, in hover the helicopter is free to move in any of three dimensions, while in cruise, the helicopter behaves like a fixed-wing airplane [2]. Hence, to test different flight conditions, I linearize the system at hover and cruise operating points. The state space equation can be written as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (11)$$

where

$$A=\begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & X_a & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & Y_b \\ 0 & 0 & Z_w & 0 & 0 & Z_r & 0 & 0 & 0 & Z_a & Z_b \\ L_u & L_v & L_w & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_b \\ M_u & M_v & M_w & 0 & 0 & 0 & 0 & 0 & 0 & M_a & 0 \\ 0 & N_v & N_w & N_p & 0 & N_r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_f & A_b/\tau_f \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & B_a/\tau_f & -1/\tau_f \end{bmatrix} \qquad (12)$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{ped} & 0 \\ 0 & 0 & 0 & Z_{col} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{col} \\ 0 & 0 & N_{ped} & N_{col} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_{lat}/\tau_f & A_{lon}/\tau_f & 0 & 0 \\ B_{lat}/\tau_f & B_{lon}/\tau_f & 0 & 0 \end{bmatrix} \qquad (13)$$

$$C=I_{11} \qquad (14)$$

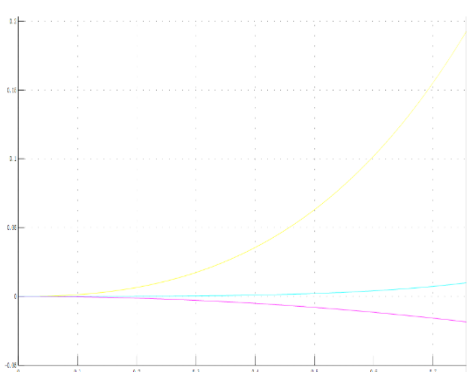

Fig. 5. *Translatory velocities of nonlinear model at hover point*

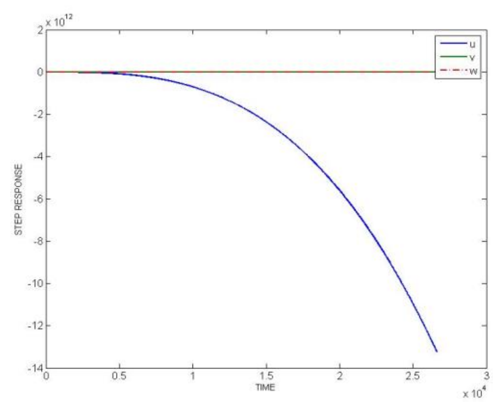

Fig. 6. *Translatory velocities of linear model at hover point*

The parameters in matrix *A* and *B* depend on the flight mode—hover or cruise. And at this point, it is assumed that there is no uncertainty in the system. The step response of linearized helicopter system at hover point is plotted in Fig. 6. As we can see, the linearized system is unstable when there is no control effort added to it.

IV. Controller

1. Linear Quadratic Regulator

In this section, it is first assumed that the all system information is given and I design a Linear Quadratic Regulator (LQR) to stabilize the system and optimized control efforts. The cost function is defined as

$$J = x^T(t_f)Hx(t_f) + \int_0^{t_f} \left[x^T(t)Qx(t) + u^T(t)Ru(t) \right] dt \qquad (15)$$

with the physical illustration: it is optimized to stay close to the origin with minimum control efforts. The optimal solution to LQR problem is in the form of optimal gain K_r, by solving the following equation

$$\dot{K}_r = K_r A - A^T K_r - Q + K_r B R^{-1} B^T K_r \qquad (16)$$

And the optimal control input is

$$u^*(t) = -R^{-1}B^T K_r x(t) = -K_{LQR}x(t) \qquad (17)$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (18)$$

$$R = \begin{bmatrix} 11 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (19)$$

To check whether LQR controller is able to keep the system stable, disturbance scenario needs to be designed. There are several disturbances in real flight test, such as wind gusts, measurement noise and offset initial state values. Here I am going to test LQR control in the presence of offset initial state values. First, given the initial condition

$$x = \begin{bmatrix} u & v & w & p & q & r & \phi & \theta & \varphi & \beta_{1c} & \beta_{1s} \end{bmatrix}^T = \begin{bmatrix} 3 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

the result in Fig. 7 shows that LQR control has satisfactory performance with this kind of disturbance and all translator velocities are regulated to zero.

