$$\hat{y} = \beta_0 + \beta_1 x \quad \Rightarrow \sum_{i=1}^{N} \beta_0 = \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} \beta_1 x_i$$

$$\Rightarrow \quad \beta_0 = \frac{1}{N}\sum_{i=1}^{N} y_i - \beta_1 \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} (\beta_1 x_i + \beta_0 - y_i)^2$$

To minimize MSE, $\frac{\partial MSE}{\partial \beta_0} = 0$

$$\Rightarrow \frac{\partial MSE}{\partial \beta_0} = \frac{2}{\cancel{N}}\sum_{i=1}^{N} (\beta_1 x_i + \beta_0 - y_i) = 0$$

$$\sum_{i=1}^{N} (\beta_1 x_i + \bar{y} - \beta_1 \bar{x} - y_i) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} \left[(x_i - \bar{x})\beta_1 + (\bar{y} - y_i)\right] = 0$$

$$\Rightarrow \quad \beta_1 \sum_{i=1}^{N} (x_i - \bar{x}) = \sum_{i=1}^{N} (y_i - \bar{y})$$

$$\beta_1 = \frac{\sum_{i=1}^{N} (y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})}$$

ile $\rho \frac{\sigma_y}{\sigma_x} = \frac{\sigma_{xy}}{\sigma_x \cancel{\sigma_y}} \frac{\cancel{\sigma_y}}{\sigma_x} = \frac{\sigma_{xy}}{(\sigma_x)^2} = \frac{\frac{1}{\cancel{N}}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{\cancel{N}}\sum_{i=1}^{N}(x_i - \bar{x})^2}$

$$= \beta_1 \; !$$