IB Mathematic Application and Approaches HL

Internal assignment

The simulation of human electrocardiograph (ECG) using Fourier Series and its applications

Table of contents

Introduction

Electrocardiogram (ECG) is a development that uses an electrocardiograph to record the adjustments in the electrical exercises of the heart during each cardiovascular cycle. The ECG is perhaps the most typically used clinical evaluations. It can record the of the human body's normal heart and help break down the information regarding heart sicknesses, for instance, arrhythmia or myocardial ischemia. An away from of the ECG outline of patients can save millions. Utilizing MATLAB and Fourier Series to reenact the human ECG signals with variable clinical boundaries can help specialists better comprehend the heart states of a patient basically from the viewpoint of the ECG graph. This article will zero in on utilizing MATLAB and existed algorisms to recreate the human ECG charts and closes with a fundamental organization of the ECG graph with real clinical parameters from some patients. This exploration has a profound root with my own life, not just due to my inclinations and abilities in MATLAB coding on PC, yet in addition as a result of its significance and importance in my family. My mother was analyzed as mild extrasystole because of hereditary reason in the family. My father, as a smoker, additionally causes harms on his lung and heart. With such foundation and climate, they captivated me with the investigation of heart and the numerical translation of human ECG.

Theoretical foundation

Fourier Series

In mathematics, there is a concept of periodic function. The periodic function is the general phrase to describe a specific type of functions which achieve the same value during the same interval. After the first introduction of the periodic function, Jean-Baptiste Joseph Fourier, a mathematician, and physicist, first introduced the idea of translating and transforming any periodic function with basic trigonometry functions. In his essay, Mémoire sur la propagation de la chaleur dans les corps solides (Treatise on the propagation of heat in solid bodies), he attempts to solve the heat conduction in metal plate. Specifically, using the concept of infinite series expansion composed of sine and cosine functions. This is the first attempts of using Fourier series. According to the further study, if a Fourier Series does exist in this function, its coefficients are

$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)dx \quad a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$, $b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$. The series can be expressed as $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$. (Serov.11)

Human electrocardiograph (ECG)

An electrocardiogram is a visual representation of potential changes recorded by an electrocardiograph from a particular piece of the body by the bioelectric flow created when the heart does work. In 1856, Klick and Miller first straightforwardly recorded the current produced by the heartbeat on the heart inside the body. In 1887, Waller found that this current could likewise be recorded on the outside of the body. In 1903, Eintofin, a Dutch physiologist, used a string galvanometer, unexpectedly, normalizing this type of recording. This technique was immediately applied to the conclusion of clinical coronary illness after resulting technological enhancements. The electrocardiogram of an ordinary individual has 5 waveforms, which are P, Q, R, S, and T waves. P wave addresses atrial depolarization, QRS complex wave addresses ventricular depolarization, T wave addresses ventricular repolarization, and P-Q stretch (from the beginning of P wave to the beginning of QRS complex) addresses the fervor between the heart's atrioventricular Conduction time. U waveform addresses possible ventricular repolarization, which is very rare. (Gacek et al. 47)