However, we may not always have good knowledge of the system. In fact, it is not easy to obtain the moment of inertia without measurement instruments or test beds. In such case, LQR can do nothing about the problem. Then, I am going to introduce adaptive control law to systems with uncertain parameters.

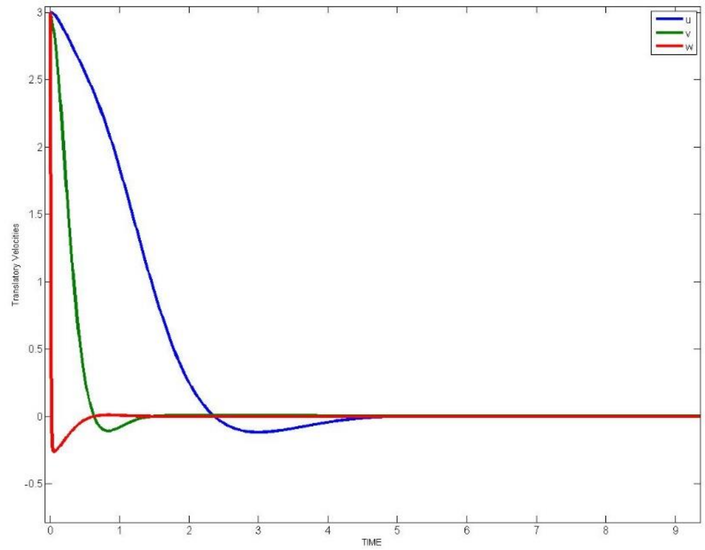

Fig. 7. *Translatory velocities response of LQR control*

2. Model Reference Adaptive Control (MRAC)

The system has known structure with unknown parameters and passive identifier based reparameterization of direct MRAC is employed to deal with such problem. Lack of information about moment of inertia, the helicopter system can be expressed as

$$\begin{cases} \dot{x}(t) = \left(A_{kn} + A_{unkn}\right)x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \qquad (20)$$

where A_{kn}, B are known, and matrix A_{unkn} contains all unknown parameters [5]. I use LQR as baseline control and then the system can be written as

$$\begin{aligned} \dot{x}(t) &= \left(A_{kn} - BK_{LQR}\right)x(t) + A_{unkn}x(t) + Bu_{ad}(t) \\ &= A_m x(t) + \theta x(t) + Bu_{ad}(t) \end{aligned} \qquad (21)$$

where $A_m = A_{kn} - BK_{LQR}$, $\theta = A_{unkn}$, and K_{LQR} is the optimal gain via LQR. We consider the following passive identifier

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + \hat{\theta}x(t) + Bu_{ad}(t) \qquad (22)$$

where $\hat{x}$ is the state of the passive identifier, also known as state predictor. By subtracting (21) from (22), we obtain prediction error dynamics

$$\dot{\tilde{x}}(t) = A_m\tilde{x}(t) + \tilde{\theta}(t)x(t) \qquad (23)$$

with $\tilde{x}(0) = 0,\ \tilde{x}(t) = \hat{x}(t) - x(t),\ \tilde{\theta}(t) = \hat{\theta}(t) - \theta$. Choosing the Lyapunov function candidate

$$V\left(\tilde{x}(t), \tilde{\theta}(t)\right) = \tilde{x}^T(t)\,\mathrm{P}\,\tilde{x}(t) + \frac{1}{\gamma}\tilde{\theta}^T(t)\tilde{\theta}(t) \qquad (24)$$

where P solves $A_m^T P + PA_m = -Q_m$ and Q_m is positive definite.

