Tariff Wars and Endogenous Growth

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Abstract

This paper develops a novel theoretical framework to examine the contentious question of whether imposing tariffs can revive a country's declining manufacturing sector in the presence of existing trade imbalances. Building on models of endogenous growth (Romer, 1990; Barro and Sala-i Martin, 1997) and international finance (Benigno et al., 2025), my model incorporates differentiated goods to study the dynamic interaction between trade policy and the long-run growth path. A tariff hike against the foreign country is designed to boost domestic demand for tradable goods. However, this action concurrently raises the relative price of the tradable consumption bundle, thereby encouraging labor reallocation toward the less productive non-tradable sector. This resource misallocation fundamentally weakens long-term growth prospects. In the canonical setting, tariffs deteriorate the trade balance and further reduce growth; however, when convenience yields are introduced, tariffs may temporarily improve the trade balance through financial reevaluation effects. My central finding is that, as the permanent growth component embedded in the utility function dominates the infinite horizon, free trade policies are always mutually beneficial: any short-term protectionist gains are outweighed by long-term welfare losses. The framework rationalizes persistent trade tensions, such as those between China and the United States, as stemming from policymakers' present bias and their impatience toward long-term growth, which make protectionist measures temporarily appealing despite their long-run inefficiency. Finally, I extend the model to a three-country framework incorporating re-exportation activities, where countries divert trade flows through intermediaries to circumvent sanctions. This extension demonstrates that escalating trade wars lead to global economic downturns, increased trade fragmentation, and permanent asymmetric welfare losses. Overall, the paper emphasizes that the tradable sector acts as the engine of growth, and policies that distort its efficiency inevitably undermine long-run national welfare.

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1 Introduction

Are tariffs ever optimal for growth? What is the optimal trade policy in the context of endogenous growth? Why do market size effects dominate in endogenous growth? If the imposition of tariffs is detrimental to long-run growth, why do policymakers continue to implement them nonetheless? What is the optimal trade policy if we allow for trade diversion and re-exportation? How does it affect long-term growth?

Recent studies have leveraged extensive empirical and theoretical work to address the endogeneity between tariffs and social welfare costs (Amiti et al. (2019); Bai et al. (2025); Boer and Rieth (2024); Barnichon and Singh (2025)) and Mayer et al. (2025). However, there remains a notable gap in the literature regarding the transmission mechanism linking tariffs to a country's long-run growth perspective. Specifically, few studies thoroughly discuss the dynamic pass-through effects of tariff aggression, particularly whether such policies are ultimately expansionary for the manufacturing sector and Total Factor Productivity (TFP) growth.

I hereby build a trade model with endogenous growth, following the frameworks of Romer (1990), Barro and Sala-i Martin (1997), and Benigno et al. (2025) featuring the growth externality[1]. Building on their contributions, I extend the model by introducing differentiated goods. I differentiate country with two sectors, tradable and non-tradable sectors. Tradable sector is subject to the monopolist competition between the home and foreign country. I herby consider only tradable sector can grow (Grossman and Helpman (1991); Matsuyama (1992); Nunn and Trefler (2010)). In general, a tariff hike against the foreign country aims to boost demand for domestic tradable goods, which may seem beneficial for a country's growth prospects. However, tariffs also raise the relative price of the tradable bundle, thereby encouraging greater consumption of non-tradable goods—precisely the outcome a policymaker tries their best to avoid. Consequently, an increase in tariffs deteriorates the trade balance and, henceforth, weakens long-run growth. Moreover, since the production of non-tradable goods is less productive, tariffs trigger inefficient labor reallocation toward this sector. Given a fixed total labor supply, such reallocation further dampens growth. The same mechanism applies symmetrically to the foreign country.

The conventional wisdom views tariffs as expansionary in a partial equilibrium setting (Monacelli (2025); Itskhoki and Mukhin (2025)). The imposition of tariffs makes domestically produced industrial goods relatively more attractive, thereby boosting demand from domestic residents. To clear the market, domestic production must expand, which in turn contributes to higher output and, potentially, long-term growth. My model, however, features a growth externality between the technology-leading country (the home country) and the technology-following country (the foreign country). In this framework, persistent trade deficits in the home country reduce the revenue of its tradable sector, thereby slowing the pace of global technological progress—a phenomenon recently referred to as a global financial resource curse (Benigno et al. (2025)). From this perspective, a moderate increase in tariffs probably

[1]In the paper Benigno et al. (2025), they present a puzzle that capital inflows associated with a TFP declines for almost all the counties (both OECD and non-OECD samples). I cherry-pick this model in order to discuss whether the restricted trade policies can reverse the financial resource curse. I will develop my argument in the section 2.

could appear to reverse this "curse" by rebalancing production incentives and stimulating global innovation. I argue hereof, however, that this interpretation is problematic, as it neglects the general equilibrium effects operating on the production side. In the long run, the aggregate labor supply is fixed, and the tradable sector is—by construction—relatively more productive. Tariff imposition, while stimulating domestic output in both tradable and non-tradable sectors, induces an unbalanced reallocation of labor, drawing disproportionately more workers from the tradable sector toward the non-tradable one. Because the aggregate labor supply curve is relatively inelastic due to the high productivity of tradable activities, this reallocation leads to a contractionary effect on the tradable sector in general equilibrium. Thus, contrary to the partial-equilibrium intuition, tariffs ultimately undermine the growth potential of the economy through adverse production-side distortions. And this effect apply to both the home and foreign country.

I thereafter introduce dynamics into my model by conjecturing the tariff shock follows an AR(1) process. Initially, the policymaker may impose tariffs, surprising the entire economy. Consider a scenario where the tariff shock is unexpected and does not immediately prompt retaliation. A tariff shock imposed by the home country will induce an immediate TFP loss for the foreign country alongside a radical expansion of its tradable and non-tradable sectors. This is because the adaptation of technology is sluggish, and the tariff surprise triggers crowding-out effects toward the research sector. Specifically, the tariff boosts the demand for both foreign domestic industrial goods and services, consequently drawing labor away from research. Gradually, as the tariff effect fades, the accumulated production capacity becomes excessive, prompting the tradable sector to cut back employment and finally converge to the steady state. The resulting dynamic is qualitatively similar to the case of a unilateral tariff shock from the foreign country. However, because the foreign country runs a trade surplus to the home country by construction, the unilateral tariffs imposed by the foreign country symbolize a market size contraction and a reduction in total trade volume, implying an immediate TFP loss for both home and foreign countries. Furthermore, I extend my framework to the context of a trade war, where I argue that countries suffer welfare losses asymmetrically. The core finding is that the market size effect dominates: an instantaneous retaliation from the foreign country will wipe out the entire transient unilateral gain of the home country.

To further enhance my argument and capture the endogenous saving motives we extend our model with convenience yield. This extension allows me to study the re-evaluation effects of real trade imbalances under different structure of durations of debts issued by the home country. When convenience yields are present, capturing the revaluation effects associated with trade imbalances, a tariff increase can temporarily improve the trade balance by offsetting these effects. Essentially, the presence of convenience yields allows us to distinguish between the different maturities of bonds issued by representative agents, thereby indirectly influencing a country's real trade balance and its future growth perspectives. According to my framework, the greater preference of the convenience yield has characterized by a higher growth perspective of the home country. And moreover, the shorter duration of bounds grant the home country a convenience of reevaluation effect of bonds and therefore it offsets partial negative effects from tariffs. However, in the long run, the growth component embedded in the utility function becomes dominant. In an infinite-horizon framework, this implies that free trade policies are ultimately mutually beneficial, as any short-term gains from protectionism are outweighed by long-term losses in growth.

Nevertheless, my advocacy for free trade, which typically maximizes unilateral utility, immediately presents a puzzle: if free trade is indeed universally beneficial, how can we rationalize the recent, intensified Sino–US trade tensions?

Generally, economic agents value long-run growth because it implies a perpetually higher level of future consumption. Our results indicate that a tariff expansion improves the Terms of Trade (ToT) immediately, thus raising agents' instantaneous utility level. However, as we argue that tariffs are contractionary to long-run growth, the tariff hike is ultimately detrimental to agents' utility over an extended horizon. This dynamic implies that the policymaker faces a stark trade-off between the short-term gain in current consumption and the long-term loss in permanent income. Therefore, under my framework, if a policymaker becomes sufficiently impatient or places a higher preference on current consumption, a tariff expansion is the most likely strategic outcome. Therefore, to address this question, I introduce several modifications to the policy-maker's problem. Specifically, policy-makers are assumed to exhibit present bias (as in Harstad (2020), Harstad and Kessler (2025); Besley and Persson (2023); Jackson and Yariv (2014); Jackson and Yariv (2015); Halac and Yared (2018, 2022, 2025) and Tabellini (1991)) and to discount future growth perspective despite the infinite-horizon setting. My results suggest that, when the policy-maker is sufficiently impatient, the emergence of tariff aggression becomes a rational outcome. Ultimately, I can conclude that the presence of tariffs in the home country may stem from a combination of factors: a partial intention to stimulate growth, higher debt limits, a decline in the foreign country's research intensity, increased dependence on the foreign economy, an excessive propensity to save abroad, a reduced preference for home bonds, and longer maturities of issued bonds.

In the final section of this paper, I extend the model to a three-country framework to account for the growing prevalence of re-exportation activities. This framework is motivated by the possible circumvention of tariffs or trade sanctions by diverting trade flows through an intermediary (re-exportation), or by characterizing offshoring, re-routing, and re-branding activities. The three-country setting can thus generate asymmetric trade patterns among the economies. In my framework, whether the third country (the Rest of the World, or RoW) benefits from the tariff war between the two major economies is crucially dependent on the degree of substitutability or complementarity between its domestic R&D and its re-exportation activities.

If re-exportation and R&D activities are substitutable, the trade war between the two major economies renders re-exportation relatively more expensive. This outcome subsequently crowds in research efforts in the third country, leading to an improved welfare outcome for the RoW. This scenario echoes the observed economic boom in ASEAN countries and Mexico following the initial phase of the Sino-US trade tensions starting in 2018. However, when re-exportation and R&D are complementary, the trade war leads to a rising cost in the tradable sector of the third country, which consequently prompts a reduced welfare outcome. This scenario aligns with reports of significant losses incurred by European and Japanese electric vehicle (EV) producers following the intensified Sino-US trade conflict that began in 2025. In conclusion, the overall world economy currently exhibits clear evidence of rising trade fragmentation.

Related Literature Since the pioneering work of Ricardo (1817), Samuelson (1948), Ohlin (1933), Metzler (1949), and Dornbusch et al. (1977), economists have emphasized the welfare gains from trade through comparative advantage and specialization. However, more recent research has shown that while trade enhances aggregate welfare, it also generates distributional conflicts within countries. The unequal distribution of gains creates political pressures for protectionism, as highlighted by Stolper and Samuelson (1941), Rodrik (1995), Ossa (2014), Autor et al. (2013), and Amiti et al. (2019).

Beyond domestic distributional concerns, the persistence of trade barriers can also be understood as a failure of international cooperation. Classical and modern analyses—such as Johnson (1953), Kennan and Riezman (1988), Bagwell and Staiger (1990), Bagwell and Staiger (1999), and Staiger and Tabellini (1987). They framed tariff policies as outcomes of a Prisoner's Dilemma: each country has an incentive to protect its own market even though mutual liberalization would improve global welfare.

Alternatively, the political economy literature highlights the decisive role of interest groups and lobbying in shaping trade policy. Mayer (1984), Grossman and Helpman (1994) and Grossman and Helpman (1995) show that trade protection emerges from the interaction between organized interest groups seeking rents and governments balancing political contributions against social welfare. Consequently, the prevalence of protectionism reflects both domestic political pressures and strategic international interactions that limit the realization of fully free trade.

Furthermore, the seminal paper by Benigno et al. (2025) has shed light on the interconnectedness between the global saving glut and productivity growth. Large capital flows from developing countries have boosted a surge in demand for the non-tradable sector of developed countries. They shifted their resources to unproductive non-tradable sectors, causing the research sector to cut investment in innovation, and therefore dampening the growth rate in the long run. In this paper, I closely follow this idea and extend their model into a three-country context. I try to argue the growth rate level under tariff wars and re-exportation. The economic growth externality in terms of international trade and finances calls for novel research focusing on protectionist activities and endogenous growth.

Past research has extensively examined optimal trade policies in settings without endogenous growth. From the classical Lerner symmetry theorem (Lerner (1936)), through the analysis of optimal industrial policies (Eaton and Grossman (1986)), to the more recent theoretical advances by Itskhoki and Mukhin (2025), the consensus is that optimal tariffs generally exist and need not be zero. However, in this paper, I extend the analysis to an infinite-horizon framework with endogenous growth. I show that when long-run growth is fully internalized, free trade policies maximize social welfare. Nonetheless, the presence of present-biased preferences—as studied in Harstad (2020) Harstad and Kessler (2025), Besley and Persson (2023) Jackson and Yariv (2014), Jackson and Yariv (2015), Halac and Yared (2018), and Tabellini (1991)—leads a policymaker with only partial intention for future growth to impose strictly positive tariffs. In such a setting, protectionism arises not from efficiency motives, but from intertemporal distortions in policymaking, where short-term welfare considerations dominate long-run growth objectives.

Most recent research has primarily focused on the pass-through of tariffs and the related policy implications. This extensive literature includes important contributions by Amiti et al. (2019); Bai et al. (2025); Boer and Rieth (2024); Barnichon and Singh (2025); Fajgelbaum et al. (2024); Mayer et al. (2025); Monacelli (2025); Bianchi and Coulibaly (2025); Werning et al. (2025); Itskhoki and Mukhin (2025); Auray et al. (2025a) and Auray et al. (2025b). However, these models typically fall short by either neglecting endogenous growth, omitting a multi-sector framework, or failing to incorporate the possibility of re-exportation. I specifically construct my model to fill this gap at the intersection of international trade, macroeconomic policies, and long-run growth. Our particular focus is on how home trade imbalances and protectionist measures propagate into the production sector and related markets.

The remainder of this paper is structured as follows:Section 2 provides readers some empirical evidence on the tariffs and TFPs. Section 3 sketches mytheoretical framework and formally proves why the imposition of tariffs is detrimental to long-run growth. Section 4 introduces dynamics, explaining why tariffs are also harmful in the short term. In Section 5, I integrate my model into a game-theoretic framework to argue why policymakers, despite understanding the inherent inefficiency of tariffs, may still be incentivized to impose them. In section 6, I introduce convenience yield to my theoretical framework. In the final section, I extend the model into a three-country framework by explicitly allowing for re-exportation. While we anticipate finding similar qualitative outcomes regarding TFP loss, I show that the third country largely benefits from the trade tensions between the primary actors.

2 Empirical Evidence

To motivate my theoretical models and provide some empirical findings, this section documents my data source, the econometric model and some preliminary results.

2.1 Data Sources

My data sources for this empirical analysis are primarily drawn from Feenstra et al. (2015), Lane and Milesi-Ferretti (2018), and Teti (2024). The Penn World Table from Feenstra et al. (2015) provides detailed real GDP comparisons across countries and over time, supplying information on relative levels of income, output, input, and productivity for 185 countries between 1950 and 2023. I specifically extract the productivity data across all samples. The External Wealth of Nations database from Lane and Milesi-Ferretti (2018) offers information regarding capital flows, foreign direct investment (FDI), country portfolio choices, and foreign exchange reserves for nearly 200 countries, beginning in 1970. From this

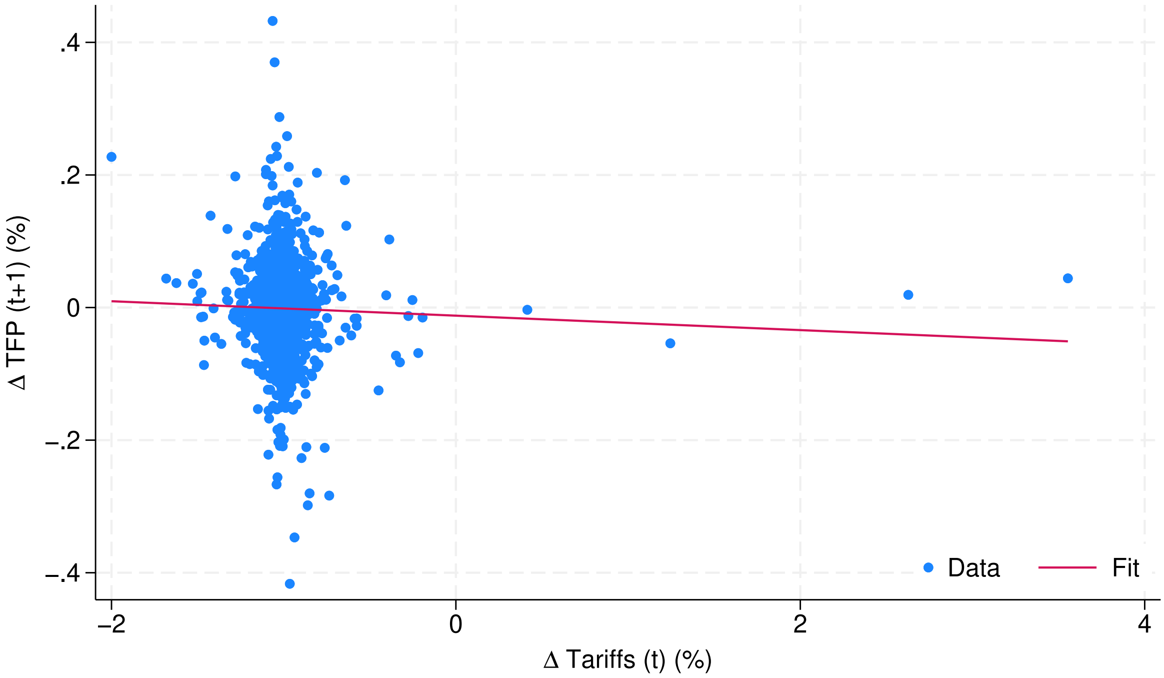

Figure 1: Scatter Plots Between the Change of Tariffs to The Lag-TFP Changes
This figure illustrates the correlation between change of tariffs to a country's TFP changes on the next period, which shows a negative correlation.

dataset, I extract data on capital inflows and current account balances. Finally, the Global Tariff Database by Teti (2024) documents yearly bilateral country import tariffs from 2000 to 2024. More specifically, this source provides average tariff data weighted by trade flows, and I use these weighted tariffs as the key independent variable in the subsequent empirical work. In conclusion, throughout the whole merged data sample, I have data from 185 countries beginning from 2000 to 2021.

According to my sample, I document a salient negative correlation between Total Factor Productivity (TFP) growth and import tariff aggression. Figure (1) presents a scatter plot illustrating this relationship, with the x-axis representing the percentage change in tariffs and the y-axis representing the country's TFP growth in the subsequent period. The sample period from 2000 to 2021 notably captures spectacular global trade liberalization. The red fitted line represents the unambiguous negative correlation between TFP growth and import tariff expansions. Empirically, a one percentage point liberalization in tariffs generally implies a 0.01% increase in TFP growth in the subsequent year.

2.2 Empirical Strategy

The empirical strategy is designed to investigate whether a change in tariffs has a significant effect on a country's Total Factor Productivity (TFP) growth across both the short and long run for a country. To execute this, I employ a linear regression model utilizing Local Projection (LP) methods. A critical assumption underlying this analysis is that the observed changes in a country's tariffs are exogenous and are not anticipated by individual and atomic firms within the economy. My econometric model is conceptualized as follows:

$$\Delta \text{TFP}_{i,t+h} = \beta_0 + \beta_h \Delta \text{Tariffs}_{i,t} + \boldsymbol{\gamma}_t \boldsymbol{X}_{i,t} + \lambda_i + \lambda_t + \varepsilon_{i,t}$$

where $\Delta\text{TFP}_{i,t+h}$ represents the growth rate of Total Factor Productivity (TFP) for country i at time $t+h$, defined as $\Delta\text{TFP}_{i,t+h} = (\text{TFP}_{i,t+h} - \text{TFP}_{i,t+h-1})/\text{TFP}_{i,t+h-1}$. Similarly, $\Delta\text{Tariffs}_{i,t}$ is defined as the percentage change in the average weighted import tariffs of country i at time t. The vector $\boldsymbol{X}_{i,t}$ represents country-specific control variables, including population, human capital, real interest rates, and current account imbalances. Furthermore, λ_i and λ_t represent the country-specific and year fixed effects, respectively. Finally, $\varepsilon_{i,t}$ represents the unobserved errors.

I execute the Local Projection (LP) model spanning up to $h = 10$ horizons. The term β_0 represents the constant intercept in the regression. My specific focus lies on the impulse response coefficient, β_h, which measures the effect of a one-unit shock (i.e., a one percentage point change) in import tariffs on the future Total Factor Productivity (TFP) growth rate at horizon h. The vector $\boldsymbol{\gamma}_t$ represents the coefficients associated with the country-specific and time-varying control variables included in the model. In conclusion, a tariff shock is potentially damaging to a country's long-run productivity. While it may transiently lead to improved output partially due to increased demand for domestic goods (expenditure switching), the tariff is ultimately detrimental to the economy in the medium and long run, even if the short-term expansionary effects are eventually neutralized.

2.3 Empirical Results

The Figure (2) illustrates the immediate negative effects following a tariff aggression for all countries. According to my estimation by controlling for fixed effects in country and year, the 1 % expansion of import tariffs will impose an immediate 0.01 % of TFP losses with a level of significance 95%. Then we observe an overshooting of TFP growth with a magnitude of 0.02% att the horizon $h = 2$, however, this pattern is followed by a further TFP declines at $h = 5$ and the effects fade away and become insignificant in the long run. This result calls for a theoretical framework to rationalize the transmission mechanism between tariffs and TFP growth.

3 The Two Country Model with Differentiated Goods

Let us assume here are two countries, a home country (denoted as H) and a foreign country (F, denoted with asterisk "$*$"). The home country dominates the technology sector, which owns the most productive technology. Meanwhile, the foreign country has an imitation technology (Barro and Sala-i Martin (1997)), which can draw on the US stock of knowledge when conducting research.

The model can be conceptualized as follows: there are two countries populated by representative

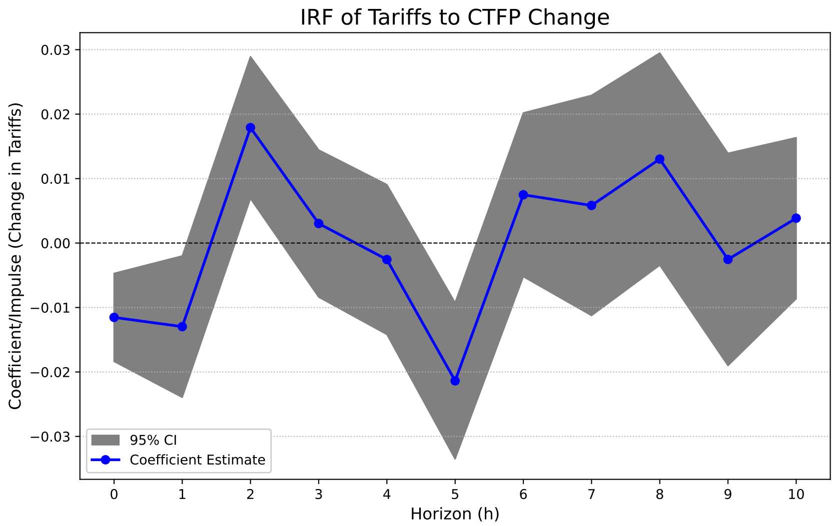

Figure 2: The Naive Regression between Tariffs and TFP
My model is $\Delta\text{TFP}_{i,t+h} = \beta_0 + \beta_h \Delta\text{Tariffs}_{i,t} + \boldsymbol{\gamma}_t \boldsymbol{X}_{i,t} + \lambda_i + \lambda_t + \varepsilon_{i,t}$. I plot the β_h and its 95% confidence interval. The vector $\boldsymbol{X}_t$ represents the macroeconomic controls including population, human capital, real interest rates, etc. I also control the country and year fixed effects. Data sources: Teti (2024); Lane and Milesi-Ferretti (2018); Feenstra et al. (2015).

agents. Representative agents residing in the home country will be denoted as H with subscripts, and similar for the representative residents in the foreign country denoted as F. I assume they live in infinite horizons with discounted windows. They decide their consumption and saving or borrowing flows every period in order to maximize their life-time utility. Within their consumption bundle, there are tradable and non-tradable goods. Non-tradable goods are produced and traded domestically, and only tradable goods can be traded across borders. Residents from the two countries are identical ex ante.

On the production side, there are two sectors: tradable and non-tradable sectors. Production in non-tradable sectors is relatively unproductive compared with the tradable sector. In the tradable sector, tradable goods are composed of labor and a continuum of differentiated intermediate goods. My statement on growth is identical to that of Grossman and Helpman (1991), Aghion and Howitt (1992), who stated that growth originates from the improvement of quality. Therefore, there are two subsectors within the tradable sectors: there is a final tradable-good producer that combines all labor and intermediaries together; and there is a research sector that hires labor to improve the quality of intermediate goods, the research is profit driven and subject to the creative destruction.

Apart from the previous two parties, I assume there is a third one: the government or the social planner. The social planner can levy taxes (import duties) or implement subsidies and rebated to agents as lump-sum. They play a key role in explaining a country's trade policies. I will explain my model more specifically in the following sections.

3.1 Representative Households

Let's assume there are infinite-lived representative households in each country that maximize their life-time utility:

$$\sum_{t=0}^{\infty} \beta^t \log(C_t) \tag{1}$$

Let's assume the households consume a bundle of goods with tradable C_t^T and non-tradable C_t^N:

$$C_t = \left(C_t^T\right)^{\omega} \left(C_t^N\right)^{1-\omega} \tag{2}$$

where ω is a weight of consumption in tradable goods. For a similar reason, the consumption of tradable goods is a bundle of tradable goods from all over the world:

$$C_t^T = \left[\nu_H^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + \nu_F^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{3}$$

where ν_j is the weight preference on tradable goods from different countries, and $j \in \{H, F\}$. Thus the index of tradable goods is given by:

$$P_t^T = \left[\nu_H \left(P_{H,t}^T\right)^{1-\rho} + \nu_F \left((1+\tau_{H,t}^T)P_{F,t}^T\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{4}$$

In these settings, households have the following budget constraint:

$$P_t^T C_t^T + P_t^N C_t^N + \frac{P_t^T B_{t+1}}{R_t^n} \leq P_t^T B_t + W_t \bar{L} + \Pi_t^T + \Pi_t^N \tag{5}$$

where B_t represents the unit of domestic bonds in terms of tradable bundles and its corresponding interest rate is R_t^n, W_t is the wage rate, $\bar{L}$ represents the fixed stock of labor in a country[2] and Π_t^i represents the profits of the sector i. We also assume there is a debt ceiling:

$$B_{t+1} \geq -\mathcal{K}_{j,t} \tag{6}$$

where $\mathcal{K}_{j,t}$ is a country-specific parameter of the debt ceiling, the debt ceiling varies from time to time, and $j \in \{H, F\}$. As we are introducing growth in my model, this debt limits will change over time. However, to make sure the balanced growth path exists I assume it increases with the same rate of growth. Symmetrically, I assume that foreign country's households have the following budget constraint:

$$P_t^{T*} C_t^{T*} + P_t^{N*} C_t^{N*} + \frac{P_t^T B_{t+1}^*}{(1+\phi_t^*)R_t^n} \leq P_t^T B_{H,t}^* + W_t^* \bar{L}_t^* + \Pi_t^{T*} + \Pi_t^{N*} - \mathcal{T}_t^* \tag{7}$$

where B^* denotes the units of international bonds held by foreign countries in terms of tradable goods, where ϕ_t^* is the subsidy of the bond denominated by tradable good bundles, and $\mathcal{T}_t^*$ is the tax. By

[2] I will discuss the case when the supply of labor is elastic in the appendix. However, the modification will not change the essential results but strengthen the argument of my paper.

balanced budget:

$$\mathcal{T}_t^* = \phi_t^* \frac{P_t^T B_{t+1}^*}{(1+\phi_t^*)R_t^n}$$

the saving subsidies implemented by the foreign government is the main driver of the saving glut of the foreign country. And it is my reduced-form way to generate trade imbalances across countries. As the tax is levied as a lump-sum, it will only distort consumers' propensity to save from the foreign country.