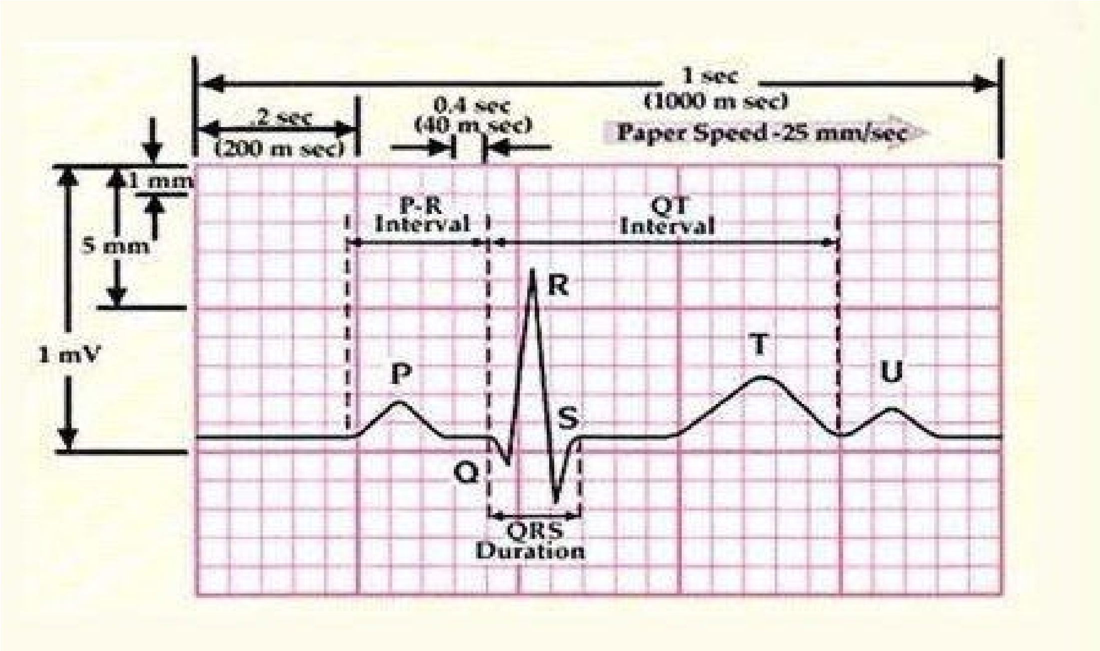


Figure 1: the sample human ECG graph of 1 heartbeat

Theoretical model of ECG simulation

From a practical and clinical perspective, the ECG signal isn't periodical. Human heart is an organ, not a motor. The mechanism is spontaneous. In any case, for arithmetic and the reason for this exploration, the ECG signal is assumed not to be spontaneous and periodic. To utilize Fourier series to address ECG signal, its design should be breaking down utilizing polynomials. The p and T waves look round with maxima, which can be drawn with trigonometric function. The QRS waveform resembles a triangle. The intervals between the waveforms can be ignored since they have flat shape with no slope. The Fourier series in this scenario only needs to predict the PTU and QRS waveforms. The intervals can be simulated by setting up different intervals between waveforms.

PTU waveform

The round and circular waveforms of P, T and U are very similar to quadratic function with its coefficient and maximum. However, waves of P, T and U cannot be simulated accurately using quadratic function. This is because quadratic function continues its function below the x-axis, which causes trouble when finalizing the overall waveform of a complete human ECG. By observing the waveform of P, T and U again, they have a very round shape, with a maximum point on the top, hence, it is very similar to basic trigonometry function such as $\sin(x)$ or $\cos(x)$. In this situation, it is very important to notice that the calculation involves integration. When the function is approaches with $\cos(x)$, its integration will not have a negative sign, which is easy to work with. Its sides are smooth and round instead of a straight line. By applying this thought in the graph, the approximate shape of the PTU waveform should look like the following diagram:

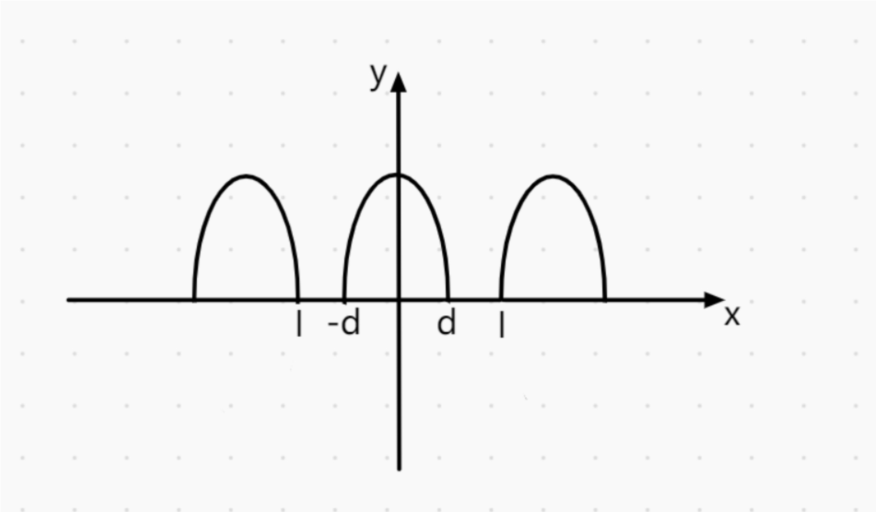


Figure 2: the general idea of PTU waveform

Making the function precise by determine the period of this function, which is $\frac{\pi}{2d}$. Hence, the function of this waveform is $f(x)=\cos\left(\frac{\pi}{2d}x\right)$. Now, using the founded theoretical model of Fourier series, this function can be approximated with precision.