$$\begin{aligned} \dot{V}\left(\tilde{x}(t), \tilde{\theta}(t)\right) &= \tilde{x}^T(t)\left(A_m^T P + PA_m\right)\tilde{x}(t) + \\ &= \left(\tilde{x}^T(t)P\tilde{\theta}(t)x(t) + x^T(t)\tilde{\theta}^T(t)P\tilde{x}(t)\right) + \frac{2}{\gamma}\tilde{\theta}^T(t)\dot{\tilde{\theta}}(t) \end{aligned} \qquad (25)$$

By selecting $\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) = -\gamma P \tilde{x}(t) x^T(t)$, we obtain

$$\dot{V}\left(\tilde{x}(t), \tilde{\theta}(t)\right) = -\tilde{x}^T Q_m \tilde{x}(t) \le 0$$

We close the loop by letting

$$Bu(t) = -\hat{\theta}(t)x(t) + k_g r \qquad (26)$$

where $k_g = -CA_m^{-1} \backslash I_{11}$, and r is the reference signal. And thus control input can be given by $u(t) = B \backslash \left(-\hat{\theta}(t)x(t) + k_g r\right)$. Then we can use Barbalat's lemma to conclude that the tracking error converges to zero asymptotically.

To better illustrate passive identifier based reparameterization of direct MRAC, a system structure of Simulink is presented as follow

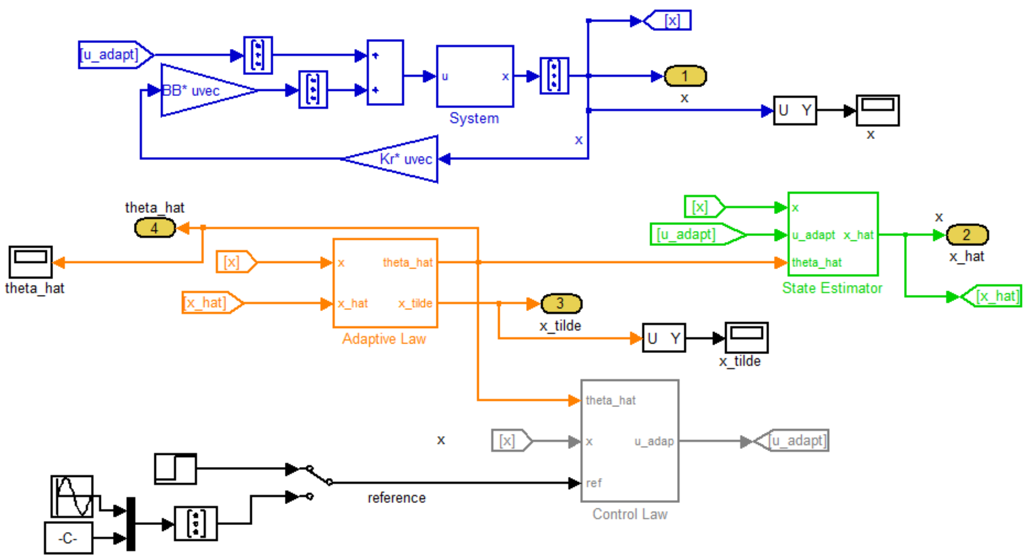

Fig. 8. *Helicopter system with MRAC*

It has a clear structure: the original system is optimized by LQR gain (Kr), the adaptive law generates the estimation of $\hat{\theta}(t)$ and send it to state estimator as well as control law. The control law gives adaptive control signal (27) and feed it back to estimator and the system. Hence, the system would follow the desired states.

Here three examples are given to show the stability and robustness of the system with MRAC. In the first test, a periodical impulse input is generated to simulate wind gusts on the helicopter. It is assumed that the helicopter is in hovering state and thus all state are initialized at the origin. And wind gusts give impulse to the system. The adaptive gain is set to 10000. As we can observe in Fig. 9 that the red dash line denotes the impulses. Translatory velocities are excited by impulses yet soon move back the origin and stay there. Besides, from the chart we can observe that the *w* component is more sensitive to wind gusts at hovering, while *u* and *v* components get less excitement in the case of impulses. In Fig. 10, All control inputs experience abrupt change when coming across impulses, but still within the reasonable range. Hence, with MRAC and LQR, the system is asymptotically stable in the presence of wind gusts.