The implicit saving subsidies deserve some comments. A vast literature has documented the phenomenon of global saving gluts (Bernanke (2005)) and the accompanying deterioration of the U.S. current account balance. The global saving glut can be interpreted through two complementary lenses. On the supply side, as emphasized by Blanchard (1985), Tabellini (1991), Halac and Yared (2018), Halac and Yared (2022), and Halac and Yared (2025), policymakers' present bias often leads to excessive issuance of public debt, effectively generating excess global savings. On the demand side, models such as Song et al. (2011) and Mendoza et al. (2009) highlight how financial frictions in emerging economies constrain the creation of safe assets, inducing households to channel savings abroad. In this paper, we introduce implicit saving subsidies as a simplifying assumption to temporarily shut down both supply- and demand-driven externalities. This device serves as a minimalist mechanism for generating home-country trade deficits without activating other confounding channels. In later sections, we relax this assumption and extend the model to explicitly incorporate both the demand- and supply-driven externalities underlying global imbalances.

3.2 Firms and Production

Assume that the production sector is perfectly competitive and that the price is fully flexible. I assume that the tradable goods are a technological combination of labor and a continuum of non-tradable intermediate goods:

$$Y_{H,t}^T = \left(L_t^T\right)^{1-\alpha} \int_s \left(A_t(s)\right)^{1-\alpha} \left(M_t(s)\right)^{\alpha} \, ds \tag{8}$$

where the $A_t(s)$ represents the quality of the intermediate goods $M_t(s)$. This assumption features the key gradient of my model: the growth originates from the amelioration of intermediaries. The incentive for agents' innovation efforts is throughout two channels: business stealing effects and monopolistic power. The former encourages new entries of innovation, and the latter guaranties the rewards of innovation. However, for the final good of the non-tradable sector, I hereby assume:

$$Y_{H,t}^N = L_t^N \tag{9}$$

We hereby assume that the non-tradable sector is relatively unproductive and its only input is labor, following the argument of Benigno, Fornaro and Wolf (2025). The non-tradable sector is perfectly competitive and implies that $P_{H,t}^N = W_t$. Moreover, α represents the ratio of intermediate goods tradable, while $1-\alpha$ represents the ratio of labor. The first-order condition of the firm's problem implies that:

$$\frac{W_t}{P_{H,t}^T} = (1-\alpha)\left(L_t^T\right)^{-\alpha} \int_s \left(A_t(s)\right)^{1-\alpha} \left(M_t(s)\right)^{\alpha} \, ds$$

$$\frac{P_{M,t}(s)}{P_{H,t}^T} = \alpha \left(L_t^T\right)^{1-\alpha} \left(A_t(s)\right)^{1-\alpha} \left(M_t(s)\right)^{\alpha-1}$$

3.2.1 Production of Intermediate Goods

I hereby assume that the production of tradable goods consists of two key components: a perfectly competitive final producer that combines labor and intermediate goods, and a research department that employs labor to enhance the quality of intermediate inputs. Following the logic of creative destruction (Aghion and Howitt (1992)), we argue that a lack of societal cooperation allows each innovator to develop higher-quality patents that effectively appropriate the profits of inferior patent holders. Accordingly, we model the innovation process as occurring in two stages: first, agents decide whether to invest or enter the market; second, they determine whether to innovate. We further assume that the intermediate good represents a 'research' or 'patent' good, that is essential for production in the tradable sector but itself non-tradable. In the first stage, we posit that the nominal cost of producing one unit of the intermediate good equals $P_{H,t}^T$; thus, the nominal price of domestic intermediate goods (assuming identical producers) is given by:

$$P_{M,t}(s) = \frac{1}{\alpha} P_{H,t}^T$$

Another angle to interpret this result is that the $1/\alpha$ represents markup or the monopolist power of the patent holder. We focus on a symmetric equilibrium, that the different varieties of intermediate goods are identically productive, this implies total supply of intermediate can be represented as follows:

$$\int_s M_t(s) \, ds = \alpha^{\frac{2}{1-\alpha}} A_t L_t^T$$

Thus, by symmetry, the net profit of the variety can be represented as follows:

$$\Pi_{M,t} = P_{H,t}^T \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} A_t L_t^T \equiv P_{H,t}^T \Delta A_t L_t^T$$

The second stage is R&D sector should determine the amount of investment in technology, with the evolution of:

$$A_{t+1} = A_t + \chi A_t L_t^R \tag{10}$$

where χ represents the intensity of the research sector. The growing quality of intermediate goods is assumed to be non-depreciating and positively related with the intensity (or productivity) of conducting research and the input of labor. An intermediate producer considers the optimal input in the research sector subject to the increase in technology (10) to maximize their profits.

$$\max_{\{L_t^R\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \Lambda_t \left(P_{H,t}^T \Delta A_t L_t^T - W_t L_t^R \right) \tag{11}$$

This implies:

$$\frac{W_t}{\chi A_t} = \Lambda_{t,t+1} \left(P_{H,t+1}^T \Delta L_{t+1}^T + \frac{W_{t+1}}{\chi A_{t+1}} \right) \tag{12}$$

where $\Lambda_{t,t+1}$ represents the stochastic discount factor. By developing an optimal level of innovation, the firm equalizes the current marginal cost of innovation to the future value of the gains it generates, followed by the classical theory from Tobin's Q (Tobin (1969); Hayashi (1982); Summers (1981)). Therefore, the total value added of the industry is:

$$Y_{H,t}^T - \int_s M_t(s)\, ds = \left(1 - \alpha^2\right) \alpha^{\frac{2\alpha}{1-\alpha}} A_t L_t^T \equiv \mathcal{Z} A_t L_t^T \tag{13}$$

Taking a stock of what we have now, net value-added of the production in the tradable sector, the profits of innovation and the growth rate are linear with the size of the tradable sector under my assumption. Therefore, the rising excessive trade imbalances will erode the size of the tradable sector, thereby exacerbating the growth perspective.

3.2.2 Foreign Production Sector

I hereby assume everything is symmetric in the foreign country: the tradable sector is the main engine of growth in this economy, whereas the non-tradable sector is unproductive. Therefore, the price of non-tradable goods is equalized to the wage by perfect competition:

$$P_{F,t}^N = W_t^* \tag{14}$$

Similarly, assume that the intermediate producer from the foreign country is identical; this implies the nominal price of tradable goods:

$$P_{F,t}^T = \frac{W_t^*}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t^*} \tag{15}$$

Intuitively, the price of foreign industrial goods is simply equalized to its marginal costs: that is, the wage of labor offset by the average quality of intermediate goods.

3.2.3 Foreign R&D, Innovation and Catch-up

Furthermore, I hereby assume that foreign countries also have a similar process of technology evolvement. The foreign country partially imitates the patents of the home country with a fraction of Ω and meanwhile develops their own:

$$A_{t+1}^* = A_t^* + \xi \left(A_t^*\right)^{1-\Omega} \left(A_t\right)^{\Omega} L_t^{R*} \tag{16}$$

where Ω and ξ represent the dependence of R&D and research intensity of the foreign country. By construction, I hereby assume $\Omega < 1$ and $\xi < \chi$ to make sure the foreign country is a technology follower. Similarly, by solving the problem of maximizing the profits, this gives the following law of motion of technologies:

$$\frac{W_t^*}{\xi\left(A_t^*\right)^{1-\Omega}\left(A_t\right)^{\Omega}} = \Lambda_{t,t+1}^*\left(P_{F,t+1}^{T*}\Delta L_{t+1}^{T*} + \frac{W_{t+1}^*}{\xi\left(A_{t+1}^*\right)^{1-\Omega}\left(A_{t+1}\right)^{\Omega}}\right) \tag{17}$$

The pace of innovation in a foreign country is determined by its counterpart in the home country. Similar to the equation of the home country, another feature of this optimal condition is that their marginal costs of innovation are decreasing with their rising proximity to the technology frontiers. Therefore, once we observe a technology decline in the home country, we can observe a more progressive convergence of technology of the foreign country.

3.3 Equilibrium

DEFINITION 1 (The Two-Country Model Equilibrium)**.** An equilibrium is a path of real allocations of $\{C_t, C_t^*, C_t^T, C_t^{T*}, C_{H,t}^N, C_{F,t}^N, C_{H,t}^T, C_{F,t}^T, C_{H,t}^{T*}, C_{F,t}^{T*} L_t^T, L_t^{T*}, L_t^N, L_t^{N*}, L_t^R, L_t^{R*}, Y_{H,t}^N, Y_{F,t}^N, Y_{H,t}^T, Y_{F,t}^T, B_{t+1}$, $B_{t+1}^*, A_{t+1}, A_{t+1}^*, R_t^n, \hat{\mu}_t, \hat{\mu}_t^*, P_t, P_t^*, P_t^T, P_t^{T*}, P_{H,t}^T, P_{F,t}^T, P_{H,t}^N, P_{F,t}^N, W_t, W_t^*\}_{t=0}^{\infty}$, given initial condition $\{B_0, B_0^*, B_0^{**}, A_0, A_0^*\}$, assuming zero domestic inflation over time $\pi_{H,t+1}^T = \pi_{F,t+1}^T = 1$, the price of home tradable good as numéraire $P_{H,t}^T = 1$, and the evolution of trade tariffs $\{\tau_{F,t}^T, \tau_{H,t}^{T*}\}$. Subject to:

(i) In equilibrium, the Euler equation determines the problem of smoothing of consumption in households:

$$\left(C_t^T\right)^{-1} = \beta\left\{\left(C_{t+1}^T\right)^{-1}(1+\phi_t^*)R_t^n + \mu_t\right\} \tag{18}$$

the foreign country has similar problem:

$$\left(C_t^{T*}\right)^{-1} = \beta\left\{\left(C_{t+1}^{T*}\right)^{-1}(1+\phi_t^*)R_t^n\left(\frac{P_{t+1}^T/P_{t+1}^{T*}}{P_t^T/P_t^{T*}}\right)^{-1} + \mu_t^*\right\} \tag{19}$$

where μ_t and μ_t^* are Lagrangian multipliers associated with the constraint of the debt ceiling. As I hereby assume a "cashless" economy and the bond is paid by real goods, the Euler equation of the foreign country is adjusted by the relative price between the home and foreign country.

(ii) The optimization problem in research sector pins down the evolution rule of technology:

$$1 = \beta\left(\frac{C_{t+1}^T}{C_t^T}\right)^{-1}\pi_{H,t+1}^T\left(\chi\alpha L_{t+1}^T + 1\right) \tag{20}$$

$$\left(\frac{A_t}{A_t^*}\right)^{\Omega} = \beta\left(\frac{C_{t+1}^{T*}}{C_t^{T*}}\right)^{-1}\pi_{F,t+1}^T\left(\xi\alpha L_{t+1}^{T*} + \left(\frac{A_{t+1}}{A_{t+1}^*}\right)^{\Omega}\right) \tag{21}$$

(iii) The non-tradable good market is cleared domestically for both home and foreign country:

$$L_t^N = Y_{H,t}^N = \frac{1-\omega}{\omega} \frac{P_t^T}{P_{H,t}^N} C_t^T \tag{22}$$

$$L_t^{N*} = Y_{H,t}^{N*} = \frac{1-\omega}{\omega} \frac{P_t^{T*}}{P_{H,t}^{N*}} C_t^{T*} \tag{23}$$

The left hand side of the equation represents the production function of the non-tradable good, while the right hand side illustrate the demands from the agents. In equilibrium, the good labor market of the non-tradable sector must clear.

(iv) The labor market is cleared, the total supply of labor is fixed and equal to the total labor demand.

(v) The tradable sector is cleared, and the resource constraint is binding.

(vi) The country's budget constraint is binding.

(vii) The world's financial market must clear, the total debt positions should be net zero over time.

The complete conditions of the equilibrium will be attached in the following. In the context of a growth model, the real variables are growing all the time. Therefore, to better characterize the equilibrium in steady state, we redefine the variables: $x_t \equiv \frac{X_t}{A_t}$ and $\kappa_{j,t} = \frac{\mathcal{K}_{j,t}}{A_t}$ as the detrended debt limit, and $a_t^* \equiv \frac{A_t^*}{A_t}$ is defined as the degree of proximity to the technological frontier. Again, to ensure that the balanced growth path exists, we need to assume that the debt limit is growing at the same rate as technology, and that the transversality condition holds for both home and foreign countries:

$$\lim_{t\to\infty} \frac{B_t}{\prod_{t=k}^{+\infty} R_t(1+\phi_t^*)} = 0 \quad \text{and} \quad \lim_{t\to\infty} \frac{B_t^*}{\prod_{t=k}^{+\infty} R_t(1+\phi_t^*)} = 0$$

Now, let's characterize the stationary equilibrium as follows:

DEFINITION 2 (The Two-Country Model Equilibrium Stationary Equilibrium)**.** A stationary equilibrium is a path of real allocations of $\{c_t^T, c_t^{T*}, c_{H,t}^T, c_{F,t}^T, c_{H,t}^{T*}, c_{F,t}^{T*}, L_t^T, L_t^{T*}, L_t^N, L_t^{N*}, L_t^R, L_t^{R*}, Y_{H,t}^N, Y_{F,t}^N, y_{H,t}^T, y_{F,t}^T, b_{t+1}, b_{t+1}^*, g_{t+1}, a_{t+1}^*, R_t^n, \hat{\mu}_t, \hat{\mu}_t^*, P_t^T, P_t^{T*}, P_{H,t}^T, P_{F,t}^T, P_{H,t}^N, P_{F,t}^N, t\}_{t=0}^{\infty}$, given initial condition $\{B_0, B_0^*, A_0, A_0^*\}$, assuming zero domestic inflation over time $\pi_{H,t+1}^T = \pi_{F,t+1}^T = 1$) and the evolution of trade tariffs $\{\tau_{F,t}^T, \tau_{H,t}^{T*}\}$. Subject to:

(i) The optimal policies for consumption and savings for both the domestic and foreign countries. Under our new assumption, the Euler is further modified as following:

$$\left(c_t^T\right)^{-1} = \beta \left\{ \left(c_{t+1}^T g_{t+1}\right)^{-1} R_t^n + \hat{\mu}_t \right\} \tag{24}$$

$$\left(c_t^{T*}\right)^{-1} = \beta \left\{ \left(c_{t+1}^{T*} g_{t+1}\right)^{-1} (1+\phi_t^*) R_t^n \left(\frac{P_{t+1}^T / P_{t+1}^{T*}}{P_t^T / P_t^{T*}} \right)^{-1} + \hat{\mu}_t^* \right\} \tag{25}$$

Similarly, in equilibrium, the debt constraint of the home country is binding and this implies $\hat{\mu}_t > 0$. Furthermore, the globalization forces the unification of world interest rates, which is apparently lower than that of under the financial autarky.

(ii) Maximization of profits, in the tradable sector, the consumer maximizes their utilities, and is a market clear of the non-tradable sector.

(iii) Maximization of profits, in the non-tradable sector, the consumer maximizes their utilities, and markets clear of tradable goods.

(iv) Maximization of profits in the research sector.

$$1 = \beta \left(\frac{c_{t+1}^T g_{t+1}}{c_t^T} \right)^{-1} \left(\chi \alpha L_{t+1}^T + 1 \right) \tag{26}$$

$$\left(a_t^*\right)^{\Omega} = \beta \left(\frac{c_{t+1}^{T*} g_{t+1}}{c_t^{T*}} \right)^{-1} \left(\xi \alpha L_{t+1}^{T*} + \left(a_{t+1}^*\right)^{\Omega} \right) \tag{27}$$

In my following section, I particularly assume the term $\pi_{H,t+1}^T = \pi_{F,t+1}^T = 1$ are completely exogenous. This assumption simplifies my analyses by only focusing on the term of trade among countries.

(v) In the clearing of the labor market, the total labor demand is equal to the fixed supply of labor. The labor demand from non-tradable sector is pinned down by the demand of non-tradable goods:

$$L_t^N = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T) \frac{P_{F,t}^T}{P_{H,t}^T} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} c_t^T \tag{28}$$

$$L_t^{N*} = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H^* \left((1+\tau_{H,t}^{T*}) \frac{P_{H,t}^T}{P_{F,t}^T} \right)^{1-\rho} + \nu_F^* \right]^{\frac{1}{1-\rho}} \frac{c_t^{T*}}{a_t^*} \tag{29}$$

The right-hand-side of the equation has a multiplier of $\frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} > 1$, which implies the production of non-tradable goods is relatively unproductive and requiring relocating unproportionately more labor into the sector in case a boosted demands on the non-tradable goods. This mechanism is the key that explain why the tariffs always lead a decline of TFP in the balanced growth path. My formal argument will follow in the following subsection.

(vi) The resource constraint is satisfied, the budget constraint is satisfied.

(vii) In terms of asset market clearing, the net supply of bonds is 0 in the world.

(viii) The debt limit constraint is binding for the home country.

$$b_{t+1} = -\kappa_{H,t+1} \tag{30}$$

Let us characterize the balanced growth path as follows:

DEFINITION 3 (Balanced Growth Path and Steady State under Financial Integration)**.** Assume, in steady state, assume the domestic inflation rate is 0[3]. The steady-state equilibrium is a series of constant real allocations of $\{g, a^*, c^T, c^{T*}, c_H^T, c_H^{T*}, c_F^T, c_F^{T*}, L^N, L^{N*}, L^T, L^{T*}, L^R, L^{R*}, \mathcal{P}, R^n,\}$, given the condition of the stationary equilibrium, the exogenous saving subsidies ϕ^*, and the fixed value of tariffs τ_F^T, τ_H^{T*}.

Under financial integration (i.e., when two countries open their financial markets and are able to trade bonds) and zero inflation, the world interest rate is equalized everywhere and determined by the foreign country's Euler equation:

$$g = \beta R^n (1 + \phi^*) \tag{31}$$

We redefine the fraction: $\mathcal{P}_t = \frac{P_{H,t}^T}{P_{F,t}^T}$, and this fraction represents the relative price of tradable goods between two countries, or can be interpreted as Terms of Trade (ToT). An increase of $\mathcal{P}$ can be interpreted as an appreciation of home products. By the optimal condition of the problem of maximizing utilities, we can get the choice of consumption bundles:

$$c_H^T = \nu_H \left[\nu_H + \nu_F \left((1 + \tau_{F,t}^T) \mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{\rho}{1-\rho}} c^T \tag{32}$$

$$c_F^T = \nu_F \left[\nu_H \left((1 + \tau_{F,t}^T)^{-1} \mathcal{P} \right)^{1-\rho} + \nu_F \right]^{\frac{\rho}{1-\rho}} c^T \tag{33}$$

$$c_H^{T*} = \nu_H^* \left[\nu_H^* + \nu_F^* \left((1 + \tau_{H,t}^{T*})^{-1} \mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{\rho}{1-\rho}} c^{T*} \tag{34}$$

$$c_F^{T*} = \nu_F^* \left[\nu_H^* \left((1 + \tau_{H,t}^{T*}) \mathcal{P} \right)^{1-\rho} + \nu_F^* \right]^{\frac{\rho}{1-\rho}} c^{T*} \tag{35}$$

The optimal condition of the research sector pins down the optimal investment of technology and its evolution rule:

$$g = \beta \left(\chi \alpha L^T + 1 \right) \tag{36}$$

$$(a^*)^{\Omega} = \frac{\beta \xi \alpha L^{T*}}{g - \beta} \tag{37}$$

In the balanced growth path, TFP growth is characterized by the size of the tradable sector. Therefore, the market size effect is the key feature for the growth of a country. Therefore, whether an imposition of a tariff is expansionary or extractionary for the tradable sector is the main driver of growth. In the long run, I hereby assume that the supply of labor is fixed; this implies that the total labor demands from each sector and each country should be equal to the fixed supply of labor:

$$L^R = \frac{g - 1}{\chi} \tag{38}$$

[3] That is, I hereby assume $\pi_{H,t}^T$ and $\pi_{F,t}^T$ are equal to 1 in steady state.

$$L^{R*} = \frac{(g-1)\,(a^*)^{\Omega}}{\xi} \tag{39}$$

$$L^N = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H + \nu_F \left((1+\tau^T_{F,t})\mathcal{P}^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} c^T \tag{40}$$

$$L^{N*} = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu^*_H \left((1+\tau^{T*}_{H,t})\mathcal{P}\right)^{1-\rho} + \nu^*_F\right]^{\frac{1}{1-\rho}} \frac{c^{T*}}{a^*} \tag{41}$$

In the long-run, for simplicity, I hereby assume the supply of labor is fixed. The labor market clear in the following conditions for both countries:

$$\bar{L} = L^N + L^T + L^R \tag{42}$$

$$\bar{L}^* = L^{N*} + L^{T*} + L^{R*} \tag{43}$$

Finally, the resource and budget constraints should bind and apply to both countries:

$$\mathcal{Z}L^T = c^T_H + c^{T*}_H \tag{44}$$

$$\mathcal{Z}a^* L^{T*} = c^T_F + c^{T*}_F \tag{45}$$

$$\mathcal{Z}L^T + \left[\nu_H + \nu_F \left((1+\tau^T_{F,t})\mathcal{P}^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \kappa_H \left(\frac{g}{R^n} - 1\right) = c^T_H + \mathcal{P}^{-1} c^T_F \tag{46}$$

PROPOSITION 1. *There exists a unique steady state given the stationary equilibrium condition, exogenous saving subsidies ϕ^*, and the fixed value of tariffs τ^T_F, τ^{T*}_H.*[4]

3.4 Benchmark: The Financial Autarky under Free Trade

The model we have built features the growth externality between two countries, following the argument from Benigno et al. (2025): Large capital flows from developing countries have boosted a surge in demand for the non-tradable sector of developed countries. The reallocation of resources to unproductive non-tradable sectors causes the research sector to cut investment in innovation and, therefore, dampens the growth rate in the long run worldwide.

For this reason, I choose the model of Benigno et al. (2025) on purpose to illustrate the growth externality puzzle. I want to address the question of whether over-consumption in the home country is the culprit of underinvestment in the research sector and whether the imposition of tariffs can reverse this puzzle. I firstly start my argument by illustrating the equilibrium under free trade and balanced trade.

PROPOSITION 2. *Under financial autarky and free trade, which means that the home country cannot raise funds from the international bond market, trade is balanced, and the interest rate of the home country is pinned*

[4] The equilibrium condition (39) may introduce an auxiliary solution of $a^* = 0$ in our system; however, as this solution is not stable, this guaranties that, as long as we impose the constraint $a^* > 0$, the solution to this system is stable and unique.

down by its Euler:

$$R_a^n = \frac{g_a}{\beta} \tag{47}$$

where the subscript a of g_a and R_a^n is denoted as the financial variable under autarky. However, because foreign subsidizes its savings, the interest rate of the foreign country is determined by:

$$R_a^{n*} = \frac{g_a}{(1+\phi^*)\,\beta} \tag{48}$$

under financial autarky, $b_t = b_t^* = 0$, in order to clear the good market, the resource and budget constraints must hold:

$$\begin{aligned} \mathcal{Z}L^T &= c_H^T + c_H^{T*} \\ \mathcal{Z}a^* L^{T*} &= c_F^T + c_F^{T*} \\ \mathcal{Z}L^T &= c_H^T + \mathcal{P}^{-1} c_F^T \end{aligned} \tag{49}$$

This system of equations of (32)-(35) along with budget and resource constraints (49) will solve the countries' consumption choice $\{c^T, c^{T*}, c_H^T, c_H^{T*}, c_F^T, c_F^{T*}\}$ and relative price $\mathcal{P}$ given a pair of L^T and L^{T*}. This relation pins down a mapping between consumption, labor, and price:

$$\mathcal{G} : \mathcal{L} \longrightarrow \mathcal{Y}, \qquad \mathcal{G}(L^T, L^{T*}) := \left(\mathcal{P}(L^T, L^{T*}),\ c^T(L^T, L^{T*}),\ c^{T*}(L^T, L^{T*})\right). \tag{50}$$

PROPOSITION 3. *The mapping satisfies the following Jacobian matrix:*

$$\mathcal{J}_{\mathcal{G}}(L^T, L^{T*}) = \begin{bmatrix} \frac{\partial \mathcal{P}}{\partial L^T} < 0 & \frac{\partial \mathcal{P}}{\partial L^{T*}} > 0 \\ \frac{\partial c^T}{\partial L^T} > 0 & \frac{\partial c^T}{\partial L^{T*}} > 0 \\ \frac{\partial c^{T*}}{\partial L^T} > 0 & \frac{\partial c^{T*}}{\partial L^{T*}} > 0 \end{bmatrix}.$$

Under financial autarky (or balanced trade), the consumer's problem boils down to the demand for the consumption bundle as well as the labor stock from both countries. To balance trade, a higher level of consumption of tradable goods must be associated with a greater ability to produce, such as a higher level of L^T. On the other hand, a greater ability of production from the opponent's country will also increase the consumption of tradable goods throughout the channel of rise in consumption of foreign goods. The relative price $\mathcal{P}$ is defined as the ratio of the home price to the foreign price. In order to clear the market, a higher production capability from the home country will result in a lower level of relative price $\mathcal{P}$, vice versa.

Moreover, combining the mapping (50) with the equilibrium condition (38) - (43) will pin down the equilibrium outcome of labor demands from tradable sectors given any pair of (g_a, a_a^*). Hereby, the subscript a denotes the growth rate and proximity to frontier under the financial autarky:

$$
\begin{aligned}
L^T &= \bar{L} - L^N(\mathcal{P}, c^T) - \frac{g_a - 1}{\chi} \\
L^{T*} &= \bar{L} - L^{N*}\left(\mathcal{P}, c^{T*}, a^*\left(L^T, L^{T*}\right)\right) - \frac{(g_a - 1)\left(a_a^*\right)^{\Omega}}{\xi}
\end{aligned}
\tag{51}
$$

PROPOSITION 4. *For home country, the mapping between consumption and relative price satisfies the following properties: $\partial L^T/\partial c^T < 0$, $\partial L^T/\partial \mathcal{P} < 0$, $\partial L^T/\partial g < 0$, $\partial L^{T*}/\partial c^{T*} < 0$, $\partial L^{T*}/\partial \mathcal{P} > 0$, $\partial L^{T*}/\partial g < 0$, $\partial L^{T*}/\partial a^* < 0$.*

From the perspective of the home producer, an increase in $\mathcal{P}$ will make its production more profitable, the tradable sector is expected to expand; an increase in consumption from the tradable sector also boosts the demand from the non-tradable sector by complementarity (the substitution elasticity between tradable goods and non-tradable is 1). Furthermore, as the labor supply is fixed, this implies that a higher growth rate or proximity to the frontier will crowd out the labor in the tradable sector. Finally, to determine the equilibrium, we combine the mapping (51) with the technology evolution equations (36) and (37) and the Euler equation (47) to obtain the equilibrium allocation in the tradable sector.

In steady state, the growth rate is determined by the Euler equation (47) under the financial autarky. As foreign country features a higher propensity of saving, its growth rate is expected to be lower. In conclusion, we get the following expression:

$$
\begin{aligned}
L^T &= (1 + \beta\alpha)^{-1}\left(\bar{L} - L^N(L^T, L^{T*}) + \frac{1}{\chi}\right) \\
L^{T*} &= \left(1 + \beta\alpha\frac{g - 1}{g - \beta}\right)^{-1}\left(\bar{L} - L^{N*}\left(L^T, L^{T*}, a^*\left(L^T, L^{T*}\right)\right)\right)
\end{aligned}
\tag{52}
$$

The mapping features an "industrial job tension" between two countries. As one country's higher labor stock in the tradable sector will result in a lower stock in the other. As the home country always performs as a leader in technology, the shrinking tradable sector will dampen the growth rate of the whole world. On the other hand, an increase of labor stock in the tradable sector from the home country will make the research process of the foreign country more expensive, as the size of the tradable sector is proportional to its technology frontier; its tradable sector is expected to shrink. After few steps of algebra, we can get the following formula:

$$
g_a = \beta\left(\frac{\alpha(\chi L + \bar{1} - \beta)}{1 + \alpha\beta + \frac{1-\omega}{\omega}(1 + \alpha)} + 1\right) \tag{53}
$$

a higher steady state growth rate is backed by a higher total labor stock $\bar{L}$, a higher intensity of research χ, a higher spending in the tradable sector ω, and a higher share of intermediate goods α. For similar reason, we can derive the technology frontier of a foreign country:

$$
\left(a_a^*\right)^{\Omega} = \frac{\alpha\beta\xi\bar{L}^*}{(g_a - \beta)\left(1 + \frac{1-\omega}{\omega}(1 + \alpha)\right) + (g_a - 1)\alpha\beta} \tag{54}
$$

a higher technology frontier of foreign country is determined by a higher total labor stock $\bar{L}^*$, a higher research intensity ξ, a higher spending in the tradable sector ω, a higher share of intermediate goods α and most importantly, a lower growth rate g. The reason is salient, as research is costly and crowding out labor from the tradable sector, a lower g implies a lower cost of research activities and therefore a larger amount of people working in the tradable sector, this results in a higher technology frontier of foreign country.