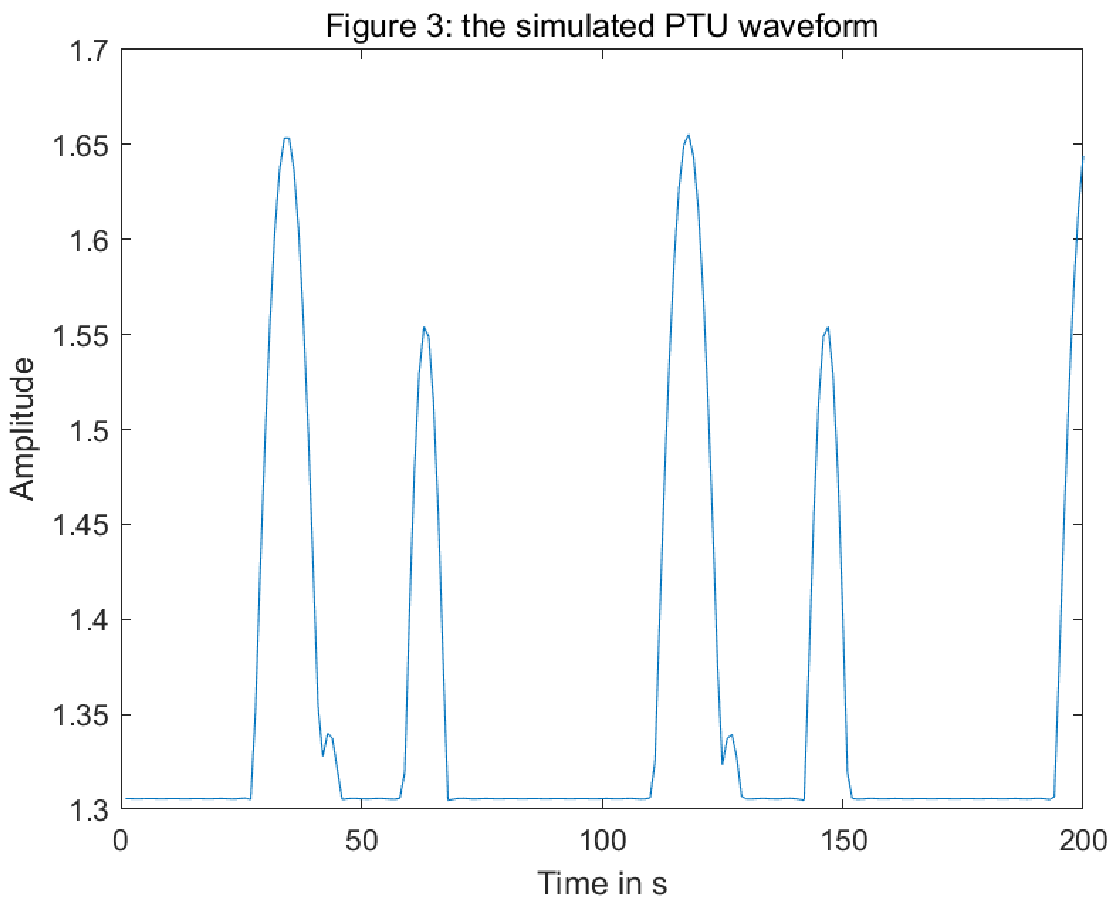
$$a_0=\frac{1}{l}\int_{-d}^{0}\cos\left(\frac{\pi}{2d}x\right)dx+\frac{1}{l}\int_{0}^{d}\cos\left(\frac{\pi}{2d}x\right)dx=\frac{2d}{\pi l}+\frac{2d}{\pi l}=\frac{4d}{\pi l}$$