Now we take a look at step response. It is assumed that the helicopter is in cruising state and a command would be given to ask the helicopter to jump to another cruising state from the initial one. The initial state is $x_0 = [15\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, and the command asks for $x_{tf} = [10\ {-5}\ 2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$. And the adaptive gain is 10000. In Fig. 11, it shows that MRAC is good for step response. The settling time for *w* component is much less than those of *u* and *v* component. But I notice that there is more or less high frequency oscillation at the beginning of green line—*w* component, which may leads to failure of actuators. In fact, as we can see in Fig. 12, such step response require high frequency control of actuators, which definitely is undesirable.

Furthermore, when time delay presents, how does MRAC behave? Again, I use the same condition of the former example but with time-delay in control signal. Just as you can see in Fig. 13, with adaptive gain 10000 and time delay 0.00001, the system with MRAC has crashed. Therefore, we have to look for more robust controller in which more time-delay margin is guaranteed.

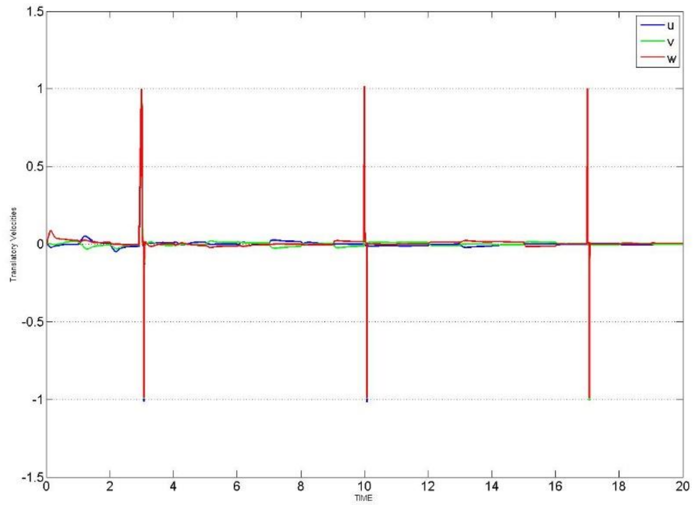

Fig. 9. *Wind gusts disturbance on the helicopter in hovering by MRAC*