3.5 The Financial Integration and Global Financial Resource Curse

ASSUMPTION 1. Assume the subsidies of saving are high enough to guarantee:

$$\frac{g}{R^n} = (1+\phi^*)\,\beta > 1$$

after financial integration. This assumption is to make sure the real trade surplus in steady state is positive, which means I hereby assume the home country cannot transfer excessive resources into the research sector and manipulate the value of the real trade surplus. In other words, this assumption requires $g > R^n$ in steady state in order to keep the public debt sustainable. This requirement has been widely discussed and verified by classical economic research (Domar (1944); Blanchard et al. (1991); Blanchard (2019); Rachel and Summers (2019)).

PROPOSITION 5. *Assume free trade. The financial integration implies a lower real interest rate as well as a lower growth rate compared with financial autarky. On the other hand, the foreign country benefits from the financial integration, whose growth rate is increasing with the rise of κ_H in steady state.*

PROOF Under free trade, after financial integration, the interest rates of both countries are equalized, and the growth rate is pinned down:

$$R_f^n = \frac{g}{(1+\phi^*)\,\beta} \tag{55}$$

and I combine this equation with our budget constraint of the home country:

$$\mathcal{Z}L_f^T + \left[\nu_H + \nu_F\left(\mathcal{P}^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}\kappa_H\left((1+\phi^*)\,\beta - 1\right) = c_H^T + \mathcal{P}c_F^T$$

under my assumption that $(1+\phi^*)\,\beta > 1$, this implies a higher level of consumption c^T of home country while a lower level of consumption in foreign country. Intuitively, the term $\left[\nu_H + \nu_F\left(\mathcal{P}^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$ $\kappa_H\left((1+\phi^*)\,\beta - 1\right)$ denotes the real value of national deficits. This mechanism increases the consumption of tradable goods from the foreign (by increasing trade deficits) and, meanwhile, increases the demand for non-tradable goods by income effects.

$$L_f^N = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}\left(\mathcal{Z}L^T + \left[\nu_H + \nu_F\left(\mathcal{P}^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}\kappa_H\left((1+\phi^*)\,\beta - 1\right)\right)$$

Thus, this dynamic reinforces the reallocation of labor into non-tradable sector, and hence causes the growth rate to go down even further. For this reason, the implicit function $L^T(L^{T*}, g_f)$ is decreasing in g_f (the growth rate under financial integration) now.

$$L_f^T = \bar{L} - \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left(\mathcal{Z} L_f^T + \left[\nu_H + \nu_F \left(\mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*) \beta - 1 \right) \right) - \frac{g_f - 1}{\chi} \tag{56}$$

After financial integration, the trade deficits have brought up a wedge into the system and caused the home country to consume arbitrarily more tradable goods. As now the rising consumption of tradable goods is largely contributed by the foreign goods, the relative price $\mathcal{P}$ will adjust and increase. Hence, the financial integration prompts the tradable goods more expensive and thus the demands of non-tradable goods must relatively increase. Since I hereby assume the labor is perfectly mobile, and its production is one-to-one to its labor employed, to clear the market, more labor is reallocated into the non-tradable sector. The determination of allocation of labor is the relative price $P_H^N/P_H^T = W_t$. As more labor is allocated into the non-tradable sector, the relative price between non-tradable and tradable goods must go down, which implies W_t has to go down. As the wage is positively correlated with the productivity A_t, the growth rate must go down. Intuitively, the financial integration between countries causes reallocation of labor within the home country; as the total labor supply is fixed and the non-tradable sector is unproductive, this reallocation is inefficient and brings down the country's real wage. As the country's growth rate is positively related to its size of tradable sector, the growing must slow down. The decomposition of effects can be interpreted numerically in figure 1. In equilibrium, the growth rate under financial integration is:

$$g_f = g_a - \frac{\alpha\beta\chi\frac{1-\omega}{\omega}(1+\alpha)}{1+\alpha\beta+\frac{1-\omega}{\omega}(1+\alpha)} \left[\nu_H + \nu_F \left(\mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*) \beta - 1 \right) \tag{57}$$

Under financial integration, there are two effects that drive the growth rate down. The first one is the direct increase of trade deficits. As I have argued, there exists an "industrial job tension" between two countries; the trade deficits will make consumers consume more goods from foreign countries. This facilitates a boom in the tradable sector from foreigners and meanwhile shrinks the home one. The second effect comes from the price adjustment of bonds. As there is an "over-supply" from the foreign tradable sector, to clear the market, the relative price $\mathcal{P}$ has to adjust and increase. Moreover, $\mathcal{P}$ can be interpreted as "term of trade": an appreciation of term of trade means an even larger consumption from foreign country F, incurring a even bigger decline of consumption from domestic tradable goods.

On the other hand, the foreign country now is under two types of amplification effects. Firstly, the global growth rate g_f is decreasing; this helps foreign countries save money from the research sector. The technology augment line shifts up, as the equation (37) has shown. On the other hand, as the demand for foreign goods starts to rise, as $\mathcal{P}$ increases. To clear the labor market, the size of the tradable sector in the foreign country must be expanding, and therefore the relative price $P_F^N/P_F^T = W_t^*$ has to rise; this

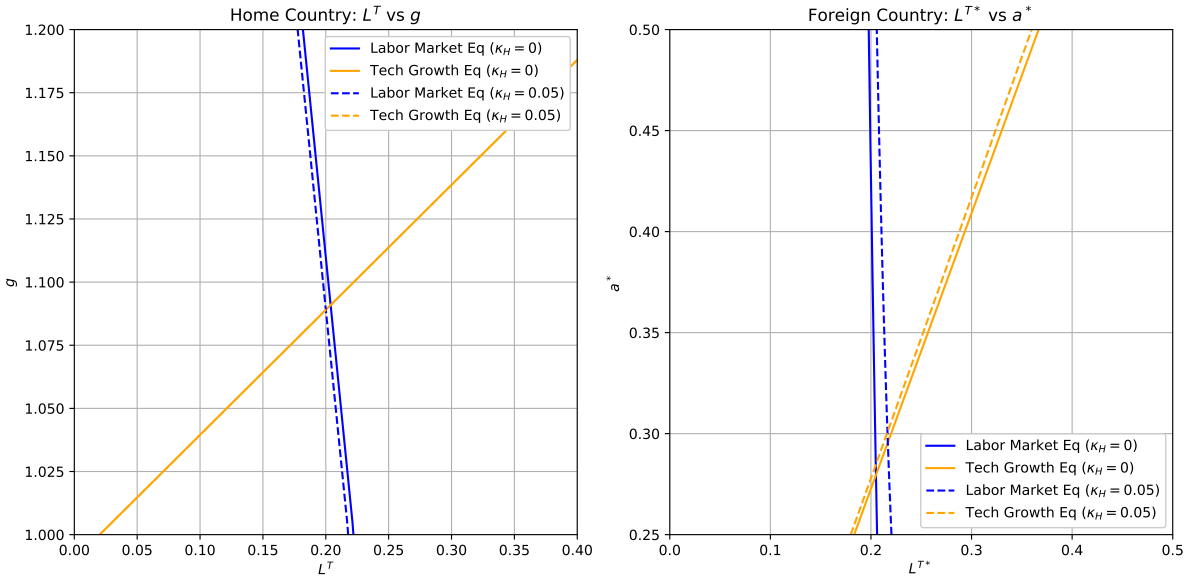

Figure 3: Dynamics under Financial Integration

shifts the labor lines rightward.

$$(a_f^*)^{\Omega} = \frac{\alpha\beta\xi\left(\bar{L}^* + \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F\right]^{\frac{1}{1-\rho}} \frac{(\kappa_H(1+\phi^*)\beta - 1)}{a_f^*}\right)}{(g_a - \beta)\left(1 + \frac{1-\omega}{\omega}(1+\alpha)\right) + (g_a - \beta)\alpha\beta} \tag{58}$$

In this equation, the wedge decreases the demands of non-tradable goods. The financial integration has brought a trade surplus to foreign countries and therefore makes foreign countries consume more tradable goods. Meanwhile, the lower global growth rate has also saved a lot of labor from the research sector and reallocated more labor to the tradable sector. In conclusion, financial integration, on the one hand, increases the total demands for foreign goods, which makes the foreign country increase the investment in the research sector; on the other hand, the decrease in the global growth rate helps the foreign country save labor from the research sector. And consequently, its proximity to the technology frontier is expected to increase.

3.6 Tariffs and Reversion of Financial Resource curse

The main idea of this framework is to discuss the controversial impacts of trade deficits. The trade deficits help the home country to enjoy more goods, whereas they bring a vicious curse of tanking growth perspective. The declining demands for domestic goods further force the research sector to cut back on investment in innovation and therefore imply a drop in global productivity growth. For this reason, a simple question needs to be asked: Is a tariff going to reverse this vicious cycle, bring back parts of the demand for home production, and therefore increase investment in innovation?

From my analysis, however, if in the long-run the real trade imbalance is fixed, the imposition of tariffs would not help the home country build-up lost manufacturing capabilities and increase investment in innovations; rather, it would result in an inefficient reallocation of labor from the tradable sector to the non-tradable sector. I argue that there are two contradictory effects present in the system. One is derived from the demand effects: the imposition of tariffs will boost the demand for domestic industrial goods. the producer internalizes this dynamic and hence scales up production by hiring more people, and the real wage increases. Another effect arises from the supply effect, which is contradictory to the demand effects: the imposition of tariffs also incurs increasing costs on the tradable bundle. Consequently, consumers will consume more non-tradable goods. the increased demand for non-tradable goods facilitates the expansion of the non-tradable sector while crowding out labor from the tradable sector. These two effects are contradictory to each other, and the net effects of tariffs—whether they are expansionary or contractionary—to the tradable sector are determined by the magnitudes of these two effects above. In this section, I argue that, in the balanced growth path, the imposition of tariffs is always contractionary to growth. I will sketch the proof in the later of this section.

THEOREM 1. *There exists a mapping between c_H^{T*} and c_F^T, and the mapping is monotonic, strictly increasing and concave (Itskhoki and Mukhin (2025)).*

PROOF Combine the equilibrium conditions (32)-(35) along with budget and resource constraints (49), the first-order condition of the consumer's problem implies that:

$$(1+\tau_{F,t}^T)\frac{\nu_H \left(c_{H,t}^T\right)^{-\frac{1}{\rho}}}{\nu_F \left(c_{F,t}^T\right)^{-\frac{1}{\rho}}} = \mathcal{P}_t = \frac{\nu_H^* \left(c_{H,t}^{T*}\right)^{-\frac{1}{\rho}}}{\nu_F^* \left(c_{F,t}^{T*}\right)^{-\frac{1}{\rho}}}(1+\tau_{H,t}^{T*})^{-1} \tag{59}$$

by rearranging the budget constraint, and this implies the following:

$$\left[\nu_H \mathcal{P}_t^{1-\rho} + \nu_F (1+\tau_{F,t}^T)^{1-\rho}\right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1\right) = c_{F,t}^T - \mathcal{P}_t c_{H,t}^{T*} \tag{60}$$

by combining equation (59) and (60), this pins down the following mapping:

$$(1+\tau_{H,t}^{T*})^{-1}\nu_H^* \left(c_{H,t}^{T*}\right)^{\frac{\rho-1}{\rho}} = \nu_F^* \left(a_t^* \mathcal{Z} L_t^{T*} - c_{F,t}^T\right)^{-\frac{1}{\rho}} \left(c_{F,t}^T - \left[\nu_H \mathcal{P}_t^{1-\rho} + \nu_F (1+\tau_{F,t}^T)^{1-\rho}\right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1\right)\right)$$

and I denote this mapping as:

$$c_{H,t}^{T*} = \mathcal{H}\left(c_{F,t}^T, \mathcal{P}_t\right) \tag{61}$$

with the property: $\partial \mathcal{H}/\partial c_F^T > 0$, $\partial \mathcal{H}/\partial \mathcal{P} < 0$. This proposition suggests that the imposition of tariffs reduces not only the exports of trading partners but also, through general equilibrium effects, the exports of the domestic economy. This mechanism constitutes the core of my model: imposing tariffs against a trading partner leads to a contraction in domestic exports, rendering the overall effect on the tradable sector ambiguous. On the one hand, tariffs stimulate domestic demand for domestically produced goods; on the other hand, they reduce foreign demand for domestic exports. If the former effect dominates, the tradable sector expands; if the latter prevails, it contracts. The impact of an import tariff on the size of the tradable sector is theoretically ambiguous and depends critically on the underlying parameterization of the model.

I combine the mapping derived from equation (61) with the resource constraint of the home country, and thus a determination of domestic labor demands of tradable sector:

$$\mathcal{Z}L_t^T = c_{H,t}^T + \mathcal{H}\left(c_{F,t}^T, \mathcal{P}_t\right) \tag{62}$$

this implies a strictly decreasing function with $\mathcal{P}_t$. As I hereby assume the linear production function, the labor demand is going one-to-one with the consumption, I total differentiate the mapping (61) and combine with equation (62), we get the following:

$$\varepsilon^{\text{demand}} = \frac{dL^T/d\mathcal{P}}{L^T/\mathcal{P}} = \rho \frac{\nu_F(1+\tau_F^T)^{1-\rho}\mathcal{P}^{\rho-1}}{\nu_H + \nu_F(1+\tau_F^T)^{1-\rho}\mathcal{P}^{\rho-1}} \frac{c_H^T}{y^T} + \frac{\rho+\mathcal{G}^*}{\rho-1}\frac{1}{\mathcal{G}} \tag{63}$$

where:

$$\mathcal{G} \equiv \frac{\mathcal{P}y^T}{c_F^T} \quad \text{and} \quad \mathcal{G}^* \equiv \frac{\mathcal{P}c_H^{T*}}{c_F^{T*}} = \frac{\nu_H^*}{\nu_F^*}(1+\tau_H^{H*})^{-\rho}\mathcal{P}^{\rho-1}$$

The proportional change in labor allocation is determined by the extent of foreign-market penetration—formally, the ratio of total value of production on home goods to total expenditure on foreign goods (or, equivalently, the inverse of foreign-goods penetration). The increase of tariffs on one hand increases the demand for home products unambiguously, while on the other hand, deteriorates the exports. The total demand of labor curve can be obtained by combining the budget constraint (46) and the mapping (62):

$$\begin{aligned} \mathcal{Z}L_t^T =& \nu_H \left(\frac{P_{H,t}^T}{P_t^T}\right)^{-\rho} \lambda_H \left(\mathcal{Z}\frac{P_{H,t}^T}{P_t^T}L_t^T + \kappa_H\left((1+\phi_t^*)\beta - 1\right)\right) \\ &+ \mathcal{H}\left(\nu_H \left(\frac{(1+\tau_{F,t}^T)P_{F,t}}{P_t^T}\right)^{-\rho} \lambda_H \left(\mathcal{Z}\frac{P_{H,t}^T}{P_t^T}L_t^T + \kappa_H\left((1+\phi_t^*)\beta - 1\right)\right)\right) \end{aligned} \tag{64}$$

ASSUMPTION 2. Residents from home and foreign countries are home-biased, this implies:

$$\nu_H > \nu_F \quad \text{and} \quad \nu_H^* < \nu_F^*$$

This assumption ensures a net increase in labor demand despite the imposition of higher import tariffs. Hence, the labor demand line is expected to shift rightwards with higher tariffs. However, the magnitude of its shift diminishes progressively as the tariff rate climbs, reflecting declining marginal effectiveness of successively higher tariff levels. Moreover, the labor market clearing condition implies a strictly increasing function with $\mathcal{P}_t$:

$$L^T = \bar{L} - \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}\left(c_H^T + (1+\tau_F^T)\mathcal{P}^{-1}c_F^T\right) - \frac{g_f - 1}{\chi} \tag{65}$$

this implies an increase in the price of home goods will make firms tend to scale up their production. I combine equation (62) and (65). This system pins down a solution of L^T and $\mathcal{P}$ by giving exogenous tariffs $\tau^T_{F,t}$ and growth rate g. A hike of tariffs against the foreign country is contracting to the tradable sector as the substitution effects. As the tariffs make the tradable bundle more expensive, the consumer would like to consume more non-tradable goods.

Under the above assumption, imposing tariffs on foreign goods shifts equation (65) leftward, while shifts equation (62) rightward. Consequently, the effect on prices is unambiguous—leading to an increase—whereas the direction of adjustment in tradable-sector labor, L^T, remains ambiguous. Moreover, the total supply of labor working in the tradable sector is determined by combining equation (62) and equation (65)[5]:

$$\begin{aligned} L_f^T = \bar{L} - \lambda_H \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} & \left(\mathcal{Z} L_f^T + \left[\nu_H + \nu_F \left((1+\tau^T_{F,t})\mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\,\beta - 1 \right) \right) \\ & - \frac{g_f - 1}{\chi} \end{aligned} \tag{66}$$

This equation features a fixed point problem on the left and right hand side with elasticity:

$$\varepsilon^{\text{supply}} = \frac{\nu_F (1+\tau_F^T)^{1-\rho}}{\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{1-\rho}} + 2(1-\rho) \frac{\nu_H \mathcal{P}^{1-\rho}}{\lambda_H + \frac{\omega}{(1-\omega)(1+\alpha)}} \tag{67}$$

comparing the elasticity between demand (63) and supply curve (67), the demand curve is fairly elastic while the supply curve is not. This happens since the substitution between tradable and non-tradable goods is inelastic with an elasticity of 1; which is assumed to be smaller than the substitution within the tradable bundles (typically, I hereby assume $\rho = 2$).

The partial equilibrium condition on price $\mathcal{P}_t$ and L_t^T is obtained by the system of equations (66) and (64) taken given the foreign technology proximity a^*, foreign size of tradable sector L^{T*} and growth rate under financial liberalization g_f. Since the demand curve is way more elastic than the supply curve, the imposition of tariffs always leads to an appreciation of home goods and a contraction of the tradable sector shown as figure (4).

The direct impact or the first-order impact of imposition of tariffs implies a greater contraction of labor supply. The intuition is simple but powerful: as the production of tradable goods is more efficient and productive, the increasing demands of non-tradable goods require a greater reallocation of labor from the tradable sector, which implies although the substitution effects within tradable bundles mark a positive spillover to the domestic tradable sector, the substitution effects between tradable and non-tradable goods signal a dominant negative effect on reallocating labor away.

[5] In this model, we can rearrange our budget constraint as:

$$c^T = \lambda_H \left(\mathcal{Z} L_f^T + \left[\nu_H + \nu_F \left((1+\tau^T_{F,t})\mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\,\beta - 1 \right) \right)$$

where $\lambda_H = \frac{\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{1-\rho}}{\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{-\rho}}$, that is a country specific wedge brought by tariffs. Consumption is increasing with tariffs if agents are home-biased.

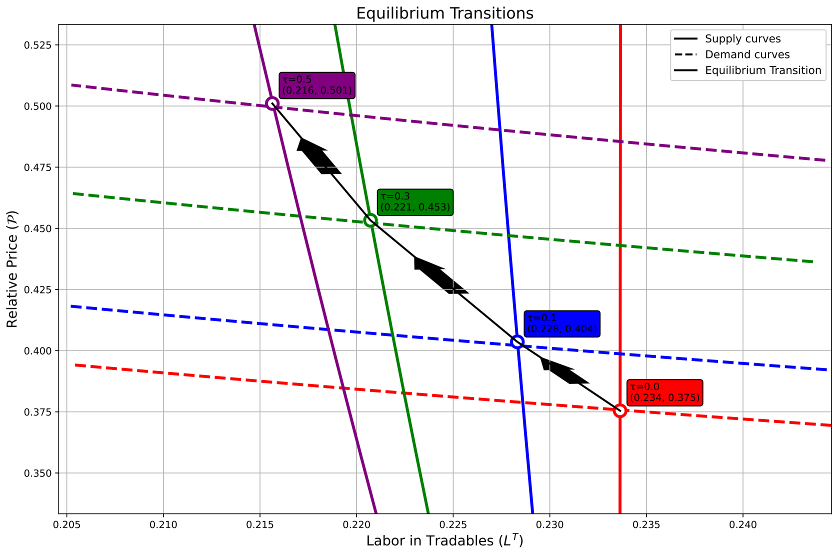

Figure 4: Equilibrium Transitions with Tariffs, Partial Equilibrium Analysis
This figure illustrates the transition of partial equilibrium given foreign country's technology proximity $a_t^* = 0.5$ and size of tradable sector $L_t^{T*} = 0.4$. The demand curve is fairly elastic while the supply curve is inelastic, their intersections pins down the partial equilibrium.

This condition embodies a fundamental mechanism of my theoretical framework: given that non-tradable production exhibits lower productivity relative to tradable production, an increase in the relative price of the tradable consumption bundle generates heightened demand for non-tradable goods and services. Consequently, this price adjustment necessitates a substantial reallocation of labor resources from the tradable to the non-tradable sector. The resulting equilibrium is characterized by a contraction in the size of the tradable sector, as labor migrates toward the relatively less productive but increasingly demanded non-tradable activities.

The amplification mechanism proceeds as follows. A rise in domestic consumption of tradable goods (c_H^T) because of tariffs generates an unproportionately greater demand for labor working in non-tradable sector; because total labor supply is fixed, additional labor shifts toward non-tradable activities, further compressing the tradable sector. As the reallocation is inefficient, the real wage of the home country adjusts downward. In equilibrium, as the home exports are destroyed by tariffs featuring an even lower production of home tradables, market clearing therefore requires an upward adjustment in the relative price $\mathcal{P}$. Consequently, the home economy forfeits export market share and increasingly directs production toward meeting domestic rather than foreign demand. However, as the growth rate is determined by the whole size of tradable sectors, this protectionist action will inevitably make the home country worse off.

Symmetrically, a higher tariff against the foreign country symbolizes shrinking foreign exports while boosting consumption of foreign tradable goods c_F^{T*} through the substitution effects, which consequently moves its demand curve rightward unambiguously. As the relative price $\mathcal{P}$ is defined as the ratio of home price to the foreign, the labor demand function is expected to be with a positive slope. The

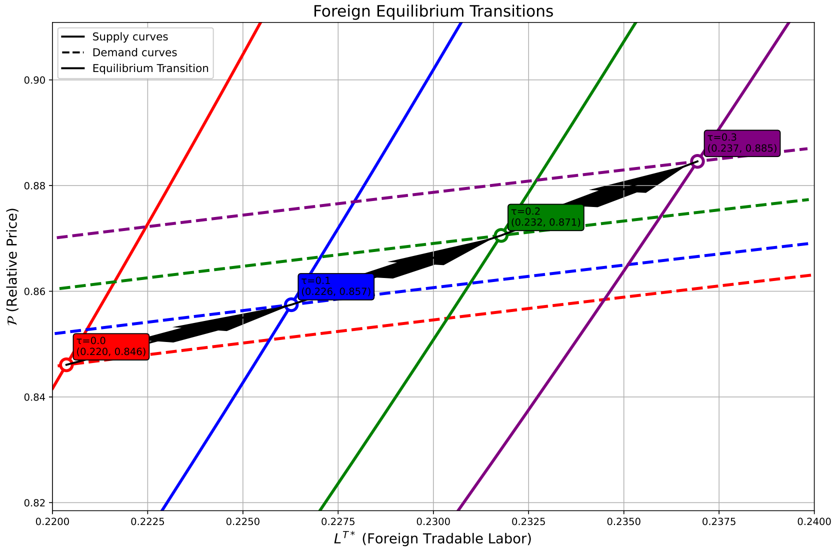

Figure 5: Equilibrium Transitions with Tariffs in Foreign County, Partial Equilibrium Analysis
This figure illustrates the transition of partial equilibrium given foreign country's technology proximity $a_t^* = 0.5$ and home size of tradable sector $L^T = 0.2$. The demand curve is fairly elastic while the supply curve is inelastic, their intersections pins down the partial equilibrium.

demand curve of the foreign country can be shown as follows:

$$
\begin{aligned}
\mathcal{Z}a^* L^{T*} =& \mathcal{H}^* \left(\nu_H^* \left(\frac{(1+\tau_H^{T*})P_H^T}{P^{T*}} \right)^{-\rho} \frac{P_F^T}{P^{T*}} \lambda_F \left(\mathcal{Z}a^* L_f^{T*} - \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_{F,t}^T)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1 \right) \right) \right. \\
& \left. + \nu_F^* \left(\frac{P_F^T}{P^{T*}} \right)^{1-\rho} \lambda_F \left(\mathcal{Z}a^* L_f^{T*} - \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_{F,t}^T)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1 \right) \right) \right)
\end{aligned}
$$

where the mapping $\mathcal{H}^*(\cdot)$ is another pair of feasibility mappings which map the foreign imports from the home country c_H^{T*} to foreign exports c_F^T; and $\lambda_F = \frac{\nu_H^* \left((1+\tau_H^{T*})\mathcal{P} \right)^{1-\rho} + \nu_F^*}{\nu_H^* (1+\tau_H^{T*})^{-\rho} \mathcal{P}^{1-\rho} + \nu_F^*}$ measures the distortions raised by impositions of tariffs. An imposition of tariffs against the foreign country therefore favors the foreign demands shown in the solid lines of figure (5). By the similar reason, we can derive the foreign labor supply curve:

$$
\begin{aligned}
L_f^{T*} = \bar{L} - \lambda_F \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} & \left(\frac{\nu_H^* ((1+\tau_H^*)\mathcal{P})^{1-\rho} + \nu_F^{T*}}{\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{1-\rho}} \right)^{\frac{1}{1-\rho}} \\
& \times \left(\mathcal{Z} L_f^{T*} - (a^*)^{-1} \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_{F,t}^T)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1 \right) \right) \\
& - \frac{(g_f - 1)\left(a_f^* \right)^{\Omega}}{\xi}
\end{aligned}
$$

In this case, the tariff imposed on the foreign country enters the system in a more subtle way. Since the home country issues real bonds denominated in its tradable bundle, the real value of trade deficits is recalibrated by foreign prices. The imposition of tariffs leads to a real depreciation of the foreign tradable bundle and a corresponding real appreciation of trade deficits, which in turn stimulates foreign

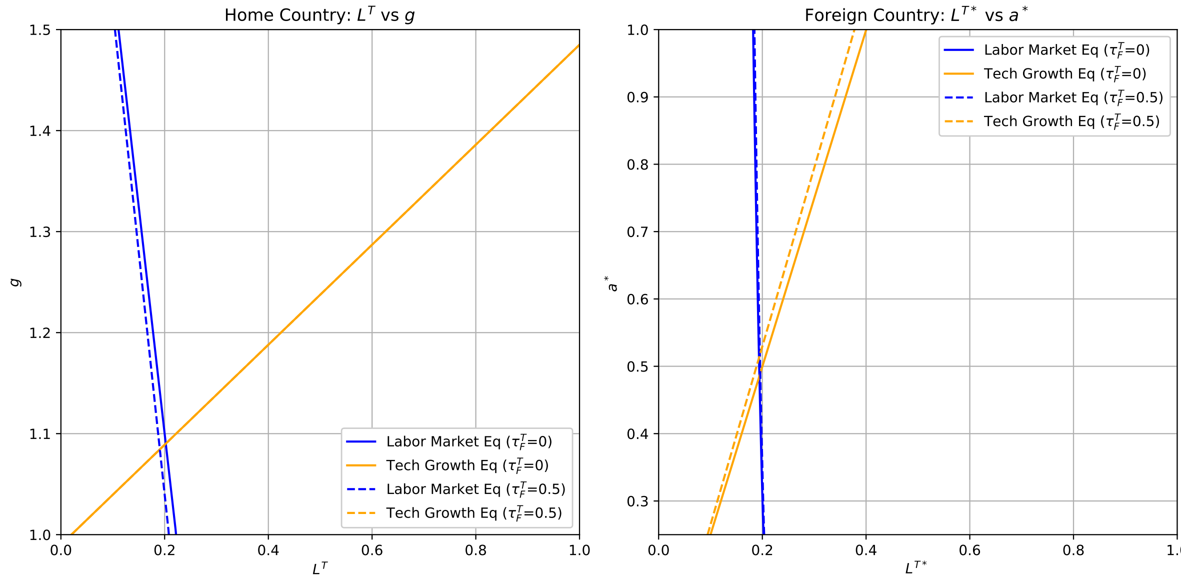

Figure 6: Breakdown the determination of growth rate under unilateral tariffs of home country

production.