$$a_n=\frac{1}{l}\int_{-d}^{0}\cos\left(\frac{\pi}{2d}x\right)\cos\left(\frac{n\pi x}{l}\right)dx+\frac{1}{l}\int_{0}^{d}\cos\left(\frac{\pi}{2d}x\right)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$=-\frac{2dl\cos\left(\frac{\pi dn}{l}\right)}{\pi\left(4d^2n^2-l^2\right)}-\frac{2dl\cos\left(\frac{\pi dn}{l}\right)}{\pi\left(4d^2n^2-l^2\right)}=-2\frac{2dl\cos\left(\frac{\pi dn}{l}\right)}{\pi\left(4d^2n^2-l^2\right)}$$

$$b_n=\frac{1}{l}\int_{-d}^{0}\cos\left(\frac{\pi}{2d}x\right)\sin\left(\frac{n\pi x}{l}\right)dx+\frac{1}{l}\int_{0}^{d}\cos\left(\frac{\pi}{2d}x\right)\sin\left(\frac{n\pi x}{l}\right)dx=0$$

$$f(x)=\frac{\frac{4d}{\pi l}}{2}+\sum_{n=1}^{\infty}-2\frac{2dl\cos\left(\frac{\pi dn}{l}\right)}{\pi\left(4d^2n^2-l^2\right)}$$



QRS waveform

The triangle shape function QRS is composed by two different linear equations with different slopes and shifts on the t-axis. But they share an elementary function $f(x) = x$. However, by closely examine the QRS waveform, the slope of the wave is clearly smaller than 1, and has discontinuity over different sections. Otherwise, the function will not have a correct frequency which differs from the required frequency as the heartbeat. Hence, the overall waveform of the QRS function should have a portion where the slope is zero, and appears as “individual triangle” on the x-axis. By applying this thought in the graph, the approximate shape of the QRS waveform should looks like the following diagram:

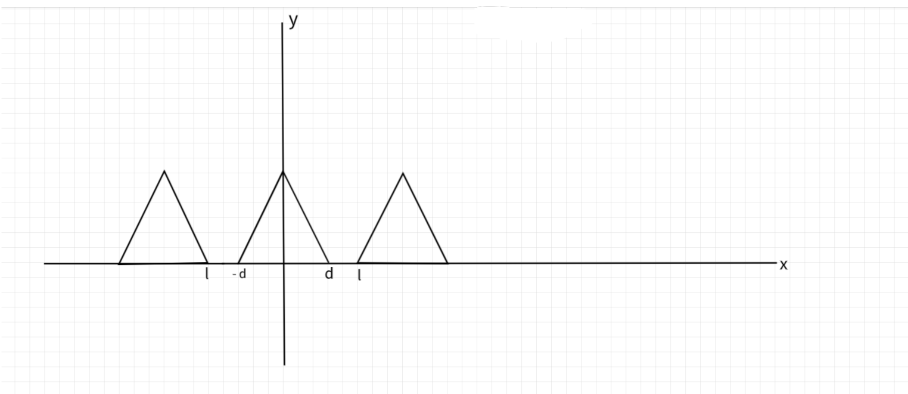


Figure 4: the general idea of QRS waveform

This graph presents the general idea of the QRS waveform. There are portions between triangles with zero slope, and the half period l. The graph has denotation of d and $-d$ to show the end points of one triangle shape. In one triangle shape, let's say that the maximum point is a, the slope of this wave is $\frac{a}{d}$, with a constant term of y-intercept of the maximum point a. Hence, this total waveform can be written as: $f(x)=\begin{cases} a+\frac{a}{d}x, -d\le x\le 0 \\ a-\frac{a}{d}x, 0\le x\le d \end{cases}$, this system of linear equations represents first, the ascending part of the QRS waveform QR, and then, the descending part, RS. Now, using the founded theoretical model of Fourier series, this function can be approximated with precision.

