Convolutional Neural Networks

ARISE 2021: ECE Machine Learning Lab

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Outline

Extending Logistic Regression

- Motivation: Feature engineering in the model
  - Removes need for domain knowledge
  - Domain knowledge often doesn't exist: ex. object recognition
- Logistic Regression Model: $\hat{y} = \sigma(W\mathrm{x} + b)$
- Replace x with $\mathrm{z} = f(W\mathrm{x} + b)$: $\hat{y} = \sigma(W\mathrm{z} + b)$
- So, $\hat{y} = \sigma(W_2 f(W_1\mathrm{x} + b_1) + b_2)$
- **Reminder**: all linear transforms can be represented as matrix multiplication
- We use non-linear function as f to give us a more expressive model
  - Recall polynomial transformations and exponential transformations of the data
  - These cannot be expressed as matrix multiplication

Extension to Neural Network

- Restrict $f(\mathrm{x})$ to non-linear function applied to all input values
  - Simplest example of a **Neural Network**
- $\hat{y} = \sigma(W_2 f_1(W_1\mathrm{x} + \mathrm{b}_1) + b_2)$
- We can optimize for both W_1, b_1 and $W_2, b_2 2$ model-parameters
  - $\nabla J = [\frac{\partial J}{\partial w_1}, \frac{\partial J}{\partial w_2}, ..., \frac{\partial J}{\partial w_P}]^T$
  - Now we're *learning* the feature engineering
- But why stop here?...

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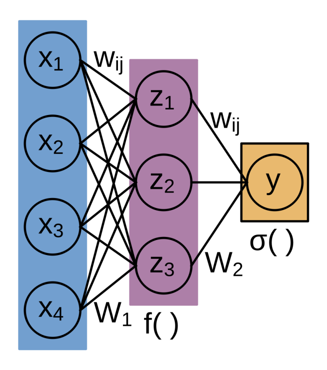

Mathematical Model: Multi-Layer Perceptron

- Model:

$$\hat{y} = f_{out}(W_{out}z_L + b_{out})$$

  - Where, $z_l = f_l(W_l z_{l-1} + b_l)$ for $1 \leq l \leq L$, $z_0 x$, and L is the number of hidden layers
  - ie. all hidden layers are non-linear activation of linear transform
  - f_{out} depends on type of ML problem: (regression: linear, classification: sigmoid/soft-max)
    - **Regression**: Linear Output
    - **Binary Classification**: Sigmoid Output
    - **Multi-Class Classification**: Soft-max Output

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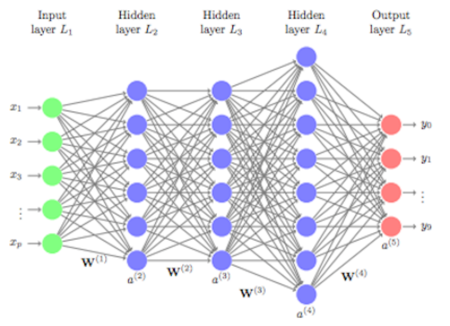
Layers

- **Input**: feature vector, x
- **Output**: target vector, ŷ
  - linear/logistic regression
- **Hidden**: intermediate vectors, z or a
  - feature extraction



Common Activation Functions

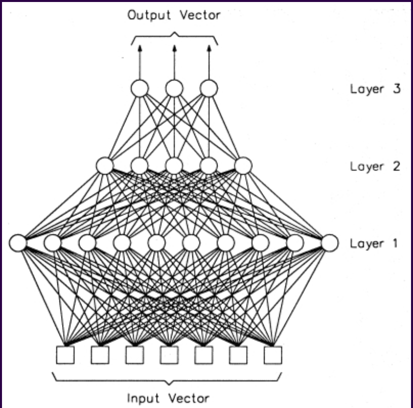
- Sigmoid: $\sigma(z) = \frac{1}{1+e^{-z}}$
  - $\sigma(z) \in (0, 1)$
- Tanh (hyperbolic tangent): $tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$
  - $tanh(z) \in [-1, 1]$
- ReLu (Rectified Linear Unit): $relu(z) = max(0, z)$
  - easy to compute, performs well in practice

Guidelines for Designing a NN

- The design space for NN is HUGE
- Hyper-parameters so far:
  - L: # of layers
  - N_L: # hidden units per layer
  - f: activation function for each layer
  - bs: batch-size
  - lr: learning-rate
  - # of epochs
  - λ: weight-regularization constant
  - J: cost/loss function
- This can be overwhelming...

Guidelines for Designing a NN

- **Start Small**: 1 or 2 layers
  - # hidden units $\sim$ 128
  - make sure code is working
  - increase size if val good
  - classification acc $\geq$ guessing
- **One activation function**
  - for all hidden layers
- **Simple MLP Arch**:
  - Pyramid
  - Expand, combine & reduce



Outline

Better performance with images

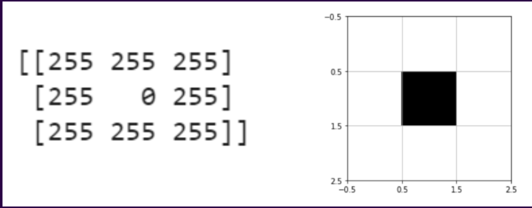
- Encoding locality
- How does an MLP see an image?
- Is this how we see images?