PROPOSITION 6. *Under financial integration and κ_H is fixed over time, however, holding the tariffs of the foreign country as constant, the effects of a unilateral increase of tariffs are harmful to its own tradable sector while favoring its opponent. A good policy should be an import subsidy rather than a tax by Lerner Symmerty.*

PROOF The growth rate under steady state is determined by the technology evolution of the supply of labor working in the tradable sector:

$$g_f = \beta \left(\chi \alpha L^T + 1 \right)$$

and the total supply of labor working in the tradable sector by combining equation (65) and equation (62):

$$L_f^T = \bar{L} - \lambda_H \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left(\mathcal{Z} L^T + \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T) \mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*) \beta - 1 \right) \right) - \frac{g_f - 1}{\chi}$$

and:

$$\lambda_H = \frac{\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{1-\rho}}{\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{-\rho}}$$

The parameter λ_H is the key explaining what is going to happen after levying tariffs on foreign goods. Under free trade $\tau_F^T = 0$, there is no distortion of prices, and $\lambda_H = 1$; therefore, we are back to the previous case. However, as unilateral tariffs start to increase, the wedge λ_H is greater than 1 unambiguously. The increasing wedge implies an increase of distortions by importing tariffs: as the imposition of tariffs might stimulate domestic consumption on domestic tradable goods, the substitution effects between tradable and non-tradable bundles are even stronger if **Assumption 1, 2** hold. This implies even more labor is reallocated into non-tradable sectors and the inefficiencies increase. The solution of g_f is pinned down by:

$$\begin{aligned} g_f =& \beta \left(\frac{\alpha(\chi \bar{L} + 1 - \beta)}{1 + \alpha\beta + \lambda_H \frac{1-\omega}{\omega}(1+\alpha)} + 1 \right) \\ &- \frac{\alpha\beta\chi\lambda_H \frac{1-\omega}{\omega}(1+\alpha)}{1 + \alpha\beta + \lambda_H \frac{1-\omega}{\omega}(1+\alpha)} \left[\nu_H + \nu_F \left((1+\tau^T_{F,t})\mathcal{P}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1 \right) \end{aligned} \tag{68}$$

This result has shed light on the close relations among financial integration, trade policies, and growth rate. The increase of tariffs has made the tradable good bundle more expensive, and by substitution effects, consumers will consume more units of goods in the non-tradable sector. On the other hand, the demand for domestic tradable goods increases. However, under the **Assumption 1, 2**, the former effect dominates. This big shift has fostered a boom in the non-tradable sector and relocated more labor into the unproductive non-tradable sector. In consequence, the size of the domestic tradable sector shrinks, the real wage declines, and it is unable to afford the current scale of the research sector. However, on the other hand, the contraction of the domestic tradable sector also shifts parts of the demand for non-tradable goods to the foreign country as the relative price $\mathcal{P}$ has gone up in steady state. In equilibrium, the foreign country employs more workers and the size of the tradable sector expands. As per my assumption, the size of the tradable sector is a growing engine of its productivity. An enlargement of the tradable sector also facilitates a technology boom; the foreign country's production increases more proportionally. To clear the market, the relative price $\mathcal{P}$ should increase even more. Furthermore, as growth rate of home country declines, which in turn helps the foreign country to save labors from research sector, the tradable sector is expected to expand even more.

Theoretically, there is never a shortage of literature explaining the acts of protectionism and the fluctuations of exchange rates (Barattieri et al. (2021), Jeanne and Son (2024)). In absence of active optimal monetary policy (Bianchi and Coulibaly (2025)), the protectionist actions have led home currency to appreciate in steady state. my model also predicts a similar outcome: A radical increase of tariffs (from 0 to 50%) will lead to massive appreciation of home currency (or in this context, a better term of trade), and in turn even generates a boom in the foreign country, the labor supply line shifts rightward even more.

PROPOSITION 7. *Holding the tariffs of the home country constant, the unilateral increase of tariffs is harmful to its tradable sector as more labor is allocated to the non-tradable sector.*

PROOF The growth rate of the foreign country is determined by the technology evolution of the supply of labor working in the tradable sector:

$$(a^*)^\Omega = \frac{\beta\xi\alpha L^{T*}}{g - \beta}$$

and the total supply of labor working in the tradable sector:

$$L_f^{T*} = \bar{L} - \lambda_F \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left(\frac{\nu_H^*((1+\tau_H^*)\mathcal{P})^{1-\rho} + \nu_F^{T*}}{\nu_H \mathcal{P}^{1-\rho} + \nu_F(1+\tau_F^T)^{1-\rho}} \right)^{\frac{1}{1-\rho}}$$
$$\times \left(\mathcal{Z} L_f^{T*} - (a^*)^{-1} \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_{F,t}^T)^{1-\rho} \right]^{\frac{1}{1-\rho}} \kappa_H \left((1+\phi^*)\beta - 1 \right) \right)$$
$$- \frac{(g_f - 1)\left(a_f^*\right)^{\Omega}}{\xi}$$

where:

$$\lambda_F = \frac{\nu_H^* \left((1+\tau_H^{T*})\mathcal{P} \right)^{1-\rho} + \nu_F^*}{\nu_H^* (1+\tau_H^{T*})^{-\rho} \mathcal{P}^{1-\rho} + \nu_F^*}$$

Intuitively, the increase in unilateral tariffs of a foreign country has very limited impacts on home consumption, but draws great impact on the foreign country. The increase of unilateral tariffs on home tradable goods implies a lower level of consumption in tradable goods. The contraction of the tradable sector causes parts of the demand for tradable goods to shift to the home country as the relative price $\mathcal{P}$ is declining. However, as the foreign country is running a trade surplus with the home country in steady state, the expanding home tradable sector is quite mild even if the home price depreciates a lot. As the κ_H is fixed in steady state, the depreciation of the home currency implies a lower level of imports from the foreign country. In the graph, this means the labor supply of the foreign country is going to shrink. The expression of the technology frontier subject to tariffs is:

$$(a_f^*)^{\Omega} = \frac{\alpha\beta\xi \left(\bar{L}^* + \lambda_F \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F \left(1+\tau_F^T\right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \frac{(\kappa_H(1+\phi^*)\beta - 1)}{a_f^*} \right)}{(g_f - \beta)\left(1 + \lambda_F \frac{1-\omega}{\omega}(1+\alpha)\right) + (g_f - 1)\alpha\beta} \tag{69}$$

However, the above discussion is based on the unilateral trade policy, which means I hereby assume the opponent will not retaliate, no matter what its opponent does. Unfortunately, this argument may not be the case in reality. A better way to keep more manufacturing jobs is not through a mercantilist approach by imposing strict positive tariffs. More manufacturing jobs require a bigger size of the tradable sector by subsidizing imports. The intuition is that the amount of goods a country can produce is positively associated with the amount of goods it can import. Moreover, an increase in the selling of tradable goods will also cause firms in the tradable sector to scale up production and hire more labor. Imposing a tariff on foreign goods will result in lower exports and, therefore, a shrinking tradable sector.

If a country has refrained from preference on tradable goods and is subject to **Assumptions 1 and 2**, the claim that tariffs yield positive outcomes for domestic industry and facilitate the repatriation of previously offshored production is internally inconsistent and analytically problematic. The surge in tariffs favors the terms of trade and appreciates the relative price. This results in an even lower level of exports from domestic industries and even greater dependence on foreign counterparts. Furthermore,

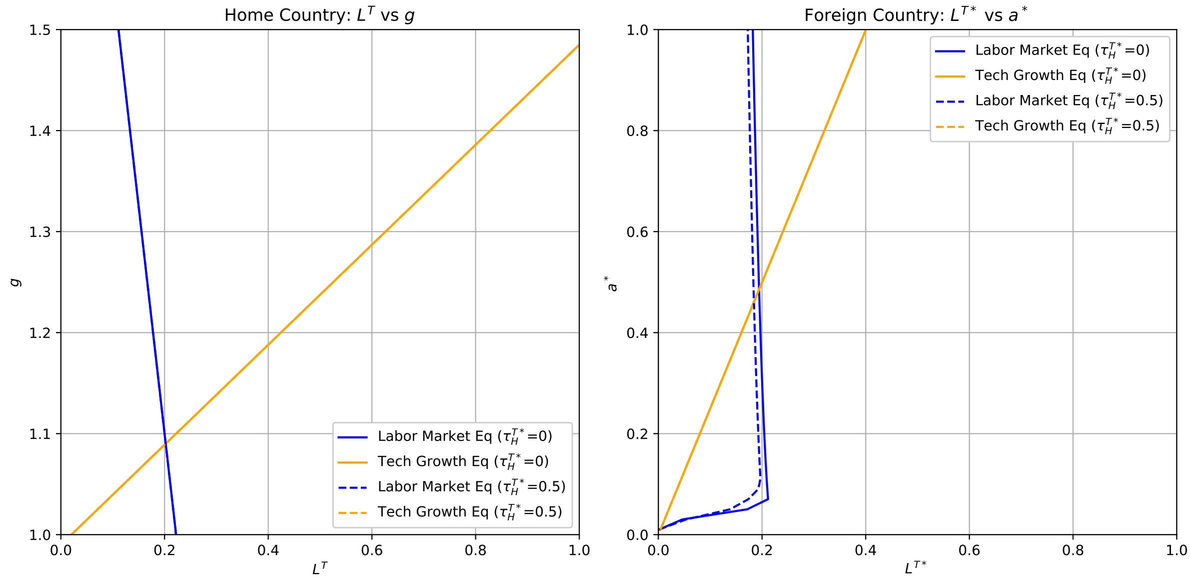

Figure 7: Breakdown the determination of growth rate under unilateral tariffs of foreign country

this argument does not consider the countermeasures and retaliatory actions of the opponents. According to my previous discussion, an equal size of retaliation in tariffs from opponents will also reduce the size of the tradable sector and dampen long-term growth.

The figure 8 illustrates the comparative statistics over different values of tariffs and trade imbalances if Assumptions **1 and 2** are satisfied. The first two rows illustrate the behavior of the key variables in steady states under varying unilateral tariff levels from home and foreign countries, respectively. These messages have been well documented and explained in previous sections: import tariffs never deliver unilateral benefits but result in pure losses in the context of endogenous growth. In the third row, I examine trade wars under a reciprocal scale of tariffs. Such trade conflicts generate a global recession and a contraction of the tradable sector, as symbolized by a slowdown of world growth and a decline in global consumption. In the last row, I examine the dynamics of growing trade imbalances under a constant tariff level, an effect characterized by the paper Benigno et al. (2025). As discussed previously, such imbalances induce a reallocation of labor from the productive to the unproductive sector within the home country, leading to downward adjustments in real wages and a sluggish expansion of consumption, thereby marking a weakening global demand.

4 Dynamics

So far, my analysis has centered on the model's steady-state equilibrium. Let us now shift our attention to its dynamic properties. Because the framework assumes perfect foresight and omits adjustment costs, the transition to a new equilibrium can, in principle, occur within a single period if the shocks are temporary. Even so, it is analytically valuable to articulate a gradual decoupling path for the two economies

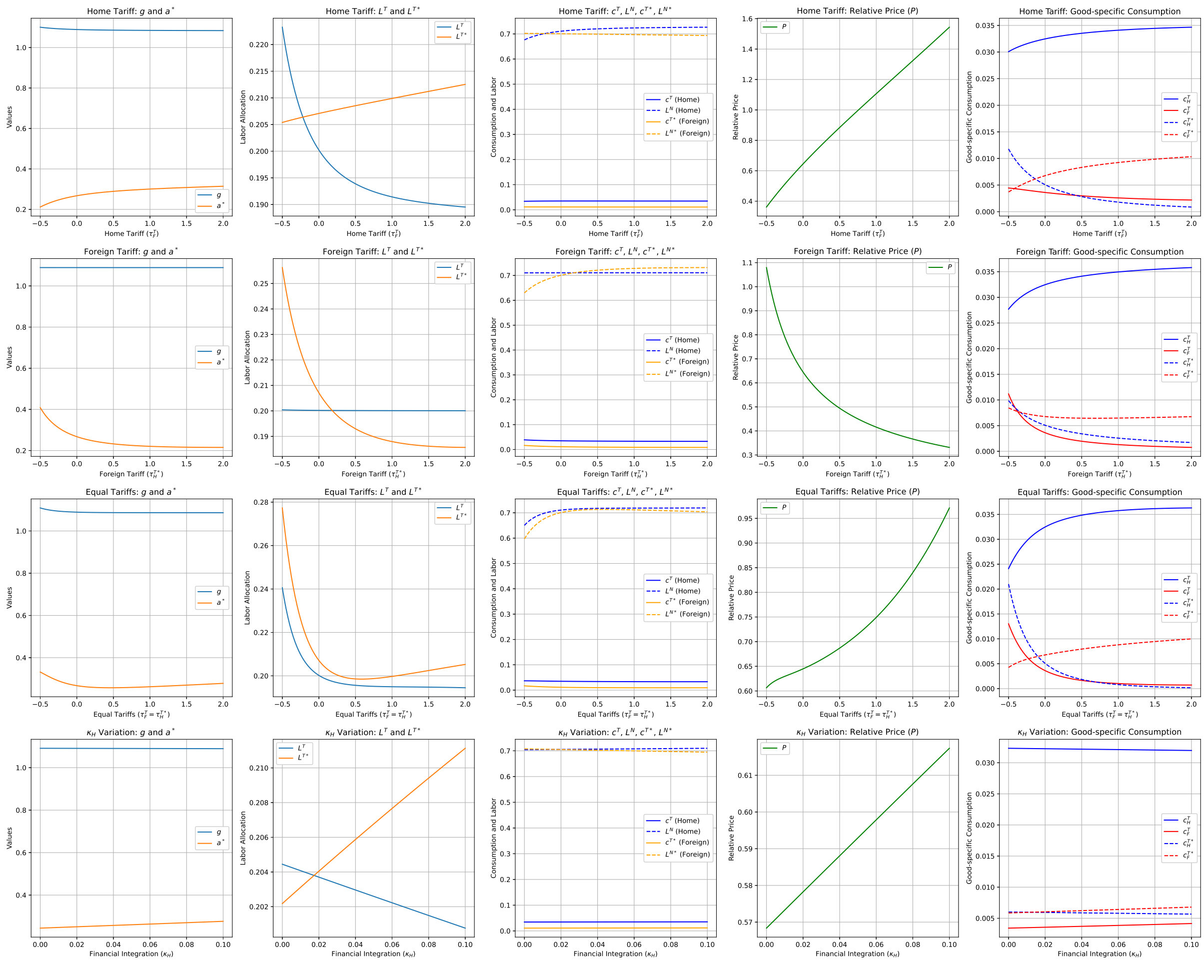

Figure 8: Technology evolution under tariffs, unilateral and bilateral

and to trace the sequence of intermediate states that such a trajectory would imply.

Now consider the social planner of the home country who plans to impose tariffs against the foreign country, she accidentally imposes innovation $\varepsilon_{H,t}$ into the environment. However, given the sluggish adjustment of R&D, the economy will gradually converge to the steady state if the shock is persistent. My analyses feature the new channel between sector allocation and labor adjustment under persistent tariff shocks echoing related studies like: Guerrieri et al. (2021). Some recent papers also illustrate the sluggish adjustment of the imposition of tariffs through the channel of uncertainty (Werning et al. (2025)), policy targeting (Bianchi and Coulibaly (2025); Monacelli (2025)), and the multiplicity of the system. In this section, I exclusively embed endogenous growth into the sollowing analysis. I will examine three cases: unilateral tariffs from either the home country or the foreign country, and a trade war.

I hereby assume that the following AR(1) process exists to characterize the unilateral tax regime from the home country:

$$\tau_{i,t}^{T} = \rho_{i,\tau}\tau_{i,t-1}^{T} + (1-\rho_{i,\tau})\tau_{i}^{T} + \varepsilon_{i,t} \tag{70}$$

where τ_i^T is assumed to be a finite constant that represents the final tariff target from the country i, and $\rho_{i,\tau}$ measures the persistence of the tariff shock. For simplicity, I hereby assume the economic environment is derived from a free-trade steady state by fixing $\tau_i^T = 0$, while being subject to unanticipated tariff shocks imposed by the social planner characterized by $\varepsilon_{i,t}$.

4.1 Transitions of Unilateral Tariff Regime

The numerical exercise is shown in Figure 9. I impose an MIT shock on $\varepsilon_{H,t}$ with a size of 1% into the environment. I hereby assume the foreign country does not respond to the tariffs, and **Assumption 1 and 2** hold. The potential threats of tariff impositions expose the foreign country to an amplification mechanism: since the foreign country's productivity is dependent on both home procurement and productivity, its R&D investment is counter-cyclical and subject to its proximity to the frontier.

I calibrate the home country's steady-state current account (CA) imbalance to 4% of GDP to align with the long-run persistent global imbalances of the USA. We normally observe a transient adjustment of the current account in response to the trade war shock from the data. Therefore, to make my model more realistic, I introduce a convex adjustment cost on the change in the country's net foreign asset position[6]. This cost prevents an instantaneous jump in the CA and instead generates a gradual transition path towards the steady state. The complete equilibrium condition of the model can be found in the following.

The imposition of a unilateral tariff exerts an immediate adverse impact on a country's Total Factor Productivity (TFP) growth. Subsequently, TFP growth exhibits an overshooting pattern, temporarily exceeding the original baseline before eventually converging to the new steady state, as illustrated in figure (9). This dynamic response mirrors the empirical findings presented in Figure (2) before.

Quantitatively, the calibration suggests that a 1% tariff shock against the foreign country induces an instantaneous loss of 0.04% in home TFP. Similar to many quantitative models incorporating investment adjustment costs (Mendoza et al. (2009); Jermann and Quadrini (2012)), my model features a gradual adaptation process in R&D activities. The unanticipated tariff shock instantly boosts demand for domestic tradable goods, reallocating labor into the production of these goods and crowding out labor specialized in research, which explains the initial drop in TFP growth. Concurrently, the sharp decline in demand for foreign goods causes a surge in the relative price of home tradable goods. This relative price shock triggers substitution effects between tradable and non-tradable bundles, shifting demand towards non-tradable goods and subsequently crowding labor into the non-tradable sector. Regarding consumption, the initial shock causes the consumption of the tradable bundle to decline due to the contraction in the consumption of foreign tradable goods. However, as labor is gradually reallocated back towards the production sector (facilitated by the R&D adjustment costs), agents' consumption of tradable goods overshoots and exceeds the baseline level before settling at the new long-run equilibrium.

[6]Hereby I strictly follow the canonical literature from Schmitt-Grohé and Uribe (2003) by introducing portfolio adjustment costs. I introduce the adjustment cost in order to allow a temporary deviation from the debt limits.

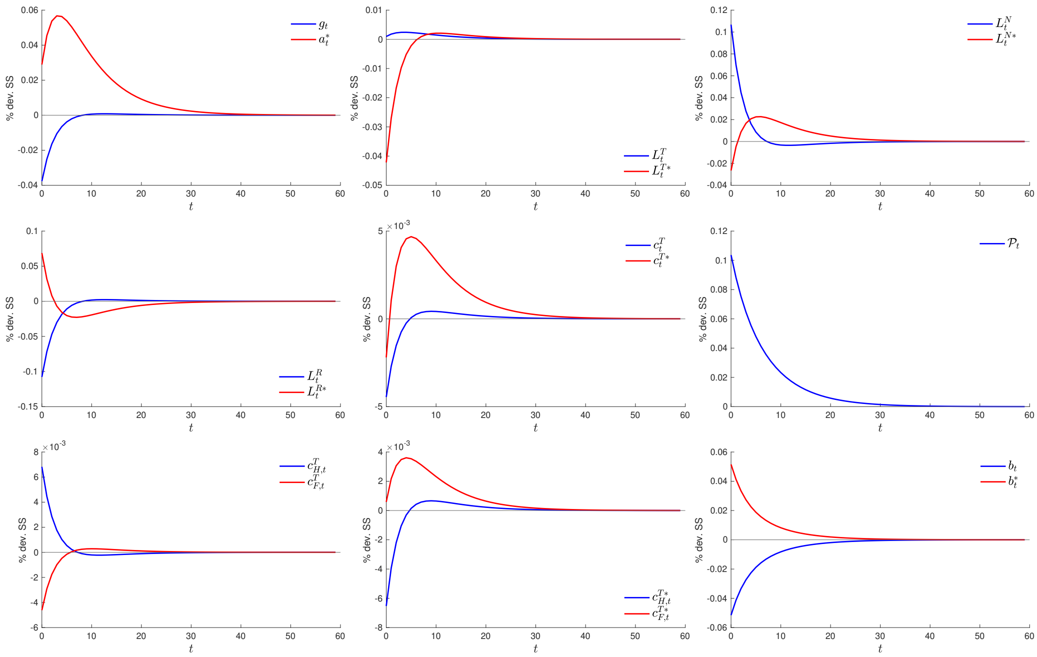

Figure 9: Dynamics under Unilateral Tariff Regime: Home Country.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{H,t} = 1\%$, I hereby assume no retaliations from the foreign country. **Assumption 1, 2** holds.

Symmetrically, the tariff shock imposed by the home country causes the foreign economy to experience an immediate contraction in its tradable sector due to the reduction in export demand. As their existing production capability becomes relatively excessive under these tariffs, labor is immediately reallocated toward the research and non-tradable sectors. A key driver of this initial adjustment is the consumption substitution effect: foreign tradable goods become relatively cheaper, leading to an expected increase in their consumption and initially crowding out labor from the non-tradable sector. However, foreign firms internalize these complex dynamics. While they may attempt to scale up production to meet the internal demand shift, the sluggishness of technological evolution, combined with the initial supply response, leads to excessive productive capability in the tradable sector over the medium term. Given that research movement in my model is profit-driven, this excessive productive capacity renders R&D less profitable. Consequently, foreign research activity slows down, causing TFP growth to slump below the baseline before eventually converging to the new long-run steady state. Finally, as the demands for foreign industrial goods decline, the trade deficit contracts.

As the figure 10 has shown, the unilateral imposition of tariffs against the home country is unlikely to grant any benefits to the foreign country. Given that the foreign country currently runs a trade surplus, such tariffs would reduce its export volume, thereby undermining its manufacturing capacity. Consequently, this contraction in external demand is expected to slow the foreign country's overall growth trajectory.

The imposition of unilateral tariffs by the foreign country initially stimulates its residents' demand for domestic industrial goods, creating a surge of expansion in the tradable sector due to substitution effects relative to the now-costlier home industrial goods. Foreign firms internalize this demand dynamic and respond by scaling up production, which initially crowds out research activities and consequently leads to a temporary increase in the price of foreign goods, thereby raising demand for non-tradable services. However, as foreign industrial goods become more expensive, the rising profitability in the tradable sector drives research to be more profitable, leading to an expected growth in the research sector thereafter. This expansion, coupled with agents realizing that prices will eventually fall and converge to the baseline, causes the demand for non-tradable goods to decline over time. The overall dynamics result in an excessive expansion of the research sector relative to the baseline, ultimately leading firms to become over-productive and cut back on employment as the economy gradually converges to the new steady state.

Symmetrically, the tariffs imposed by the foreign country undermine exports from the home country, causing an immediate shrinkage of demand for home tradable goods. Despite the resulting higher prices of foreign goods, the reduction in export volume forces the home country to cut back on employment in its tradable sector. Concurrently, firms internalize the expectation of weaker global aggregate demand and downsize, leading to a significant reallocation of labor into the non-tradable sector. This labor reallocation, being structurally inefficient in the context of the shock, depresses the country's real wage rate, which in turn causes an even weaker aggregate demand domestically. The combination of depressed demand and inefficient resource allocation exacerbates the country's external position. As shown in Figure (10), the home country's trade imbalances widen, necessitating a much longer recovery time for domestic consumption to return to its baseline level.

4.2 Trade War and Retaliations

Now, I hereby assume the world starts a trade war led by the home country. The transient tariff shock starts at $t = 0$, and there is no anticipation of the opponent's retaliation at that time. I hereby assume the foreign country retaliates with the exact same scale but lags by one period.

The dynamics of a bilateral trade war is plotted in figure (11)[7]. The bilateral tariff war generates considerable TFP losses primarily derived from market size effects. The tariffs imposed by the home country initially stimulate domestic production, symbolizing a transient expansion of its tradable sector due to immediate substitution effects. However, subsequent unexpected foreign retaliation effectively wipes out this potential growth momentum, trapping the global economy in a deep recession. Home country firms, recognizing the severely shrinking market size, suddenly reduce their scale in response to the intensifies trade tensions. In contrast, a foreign country experiences a partial, temporary benefit due to a decline in the marginal costs of conducting research, a gain that subsequently boosts foreign consumption for several periods. In conclusion, initiating a trade war is the worst strategic choice, as it yields only a very transient boom in the home tradable sector, while ultimately resulting in substantial

[7]The simultaneous retaliation is reported in Figure (19)

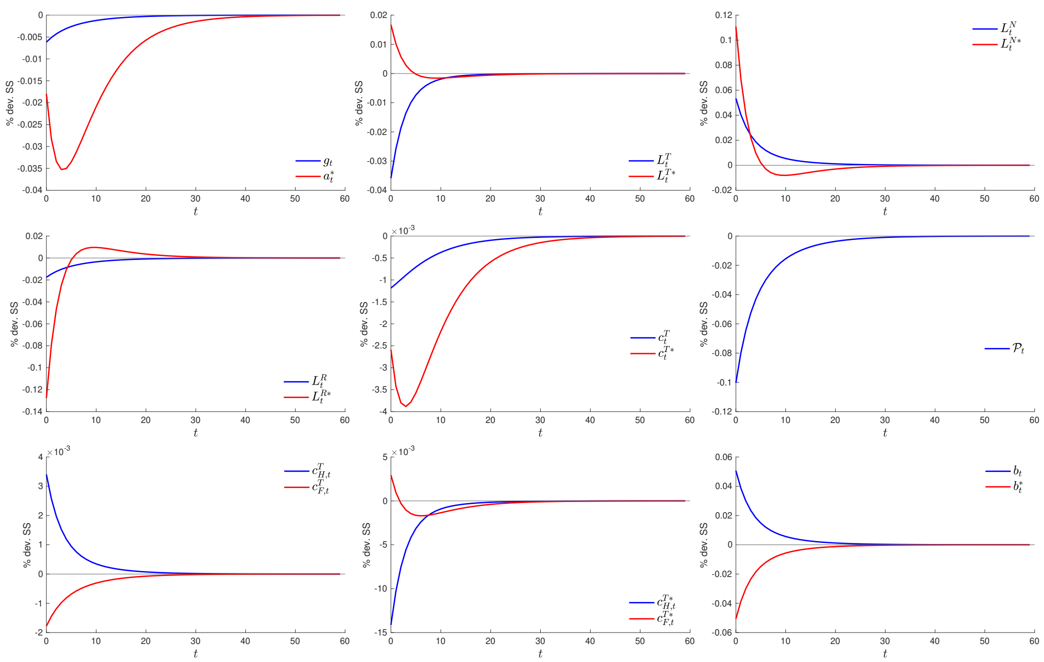

Figure 10: Dynamics under Unilateral Tariff Regime: Foreign Country.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{F,t} = 1\%$, I hereby assume no retaliations from the foreign country. **Assumption 1, 2** holds.

TFP losses and a persistent and deep decline across the global tradable sector.

COROLLARY 1. *The foreign country is attempting to retaliate against the home country's trade aggression, provided that the time horizon for consideration is the short run and the imposed tariffs are perceived as transient.*

5 Nash Equilibrium and Social Planner Solution

The persistent existence of both visible and invisible trade barriers worldwide presents a puzzle that challenges the typical advocacy for free trade. One possible explanation for this phenomenon is the failure of cooperation among countries (Bagwell and Staiger (1990); Bagwell and Staiger (1999); Staiger and Tabellini (1987)). My prior analysis suggests that the imposition of permanent tariffs may transiently result in a higher level of future consumption for both tradable and non-tradable goods. However, this outcome requires that the social planner makes a critical trade-off between this rising consumption and a permanent decline in productivity growth.