$$a_0=\frac{1}{l}\int_{-d}^{d} f(x)dx=\frac{1}{l}\int_{-d}^{0}\left(a+\frac{a}{d}x\right)dx+\frac{1}{l}\int_{0}^{d}\left(a-\frac{a}{d}x\right)dx=\frac{ad}{2l}+\frac{ad}{2l}=\frac{2ad}{2l}=\frac{ad}{l}$$

$$a_n=\frac{1}{l}\int_{-d}^{d} f(x)\cos\left(\frac{n\pi x}{l}\right)dx=\frac{1}{l}\int_{-d}^{0}\left(a+\frac{a}{d}x\right)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$+\frac{1}{l}\int_{0}^{d}\left(a-\frac{a}{d}x\right)\cos\left(\frac{n\pi x}{l}\right)dx=-\frac{al\left[\cos\left(\frac{\pi dn}{l}\right)-1\right]}{\pi^2 dn^2}-\frac{al\left[\cos\left(\frac{\pi dn}{l}\right)-1\right]}{\pi^2 dn^2}$$

$$=-2\frac{al\left[\cos\left(\frac{\pi dn}{l}\right)-1\right]}{\pi^2 dn^2}$$

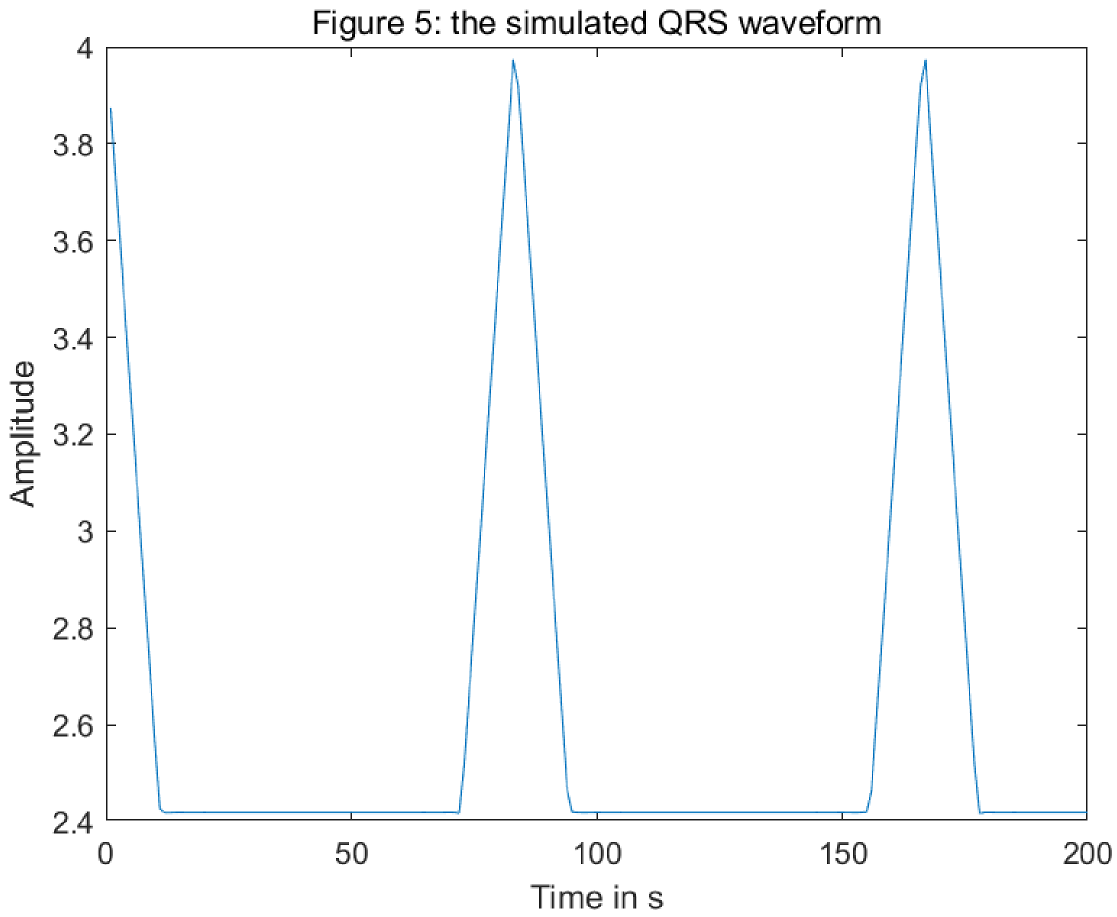
$$b_n=\frac{1}{l}\int_{-d}^{d} f(x)\sin\left(\frac{n\pi x}{l}\right)dx=\frac{1}{l}\int_{-d}^{0}\left(a+\frac{a}{d}x\right)\sin\left(\frac{n\pi x}{l}\right)dx+\frac{1}{l}\int_{0}^{d}\left(a-\frac{a}{d}x\right)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$=\frac{a\left[l\sin\left(\frac{\pi dn}{l}\right)-\pi dn\right]}{\pi^2 dn^2}-\frac{a\left[l\sin\left(\frac{\pi dn}{l}\right)-\pi dn\right]}{\pi^2 dn^2}=0$$