Examples: Lena & Mandrill

Outline

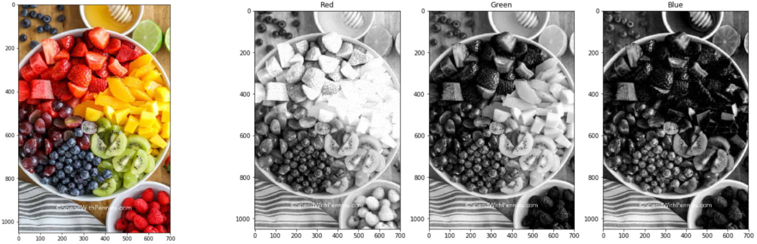
Images in Computer

- Images are stored as arrays of quantized numbers in computers
- Gray scale image: 2D matrices with each entry specifying the intensity (brightness) of a pixel
  - Pixel values range from 0 to 255, 0 being the darkest, 255 being the brightest



Color Images

- Color image: 3D array, 2 dimensions for space, 1 dimension for color
  - Can be thought of as three 2D matrices stacked together into a cube, each 2D matrix specify the amount of each color: Red ,Green ,Blue value at each pixel



- Shape of this image: (1050,700,3)
- There are 1050x700 pixels, 3 channels: R,G,B

Outline

Limitations of Fully Connected Network

- In Fashion MNIST, we used a fully connected network, in which each neuron in the hidden layer is connected to all 28x28 $= 784$ pixels
- Higher definition images often contain millions of pixels $\rightarrow$ It is not practical to use fully connected network
- Fully connected network treat each individual pixel as a feature, it does not utilize the positional relationship between pixels

Convolution

- Introducing a new operation: Convolution

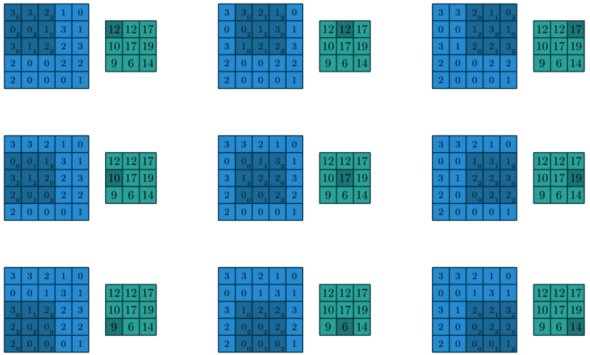
- An operation on an image(matrix) X with a kernel W
- $Z = X \circledast W$

- Equation:

At each offset (j_1, j_2) compute:

$$Z[j_1, j_2] = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} W[k_1, k_2] X[j_1 + k_1, j_2 + k_2]$$

Example of a Convolultion

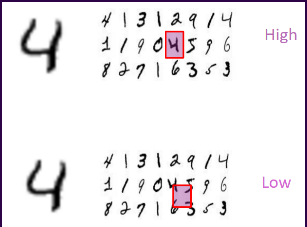


Kernel

$$W = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Why Convolution?

- With convolution, each output pixel depends on only the neighboring pixels in the input
- This allows us to learn the positional relationship between pixels
- Use of different kernels allows us to detect features



Convolution for Multiple Channels

- A kernel for each channel. Could be same kernel, or different
- Perform a convolution for each of the channel, with the respective kernel
- Sum the results

Outline

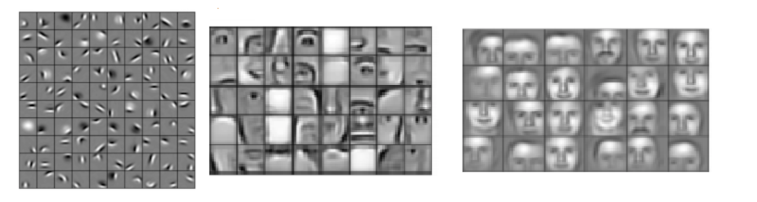
Averaging Kernels

- Uniform Kernel: $\frac{1}{K_x K_y}\begin{bmatrix}1 & .. & 1\\1 & .. & 1\\1 & .. & 1\end{bmatrix}$

  K_x = Number of Columns
  K_y = Number of Rows
- Gaussian Kernel is a blurring kernel too.

Edge Detection

- Initial layers in a deep neural networks detect small patterns like lines, curves or edges.
- Subsequent layers combine these local features to create more complex features.



Edge Detection

- Using Sobel filters:
  - Vertical Edge Detection $G_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$
  - Horizontal Edge Detection $G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Thank You!

- Next Class: Deep Learning and Applications of CNNs