In this section, I integrate my theoretical framework into game theory to rationalize the seemingly inefficient imposition of tariffs. I specifically decompose the agent's utility function into two parts: consumption and growth. I aim to solve for the countries' Nash equilibrium under scenarios of full and partial attention regarding long-run growth. I argue that if a social planner is sufficiently patient and places full weight on future growth (full attention regarding growth), then free trade is the strategy that always maximizes the country's utility. Conversely, if the policymaker is impatient, they will always be

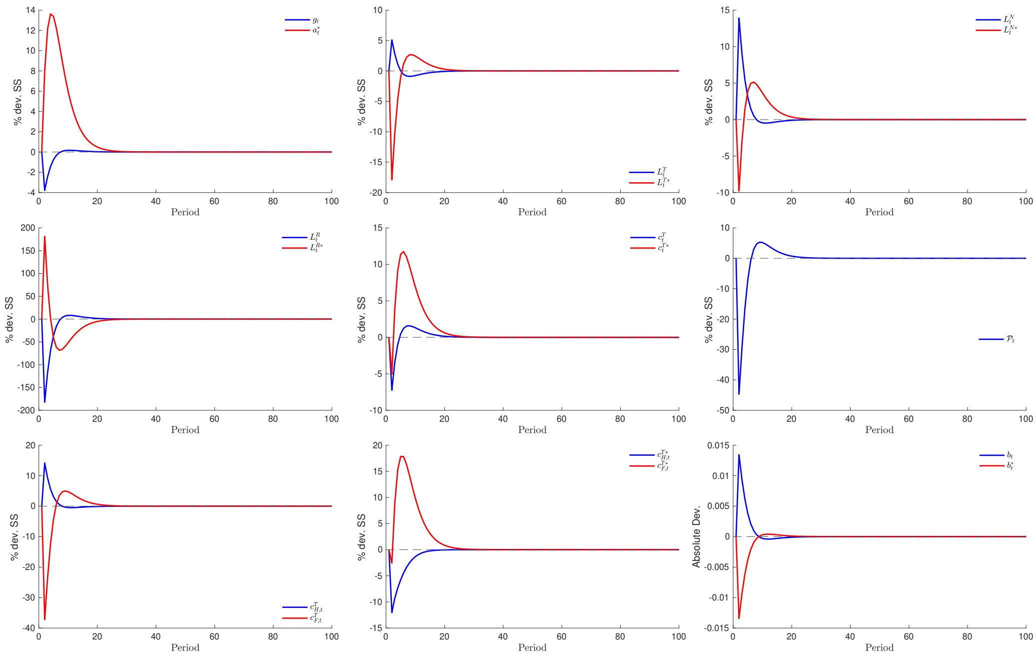

Figure 11: Dynamics under a Trade War with Sequential Retaliation
This figure illustrates the adjustment dynamics of key macroeconomic variables following the tariff war. I hereby assume $\rho_\tau = 0.8$, $\varepsilon_{H,t} = 1\%$, and $\rho_\tau^* = 0.8$, $\varepsilon_{F,t} = 1\%$. **Assumption 1, 2** holds.

incentivized to impose tariffs. Given that TFP growth enters the system in a non-trivial and subtle manner, the policymaker may exhibit a greater preference for current consumption while being inattentive to the consequences for long-term growth. Finally, I hereby consider the case of trade and technology decoupling. I will consider two cases: the impacts of increasing research intensity (ξ) and technology independence ($1-\Omega$) on tariffs.

5.1 Nash Equilibrium with Full Intention on Growth

First, I try to modify the agent's utility function in steady state.

LEMMA 1. *Assume there is no gradual regime of imposing tariffs and A_0 given, the utility function can be modified as follows:*

$$\mathcal{U}\left(\{C_t\}_{t=0}^{\infty}\right) \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t \log\left(C_t\right) - \omega \log A_0 = (1-\omega)\log C^N + \omega \log c^T + \frac{\omega\beta}{1-\beta}\log g \tag{71}$$

Where the variables C^N, c^T, and g refer to the values of the variables under steady state subject to the pair of tariffs $\{\tau_F^T, \tau_H^{T*}\}$ and the implicit saving subsidies ϕ^*. As my model shows, there is no adjustment cost; thus, an imposed tariff will imply an immediate transition; For this reason, we are safe to drop the time variable t.

LEMMA 2. *Symmetrically, the foreign country's welfare function can be modified as:*

$$\mathcal{U}\left(\{C_t^*\}_{t=0}^{\infty}\right) = (1-\omega)\log C^{N*} + \omega \log c^{T*} \tag{72}$$

The technology proximity does not appear in the modified system, as it cancels out in the steady state. Moreover, since the foreign country does not directly determine R&D decisions in the home country and takes g as given, the TFP term will not appear in its welfare function either. Furthermore, since imposing additional tariffs may harm its own tradable sector and consumption, the foreign country will also prefer the free trade policy. The modified welfare function (71) and (72) have upper and lower bounds for a pair of tariffs $\{\tau_F^T, \tau_H^{T*}\} \in [0, \infty)$. In other words, the Nash equilibrium must exist and be unique. Since the welfare function is continuous, monotonic, and the welfare function under free trade and fully autarky (tariffs go to infinity) is finite, the global maximization and minimization must exist no matter the shape of the curves.

COROLLARY 2. *Under the steady-state equilibrium conditions, free trade policies in both countries maximize the modified welfare functions (71) and (72).*

PROOF This happens as the term $\frac{\beta}{1-\beta}$ makes the growth rate $g > 1$ dominate in the welfare function. As I have previously proved, positive tariffs always have contraction effects on the domestic tradable sector. If a country is forward-looking and fully intentional about long-term growth, the free trade equilibrium is always optimal.

COROLLARY 3. *In the Nash equilibrium, the foreign country treats the growth rate g as exogenous and, consequently, refrains from any retaliatory behavior. Specifically, it sets its tariff rate to zero, $\tau_{H,t}^T = 0$, for all t.*

PROOF The intuition behind this result is as follows: the imposition of tariffs raises the relative price of foreign goods, thereby worsening the foreign country's trade surplus. Consequently, since a unilateral tariff yields no net welfare gain from any perspective, the foreign country has no incentive to impose a positive tariff in the first place.

5.2 Nash Equilibrium with Partial Intention on Growth

TFP growth enters the given economy in a non-trivial yet subtle manner. Impatient politicians may impose excessive tariffs to secure short-term welfare gains, but this comes at the expense of long-run productivity growth. Seminal contributions—including Cukierman and Harstad (2020, 2025); Farhi and Tirole (2012); Jackson and Yariv (2014, 2015); Meltzer (1989); and Tabellini (1991)—have emphasized that politicians tend to exhibit present bias and pursue time-inconsistent policies. This behavior arises either because the temporary concentration of power generates additional unilateral benefits for incumbents who engage in radical and contentious policies, or because successive policymakers lack the mechanisms to hold their predecessors accountable. Therefore, in this section, I re-examine the Nash

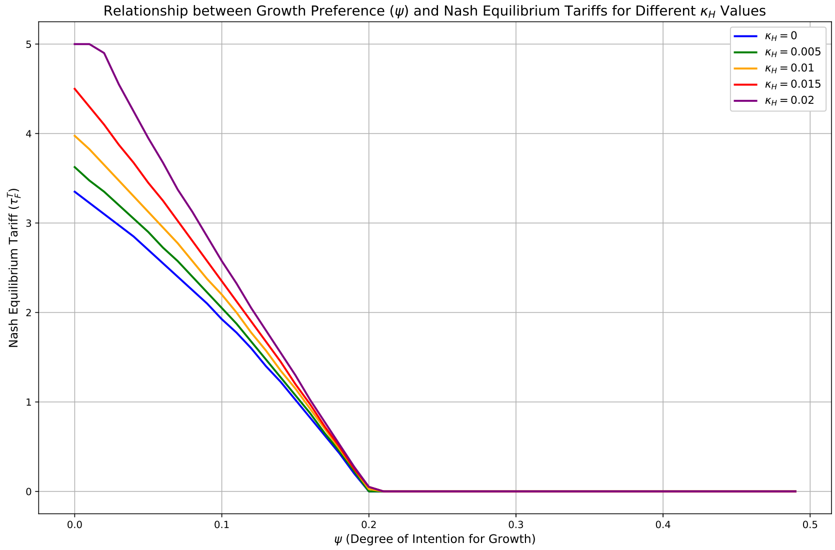

Figure 12: Nash Equilibrium with Full and Partial Intention on Growth: Varied Trade Imbalances
This figure illustrates the solutions of Nash Equilibrium under different degrees of intention ψ, and different levels of trade imbalances (κ_H). A deterioration of trade deficits of the home country will induce them to increase the tariffs progressively. In this figure, I put a upper bound of tariff of 500%.

equilibrium if the politicians in the home country have partial intention on growth. I further modify the home country welfare function (71) as:

$$\widetilde{\mathcal{U}}\left(\{C_t\}_{t=0}^{\infty}, \kappa_H\right) = (1-\omega)\log C^N + \omega \log c^T + \psi \frac{\omega\beta}{1-\beta}\log g \tag{73}$$

where $0 < \psi < 1$ measures the politician's degree of intention to the long-run growth.[8] The figure (12) shows the relationship between the degree of intention on growth (growth preference, ψ) and the tariff level imposed by the home country varied by different levels of trade imbalances. When trade is balanced (depicted by the blue line), the growth externality channel is shut down, productivity growth is maximized, and the home country prefers a lower level of tariffs relative to the case of imbalanced trade. However, as trade imbalances become more pronounced, tariffs rise progressively. This outcome is intuitive: as politicians in the home country grow increasingly impatient while trade imbalances spiral out of control, they resort to imposing ever more radical and progressive tariff levels. If we consider the home country exhibiting a trade imbalance of approximately 4% (that is, $\kappa = 0.005$), and maintaining an average tariff rate of 30% against the foreign country, its policy stance toward growth can be characterized by $\psi = 0.18$. This value implies that, compared to a fully forward-looking policymaker who internalizes growth over an infinite horizon, the politician effectively cares about only 77 periods of future growth.

[8] Alternatively, we can consider the policymaker has following discounted flow on growth: $\psi\beta$, $\psi\beta^2$, $\psi\beta^3$, $\psi\beta^4$, ... As the growth rate only matters for the consumption to the next period, the whole summation of growth can be rearranged as $\psi\sum_{t=1}^{\infty}\log g$.

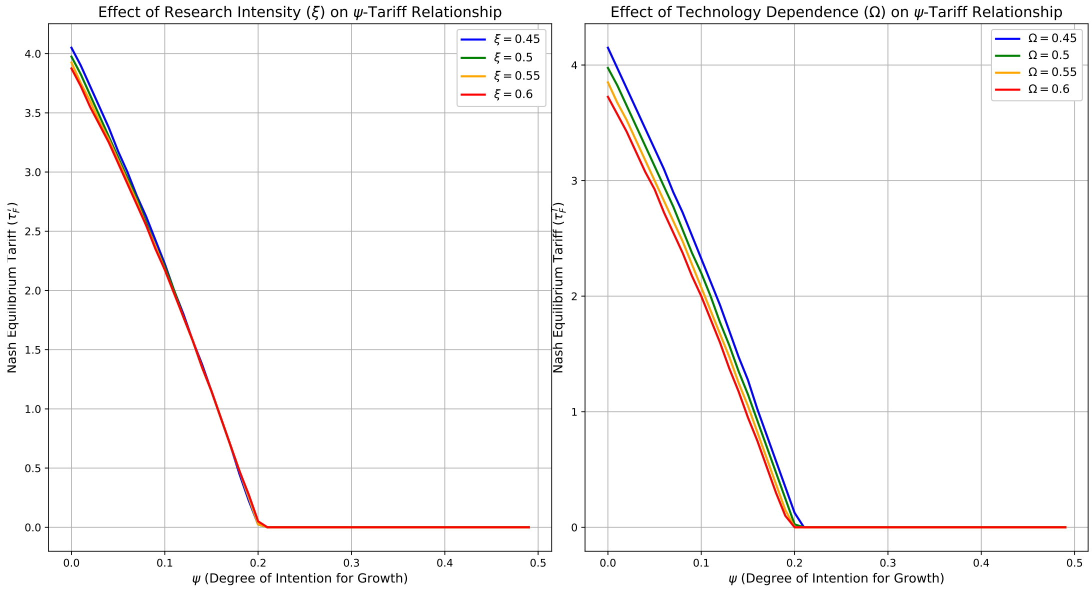

Figure 13: Nash Equilibrium with Full and Partial Intention on Growth: Varied Research Intensity and Technology Dependence

5.3 Trade and Technology Decoupling

The broader backdrop is an epochal shift in which the two dominant economies are deliberately pursuing decoupling. This process reflects not only increasing disruptions to global supply chains and growing trade fragmentation, but also a deeper separation in technology and even cultural interactions. There is a booming field of literature documenting trade fragmentation and growing supply chain disruptions: Autor et al. (2013); Amiti et al. (2019); Barrot and Sauvagnat (2016); Boehm et al. (2020); Carvalho et al. (2021); Fajgelbaum et al. (2020). In this subsection, I discuss the impacts of increasing research intensity (ξ) and technology independence ($1 - \Omega$) on tariffs.

The figure (13) depicts the tariff response of the home country under variable levels of research intensity (ξ, shown in the left panel of figure 39) and technology independence ($1 - \Omega$ shown in the right panel of figure 39). In my model, as the debt limit κ_H is fixed, the growing productivity a^* of the foreign country comes with an increasing level of international trade, thereof a growing market size of the tradable sector, which in turn increases the productivity growth of the home country. The increase of research intensity (ξ) from the foreign country marks an improvement in welfare for all countries: as research activity is more productive than before, fewer people are expected to work in the research and hence the tradable sector. Further, this also implies growing trade activities worldwide; exports of the home country are increasing, and every party benefits from this process. However, if the technology independence rises, which implies the flipped story: the foreign country relies more heavily on its own technology, this could possibly decrease the technology proximity of the foreign country[9] and signal a

[9]This argument requires further clarification: it may seem counterintuitive to claim that a country's productivity declines as its dependence on the home country diminishes. The explanation lies in the one-dimensional evaluation of technology. Because productivity is always benchmarked against the home country, any deviation from this benchmark is measured as a relative decline. In a two-country model, such a representation is unavoidable for quantification. Alternatively, one may reinterpret the

larger tradable sector in the foreign country, which in turn intensifies the competition and prompts a lower productivity growth of the home country.

6 The Two Country Model with Convenience Yield

There exists a well-documented strand of literature emphasizing the emergence of global saving gluts (Bernanke (2005)) and the associated imperative of safe assets in international financial markets. The general shortage in the global supply of safe assets (Caballero (2006); Caballero et al. (2008); Gourinchas and Rey (2007)) has been identified as a major driver of persistently low interest rates and the accumulation of substantial global trade imbalances, particularly involving the United States as a supplier of such assets. This environment has also intensified concerns over liquidity traps and their interaction with resource misallocation. Recent studies (Gopinath et al. (2017)) highlighted the strong link between low interest rates, capital misallocation, and productivity dispersion across firms — a context that is closely related to this paper.

Returning to my model, the internationalization of the U.S. dollar comes at the cost of worsening trade deficits, which in turn raises global concerns regarding the sustainability of U.S. government debt. The resulting accumulation of national debt has fueled political movements such as "Make America Great Again (MAGA)", emphasizing the need to revitalize domestic manufacturing and reduce external dependence. To connect my analysis with the traditional literature in international finance, I extend our framework by allowing the foreign country to exhibit a preference for convenience yield, which is a non-pecuniary benefit derived from holding U.S. safe assets. The inclusion of this feature introduces an additional externality through the revaluation of financial assets, enriching our understanding of how tariffs, global imbalances, and growth interact. In particular, the convenience yield alters the elasticity of foreign demand for U.S. debt, thereby changing the general equilibrium response of interest rates, capital flows, and long-run technological dynamics.

An insightful prediction of my model is that the imposition of tariffs, rather than mitigating global imbalances, may actually exacerbate them. This outcome arises because the demand for assets is fairly inelastic as characterized in the equation (31). And therefore a unilateral imposition of tariffs will appreciate the real value of debts and hence exacerbates the national current account. However, this prediction may not hold if we extend the model to incorporate a convenience yield in the preferences of the foreign country. In my baseline framework, the demand for U.S. debt is assumed to be perfectly elastic. Introducing a convenience yield would instead generate a downward-sloping demand curve for safe assets, potentially altering the equilibrium relationship between tariffs, interest rates, and trade balances. I follow the literature of Caballero et al. (2008), Krishnamurthy and Vissing-Jorgensen (2012), Jiang et al.

argument by considering a scenario in which the foreign country invents cutting-edge products outside the technological scope of the home country. In that case, the foreign innovations would imply a relatively smaller market when viewed from the perspective of the home country.

(2024), suppose that now I modify the preference of the foreign country's preference as follows:

$$v^*(C_t^*, B_t^*) = \log(C_t^*) + \vartheta \log(B_t^*)$$

while keeping the preference of the home country unchanged. I further modify the budget constraint of the home country to allow for long maturities:

$$P_t^T C_t^T + P_t^N C_t^N + Q_t B_{t+1} \leq (\delta Q_t + P_t^T) B_t + W_t \bar{L} + \Pi_t^T + \Pi_t^N \tag{74}$$

The bond is sold with a price of Q_t and pays P_t^T per unit upon maturing, where the Q_t represents the price of bonds, $1-\delta$ represents the fraction of bonds maturing in each period. By symmetry, the foreign country's budget constraint can be rearranged as:

$$P_t^{T*} C_t^{T*} + P_t^{N*} C_t^{N*} + Q_t B_{t+1} \leq (1+\phi^*)(\delta Q_t + P_t^T) B_t + W_t^* \bar{L}_t^* + \Pi_t^{T*} + \Pi_t^{N*} - \mathcal{T}_t^* \tag{75}$$

by balanced budget:

$$\mathcal{T}_t^* = \phi^*(\delta Q_t + P_t^T) B_t$$

This modification brings us a new Euler equation in optimum:

$$Q_t = \beta(1+\phi_t^*) \frac{u'(C_{t+1}^*)}{u'(C_t^*)} \frac{\delta Q_{t+1} + P_{t+1}^T}{P_{t+1}^*/P_t^*} + \beta \frac{v'(B_{t+1}^*)}{u'(C_t^*)/P_t^*} \tag{76}$$

As now we have an additional part of convenience yield, the **Assumption 1** is not necessarily to be satisfied. However, it may provide us with more insights in equilibrium, I will bring more analyses in the later section. To obtain the steady state, I define $\mathcal{Q} = \frac{Q}{P_F^T}$.

PROPOSITION 8. *The steady state implies the following condition with convenience yield:*

$$g = \beta(1+\phi^*) \frac{\delta \mathcal{Q} + \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\mathcal{Q}} + \vartheta \frac{\beta}{\omega} \frac{\left[\nu_H^* \left((1+\tau_H^{T*})\mathcal{P}\right)^{1-\rho} + \nu_F^*\right]^{\frac{1}{1-\rho}}}{\mathcal{Q}} \frac{c^{T*}/a^*}{\kappa_H} \tag{77}$$

equivalently, the term $R = \frac{\delta \mathcal{Q} + \left[\nu_H \mathcal{P}^{1-\rho} + \nu_F (1+\tau_F^T)^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\mathcal{Q}}$ represents the gross interest rate like before. The convenience yield brings us an additional term that raises the bond prices $\mathcal{Q}$ in equilibrium while reducing the gross interest rate R. Moreover, an increase of preference on productivity also implies a higher growth, and the intuition is salient: a higher demand on home's bonds require a higher growth for compensation. Now, the equation of balance of payment (60) can be modified as follows:

$$\begin{aligned} c_F^T - \mathcal{P}c_H^{T*} = \vartheta \frac{g-\delta}{g-(1+\phi^*)\beta\delta}\frac{\beta}{\omega}\left[\nu_H^*\left((1+\tau_H^{T*})\mathcal{P}\right)^{1-\rho}+\nu_F^*\right]^{\frac{1}{1-\rho}}\frac{c^{T*}}{a^*} \\ -\frac{(1-\beta(1+\phi^*))\,g}{g-\beta(1+\phi^*)\delta}\left[\nu_H\mathcal{P}^{1-\rho}+\nu_F(1+\tau_F^T)^{1-\rho}\right]^{\frac{1}{1-\rho}}\kappa_H \end{aligned} \tag{78}$$

The maturity of bonds and the convenience yield provide the home country with an additional channel to mitigate trade imbalances. The left-hand side of the equation captures the trade deficit of the home country, while the right-hand side represents its net foreign asset position. The first term reflects the reevaluation effects arising from the convenience yield. Unlike the prediction of Itskhoki and Mukhin (2025), these reevaluation effects do not vanish even when $\delta = 1$. As δ decreases—implying that a larger fraction of bonds matures—the magnitude of these effects increases. The second term corresponds to the net bond payments, which partially offset trade imbalances. Specifically, under the condition $\beta(1+\phi^*) < 1$, the reevaluation effects from the convenience yield are minimized as $\delta \to 1$ and maximized as $\delta \to 0$. However, when $\beta(1+\phi^*) = 1$, bond maturity no longer affects the outcome. Under **Assumption 1**, where $\beta(1+\phi^*) > 1$, the term net bond payment turns positive, and a longer maturity of bonds will introduce greater trade imbalances.

DEFINITION 4 (Balanced Growth Path and Steady State with Convenience Yield)**.** Assume, in steady state, assume the domestic inflation rate is 0. The steady-state equilibrium is a series of constant real allocations of $\{g, a^*, c^T, c^{T*}, c_H^T, c_H^{T*}, c_F^T, c_F^{T*}, L^N, L^{N*}, L^T, L^{T*}, L^R, L^{R*}, \mathcal{P}, \mathcal{Q},\}$, given the condition of the stationary equilibrium, and the fixed value of tariffs τ_F^T, τ_H^{T*}.

COROLLARY 4. *Excessive saving regimes from foreign countries will induce greater aggression in tariffs imposed by the home country.*

Another insightful observation from equation (78) is that closing trade imbalances does not necessarily require higher growth. The presence of implicit saving subsidies introduces an additional confounding mechanism into the analysis. Higher growth becomes necessary only when the foreign country's propensity to save is relatively low, that is, when $\beta(1+\phi^*) < 1$. However, once domestic savings become excessive, $\beta(1+\phi^*) > 1$, we return to the case discussed in the previous section: there is no unilateral gain from imposing tariffs, and any positive tariff instead amplifies trade imbalances and reduces the growth rate. Higher growth, on the one hand, lowers the reevaluation effects arising from the convenience yield; on the other hand, it increases the net payment on bonds. These opposing forces highlight the delicate interaction between growth, external balances, and financial frictions in determining the optimal trade policy.

I replicate the exercise from Section 4.2 with the inclusion of the convenience yield. Figure 14 presents the numerical analysis under different saving-subsidy regimes. An increase in the country's preference for foreign assets, denoted by ϑ, generates substantial reevaluation effects on the trade balance and the net foreign asset position. Intuitively, a higher ϑ implies a stronger propensity to save, which, in turn, ex-

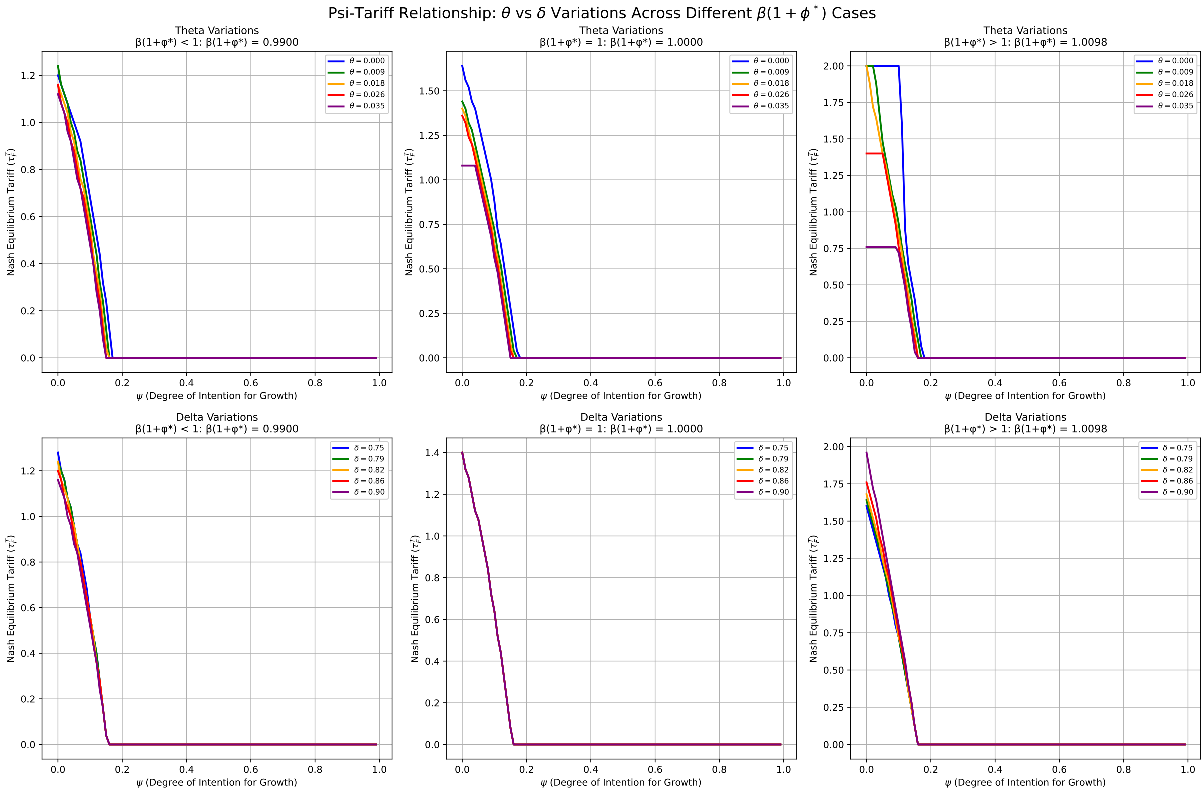

Figure 14: Nash Equilibrium with Full and Partial Intention on Growth: Varied Preference on Home Asset and Bond Maturities

acerbates the home country's trade imbalance. However, according to equation (77), a higher preference for foreign assets also translates into a higher growth rate. Since ϑ is relatively small in my calibration (below 0.05), the latter effect dominates the former, implying that higher ϑ leads to a higher equilibrium growth rate. As argued previously, faster growth facilitates the closing of trade imbalances and, consequently, implies lower optimal tariffs when policymakers partially internalize growth considerations. Nevertheless, once the foreign saving becomes excessive and **Assumption 1** holds, debt repayments rise sharply, calling for a more aggressive imposition of tariffs to stabilize external positions.

The maturity of bonds plays another important role in shaping the equilibrium dynamics. When a smaller fraction of bonds matures within a short period, the implied interest rate rises, increasing debt-servicing costs and thereby worsening trade imbalances. As these imbalances become excessive, the long-term growth outlook deteriorates, prompting a more aggressive imposition of tariffs. The special case is when $\beta(1+\phi^*) = 1$, the evaluation effects through the channel of bond maturity have vanished, and all equilibrium outcomes coincide. Moreover, similarly, as the saving subsidies from the foreign country become excessive, the trade balance is eroded, the growth perspective of the home country is loomed, and calling for aggressive imposition of tariffs.

Taking stock of the previous analyses, the aggressiveness of the home country in imposing tariffs is determined by several factors: partial intention to promote growth, higher debt limits, a deterioration in the foreign country's research intensity, increased dependence on the foreign economy, an excessive propensity to save abroad, a lower preference for home bonds, and longer maturities of issued bonds. These mechanisms jointly shape the incentives for protectionist policies. The potential explanations are mixed and complex, making it difficult to fully understand why the Trump administration adopted a particularly hawkish stance toward trade imbalances and showed an increasing inclination to impose tariffs. Uncovering their true intentions requires further investigation and more related evidence.

7 The World Economy Model

In this section, I aim to contextualize the model within a three-country framework, I extend my model in order to study the welfare and growth perspective of countries once the re-exportation is allowed. Different from the previous two-country case, the three-country model features the possibility of "rerouting" goods, and thereby provides the foreign country with a goods "hedge" against the threats of tariff impositions. Normally, countries under sanctions reroute their goods through a third country in order to circumvent possible seizure and penalties (Haidar (2017), Crozet and Hinz (2020), Tyazhelnikov and Romalis (2024), Shingal (2024), Scheckenhofer et al. (2025)). Under this context, I would like to illustrate that an intensifying trade war will make the third country an intermediate stop. The home country wants to reduce excessive dependence on foreign goods; therefore, it imposes tariffs on foreign products and encourages diversification of the consumption bundle. The foreign country, which has persistently run trade deficits with the entire world, dumps massive amounts of goods into the rest of the world and reroutes them to the home country.