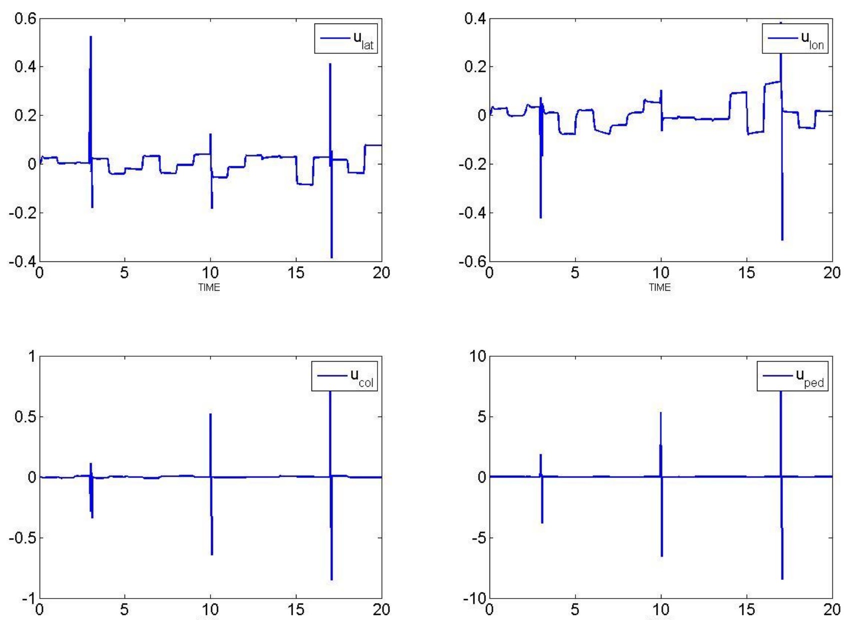

Fig. 10. *Control inputs with wind gusts disturbance by MRAC*

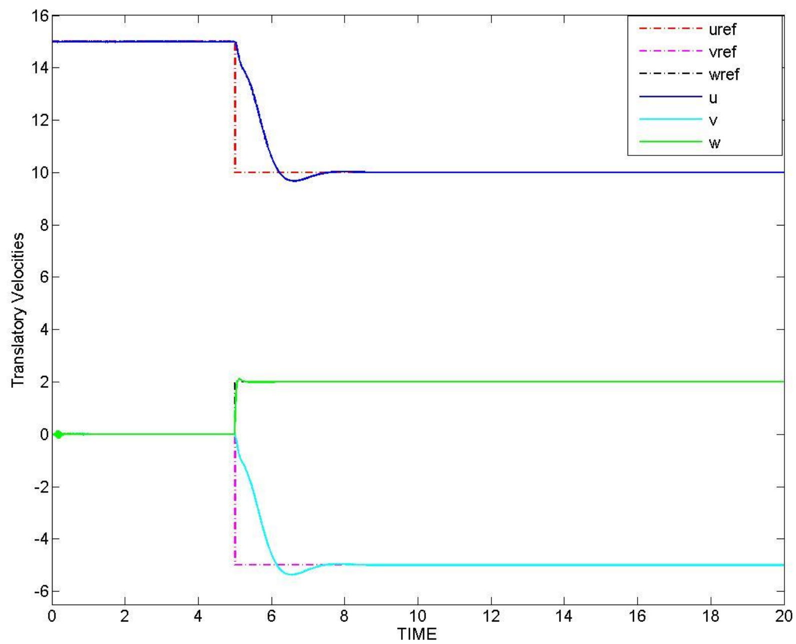

Fig. 11. *Step response of the helicopter by MRAC*

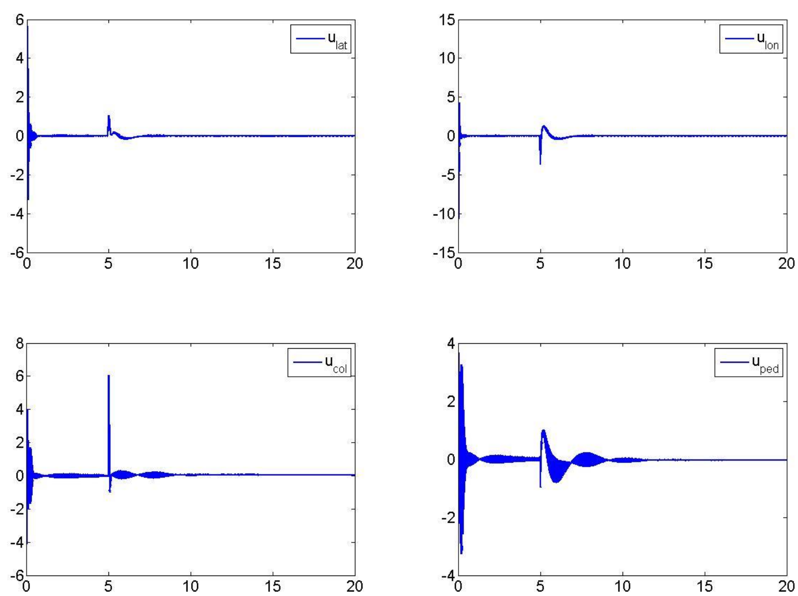

Fig. 12. *Control inputs in step response by MRAC*

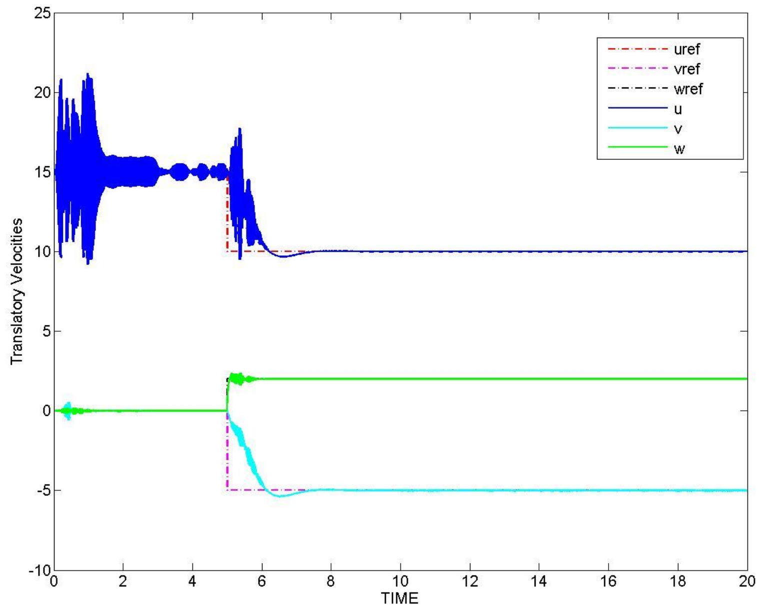

Fig. 11. *Step response of the helicopter with time delay by MRAC*

3. L1 Adaptive Control [5]

Now we consider L1 adaptive controller with the control signal

$$u(t) = C(s)\left(B \backslash \left(-\hat{\theta}(t)x(t) + k_g r\right)\right) \qquad (28)$$

where $C(s) = \dfrac{\omega}{s+\omega}$

Consider the Lyapunov function candidate as before

$$V\left(\tilde{x}(t), \tilde{\theta}(t)\right) = \tilde{x}^T(t)\,\mathrm{P}\,\tilde{x}(t) + \frac{1}{\gamma}\tilde{\theta}^T(t)\tilde{\theta}(t) \qquad (29)$$

And we have proved that by selecting

$$\dot{\tilde{\theta}}(t) = \dot{\hat{\theta}}(t) = \gamma \mathrm{Proj}\left(x(t)\tilde{x}^T(t)\,\mathrm{Pb}, \hat{\theta}(t)\right) \qquad (30)$$

we obtain

$$\dot{V}\left(\tilde{x}(t), \tilde{\theta}(t)\right) = -\tilde{x}^T Q_m \tilde{x}(t) \le 0 \qquad (31)$$

which is independent of control input, whether it is filtered or not [5]. Before applying Barbalat's lemma, we have to show the boundedness of the state of estimator first. In fact, given Theorem 17.4 in ME 562 lecture, with adaptation law (30) and

$$\left\| C(sI - A_m)^{-1} B\left(C(s) - 1\right)\right\|_{L_1} \theta_{\max} < 1 \qquad (32)$$

the tracking error $\tilde{x}(t) \to 0$ asymptotically.

To compare the robustness of MRAC and L1 adaptive control, we use the same step response to L1 case. It is assumed that the helicopter is in cruising state and a command would be given to ask the helicopter to jump to another cruising sate from the initial one. The initial state is

$x_0 = [15\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, and the command asks for $x_{tf} = [10\ -5\ 2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$.

I still set the adaptive gain as 10000. Furthermore the bandwidth of low-pass filter C(s) is 30 and time delay is as large as 0.01. As we can observe in Fig. 12, it has quite clean response with L1 adaptive control, even if the time delay is 1000 times larger than that has been applied to MRAC. Also, actuators response lies in a reasonable range. This obviously demonstrates the advantage of L1 adaptive control since it has far larger time-delay margin than MRAC does. Hence L1 is more robust and we can employ higher adaptive gain without system crashing.

