

# 标准 DH 表

i	d <sub>i</sub>	a <sub>i</sub>	α <sub>i</sub>
1	d <sub>1</sub>	-9.55	90°
2	0	-104	0
3	0	-96.4	0
4	0	0	90°
5	80.66	0	-90°
6	27.5	0	0

$${}_{i-1}^i T = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & \alpha_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & \alpha_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} R^T & -R^T \cdot P \\ 0 & 1 \end{bmatrix}$$

$${}^0T = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & a_1 c\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & a_1 s\theta_1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 c\theta_2 \\ s\theta_3 & c\theta_3 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3T = \begin{bmatrix} c\theta_4 & 0 & s\theta_4 & 0 \\ s\theta_4 & 0 & -c\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4T = \begin{bmatrix} c\theta_5 & 0 & -s\theta_5 & 0 \\ s\theta_5 & 0 & c\theta_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^5T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3T = {}^1T {}^2T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_3 c_{23} + a_2 c_2 \\ s_{23} & c_{23} & 0 & a_3 s_{23} + a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4T = {}^1T {}^3T = \begin{bmatrix} c_{23}c_4 - s_{23}s_4 & 0 & c_{23}s_4 + s_{23}c_4 & a_3c_{23} + a_2c_2 \\ s_{23}c_4 + c_{23}s_4 & 0 & s_{23}s_4 - c_{23}c_4 & a_3s_{23} + a_2s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-\begin{bmatrix} c_1 & s_1 & 0 \\ 0 & 0 & 1 \\ s_1 & -c_1 & 0 \end{bmatrix} \begin{bmatrix} a_4c_1 \\ a_5s_1 \\ d_1 \end{bmatrix} \quad \begin{bmatrix} a_3c_{23} + a_2c_2 \\ a_3s_{23} + a_2s_2 \\ 0 \\ 1 \end{bmatrix}$$

$${}^1_5 T = {}^1_4 T {}^4_5 T = \begin{bmatrix} (C_{23}C_4 - S_{23}S_4)C_5 & -C_{23}S_4 - S_{23}C_4 & (-C_{23}C_4 + S_{23}S_4)S_5 \\ (S_{23}C_4 + C_{23}S_4)C_5 & -S_{23}S_4 + C_{23}C_4 & (-S_{23}C_4 - C_{23}S_4)S_5 \\ S_5 & 0 & C_5 \\ 0 & 0 & 0 \end{bmatrix}$$

$${}^0_1 T^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & -d_1 \\ 0 & 0 & 1 & -d_1 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^5_6 T^{-1} = \begin{bmatrix} c_6 & s_6 & 0 & 0 \\ -s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & -d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{全 } T = \begin{bmatrix} n_x & 0_x & \alpha_x & p_x \\ n_y & 0_y & \alpha_y & p_y \\ n_z & 0_z & \alpha_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^0_1 T^{-1} {}^1_5 T {}^5_6 T^{-1} = {}^0_5 T$$

$${}^0_1 T^{-1} {}^1_5 T {}^5_6 T^{-1} = \begin{bmatrix} c_1 n_x + s_1 n_y & c_1 0_x + s_1 0_y & c_1 \alpha_x + s_1 \alpha_y & c_1 p_x + s_1 p_y - d_1 \\ n_z & 0_z & \alpha_z & p_z - d_1 \\ s_1 n_x - c_1 n_y & s_1 0_x - c_1 0_y & s_1 \alpha_x - c_1 \alpha_y & s_1 p_x - c_1 p_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (c_1 n_x + s_1 n_y) c_6 - (c_1 0_x + s_1 0_y) s_6 & (c_1 n_x + s_1 n_y) s_6 + (c_1 0_x + s_1 0_y) c_6 & c_1 \alpha_x + s_1 \alpha_y & (-d_6(c_1 \alpha_x + s_1 \alpha_y) \\ & & & (+c_1 p_x + s_1 p_y - d_1)) \\ n_z c_6 - 0_z s_6 & n_z s_6 + 0_z c_6 & \alpha_z & -d_6 \alpha_z + p_z - d_1 \\ (s_1 n_x - c_1 n_y) c_6 - (s_1 0_x - c_1 0_y) s_6 & (s_1 n_x - c_1 n_y) s_6 + (s_1 0_x - c_1 0_y) c_6 & s_1 \alpha_x - c_1 \alpha_y & (-d_6(s_1 \alpha_x - c_1 \alpha_y) \\ & & & (+s_1 p_x - c_1 p_y)) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^T S = \begin{bmatrix} (C_{23}C_4 - S_{23}S_4)C_5 & -C_{23}S_4 - S_{23}C_4 & (-C_{23}C_4 + S_{23}S_4)S_5 & C_3C_{23} + C_3C_2 \\ (S_{23}C_4 + C_{23}S_4)C_5 & -S_{23}S_4 + C_{23}C_4 & (-S_{23}C_4 - C_{23}S_4)S_5 & C_3S_{23} + C_3S_2 \\ S_5 & 0 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-d_6(S_1\alpha_x - C_1\alpha_y) + S_1p_x - C_1p_y = 0$$

$$d_6(\tan_1\alpha_x - \alpha_y) = \tan_1p_x - p_y$$

$$\tan_1 = \frac{\alpha_y d_6 - p_y}{d_6 \alpha_x - p_x} \Rightarrow \underline{\underline{\theta_1}}$$

$$S_1\alpha_x - C_1\alpha_y = C_5 \Rightarrow \underline{\underline{\theta_5}}$$

$$(S_1n_x - C_1n_y)S_6 + (S_1O_x - C_1O_y)C_6 = 0 \Rightarrow \underline{\underline{\theta_6}}$$

$${}^4T = {}^1T {}^3T {}^4T = \begin{bmatrix} C_{23}C_4 - S_{23}S_4 & 0 & C_{23}S_4 + S_{23}C_4 & \alpha_3C_3 + \alpha_2C_2 \\ S_{23}C_4 + C_{23}S_4 & 0 & S_{23}S_4 - C_{23}C_4 & \alpha_3S_{23} + \alpha_2S_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T^{-1}T {}^5T {}^6T^{-1} {}^6T^{-1} = {}^1T \quad {}^4T^{-1} = \begin{bmatrix} C_5 & S_5 & 0 & 0 \\ 0 & 0 & -1 & d_5 \\ -S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{d_5((C_1n_x + S_1n_y)S_6 + (C_1O_x + S_1O_y)C_6)}_m - d_6(C_1a_x + S_1a_y) + C_1p_x + S_1p_y - \alpha_1 = \alpha_3C_{23} + \alpha_2C_2$$

$$\underbrace{d_5(n_zS_6 + O_zS_6)}_n - d_6\alpha_z + p_z - \alpha_1 = \alpha_3S_{23} + \alpha_2S_2$$

$$\theta_3 = \pm \arccos \frac{m^2 + n^2 - \alpha_2^2 - \alpha_3^2}{2\alpha_2\alpha_3}$$

$$\theta_2 = \arccos \frac{n(\alpha_2 + \alpha_3C_3) - m\alpha_3S_3}{m(\alpha_2 + \alpha_3C_2) + n\alpha_3S_2}$$

$$\theta_2 + \theta_3 + \theta_4 = \arccos \frac{(n_zC_6 - O_zS_6)C_5 - \alpha_2S_5}{[(C_1n_x + S_1n_y)C_6 - (C_1O_x + S_1O_y)S_6]C_5 - (C_1a_x + S_1a_y)S_5}$$