Another common criticism of the standard two-country framework stems from its symmetric asset market, where a deterioration in one country's net foreign asset (NFA) position strictly implies a corresponding gain for its opponent. However, within our three-country framework, the third country—the Rest of the World (RoW)—plays a crucial role as an absorber of shocks and trade imbalances. My subsequent analysis demonstrates how this three-country setting helps align the model with real-world dynamics: specifically, the spectacular trade diversion of China toward ASEAN, BRICS, and Latin American countries, concurrent with a sharp slump in direct trade volumes with the U.S. Crucially, China's global trade surplus has continued to increase despite its exacerbating economic relations with the U.S., a phenomenon that the two-country setting fails to capture.

Now I introduce a third entity, the Rest of the World (RoW), which I assume is composed of a continuum of small open economies sharing the same fundamental economic structure as the home and foreign countries. Crucially, the RoW possesses an additional key function: facilitating the re-exportation of goods from the foreign country to circumvent economic sanctions. Since the welfare of a country can be characterized by its level of technology, which is a sufficient statistic for the real wage, the core objective of this section is to investigate whether the increased volume of re-exportation from the foreign country leads to a technology expansion in the RoW. Motivated by this dynamic, the relationship between the RoW's traditional research-driven tradable sector and its re-exportation activities becomes paramount. This relationship exclusively determines the outcome: if the two sectors are substitutable, a rising cost of re-exportation would stimulate an expansion of the research sector, thereby leading to improved welfare in the RoW. Conversely, if the two sectors are complementary, the increased cost of re-exportation triggers a surge in the overall cost of the tradable sector, signaling a reduction in RoW welfare.

As I argued previously, global technological progress is closely linked to the scale of the tradable sector—that is, to the expanded markets in which tradable goods can be traded. A policy-induced "decoupling" of trading blocs is therefore not neutral: it imposes sizable welfare and growth costs. By contracting market size, decoupling diverts a non-trivial share of labor toward unproductive non-tradable activities, thereby depressing aggregate efficiency. At the same time, intermediary economies positioned along the new trading periphery can take advantage of a portion of the surplus, as their import and export volumes expand to accommodate detoured trade flows.

7.1 Trade War and Re-exportation Trade

Now I hereby assume that the world has two hegemons: country H and F. And there exists a mass of representative small open economies, denoted as RoW, in the following model their variables are denoted with double asterisks $**$. These small countries do not have an innovation sector. They import final goods from home and foreign countries, and consume a bundle of tradable goods. Most importantly, they have a "re-exportation" (or "reroute") technology, which means they can import final goods from foreign countries and thereafter reprocess them as a new final good. I hereby assume the reverse process is not possible. Under this assumption, the wage of the rest of the world W_t^{**} is a function of the amount of re-exportation goods. The escalation of trade war diverts direct trade flows from the foreign to the home country, and meanwhile reinforces the importance of the rest of the world as an intermediary, and therefore increases the welfare of the rest of the world.

7.1.1 The Re-exportation Technology

Assume now, the tradable sector of the rest of the world is composed of two subsectors: a traditional one specializing in ordinary industrial products, and a re-exportation one specializing in re-routing and rebranding final goods from the foreign country. To simplify the denotations, I hereby assume the re-exportation activities are alike the system of the supply chain. The rest of the world imports the final good $Y_{F,t}^{Re}$ as an intermediate good and outputs their own final goods. I hereby assume these goods are non-distinguishable from their own goods [10], the technology is:

$$Y_{RoW,t}^{T} = \left(L_t^{T**}\right)^{1-\alpha} \int_s \left(\gamma \left(A_t^{**}(s)\right)^{1-\eta} + (1-\gamma)\left((1-\delta^T)Y_{F,t}^{T,Re}(s)\right)^{1-\eta}\right)^{\frac{1-\alpha}{1-\eta}} \left(M_t^{**}(s)\right)^{\alpha} ds \tag{79}$$

[10] If the home country can distinguish the re-exportation good, it can otherwise propose another series of tariff regarding re-exportation goods. That consequently makes the re-exportation trade fail at the first place. However, since the information is complete, and the home country knows partial goods from the rest of world are from foreign country, it will increase the tariff against the rest of the world as well.

where $Y_{F,t}^{Re}$ represents the re-exportation of goods from the foreign country. The α represents the share of intermediate goods, and $1-\delta^T$ represents the efficiency of re-exportation of the rest of the world. This assumption implies that the re-exportation sector obtains an inferior technology.[11] The parameter η represents the inverse elasticity between the traditional tradable sector and the re-exportation sector, this is the key parameter governs the welfare implications under trade wars. Therefore, the RoW executes two parallel tasks: they can either produce traditional tradable goods and invest in R&D like what home and foreign county do, or they re-export and launder the origin of final goods from a third country. Similarly, as I assume hereby that there is a continuum of small open economies, the tradable sector of the rest of the world RoW is perfectly competitive. In order to maximize their profits, the price of the intermediate is set as:

$$P_{M^{**},t}(s) = \frac{1}{\alpha} P_{RoW,t}^T$$

I hereby assume small economies are identical and symmetric; this implies the total supply of intermediate goods is:

$$\int_s M_t^{**}(s) = \alpha^{\frac{2}{1-\alpha}} \left(\gamma \left(A_t^{**}(s) \right)^{1-\eta} + (1-\gamma) \left((1-\delta^T) Y_{F,t}^{T,Re}(s) \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} L_t^{T**}$$

Therefore, the net value-added of tradable goods including re-exportation from the rest of the world is:

$$Y_{RoW,t}^T = \mathcal{Z} \left(\gamma \left(A_t^{**}(s) \right)^{1-\eta} + (1-\gamma) \left((1-\delta^T) Y_{F,t}^{T,Re}(s) \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} L_t^{T**}$$

Further, I can detrend this equation by dividing A_t at both hand-side in order to capture the economy in the balanced growth path:

$$y_{RoW,t}^T = \mathcal{Z} \left(\gamma (a_t^{**})^{1-\eta} + (1-\gamma) \left((1-\delta^T) y_{F,t}^{T,Re} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} L_t^{T**} \tag{80}$$

where $a^{**} \equiv A_t^{**}/A_t$ represents the technological frontier of RoW. Further, this implies the real wage of the rest of the world is:

$$\frac{W_t^{**}}{P_{RoW,t}^T} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \left(\gamma \left(A_t^{**}(s) \right)^{1-\eta} + (1-\gamma) \left((1-\delta^T) Y_{F,t}^{T,Re}(s) \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{81}$$

and the input of final goods produced by the foreign country:

$$\frac{P_{F,t}^T}{P_{RoW,t}^T} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} (1-\gamma)(1-\delta^T) \left(\gamma \left(\frac{a_t^{**}}{(1-\delta^T) y_{F,t}^{T,Re}} \right)^{1-\eta} + (1-\gamma) \right)^{\frac{\eta}{1-\eta}} L_t^{T**} \tag{82}$$

[11] Alternatively, we can think of the re-exportation sector of the RoW as substitutable for its traditional tradable sector. The parameter γ measures the unit loss of goods under rerouting. Intuitively, $1-\delta^T$ captures the efficiency of its re-exportation sector, and it is assumed to be exogenous.

As now we can see from the two equations above, the expanding trading activities of the third country will boost the real payment of salaries to consumers. From another perspective, the trade will devalue the foreign goods, which in turn increases the imports from the foreign country. The increase of imports associated with escalating trade wars expands the country's trading activities crazily, which benefits the rest of the world further.

Similarly, I also assume the RoW has a symmetric research sector akin to the foreign country. Its optimal technology process is following:

$$(a_t^{**})^{\Omega} = \beta \left(\frac{c_{t+1}^{T**} g_{t+1}}{c_t^{T**}} \right)^{-1} \pi_{W,t+1} \left(\zeta \alpha L_{t+1}^{T**} + (a_{t+1}^{**})^{\Omega} \right) \tag{83}$$

From equation (80), an increase in re-exportation activities has mixed effects on the productivity level of the rest of the world (RoW). When $\eta < 1$, the two sectors are substitutable, the increasing cost of re-exportation activity crowds in research efforts from the domestic production sector. Conversely, when $\eta > 1$, the sectors are complementary, increasing cost of re-exportation activity crowds out the research efforts, and signaling a contraction of the tradable sector.

This assumption introduces a critical externality into the system. When the elasticity parameter $\eta < 1$, the sectors are substitutable. In this case, an increase in the cost of re-exportation activities effectively crowds in research efforts in the RoW. This mechanism simultaneously increases the real wage rate and expands the market size for the RoW, thereby improving its overall welfare. In reality, when the home country imposes aggressive tariffs against both the foreign country and the RoW (as exemplified by tariffs implemented during the Trump Administration), such actions may inadvertently foster an economic boom in the RoW, signaling an increasingly competitive relationship between the foreign country and the RoW. Conversely, when re-exportation and R&D are complementary ($\eta > 1$), the trade war between the home and foreign country leads to an increasing cost of production in the RoW's tradable sector, resulting in a worsening welfare condition for the RoW.

DEFINITION 5 (The Stationary Equilibrium under Re-exportation)**.** A stationary equilibrium is a path of real allocations of $\{c_t^T, c_t^{T*}, c_t^{T**}, c_{H,t}^T, c_{F,t}^T, c_{RoW,t}^T, c_{H,t}^{T*}, c_{F,t}^{T*}, c_{RoW,t}^{T*}, c_{H,t}^{T**}, c_{F,t}^{T**}, c_{RoW,t}^{T**}, L_t^T, L_t^{T*}, L_t^{T**},$ $L_t^N, L_t^{N*}, L_t^{N**}, L_t^R, L_t^{R*}, L_t^{R**}, Y_{H,t}^N, Y_{F,t}^N, Y_{RoW,t}^N, y_{H,t}^T, y_{F,t}^T, y_{RoW,t}^T, y_{F,t}^{T,Re}, b_{t+1}, b_{t+1}^*, b_{t+1}^{**}, g_{t+1}, a_{t+1}^*, a_{t+1}^{**},$ $R_t^n, \hat{\mu}_t, \hat{\mu}_t^*, \hat{\mu}_t^{**}, \mathcal{P}_{1,t}, \mathcal{P}_{2,t}, \mathcal{P}_{3,t}\}_{t=0}^{\infty}$, given initial condition $\{B_0, B_0^*, B_0^{**}, A_0, A_0^*\}$, assuming zero domestic inflation over time $\pi_{H,t+1}^T = \pi_{F,t+1}^T = \pi_{W,t+1}^T = 1$ and the evolution of trade tariffs $\{\tau_{F,t}^T, \tau_{RoW,t}^T, \tau_{H,t}^{T*},$ $\tau_{RoW,t}^{T*}, \tau_{H,t}^{T**}, \tau_{F,t}^{T**}\}$ subject to:

(i) The optimal policies for consumption and savings for every country. Here, I hereby assume only foreign country does subsidize the savings, therefore, in steady state, the borrowing constraint of the rest of the world is binding, $\hat{\mu}_t^{**} > 0$:

$$\left(c_t^{T**}\right)^{-1} = \beta \left\{ \left(c_{t+1}^{T**} g_{t+1}\right)^{-1} R_t^n + \hat{\mu}_t^{**} \right\} \tag{84}$$

(ii) Maximization of profits, in the tradable sector, the consumer maximizes their utilities, and is a market clear of the non-tradable sector. In addition, the firm's profits maximization problem will pin down the demand of re-exportation goods.

(iii) Maximization of profits, in the non-tradable sector, the consumer maximizes their utilities, and markets clear of tradable goods.

(iv) Maximization of profits in the research sector.

(v) In the clearing of the labor market, the total labor demand is equal to the fixed supply of labor. The labor demand from non-tradable sector is pinned down by the demand of non-tradable goods:

$$L_t^{N**} = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H^{**} \left((1+\tau_{H,t}^{T**}) \frac{P_{H,t}^T}{P_{RoW,t}^T} \right)^{1-\rho} + \nu_F^{**} \left((1+\tau_{F,t}^{T**}) \frac{P_{F,t}^T}{P_{RoW,t}^T} \right)^{1-\rho} + \nu_W^{**} \right]^{\frac{1}{1-\rho}} \times \frac{c_t^{T**}}{\left((a_t^{**})^{1-\eta} + (1-\gamma) \left((1-\delta^T) y_{F,t}^{T,Re} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}} \tag{85}$$

The structure of the determination of the size of the non-tradable sector in RoW is similar to the previous section in equations (28) and (29). Therefore, the size of the non-tradable sector in the RoW depends on its own R&D efforts, the volume of re-exportation, and the elasticity between the two variables.

(vi) The resource constraint is satisfied, the budget constraint is satisfied.

$$\left((a_t^{**})^{1-\eta} + (1-\gamma) \left((1-\delta^T) y_{F,t}^{T,Re} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} P_{RoW,t}^T \mathcal{Z} L_t^{T**} + P_t^{T**} b_t^{**} = P_{H,t}^T c_{H,t}^{T**} + P_{F,t}^T (c_{F,t}^{T**} + y_{F,t}^{T,Re}) + P_{RoW,t}^{T**} c_{RoW,t}^{T**} + \frac{P_t^{T**} b_{t+1}^{T**}}{R_t^n} \tag{86}$$

(vii) In terms of asset market clearing, the net supply of bonds is 0 in the world.

In order to avoid the Ponzi scheme, similar to the home country and foreign country, the transversality condition of the rest of the world must be satisfied.

$$\lim_{t\to\infty} \frac{B_t^{**}}{\prod_{t=k}^{+\infty} R_t(1+\phi_t^*)} = 0$$

I redefine the relative price as: $\mathcal{P}_1 = P_H^T/P_F^T$, $\mathcal{P}_2 = P_H^T/P_{RoW}^T$, $\mathcal{P}_3 = P_{RoW}^T/P_F^T$. Detailed equilibrium conditions are defined in the appendix section.

DEFINITION 6 (Steady State under Trade Re-exportation)**.** An equilibrium in steady state under trade re-exportation is a series of constant real allocation of $\{g, a^*, c^T, c^{T*}, c^{T**}, c_H^T, c_H^{T*}, c_H^{T**}, c_F^T, c_F^{T*}, c_F^{T**}, c_W, c_W^{T*}, c_W^{T**}, y_F^{T,Re}, L^N, L^{N*}, L^{N**}, L^T, L^{T*}, L^{T**}, L^R, L^{R*}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, R^n\}$ given the condition of the

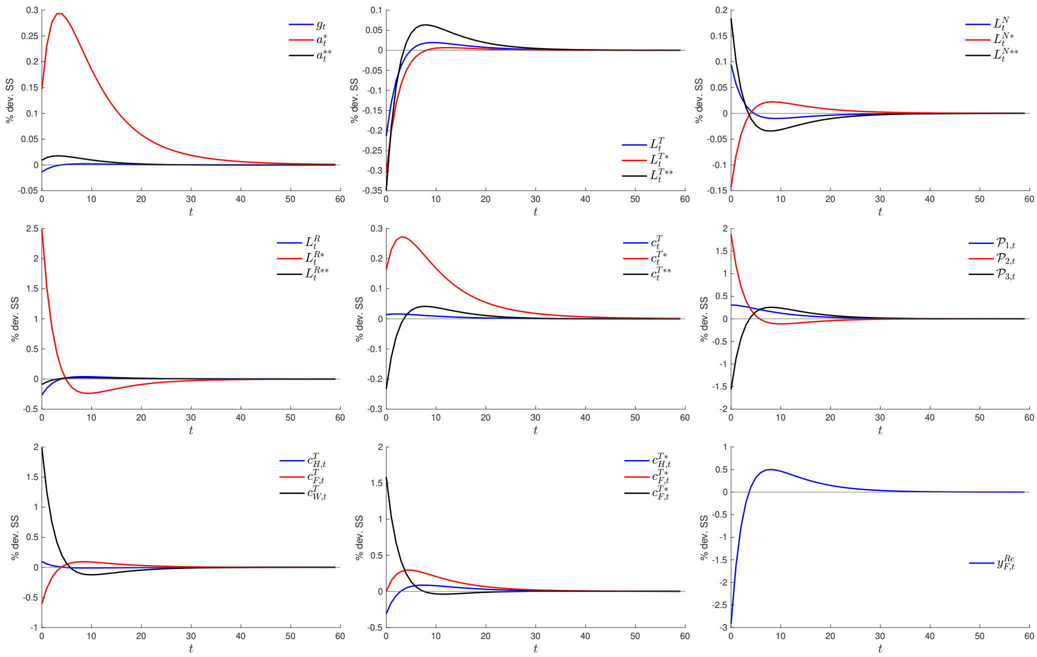

Figure 15: Dynamics under Unilateral Tariff Regime: Home Country against the Foreign Country, Three Country Cases.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{HF,t} = 1\%$, I hereby assume no retaliations from the foreign country and $\eta = 0.67$. **Assumption 1, 2** holds.

stationary equilibrium initial condition $\{B_0, B_0^*, B_0^{**}, A_0, A_0^*\}$, assuming zero domestic inflation over time ($\pi^T_{H,t+1} = \pi^T_{F,t+1} = \pi^T_{W,t+1} = 1$), and the evolution of trade tariffs $\{\tau^T_{F,t}, \tau^T_{RoW,t}, \tau^{T*}_{H,t}, \tau^{T*}_{RoW,t}, \tau^{T**}_{H,t}, \tau^{T**}_{F,t}\}$. The complete equilibrium conditions will be displayed in the following.

7.2 Numerical Analyses under The World Economy Model

Similarly, I follow the exercise that I conducted in the two country case. I hereby assume tariff shocks are persistent and are subject to an AR(1) process with a mean of 0:

$$\tau^T_{ij,t} = \rho_i \tau^T_{ij,t-1} + \varepsilon^T_{ij,t}$$

where $\tau^T_{ij,t}$ represents the tariff rates that country i imposes on country j at time t, and ρ_i measures the persistence of the tariff shock. Similarly, I calibrate the home country's steady-state current account (CA) imbalance to 4% of GDP to align with the real data.

Figure (15) illustrates the dynamics of key macroeconomic variables subjected to a 1% tariff shock imposed by the home country on the foreign country, assuming the home country currently maintains free trade with the Rest of the World (RoW). my model suggest the similar results as Fajgelbaum et al. (2024). my model suggests an immediate 0.04% TFP decline following the unilateral tariffs. The three-country setting introduces more intricate dynamics into my analysis. The tariffs imposed by the home

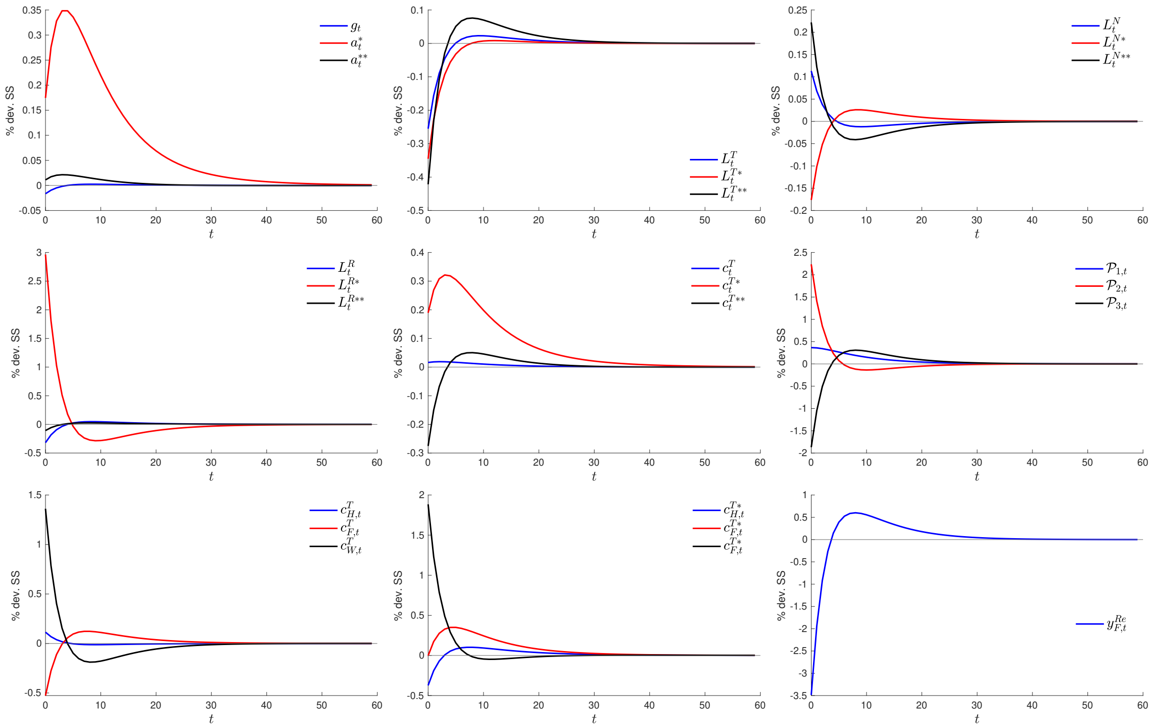

Figure 16: Dynamics under Unilateral Tariff Regime: Home Country against the Foreign Country and RoW, Three Country Cases.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{HF,t} = \varepsilon_{HW,t} = 1\%$, I hereby assume no retaliations from the foreign country and RoW, and $\eta = 0.67$. **Assumption 1, 2** holds.

country facilitate a boom in its tradable and non-tradable sectors, leading to a more favorable terms of trade with the foreign country. Concurrently, the shock relatively devalues home country exports to the RoW, increasing the volume of re-exportation through the RoW. This dynamic, in turn, crowds in research efforts from the RoW and signals a worsened trade imbalance for the RoW. However, this does not imply the welfare of roW is reduced. As a country's real wage is determined by the advancement of research sector, a boom in re-exportation activities will facilitate a surge of wages, signaling an economic boom. This outcome mirrors observed recent trade dynamics, such as the massive export of intermediates from the foreign country (China) to ASEAN, BRICS, and Latin American countries (RoW) while its direct exports to the home country (USA) are declining. From the foreign country's perspective, the tariff surprise facilitates re-exportation activities, which drives a significant investment in innovation and a boom in its tradable sector. The foreign country is benefited from this dynamic as its final production is devaluing, and this fostering its growth of exports. The RoW, although subject to the trade turbulence, also experiences an increase in investment in innovation due to the rising price of its final goods.

In essence, the overshooting feature observed in our model is generated by the interplay between the sluggish adjustment of technology (R&D) and the fixed supply of labor. A surprise increase in tariffs instantly boosts the demand for domestic industrial and service goods. However, this demand shift crowds out labor from the research sector, causing an immediate loss in TFP growth. In the case of a possible permanent tariff shock, the initial crowding out of research labor persists, but TFP growth later overshoots the baseline due to the continued surge in domestic industrial goods demand (the expenditure switching effect). Ultimately, however, as the market size effects come to dominate the growth engine of the economy, TFP growth falls permanently below the baseline, characterizing a sustained, permanent TFP loss.

Figure (16) illustrates a scenario where the home country imposes tariffs on both the foreign country and the RoW. While this does not fundamentally alter the dynamic pattern, we observe a greater boom in inter-trade between the foreign country and the RoW. Compared with the simulation results from Figure (15), the trade wars characterize a more spectacular trade cooperation between the foreign country and the RoW. These simulations also echo real-world dynamics: since Liberation Day on April 2, 2025, China's exports to the U.S. have declined by 33%, while its exports to the rest of the world have simultaneously risen by 11% through August 31. China's net foreign asset position has been further strengthened, as evidenced by a 6% reduction in its holdings of U.S. treasury bonds combined with its active issuance of debt denominated in U.S. dollars and Euros (Data sources: Bloomberg and the U.S. Department of the Treasury). Crucially, the intensifying trade war has also incentivized the RoW to invest in innovation, leading to a characteristic rise in their real wages and an improved welfare outcome.

Another critical concern for our model involves the calibration of the elasticity parameter, η. We conduct a similar exercise for the case where $\eta = 1.5$, as illustrated in Figures (23) and (24) attached in our appendix. In my model specification, η governs the complementarity or substitutability between R&D intermediaries and re-exportation goods. Under my current construction, the foreign country is always incentivized to reroute its products through the third country (RoW). However, subject to the tariff shock, the home country's demand for industrial goods from the RoW increases disproportionately, which further raises the price of RoW goods, $\mathcal{P}_2$. To clear the market, the foreign country's price, $\mathcal{P}_3$, must adjust downward, implying a relative appreciation of foreign goods initially. Given $\eta < 1$, R&D innovation in the RoW and re-exportation activities are treated as substitutable inputs. Therefore, the home country's tariff shock renders re-exportation goods relatively more expensive, thereby incentivizing greater research efforts thereafter in the RoW and meanwhile crowds out labor in the production sector. Consequently, we observe an improvement in welfare for countries that serve as intermediary hubs for foreign products, such as most ASEAN countries and Mexico documented by Fajgelbaum et al. (2024).

Alternatively, re-exportation and R&D are regarded as complementary if $\eta > 1$. Subject to the tariff shock, the home country now requires a greater volume of goods from the RoW, which initially raises the price $\mathcal{P}_2$. The rise in $\mathcal{P}_2$ signals an increasing requirement for re-exportation from the foreign country through the model's construction, subsequently boosting the expansion of the research sector, given the defined complementarity. However, as the proximity to the technological frontier is dependent on the magnitude of the research sector, this complementarity makes R&D more expensive by construction and therefore relocates more labor into the non-tradable sector. This labor reallocation is inefficient and subsequently leads to a deterioration in the technology level of the RoW. This scenario corresponds to real-world supply chain disruptions under the intensifying Sino-US trade tensions, such as the case of Nexperia, which subsequently triggered substantial losses for European car producers; And relevant EV car producers suffer a sales plunge, such as General Motors, Ford, Hyundai, etc. Given that the relevant economic mechanisms may vary across countries, we leave the choice of the appropriate η value to the readers.

8 Conclusion

The implementation of positive tariffs on foreign goods generates significant welfare costs that extend beyond the immediate bilateral trade relationship. While such protectionist measures may confer short-term comparative advantages to the imposing country through terms-of-trade improvements and expansion of industrial sectors, they ultimately precipitate long-run economic contraction through several transmission mechanisms. First, tariffs induce resource misallocation by shifting labor from productive tradable sectors toward less efficient non-tradable activities, thereby reducing aggregate productivity and dampening growth prospects. Second, the resulting contraction in global trade volumes creates negative spillover effects that adversely impact third-party economies, including the rest of the world (RoW), despite their non-participation in the trade dispute. These countries experience welfare losses through reduced export opportunities, disrupted supply chain networks, and diminished economies of scale in global markets. Consequently, tariff wars represent a negative-sum game that imposes systemic costs on the international economic system, undermining long-term growth and global welfare even for countries that maintain neutral trade policies.

Against the backdrop of Sino–U.S. decoupling, the world is witnessing a paradigm shift characterized by rising trade fragmentation and intensifying economic nationalism. The role of the World Trade Organization (WTO) has been increasingly questioned, and the international community is calling for a new framework to manage trade disputes in this evolving landscape. A central puzzle of this process lies in the fact that cooperation between the two countries once generated remarkable global growth and unprecedented technological advancement. The ongoing decoupling appears to signal the end of the "Chi–America" era and the emergence of a fractured pattern of globalization. Future research should focus on uncovering the origins of the divisions between these two superpowers and exploring their macroeconomic and technological implications for the global economy.