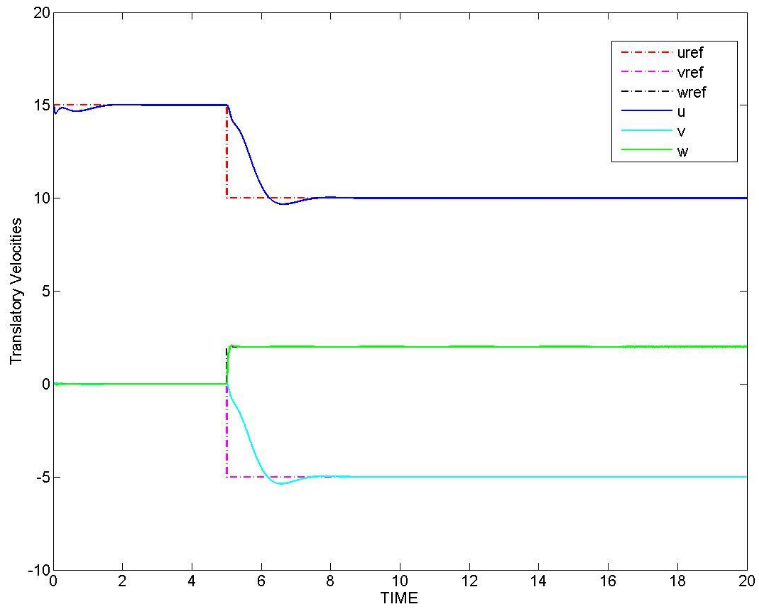

Fig. 12. *Step response of the helicopter with time delay by L1*

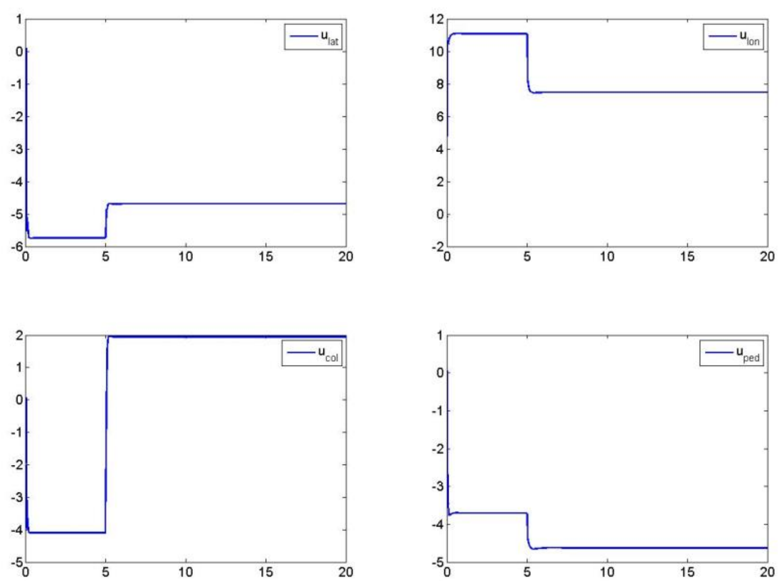

Fig. 13. *Control inputs in step response of the helicopter with time delay by L1*

Moreover, we take a look at path following of L1 adaptive control with time delay, wind gusts and measurement noise, with respect to cruise mode. Specifically, I let the guidance trajectory follow a spiral and set time delay as 0.01, periodical wind gusts impulse every 7 seconds with magnitude of 1m/s in all three directions of translatory velocities, and Gaussian noise with mean value 0 and magnitude of 0.1. For the choice of low-pass filter C(s), it is a trade-off between stability and capability of filtering high frequency signals. One-order and three-order filters have been tested. It turns out that $C(s) = \left(3\omega^2 s + \omega^3\right) / \left(s + \omega\right)^3$ with $\omega = 10$ has the best performance. As we can see in Fig. 14

and Fig. 15, despite of all those unexpected disturbances and time delay, the helicopter perfectly follows the reference path with L1 adaptive control, with good actuators response, showing fast adaptation and great robustness.