$$f(x)=\frac{\frac{ad}{l}}{2}+\sum_{n=1}^{\infty}-2\frac{al\left[\cos\left(\frac{\pi dn}{l}\right)-1\right]}{\pi^2 dn^2}\cos\left(\frac{n\pi x}{l}\right)=\frac{ad}{2l}$$

$$+\sum_{n=1}^{\infty}-2\frac{al\left[\cos\left(\frac{\pi dn}{l}\right)-1\right]}{\pi^2 dn^2}\cos\left(\frac{n\pi x}{l}\right)$$

The constant $l=\frac{30\text{seconds}}{\text{number of heartbeats}}$, which is the half period. The constant $d=\text{the duration in seconds}$, and $a=\text{Amplitude}$. This equation not only give us the correct waveform of QRS, but also customization properties of the waveform according to the physical condition of the heart. Hence, the purpose of according simulation is fulfilled.



Simulation of ECG

Since all the sections have been studied individually, the complete ECG signal is simply the assembly of PTU and QRS section:

$$f_{PTU}(x) = \frac{\frac{4d}{\pi l}}{2} + \sum_{n=1}^{\infty} -2\frac{2dl\cos\left(\frac{\pi dn}{l}\right)}{\pi\left(4d^2n^2 - l^2\right)}$$

$$f_{QRS}(x) = \frac{ad}{2l} + \sum_{n=1}^{\infty} -2\frac{al\left[\cos\left(\frac{\pi dn}{l}\right) - 1\right]}{\pi^2 dn^2}\cos\left(\frac{n\pi x}{l}\right)$$

In those two equations, a means the amplitude of the waves and d is the duration. An important feature must also be introduced in this essay, which is t, the interval of each waves. Just as I mentioned previously, there are intervals between waves with zero slope. They are practically a straight line with a finite length on the x axis. In order to simulate the general shape of the ECG and each section correctly. An idea of interval is important. Essentially, an ECG starts with the P wave, QRS later comes. In between those two waves, there is an interval. With the known duration of P wave, the end of the P wave can be deduced correctly. The duration of the QRS waveform is also available. Hence, the correct way to simulate the interval is to leave the parts between the end point of P wave and starting of QRS wave blank. Using this idea, the ECG can be simulated with greater accuracy. Using the online sources, the ranges of clinical values of the ECG details can be obtained:

	Clinical parameters	
	Duration d in seconds	Amplitude a in v
P wave	$0 \le d \le 0.11$	$0 \le a \le 0.3$
Q wave	$0 \le d \le 0.2$	$0 \le a \le 0.4$
QRS waveform	$0.08 \le d \le 0.1$	$0 \le a \le 0.5$
S wave	$0.12 \le d \le 0.2$	$0 \le a \le 0.7$
T wave	$0.1 \le d \le 0.25$	$0 \le a \le 0.8$
U wave	$0.025 \le d \le 0.0625$	$0.1 \le a \le 0.33$

(Gacek et al. 21)