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Appendix

Two Country Model with Elastic Supply of Labor

Now, consider modifying the utility function of agents as follows:

$$u(C_t, L_t) = \log C_t - \varrho \frac{L_t^{1+\varphi}}{1+\varphi} \tag{87}$$

where ϱ is a preference parameter to weight people's disutility to the working. We assume there are a continuum of identical agents in this economy, agents spend an amount of L_t^T efforts to produce tradable goods, L_t^N amount of time to produce non-tradable goods, and L_t^R to conduct the research. This also modifies the budget constraint:

$$P_t^T C_t^T + P_t^N C_t^N + \frac{P_t^T B_{t+1}}{R_t^n} \leq P_t^T B_t + W_t L_t + \Pi_t^T + \Pi_t^N$$

The essential first order condition captures the determination of labor supply:

$$\omega \frac{W_t}{P_t^T} = \varrho C_t^T L_t^{\varphi} \tag{88}$$

The detrended equation is:

$$L_t^{\varphi} = \frac{\omega}{\varrho} \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\left(\nu_H + \nu_F \left((1+\tau_{F,t}^T)\mathcal{P}_t^{-1}\right)^{1-\rho}\right)^{\frac{1}{1-\rho}} c_t^T} \tag{89}$$

Similarly, the labor supply of the foreign agent can be conceptualize as follows:

$$(L_t^*)^{\varphi} = \frac{\omega}{\varrho} \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{\left(\nu_H \left((1+\tau_{H,t}^{T*})\mathcal{P}_t\right)^{1-\rho} + \nu_F\right)^{\frac{1}{1-\rho}} c_t^{T*}/a_t^*} \tag{90}$$

By keeping the rest of the structure of my model unchanged, this brings us the new equilibrium condition:

$$L_t = L_t^N + L_t^T + L_t^R$$

$$L_t^* = L_t^{N*} + L_t^{T*} + L_t^{R*}$$

DEFINITION 7 (The Two-Country Model Equilibrium Stationary Equilibrium with Elastic Supply of Labor)**.** A stationary equilibrium is a path of real allocations of $\{c_t^T$, c_t^{T*}, $c_{H,t}^T$, $c_{F,t}^T$, $c_{H,t}^{T*}$, $c_{F,t}^{T*}$, L_t, L_t, L_t^T, L_t^{T*}, L_t^N, L_t^{N*}, L_t^R, L_t^{R*}, $Y_{H,t}^N$, $Y_{F,t}^N$, $y_{H,t}^T$, $y_{F,t}^T$, b_{t+1}, b_{t+1}^*, g_{t+1}, a_{t+1}^*, R_t^n, $\tilde{\mu}_t$, $\tilde{\mu}_t^*$, $\mathcal{P}_t\}_{t=0}^{\infty}$, given initial condition $\{B_0$, B_0^*, A_0, $A_0^*\}$, assuming zero domestic inflation over time ($\pi_{H,t+1}^T$, $\pi_{F,t+1}^T$) and the evolution of trade tariffs $\{\tau_{F,t}^T$, $\tau_{H,t}^{T*}\}$.

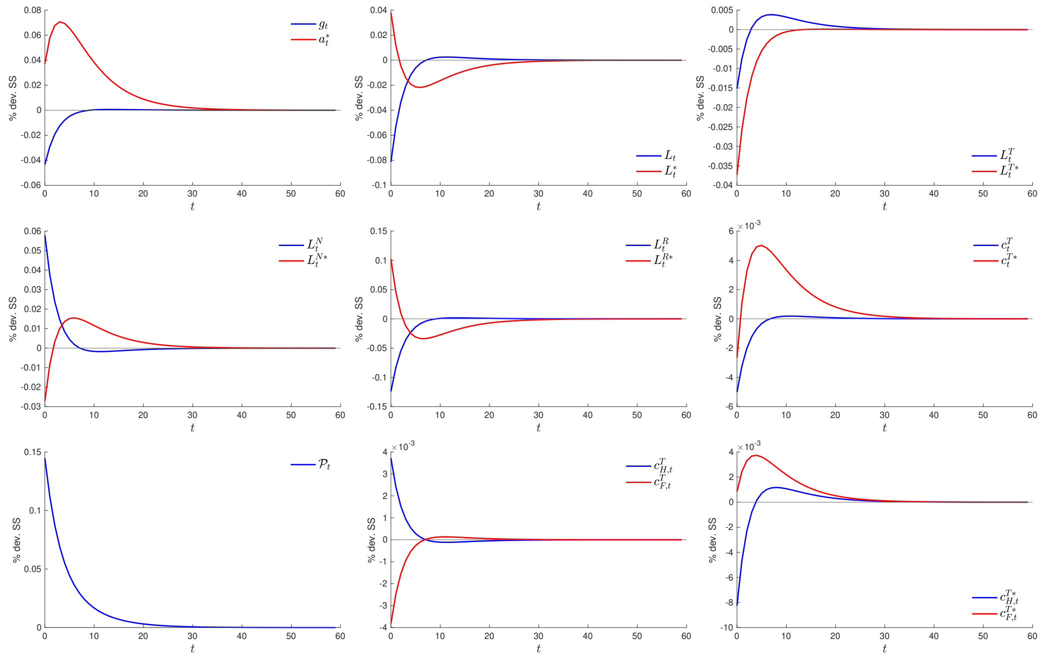

Figure 17: Dynamics under Unilateral Tariff Regime: Home Country.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{H,t} = 1\%$, I hereby assume no retaliations from the foreign country. **Assumption 1, 2** holds.

It is clear to say that the increase in consumption due to financial liberalization clearly reduces the labor supply of the residents of the home country according to equation (89). This finding is confirmed by the paper Ramey and Francis (2009). However, financial liberalization increases the working hours of the foreign country through the channel of increasing proximity to the frontier according to equation (90).

Again, I calibrate my model to match the long-term trade imbalances of the U.S. as 4% of its GDP. The IRFs of the modified model subject to the AR(1) tariff shock are shown in Figure (17). The long-term decline in labor supply led by financial liberalization in the home country even strengthens my analyzes. Figure (17) implies the dominance of the market size effects and improved term of trade (Monacelli (2025)), the supply of labor is shrinking regardless of the higher prices of domestic goods. Similarly, the IRFs are similar to the fixed labor supply case for the foreign country illustrated at the Figure (18).

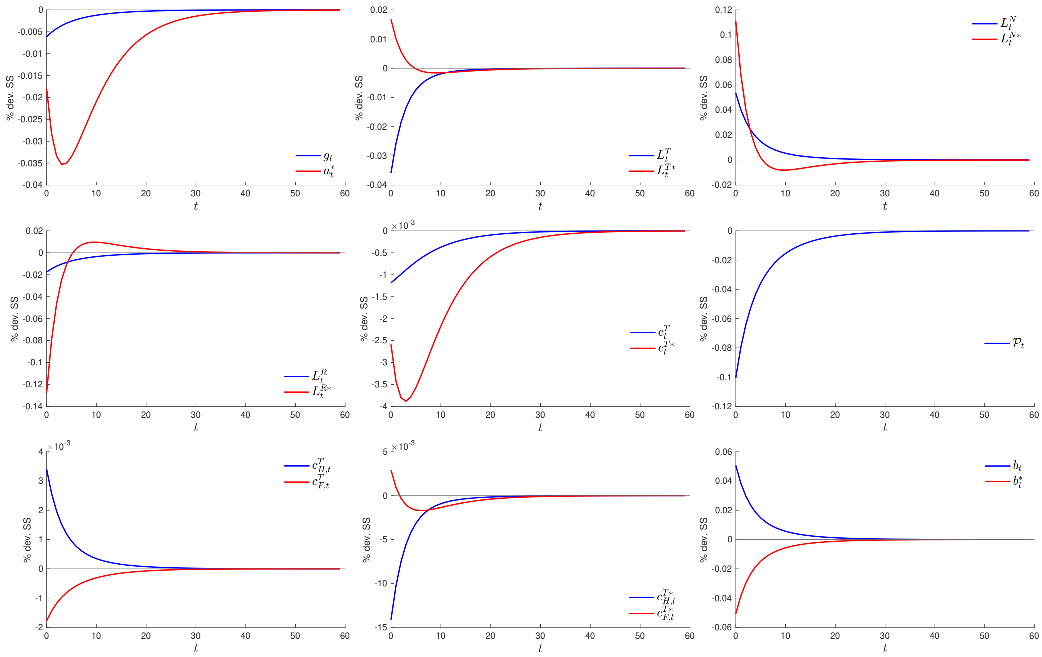

Figure 18: Dynamics under Unilateral Tariff Regime: Foreign Country.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{F,t} = 1\%$, I hereby assume no retaliations from the foreign country. **Assumption 1, 2** holds.

Parameter Value: Two Country Model

Parameter	Description	Values
β	discount factor	0.995
$\bar{L}$	labor stock in home country	1
$\bar{L}^*$	labor stock in foreign country	1
α	intermediate share	0.33
ν_H	home preference on consuming home goods	0.65
ν_F	home preference on consuming foreign goods	0.35
ν_H^*	foreign preference on consuming home goods	0.25
ν_F^*	foreign preference on consuming foreign goods	0.75
ρ	elasticity of substitution of tradable goods	2
κ_F	debt limit of foreign country	0.001
χ	research intensity of home country	1
ξ	research intensity of foreign country	0.5
ϕ^*	saving subsidies	0.01
Ω	research dependence	0.5
ω	share of tradable goods	0.3

Table 1: Table of Parameter Values

Equilibrium Condition

The Two-Country Model Equilibrium: An equilibrium is a path of real allocations of $\{C_t$, C_t^*, C_t^T, C_t^{T*}, $C_{H,t}^N$, $C_{F,t}^N$, $C_{H,t}^T$, $C_{F,t}^T$, $C_{H,t}^{T*}$, $C_{F,t}^{T*}$ L_t^T, L_t^{T*}, L_t^N, L_t^{N*}, L_t^R, L_t^{R*}, $Y_{H,t}^N$, $Y_{F,t}^N$, $Y_{H,t}^T$, $Y_{F,t}^T$, B_{t+1}, B_{t+1}^*, A_{t+1},

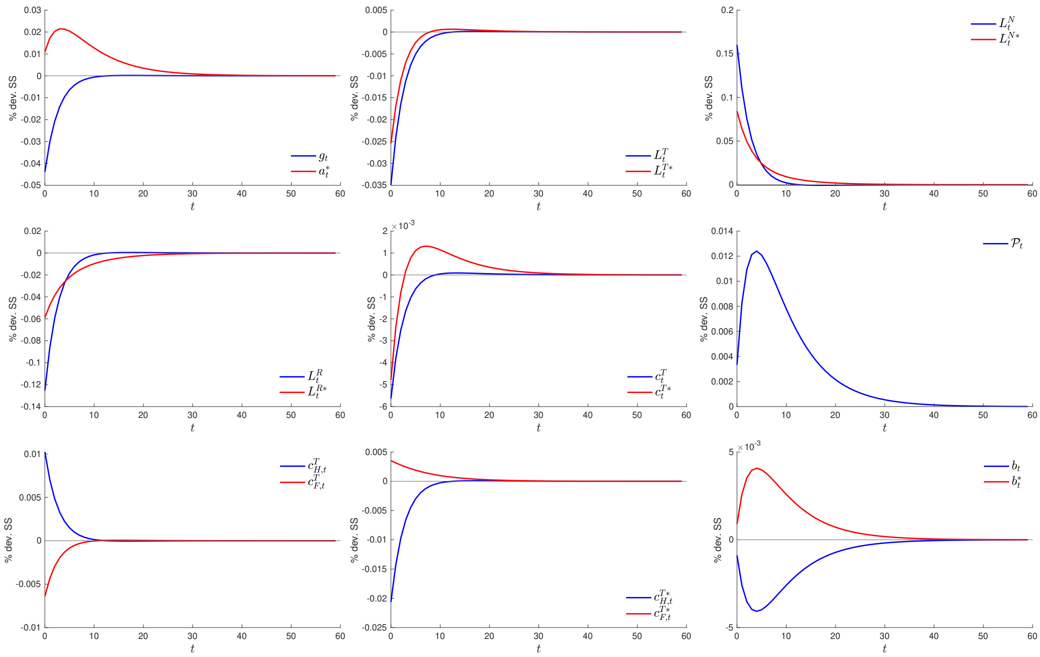

Figure 19: Dynamics under a Trade War with Simultaneous Retaliation
This figure illustrates the adjustment dynamics of key macroeconomic variables following the tariff war. I hereby assume $\rho_\tau = 0.8$, $\varepsilon_{H,t} = 1\%$, and $\rho_\tau^* = 0.8$, $\varepsilon_{F,t} = 1\%$. **Assumption 1, 2** holds.

A_{t+1}^*, R_t^n, $\hat{\mu}_t$, $\hat{\mu}_t^*$, P_t, P_t^*, P_t^T, P_t^{T*}, $P_{H,t}^T$, $P_{F,t}^T$, $P_{H,t}^N$, $P_{F,t}^N$, W_t, $W_t^*\}_{t=0}^{\infty}$, given initial condition $\{B_0, B_0^*, B_0^{**}, A_0, A_0^*\}$, assuming zero domestic inflation over time ($\pi_{H,t+1}^T, \pi_{F,t+1}^T$), the price of home tradable good as numéraire $P_{H,t}^T = 1$, and the evolution of trade tariffs $\{\tau_{F,t}^T, \tau_{H,t}^{T*}\}$. Subject to:

(i) The optimal policies for consumption and saving for both domestic and foreign country:

$$\left(C_t^T\right)^{-1} = \beta\left\{\left(C_{t+1}^T\right)^{-1}(1+\phi_t^*)R_t^n + \hat{\mu}_t\right\}$$

$$\left(C_t^{T*}\right)^{-1} = \beta\left\{\left(C_{t+1}^{T*}\right)^{-1}(1+\phi^*)R_t^n \frac{P_{t+1}^T/P_{t+1}^{T*}}{P_t^T/P_t^{T*}} + \hat{\mu}_t^*\right\}$$

(ii) Demand of non-tradable sector

$$C_{H,t}^N = \frac{1-\omega}{\omega}\frac{P_t^T}{P_{H,t}^N}C_t^T$$

$$C_{F,t}^N = \frac{1-\omega}{\omega}\frac{P_t^{T*}}{P_{F,t}^N}C_t^{T*}$$

(iii) Demand of tradable goods:

$$C_{H,t}^T = \nu_H\left(\frac{P_{H,t}^T}{P_t^T}\right)^{-\rho}C_t^T$$

$$C_{F,t}^T = \nu_F\left(\frac{(1+\tau_{F,t}^T)P_{F,t}^T}{P_t^T}\right)^{-\rho}C_t^T$$

$$C_{H,t}^{T*} = \nu_H^* \left(\frac{(1+\tau_{H,t}^{T*}) P_{H,t}^{T}}{P_t^{T*}} \right)^{-\rho} C_t^{T*}$$

$$C_{F,t}^{T*} = \nu_F^* \left(\frac{P_{F,t}^{T}}{P_t^{T*}} \right)^{-\rho} C_t^{T*}$$

(iv) Optimal condition in research sector:

$$1 = \beta \left(\frac{C_{t+1}^{T}}{C_t^{T}} \right)^{-1} \frac{P_{H,t+1}^{T}}{P_{H,t}^{T}} \left(\chi \alpha L_{t+1}^{T} + 1 \right)$$

$$\left(\frac{A_t}{A_t^*} \right)^{\Omega} = \beta \left(\frac{C_{t+1}^{T*}}{C_t^{T*}} \right)^{-1} \frac{P_{F,t+1}^{T}}{P_{F,t}^{T}} \left(\xi \alpha L_{t+1}^{T*} + \left(\frac{A_{t+1}}{A_{t+1}^*} \right)^{\Omega} \right)$$

$$A_{t+1} = A_t + \chi A_t L_t^R$$

$$A_{t+1}^* = A_t^* + \xi \left(A_t^* \right)^{1-\Omega} \left(A_t \right)^{\Omega} L_t^{R*}$$

(v) Supply of non-tradable sector:

$$Y_{H,t}^{N} = L_t^N$$

$$Y_{F,t}^{N} = L_t^{N*}$$

(vi) Supply of tradable sector:

$$Y_{H,t}^{T} = \mathcal{Z} A_t L_t^T$$

$$Y_{F,t}^{T} = \mathcal{Z} A_t^* L_t^{T*}$$

(vii) Non-tradable good market clearing:

$$Y_{H,t}^{N} = C_{H,t}^{N}$$

$$Y_{F,t}^{N} = C_{F,t}^{N}$$

(viii) Budget constraint:

$$P_{H,t}^{T} \mathcal{Z} A_t L_t^T + P_t^T B_t = P_{H,t}^{T} C_{H,t}^{T} + P_{F,t}^{T} C_{F,t}^{T} + \frac{P_t^T B_{t+1}}{R_t^n}$$

(ix) Labor market clearing

$$\bar{L} = L_t^N + L_t^T + L_t^R$$

$$\bar{L}^* = L_t^{N*} + L_t^{T*} + L_t^{R*}$$

(x) Resource constraint

$$C_{H,t}^{T} + C_{H,t}^{T*} = \mathcal{Z} A_t L_t^T$$

$$C_{F,t}^{T} + C_{F,t}^{T*} = \mathcal{Z} A_t^* L_t^{T*}$$

(xi) Prices

$$P_{H,t}^{T} = \frac{W_t}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t}$$

$$P_{F,t}^{T} = \frac{W_t^*}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t^*}$$

$$P_{H,t}^{N} = W_t$$

$$P_{F,t}^{N} = W_t^*$$

(xii) Asset market clearing:

$$B_t + B_t^* = 0$$

(xiii) Slackness condition of borrowing:

$$\hat{\mu}_t \left(B_t + \mathcal{K}_{H,t}\right) = 0$$

$$\hat{\mu}_t^* \left(B_t^* + \mathcal{K}_{F,t}\right) = 0$$

(xiv) Price Aggregator

$$P_t^T = \left[\nu_H \left(P_{H,t}^T\right)^{1-\rho} + \nu_F \left((1+\tau_F^T) P_{F,t}^T\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

$$P_t^{T*} = \left[\nu_H^* \left((1+\tau_H^{T*}) P_{H,t}^T\right)^{1-\rho} + \nu_F^* \left(P_{F,t}^T\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

The Two-Country Model Equilibrium Stationary Equilibrium: A stationary equilibrium is a path of real allocations of $\{c_t^T, c_t^{T*}, c_{H,t}^T, c_{F,t}^T, c_{H,t}^{T*}, c_{F,t}^{T*}, L_t^T, L_t^{T*}, L_t^N, L_t^{N*}, L_t^R, L_t^{R*}, Y_{H,t}^N, Y_{F,t}^N, y_{H,t}^T, y_{F,t}^T, b_{t+1}, b_{t+1}^*,$ $g_{t+1}, a_{t+1}^*, R_t^n, \tilde{\mu}_t, \tilde{\mu}_t^*, \mathcal{P}_t\}_{t=0}^{\infty}$, given initial condition $\{B_0, B_0^*, A_0, A_0^*\}$, assuming zero domestic inflation over time $(\pi_{H,t+1}^T, \pi_{F,t+1}^T)$ and the evolution of trade tariffs $\{\tau_{F,t}^T, \tau_{H,t}^{T*}\}$. Subject to:

(i) The optimal policies for consumption and saving for both domestic and foreign country:

$$\left(c_t^T\right)^{-1} = \beta \left\{ \left(c_{t+1}^T g_{t+1}\right)^{-1} (1+\phi_t^*) R_t^n + \tilde{\mu}_t \right\}$$

$$\left(c_t^{T*}\right)^{-1} = \beta \left\{ \left(c_{t+1}^{T*} g_{t+1}\right)^{-1} (1+\phi^*) R_t^n \left(\frac{\left[\frac{\nu_H + \nu_F \left((1+\tau_{F,t+1}^T)\mathcal{P}_{t+1}^{-1}\right)^{1-\rho}}{\nu_H^* (1+\tau_{H,t+1}^{T*})^{1-\rho} + \nu_F^* \left(\mathcal{P}_{t+1}^{-1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}}{\left[\frac{\nu_H + \nu_F \left((1+\tau_{F,t}^T)\mathcal{P}_{t}^{-1}\right)^{1-\rho}}{\nu_H^* (1+\tau_{H,t}^{T*})^{1-\rho} + \nu_F^* \left(\mathcal{P}_{t}^{-1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}} \right)^{-1} + \tilde{\mu}_t^* \right\}$$

(ii) Demand of tradable goods:

$$c_{H,t}^T = \nu_H \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T)\mathcal{P}_t^{-1}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}} c_t^T$$

$$c_{F,t}^T = \nu_F \left[\nu_H \left((1+\tau_{F,t}^T)^{-1}\mathcal{P}_t\right)^{1-\rho} + \nu_F\right]^{\frac{\rho}{1-\rho}} c_t^T$$

$$c_{H,t}^{T*} = \nu_H^* \left[\nu_H^* + \nu_F^* \left((1+\tau_{H,t}^T)^{-1}\mathcal{P}_t^{-1}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}} c_t^{T*}$$

$$c_{F,t}^{T*} = \nu_F^* \left[\nu_H^* \left((1+\tau_{H,t}^{T*})\mathcal{P}_t\right)^{1-\rho} + \nu_F^*\right]^{\frac{\rho}{1-\rho}} c_t^{T*}$$

(iii) Optimal condition in research sector:

$$1 = \beta \left(\frac{c_{t+1}^T g_{t+1}}{c_t^T}\right)^{-1} \left(\chi\alpha L_{t+1}^T + 1\right)$$

$$\left(a_t^*\right)^{\Omega} = \beta \left(\frac{c_{t+1}^{T*} g_{t+1}}{c_t^{T*}}\right)^{-1} \left(\xi\alpha L_{t+1}^{T*} + \left(a_{t+1}^*\right)^{\Omega}\right)$$

$$L_t^R = \frac{g_{t+1} - 1}{\chi}$$

$$L_t^{R*} = \frac{g_{t+1} a_{t+1}^* - a_t^*}{\xi \left(a_{t+1}^*\right)^{1-\Omega}}$$

(iv) Labor demand in non-tradable sector

$$L^N = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T)\mathcal{P}^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} c_t^T$$

$$L^{N*} = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H^* \left((1+\tau_{H,t}^{T*})\mathcal{P}\right)^{1-\rho} + \nu_F^*\right]^{\frac{1}{1-\rho}} \frac{c_t^{T*}}{a_t^*} \tag{91}$$

(v) Supply of tradable sector:

$$y_{H,t}^T = \mathcal{Z} L_t^T$$

$$y_{F,t}^T = \mathcal{Z} a_t^* L_t^{T*}$$

(vi) Supply of non-tradable sector:

$$Y_{H,t}^N = L_t^N$$

$$Y_{F,t}^N = L_t^{N*}$$

(vii) Tradable good market clearing:

$$y_{H,t}^T = c_{H,t}^T + c_{H,t}^{T*}$$

$$y_{F,t}^T = c_{F,t}^T + c_{F,t}^{T*}$$

(viii) Labor market clearing

$$\bar{L} = L_t^N + L_t^T + L_t^R$$

$$\bar{L}^* = L_t^{N*} + L_t^{T*} + L_t^{R*}$$

(ix) Asset market clearing:

$$P_t^T b_t + P_t^{T*} b_t^* = 0$$

(x) Resource constraint

$$\mathcal{Z}L_t^T + \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T)\mathcal{P}_t^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} b_t = c_{H,t}^T + \mathcal{P}_t^{-1} c_{F,t}^T + \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T)\mathcal{P}_t^{-1}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \frac{g_{t+1} b_{t+1}}{R_t^n}$$

(xi) Slackness condition of borrowing:

$$\hat{\mu}_t \left(b_t + \kappa_H\right) = 0$$

$$\hat{\mu}_t^* \left(b_t^* + \kappa_F\right) = 0$$

Parameter Value: Three Country Model

Parameter	Description	Values
β	discount factor	0.995
$\bar{L}$	labor stock in home country	1
$\bar{L}^*$	labor stock in foreign country	1
$\bar{L}^{**}$	labor stock in rest of the world	1
α	intermediate share	0.33
ν_H	home preference on consuming home goods	0.75
ν_F	home preference on consuming foreign goods	0.20
ν_W	home preference on consuming rest of the world goods	0.05
ν_H^*	foreign preference on consuming home goods	0.05
ν_F^*	foreign preference on consuming foreign goods	0.80
ν_W^*	foreign preference on consuming rest of the world goods	0.15
ν_H^{**}	rest of the world preference on consuming home goods	0.05
ν_F^{**}	rest of the world preference on consuming foreign goods	0.10
ν_W^{**}	rest of the world preference on consuming rest of the world goods	0.85
ρ	elasticity of substitution of tradable goods	1.5
κ_H	debt limit of home country	-0.001
κ_F	debt limit of foreign country	0.0016
κ_W	debt limit of rest of the world	-0.0006
χ	research intensity of home country	1
ξ	research intensity of foreign country	0.5
ζ	technology gap of rest of the world	0.3
ϕ^*	saving subsidies	0.01
Ω	research dependence	0.5
ω	share of tradable goods	0.3

Table 2: Table of Parameter Values

Three Country Model Equilibrium Condition

The Stationary Equilibrium under Re-exportation: A stationary equilibrium is a path of real allocations of $\{c_t^T,\, c_t^{T*},\, c_t^{T**},\, c_{H,t}^T,\, c_{F,t}^T,\, c_{RoW,t}^T,\, c_{H,t}^{T*},\, c_{F,t}^{T*},\, c_{RoW,t}^{T*},\, c_{H,t}^{T**},\, c_{F,t}^{T**},\, c_{RoW,t}^{T**},\, L_t^T,\, L_t^{T*},\, L_t^{T**},\, L_t^N,\, L_t^{N*},$ $L_t^{N**},\, L_t^R,\, L_t^{R*},\, Y_{H,t}^N,\, Y_{F,t}^N,\, Y_{RoW,t}^N,\, y_{H,t}^T,\, y_{F,t}^T,\, y_{RoW,t}^T,\, y_{F,t}^{T,Re},\, b_{t+1},\, b_{t+1}^*,\, b_{t+1}^{**},\, g_{t+1},\, a_{t+1}^*,\, R_t^n,\, \hat{\mu}_t,\, \hat{\mu}_t^*,\, \hat{\mu}_t^{**},\, \mathcal{P}_{1,t},$ $\mathcal{P}_{2,t},\, \mathcal{P}_{3,t}\}_{t=0}^{\infty}$, given initial condition $\{B_0,\, B_0^*,\, B_0^{**},\, A_0,\, A_0^*\}$, assuming zero domestic inflation over time ($\pi_{H,t+1}^T = \pi_{F,t+1}^T = \pi_{W,t+1}^T = 1$) and the evolution of trade tariffs $\{\tau_{F,t}^T,\, \tau_{RoW,t}^T,\, \tau_{H,t}^{T*},\, \tau_{RoW,t}^{T*},\, \tau_{H,t}^{T**},\, \tau_{F,t}^{T**}\}$ subject to:

(i) The optimal policies for consumption and saving for both domestic and foreign country

$$\left(c_t^T\right)^{-1} = \beta\left\{\left(\left(c_{t+1}^T g_{t+1}\right)^{-1}\right)(1+\phi_t^*)R_t^n + \tilde{\mu}_t\right\}$$

$$\left(c_t^{T*}\right)^{-1} = \beta\left\{\left(c_{t+1}^{T*} g_{t+1}\right)^{-1}(1+\phi^*)R_t^n\left[\frac{\left[\frac{\nu_H+\nu_F\left((1+\tau_{F,t+1}^T)\mathcal{P}_{1,t+1}^{-1}\right)^{1-\rho}+\nu_W\left((1+\tau_{W,t+1}^T)\mathcal{P}_{2,t+1}^{-1}\right)^{1-\rho}}{\nu_H^*(1+\tau_{H,t+1}^{T*})^{1-\rho}+\nu_F^*\left(\mathcal{P}_{1,t+1}^{-1}\right)^{1-\rho}+\nu_W^*\left((1+\tau_{W,t+1}^{T*})\mathcal{P}_{2,t+1}^{-1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}}{\left[\frac{\nu_H+\nu_F\left((1+\tau_{F,t}^T)\mathcal{P}_{1,t}^{-1}\right)^{1-\rho}+\nu_W\left((1+\tau_{RoW,t}^T)\mathcal{P}_{2,t}^{-1}\right)^{1-\rho}}{\nu_H^*(1+\tau_{H,t}^{T*})^{1-\rho}+\nu_F^*\left(\mathcal{P}_{1,t}^{-1}\right)^{1-\rho}+\nu_W^*\left((1+\tau_{RoW,t}^{T*})\mathcal{P}_{2,t}^{-1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}}\right]^{-1} + \tilde{\mu}_t^*\right\}$$

$$\left(c_t^{T**}\right)^{-1} = \beta\left\{\left(c_{t+1}^{T**} g_{t+1}\right)^{-1}(1+\phi_t^*)R_t^n\left[\frac{\left[\frac{\nu_H+\nu_F\left((1+\tau_{F,t+1}^T)\mathcal{P}_{1,t+1}^{-1}\right)^{1-\rho}+\nu_W\left((1+\tau_{W,t+1}^T)\mathcal{P}_{2,t+1}^{-1}\right)^{1-\rho}}{\nu_H^{**}(1+\tau_{H,t+1}^{T**})^{1-\rho}+\nu_F^{**}\left((1+\tau_{F,t+1}^{T**})\mathcal{P}_{1,t+1}^{-1}\right)^{1-\rho}+\nu_W^{**}\left(\mathcal{P}_{2,t+1}^{-1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}}{\left[\frac{\nu_H+\nu_F\left((1+\tau_{F,t}^T)\mathcal{P}_{1,t}^{-1}\right)^{1-\rho}+\nu_W\left((1+\tau_{RoW,t}^T)\mathcal{P}_{2,t}^{-1}\right)^{1-\rho}}{\nu_H^{**}(1+\tau_{H,t}^{T**})^{1-\rho}+\nu_F^{**}\left((1+\tau_{F,t}^{T**})\mathcal{P}_{1,t}^{-1}\right)^{1-\rho}+\nu_W^{**}\left(\mathcal{P}_{2,t}^{-1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}}\right]^{-1} + \tilde{\mu}_t^{**}\right\}$$