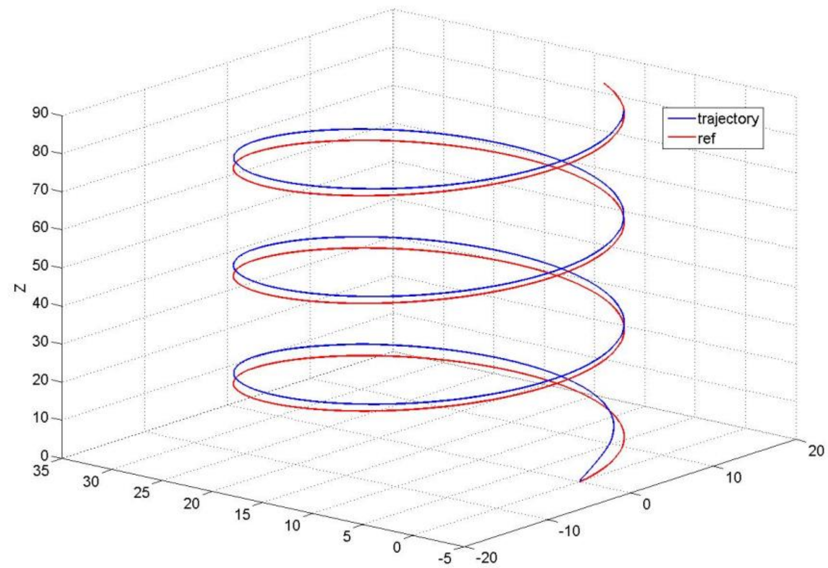

Fig. 14. *Path following of the helicopter by L1*

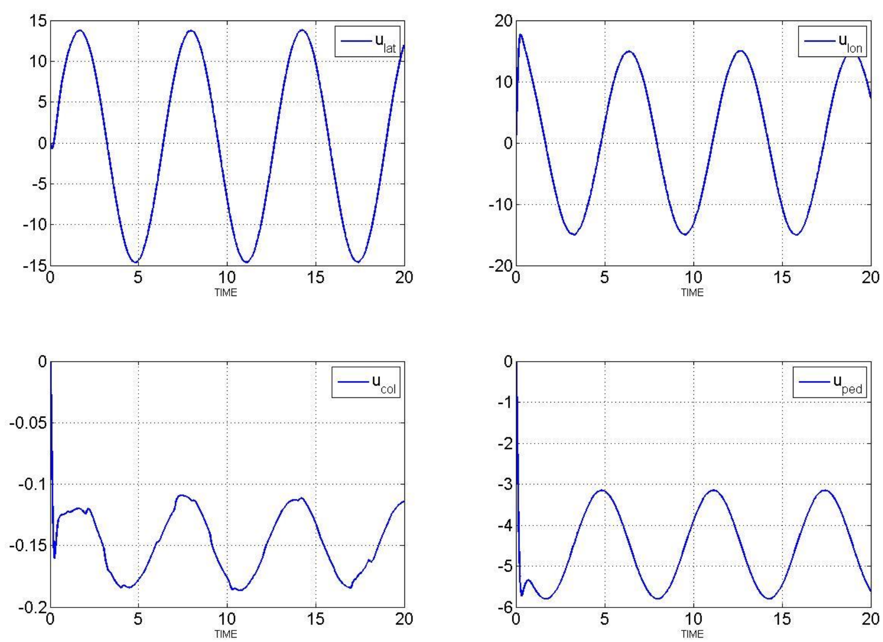

Fig. 15. *Control inputs in path following of the helicopter by L1*

V. Conclusion

In this report, I build the nonlinear model of Maxi Joker 3 and implement it in Simulink. The nonlinear model is coupled and inherently unstable. Therefore the system is linearized at hover and cruise operating points. I first employ

LQG to stabilize the helicopter system and optimize control effort. However, it is not easy to get all information of the helicopter system. Lack of data about the moment of inertia, I use MRAC and L1 adaptive control to estimate these system uncertainties and achieve stability and transient performance. MRAC and L1 adaptive control are compared in the presence of time delay, and L1 has time-delay margin 1000 times larger than that MRAC does. Given extreme situation, including time delay, measurement noise and periodical wind gusts, the helicopter with L1 adaptive controller is able to stay stable and follow the desired path with excellent transient performance.

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