Simulation of my personal's ECG

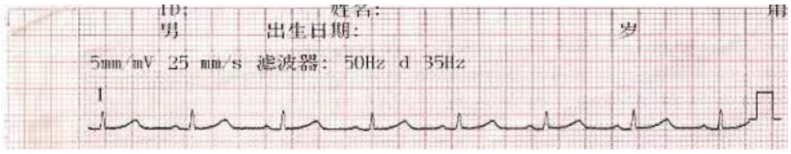


Figure 6: The ECG graph of me during 2019

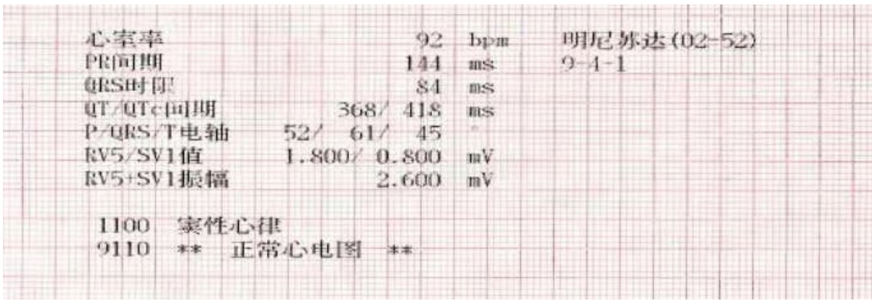
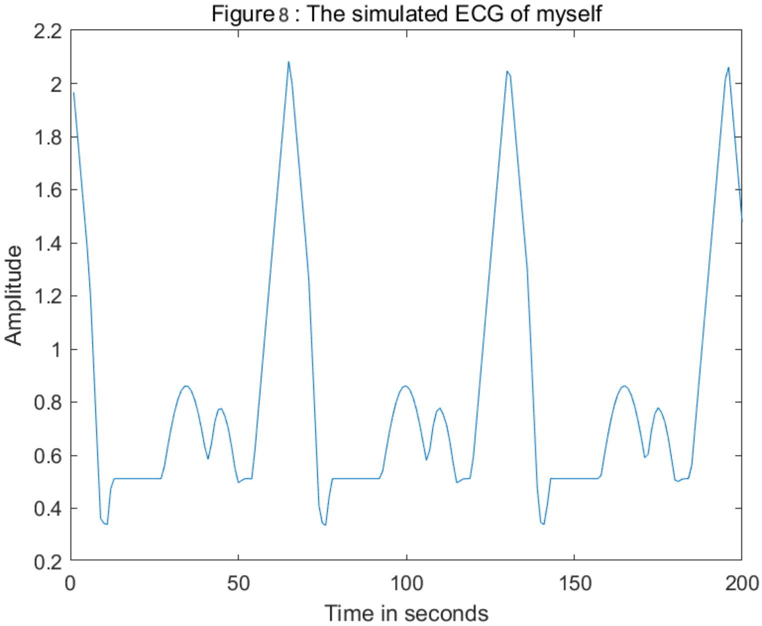


Figure 7: The ECG graph details

In this ECG, the P duration is about 144 ms, the QRS duration is about 84 ms, the T duration is about 368 ms. The amplitudes of P are 52mv, QRS is 61mv, and P is 45mv. Using the algorithm designed previously, the simulated ECG has the following outlook:

Figure 8 : The simulated ECG of myself



This simulated ECG shows some similarities with the actual ECG. There is still difference between the P to QRS sections, where the actual ECG shows a overlap, but the simulated ECG shows distinguishable intervals. Hence, this program has limitations in simulating my ECG. It is largely because of my constant changing heartbeat during the actual measurement, while the program only allows the entry of heartbeat counts in a minute.

Simulation of my mother' s ECG

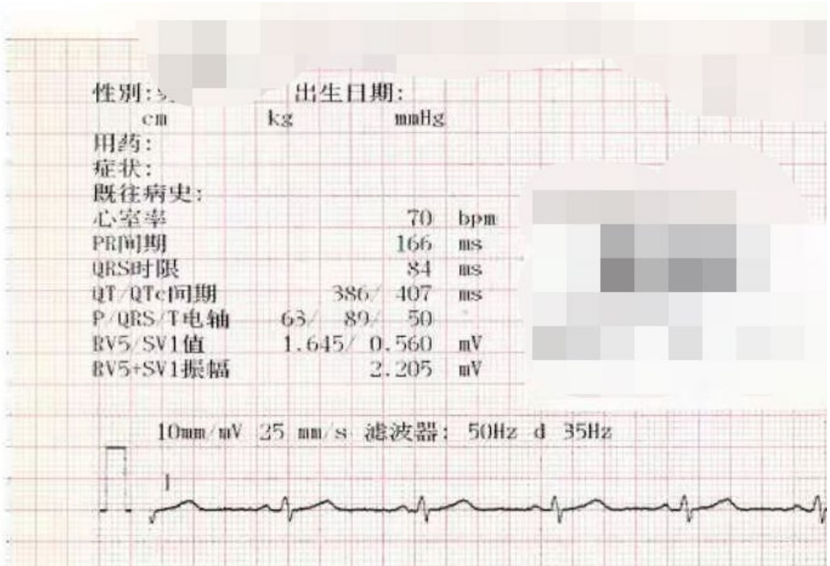
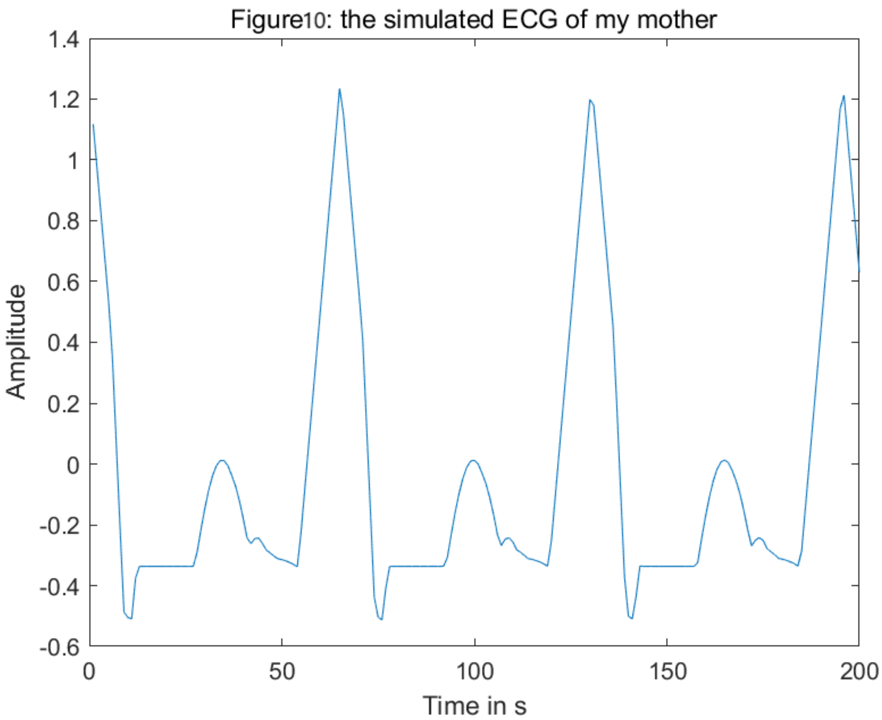


Figure 9: The Actual ECG and ECG details of my mother

In this ECG, the P duration is about 166 ms, the QRS duration is about 84 ms, the T duration is about 386 ms. The amplitudes of P are 63mv, QRS is 89mv, and P is 50mv. Using the algorithm designed previously, the simulated ECG has the following outlook:



This simulated ECG shows some similarities with the actual ECG. There is still difference between the T to U section, where the actual ECG shows a smoother line, but the simulated ECG shows distinguishable small rises. Hence, this program has limitations in simulating my ECG. Those differences suggest a defect in the program in which physical uncertainties (instrumental errors, small heartbeat disturbances).

Conclusion

By using Fourier series to simulate the interval and ECG waveform, a simulation program that can be customized based on clinical data can be successfully designed. By getting the heart rate for one minute, half the frequency can be obtained. Secondly, input all the obtained clinical ECG data into the program, and a simulated ECG waveform can be obtained. However, this waveform is not very accurate. Compared with the actual waveform, the simulated waveform is very mechanized, and many small details cannot be displayed by the simulated waveform. All in all, although this paper successfully designed the simulation program, it is still different from the actual clinical ECG waveform. This program is not suitable for medical purposes, and its accuracy still needs to be improved.

Works cited

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Appendix:

Coding details can be viewed in this website: https://yualex1234.wixsite.com/mysite