(ii) Demand of tradable goods:

$$c_{H,t}^T = \nu_H\left[\nu_H + \nu_F\left((1+\tau_{F,t}^T)\mathcal{P}_{1,t}^{-1}\right)^{1-\rho} + \nu_W\left((1+\tau_{RoW,t}^T)\mathcal{P}_{2,t}^{-1}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}} c_t^T$$

$$c_{F,t}^T = \nu_F\left[\nu_H\left((1+\tau_{F,t}^T)^{-1}\mathcal{P}_{1,t}\right)^{1-\rho} + \nu_F + \nu_W\left(\frac{1+\tau_{RoW,t}^T}{1+\tau_{F,t}^T}\mathcal{P}_{3,t}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}} c_t^T$$

$$c_{RoW,t}^T = \nu_W\left[\nu_H\left((1+\tau_{RoW,t}^T)^{-1}\mathcal{P}_{2,t}\right)^{1-\rho} + \nu_F\left(\frac{1+\tau_{F,t}^T}{1+\tau_{RoW,t}^T}\mathcal{P}_{3,t}^{-1}\right)^{1-\rho} + \nu_W\right]^{\frac{\rho}{1-\rho}} c_t^T$$

$$c_{H,t}^{T*} = \nu_H^*\left[\nu_H^* + \nu_F^*\left((1+\tau_{H,t}^{T*})^{-1}\mathcal{P}_{1,t}^{-1}\right)^{1-\rho} + \nu_W^*\left(\frac{1+\tau_{RoW,t}^{T*}}{1+\tau_{H,t}^{T*}}\mathcal{P}_{2,t}^{-1}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}} c_t^{T*}$$

$$c_{F,t}^{T*} = \nu_F^*\left[\nu_H^*\left((1+\tau_{H,t}^{T*})\mathcal{P}_{1,t}\right)^{1-\rho} + \nu_F^* + \nu_W^*\left((1+\tau_{RoW,t}^{T*})\mathcal{P}_{3,t}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}} c_t^{T*}$$

$$c_{RoW,t}^{T*} = \nu_W^*\left[\nu_H^*\left(\frac{1+\tau_{H,t}^{T*}}{1+\tau_{RoW,t}^{T*}}\mathcal{P}_{2,t}\right)^{1-\rho} + \nu_F^*\left((1+\tau_{RoW,t}^{T*})^{-1}\mathcal{P}_{3,t}^{-1}\right)^{1-\rho} + \nu_W^*\right]^{\frac{1}{1-\rho}} c_t^{T*}$$

$$c_{H,t}^{T**} = \nu_H^{**} \left[\nu_H^{**} + \nu_F^{**} \left(\frac{1+\tau_{F,t}^{T**}}{1+\tau_{H,t}^{T**}} \mathcal{P}_{1,t}^{-1} \right)^{1-\rho} + \nu_W^{**} \left((1+\tau_{H,t}^{T**})^{-1} \mathcal{P}_{2,t}^{-1} \right)^{1-\rho} \right]^{\frac{\rho}{1-\rho}} c_t^{T**}$$

$$c_{F,t}^{T**} = \nu_F^{**} \left[\nu_H^{**} \left(\frac{1+\tau_{H,t}^{T**}}{1+\tau_{F,t}^{T**}} \mathcal{P}_{1,t} \right)^{1-\rho} + \nu_F^{*} + \nu_W^{*} \left((1+\tau_{F,t}^{T*})^{-1} \mathcal{P}_{3,t} \right)^{1-\rho} \right]^{\frac{\rho}{1-\rho}} c_t^{T**}$$

$$c_{RoW,t}^{T**} = \nu_W^{**} \left[\nu_H^{**} \left((1+\tau_{H,t}^{T**}) \mathcal{P}_{2,t} \right)^{1-\rho} + \nu_F^{**} \left((1+\tau_{F,t}^{T**}) \mathcal{P}_{3,t}^{-1} \right)^{1-\rho} + \nu_W^{**} \right]^{\frac{\rho}{1-\rho}} c_t^{T**}$$

(iii) Optimal condition in research sector:

$$g_{t+1} = \beta \left(\frac{c_{t+1}^T}{c_t^T} \right)^{-1} \frac{P_{H,t+1}^T}{P_{H,t}^T} \left(\chi \alpha L_{t+1}^T + 1 \right)$$

$$(a_t^*)^\Omega = \beta \left(\frac{c_{t+1}^{T*} g_{t+1}}{c_t^{T*}} \right)^{-1} \frac{P_{F,t+1}^T}{P_{F,t}^T} \left(\xi \alpha L_{t+1}^{T*} + \left(a_{t+1}^*\right)^\Omega \right)$$

$$(a_t^{**})^\Omega = \beta \left(\frac{c_{t+1}^{T**} g_{t+1}}{c_t^{T**}} \right)^{-1} \frac{P_{RoW,t+1}^T}{P_{RoW,t}^T} \left(\zeta \alpha L_{t+1}^{T*} + \left(a_{t+1}^{**}\right)^\Omega \right)$$

$$L_t^R = \frac{g_{t+1} - 1}{\chi}$$

$$L_t^{R*} = \frac{g_{t+1} a_{t+1}^* - a_t^*}{\xi \left(a_{t+1}^*\right)^{1-\Omega}}$$

$$L_t^{R**} = \frac{g_{t+1} a_{t+1}^{**} - a_t^{**}}{\zeta \left(a_{t+1}^*\right)^{1-\Omega}}$$

(iv) Labor demand in non-tradable sector

$$L_t^N = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H + \nu_F \left((1+\tau_{F,t}^T) \mathcal{P}_{1,t}^{-1} \right)^{1-\rho} + \nu_W \left((1+\tau_{RoW,t}^T) \mathcal{P}_{2,t}^{-1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} c_t^T$$

$$L_t^{N*} = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H^* \left((1+\tau_{H,t}^{T*}) \mathcal{P}_{1,t} \right)^{1-\rho} + \nu_F^* + \nu_W^* \left((1+\tau_{RoW,t}^{T*}) \mathcal{P}_{3,t} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \frac{c_t^{T*}}{a_t^*}$$

$$L_t^{N**} = \frac{1-\omega}{\omega(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}} \left[\nu_H^{**} \left((1+\tau_{H,t}^{T**}) \mathcal{P}_{2,t} \right)^{1-\rho} + \nu_F^{**} \left((1+\tau_{F,t}^{T**}) \mathcal{P}_{3,t}^{-1} \right)^{1-\rho} + \nu_W^{**} \right]^{\frac{1}{1-\rho}}$$
$$\times \frac{c_t^{T**}}{\left((a_t^{**})^{1-\eta} + (1-\gamma) \left((1-\delta^T) y_{F,t}^{T,Re} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}}$$

(v) Supply of tradable sector:

$$y_{H,t}^T = \mathcal{Z} L_t^T$$

$$y_{F,t}^T = \mathcal{Z} a_t^* L_t^{T*}$$

$$y_{RoW,t}^T = \mathcal{Z} \left(\left(a_t^{**}\right)^{1-\eta} + (1-\gamma) \left((1-\delta^T) y_{F,t}^{T,Re} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} L_{RoW,t}^{T**}$$

(vi) Supply of non-tradable sector:

$$Y^N_{H,t} = L^N_t$$

$$Y^N_{F,t} = L^{N*}_t$$

$$Y^N_{RoW,t} = L^{N**}_t$$

(vii) Demands from re-exportation goods from foreign country:

$$\mathcal{P}_3^{-1} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(1-\gamma)(1-\delta^T)\left(\gamma\left(\frac{a^{**}_t}{(1-\delta^T)y^{T,Re}_{F,t}}\right)^{1-\eta} + (1-\gamma)\right)^{\frac{\eta}{1-\eta}} L^{T**}_t$$

(viii) Tradable good market clearing:

$$y^T_{H,t} = c^T_{H,t} + c^{T*}_{H,t} + c^{T**}_{H,t}$$

$$y^T_{F,t} = c^T_{F,t} + c^{T*}_{F,t} + c^{T**}_{F,t} + y^{T,Re}_{F,t}$$

$$y^T_{RoW,t} = c^T_{RoW,t} + c^{T*}_{RoW,t} + c^{T**}_{RoW,t}$$

(ix) Labor market clearing

$$\bar{L} = L^N_t + L^T_t + L^R_t$$

$$\bar{L}^* = L^{N*}_t + L^{T*}_t + L^{R*}_t$$

$$\bar{L}^{**} = L^{N**}_t + L^{T**}_t + L^{R**}_t$$

(x) Asset market clearing:

$$b_t + b^*_t + b^{**}_t = 0$$

(xi) Slackness condition of borrowing:

$$\hat{\mu}_t\left(b_t + \kappa_H\right) = 0$$

$$\tilde{\mu}^*_t\left(b^*_t + \kappa_F\right) = 0$$

$$\tilde{\mu}^{**}_t\left(b^{**}_t + \kappa_W\right) = 0$$

(xii) Prices, no arbitrage condition

$$\mathcal{P}_{3,t} = \frac{\mathcal{P}_{1,t}}{\mathcal{P}_{2,t}}$$

(xiii) Budget constraint:

$$\begin{aligned}
&\mathcal{Z}L^T_t + \left[\nu_H + \nu_F\left((1+\tau^T_{F,t})\mathcal{P}^{-1}_{1,t}\right)^{1-\rho} + \nu_W\left((1+\tau^T_{RoW,t})\mathcal{P}^{-1}_{2,t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} b_t \\
&\quad = c^T_{H,t} + \mathcal{P}^{-1}_{1,t}c^T_{F,t} + \mathcal{P}^{-1}_{2,t}c^T_{RoW,t} + \left[\nu_H + \nu_F\left((1+\tau^T_{F,t})\mathcal{P}^{-1}_{1,t}\right)^{1-\rho} + \nu_W\left((1+\tau^T_{RoW,t})\mathcal{P}^{-1}_{2,t}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} \frac{b_{t+1}}{R^n_t}
\end{aligned}$$

$$
\left((a_t^{**})^{1-\eta}+(1-\gamma)\left((1-\delta^T)y_{F,t}^{T,Re}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\mathcal{Z}L_t^{T**}+\Big[\nu_H^{**}\left((1+\tau_{H,t}^{T**})\mathcal{P}_{2,t}\right)^{1-\rho}+\nu_F^{**}\left((1+\tau_{F,t}^{T**})\mathcal{P}_{3,t}^{-1}\right)^{1-\rho}+\nu_W^{**}\Big]
$$

$$
=\mathcal{P}_{2,t}c_{H,t}^{T**}+\mathcal{P}_{3,t}^{-1}(c_{F,t}^{T**}+y_{F,t}^{T,Re})+c_{RoW,t}^{T**}+\Big[\nu_H^{**}\left((1+\tau_{H,t}^{T**})\mathcal{P}_{2,t}\right)^{1-\rho}+\nu_F^{**}\left((1+\tau_{F,t}^{T**})\mathcal{P}_{3,t}^{-1}\right)^{1-\rho}+\nu_W^{**}\Big]^{\frac{1}{1-\rho}}\frac{b_{t+1}}{R_t^n}
$$

Log-linearization of the Three Country Model

The Euler Equation

$$
-\widehat{c}_t^{T*}=\mathbb{E}_t\Big[\widehat{c}_{t+1}^{T*}+\hat{g}_{t+1}-\hat{r}_t^n-(\hat{q}_{t+1}-\hat{q}_t)\Big]
$$

where:

$$
\begin{aligned}
\hat{q}_t=&\frac{\nu_F\left((1+\tau_{F,t}^T)\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}}{\nu_H+\nu_F\left((1+\tau_{F,t}^T)\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}+\nu_W\left((1+\tau_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}}\left(\ln\widehat{(1+\tau_{F,t}^T)}-\hat{p}_{1,t}\right)\\
&+\frac{\left((1+\tau_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}}{\nu_H+\nu_F\left((1+\tau_{F,t}^T)\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}+\nu_W\left((1+\tau_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}}\left(\ln\left(\widehat{1+\tau_{RoW,t}^T}\right)-\hat{p}_{2,t}\right)\\
&-\Bigg[\frac{\nu_H^*(1+\tau_H^{T*})^{1-\rho}}{\nu_H^*(1+\tau_{H,t}^{T*})^{1-\rho}+\nu_F^*\left(\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}+\nu_W^*\left((1+\tau_{RoW}^{T*})\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}}\ln\left(\widehat{1+\tau_{H,t}^{T*}}\right)\\
&+\frac{\nu_F^*\left(\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}}{\nu_H^*(1+\tau_H^{T*})^{1-\rho}+\nu_F^*\left(\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}+\nu_W^*\left((1+\tau_{RoW}^{T*})\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}}(-\hat{p}_{1,t})\\
&+\frac{\nu_W^*\left((1+\tau_{RoW}^{T*})\,\mathcal{P}_2^{-1}\right)^{1-\rho}}{\nu_H^*(1+\tau_{H,t}^{T*})^{1-\rho}+\nu_F^*\left(\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho}+\nu_W^*\left((1+\tau_{RoW}^{T*})\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}}\left(\ln\left(\widehat{1+\tau_{RoW,t}^{T*}}\right)-\hat{p}_{2,t}\right)\Bigg]
\end{aligned}
$$

Home economy

$$
\begin{aligned}
\hat{c}_{H,t}^T=\hat{c}_t^T+\rho\Bigg\{&\frac{\nu_F\big((1+\bar{\tau}_F^T)\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}}{\nu_H+\nu_F\big((1+\bar{\tau}_F^T)\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}+\nu_W\big((1+\bar{\tau}_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{F,t}^T})-\hat{p}_{1,t}\Big)\\
&+\frac{\nu_W\big((1+\bar{\tau}_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}{\nu_H+\nu_F\big((1+\bar{\tau}_F^T)\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}+\nu_W\big((1+\bar{\tau}_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{RoW,t}^T})-\hat{p}_{2,t}\Big)\Bigg\}.
\end{aligned}
$$

$$
\begin{aligned}
\hat{c}_{F,t}^T=\hat{c}_t^T+\rho\Bigg\{&\frac{\nu_H\big((1+\bar{\tau}_F^T)^{-1}\bar{\mathcal{P}}_1\big)^{1-\rho}}{\nu_H\big((1+\bar{\tau}_F^T)^{-1}\bar{\mathcal{P}}_1\big)^{1-\rho}+\nu_F+\nu_W\big(\frac{1+\bar{\tau}_{RoW}^T}{1+\bar{\tau}_F^T}\bar{\mathcal{P}}_3\big)^{1-\rho}}\Big(-\ln(\widehat{1+\tau_{F,t}^T})+\hat{p}_{1,t}\Big)\\
&+\frac{\nu_W\big(\frac{1+\bar{\tau}_{RoW}^T}{1+\bar{\tau}_F^T}\bar{\mathcal{P}}_3\big)^{1-\rho}}{\nu_H\big((1+\bar{\tau}_F^T)^{-1}\bar{\mathcal{P}}_1\big)^{1-\rho}+\nu_F+\nu_W\big(\frac{1+\bar{\tau}_{RoW}^T}{1+\bar{\tau}_F^T}\bar{\mathcal{P}}_3\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{RoW,t}^T})-\ln(\widehat{1+\tau_{F,t}^T})+\hat{p}_{3,t}\Big)\Bigg\}.
\end{aligned}
$$

$$
\hat{c}_{RoW,t}^T=\hat{c}_t^T+\rho\Bigg\{\frac{\nu_H\big((1+\bar{\tau}_{RoW}^T)^{-1}\bar{\mathcal{P}}_2\big)^{1-\rho}}{\nu_H\big((1+\bar{\tau}_{RoW}^T)^{-1}\bar{\mathcal{P}}_2\big)^{1-\rho}+\nu_F\big(\frac{1+\bar{\tau}_F^T}{1+\bar{\tau}_{RoW}^T}\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}+\nu_W}\Big(-\ln(\widehat{1+\tau_{RoW,t}^T})+\hat{p}_{2,t}\Big)
$$

$$+\frac{\nu_F\big(\frac{1+\bar{\tau}_F^T}{1+\bar{\tau}_{RoW}^T}\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}}{\nu_H\big((1+\bar{\tau}_{RoW}^T)^{-1}\bar{\mathcal{P}}_2\big)^{1-\rho}+\nu_F\big(\frac{1+\bar{\tau}_F^T}{1+\bar{\tau}_{RoW}^T}\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}+\nu_W}\Big(\ln(\widehat{1+\tau_{F,t}^T})-\ln(\widehat{1+\tau_{RoW,t}^T})-\hat{p}_{3,t}\Big)\Bigg\}.$$

Foreign economy ($*$)

$$\hat{c}_{H,t}^{T*}=\hat{c}_t^{T*}+\rho\Bigg\{\frac{\nu_F^*\big((1+\bar{\tau}_H^{T*})^{-1}\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}}{\nu_H^*+\nu_F^*\big((1+\bar{\tau}_H^{T*})^{-1}\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}+\nu_W^*\big(\frac{1+\bar{\tau}_{RoW}^{T*}}{1+\bar{\tau}_H^{T*}}\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}\Big(-\ln(\widehat{1+\tau_{H,t}^{T*}})-\hat{p}_{1,t}\Big)$$

$$+\frac{\nu_W^*\big(\frac{1+\bar{\tau}_{RoW}^{T*}}{1+\bar{\tau}_H^{T*}}\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}{\nu_H^*+\nu_F^*\big((1+\bar{\tau}_H^{T*})^{-1}\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}+\nu_W^*\big(\frac{1+\bar{\tau}_{RoW}^{T*}}{1+\bar{\tau}_H^{T*}}\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{RoW,t}^{T*}})-\ln(\widehat{1+\tau_{H,t}^{T*}})-\hat{p}_{2,t}\Big)\Bigg\}.$$

$$\hat{c}_{F,t}^{T*}=\hat{c}_t^{T*}+\rho\Bigg\{\frac{\nu_H^*\big((1+\bar{\tau}_H^{T*})\bar{\mathcal{P}}_1\big)^{1-\rho}}{\nu_H^*\big((1+\bar{\tau}_H^{T*})\bar{\mathcal{P}}_1\big)^{1-\rho}+\nu_F^*+\nu_W^*\big((1+\bar{\tau}_{RoW}^{T*})\bar{\mathcal{P}}_3\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{H,t}^{T*}})+\hat{p}_{1,t}\Big)$$

$$+\frac{\nu_W^*\big((1+\bar{\tau}_{RoW}^{T*})\bar{\mathcal{P}}_3\big)^{1-\rho}}{\nu_H^*\big((1+\bar{\tau}_H^{T*})\bar{\mathcal{P}}_1\big)^{1-\rho}+\nu_F^*+\nu_W^*\big((1+\bar{\tau}_{RoW}^{T*})\bar{\mathcal{P}}_3\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{RoW,t}^{T*}})+\hat{p}_{3,t}\Big)\Bigg\}.$$

$$\hat{c}_{RoW,t}^{T*}=\hat{c}_t^{T*}+\rho\Bigg\{\frac{\nu_H^*\big(\frac{1+\bar{\tau}_H^{T*}}{1+\bar{\tau}_{RoW}^{T*}}\bar{\mathcal{P}}_2\big)^{1-\rho}}{\nu_H^*\big(\frac{1+\bar{\tau}_H^{T*}}{1+\bar{\tau}_{RoW}^{T*}}\bar{\mathcal{P}}_2\big)^{1-\rho}+\nu_F^*\big((1+\bar{\tau}_{RoW}^{T*})^{-1}\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}+\nu_W^*}\Big(\ln(\widehat{1+\tau_{H,t}^{T*}})-\ln(\widehat{1+\tau_{RoW,t}^{T*}})+\hat{p}_{2,t}\Big)$$

$$+\frac{\nu_F^*\big((1+\bar{\tau}_{RoW}^{T*})^{-1}\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}}{\nu_H^*\big(\frac{1+\bar{\tau}_H^{T*}}{1+\bar{\tau}_{RoW}^{T*}}\bar{\mathcal{P}}_2\big)^{1-\rho}+\nu_F^*\big((1+\bar{\tau}_{RoW}^{T*})^{-1}\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}+\nu_W^*}\Big(-\ln(\widehat{1+\tau_{RoW,t}^{T*}})-\hat{p}_{3,t}\Big)\Bigg\}.$$

Second foreign economy ($**$)

$$\hat{c}_{H,t}^{T**}=\hat{c}_t^{T**}+\rho\Bigg\{\frac{\nu_F^{**}\big(\frac{1+\bar{\tau}_F^{T**}}{1+\bar{\tau}_H^{T**}}\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}}{\nu_H^{**}+\nu_F^{**}\big(\frac{1+\bar{\tau}_F^{T**}}{1+\bar{\tau}_H^{T**}}\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}+\nu_W^{**}\big((1+\bar{\tau}_H^{T**})^{-1}\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{F,t}^{T**}})-\ln(\widehat{1+\tau_{H,t}^{T**}})-\hat{p}_{1,t}\Big)$$

$$+\frac{\nu_W^{**}\big((1+\bar{\tau}_H^{T**})^{-1}\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}{\nu_H^{**}+\nu_F^{**}\big(\frac{1+\bar{\tau}_F^{T**}}{1+\bar{\tau}_H^{T**}}\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}+\nu_W^{**}\big((1+\bar{\tau}_H^{T**})^{-1}\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}\Big(-\ln(\widehat{1+\tau_{H,t}^{T**}})-\hat{p}_{2,t}\Big)\Bigg\}.$$

$$\hat{c}_{F,t}^{T**}=\hat{c}_t^{T**}+\rho\Bigg\{\frac{\nu_H^{**}\big(\frac{1+\bar{\tau}_H^{T**}}{1+\bar{\tau}_F^{T**}}\bar{\mathcal{P}}_1\big)^{1-\rho}}{\nu_H^{**}\big(\frac{1+\bar{\tau}_H^{T**}}{1+\bar{\tau}_F^{T**}}\bar{\mathcal{P}}_1\big)^{1-\rho}+\nu_F^{**}+\nu_W^{**}\big((1+\bar{\tau}_F^{T**})^{-1}\bar{\mathcal{P}}_3\big)^{1-\rho}}\Big(\ln(\widehat{1+\tau_{H,t}^{T**}})-\ln(\widehat{1+\tau_{F,t}^{T**}})+\hat{p}_{1,t}\Big)$$

$$+\frac{\nu_W^{**}\big((1+\bar{\tau}_F^{T**})^{-1}\bar{\mathcal{P}}_3\big)^{1-\rho}}{\nu_H^{**}\big(\frac{1+\bar{\tau}_H^{T**}}{1+\bar{\tau}_F^{T**}}\bar{\mathcal{P}}_1\big)^{1-\rho}+\nu_F^{**}+\nu_W^{**}\big((1+\bar{\tau}_F^{T**})^{-1}\bar{\mathcal{P}}_3\big)^{1-\rho}}\Big(-\ln(\widehat{1+\tau_{F,t}^{T**}})+\hat{p}_{3,t}\Big)\Bigg\}.$$

$$\hat{c}^{T**}_{RoW,t} = \hat{c}^{T**}_t + \rho \left\{ \frac{\nu^{**}_H \big((1+\bar{\tau}^{T**}_H)\bar{\mathcal{P}}_2\big)^{1-\rho}}{\nu^{**}_H \big((1+\bar{\tau}^{T**}_H)\bar{\mathcal{P}}_2\big)^{1-\rho} + \nu^{**}_F \big((1+\bar{\tau}^{T**}_F)\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho} + \nu^{**}_W} \Big(\ln(\widehat{1+\tau^{T**}_{H,t}}) + \hat{p}_{2,t} \Big) \right.$$

$$\left. + \frac{\nu^{**}_F \big((1+\bar{\tau}^{T**}_F)\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho}}{\nu^{**}_H \big((1+\bar{\tau}^{T**}_H)\bar{\mathcal{P}}_2\big)^{1-\rho} + \nu^{**}_F \big((1+\bar{\tau}^{T**}_F)\bar{\mathcal{P}}_3^{-1}\big)^{1-\rho} + \nu^{**}_W} \Big(\ln(\widehat{1+\tau^{T**}_{F,t}}) - \hat{p}_{3,t} \Big) \right\}.$$

The Optimal Innovation Conditions

$$\hat{g}_{t+1} = -\big(\hat{c}^T_{t+1} - \hat{c}^T_t\big) + \frac{\chi\alpha\bar{L}^T}{1+\chi\alpha\bar{L}^T}\hat{L}^T_{t+1}$$

$$\Omega\,\hat{a}^*_t = -\big(\hat{c}^{T*}_{t+1} + \hat{g}_{t+1} - \hat{c}^{T*}_t\big) + \frac{\xi\alpha\bar{L}^{T*}}{\xi\alpha\bar{L}^{T*} + (\bar{a}^*)^\Omega}\,\hat{L}^{T*}_{t+1} + \frac{\Omega\,(\bar{a}^*)^\Omega}{\xi\alpha\bar{L}^{T*} + (\bar{a}^*)^\Omega}\,\hat{a}^*_{t+1}$$

$$\Omega\,\hat{a}^{**}_t = -\big(\hat{c}^{T**}_{t+1} + \hat{g}_{t+1} - \hat{c}^{T**}_t\big) + \frac{\zeta\alpha\bar{L}^{T*}}{\zeta\alpha\bar{L}^{T*} + (\bar{a}^{**})^\Omega}\,\hat{L}^{T*}_{t+1} + \frac{\Omega\,(\bar{a}^{**})^\Omega}{\zeta\alpha\bar{L}^{T*} + (\bar{a}^{**})^\Omega}\,\hat{a}^{**}_{t+1}$$

$$\hat{L}^R_t = \frac{\bar{g}}{\bar{g}-1}\,\hat{g}_{t+1}$$

$$\hat{L}^{R*}_t = \frac{\bar{g}}{\bar{g}-1}\,\hat{g}_{t+1} + \left[\frac{\bar{g}}{\bar{g}-1} - (1-\Omega)\right]\hat{a}^*_{t+1} - \frac{1}{\bar{g}-1}\,\hat{a}^*_t$$

$$\hat{L}^{R**}_t = \frac{\bar{g}}{\bar{g}-1}\,\hat{g}_{t+1} + \frac{\bar{g}}{\bar{g}-1}\,\hat{a}^{**}_{t+1} - \frac{1}{\bar{g}-1}\,\hat{a}^{**}_t - (1-\Omega)\,\hat{a}^*_{t+1}$$

The Labor Market Conditions

$$\begin{aligned}\hat{L}^N_t =& \hat{c}^T_t + \frac{\nu_F\big((1+\bar{\tau}^T_F)\,\bar{\mathcal{P}}_1^{-1}\big)^{1-\rho}}{\nu_H + \nu_F\Big((1+\bar{\tau}^T_F)\,\bar{\mathcal{P}}_1^{-1}\Big)^{1-\rho} + \nu_W\Big((1+\bar{\tau}^T_{RoW})\,\bar{\mathcal{P}}_2^{-1}\Big)^{1-\rho}}\Big(\ln(\widehat{1+\tau^T_{F,t}}) - \hat{p}_{1,t}\Big) \\ &+ \frac{\nu_W\big((1+\bar{\tau}^T_{RoW})\,\bar{\mathcal{P}}_2^{-1}\big)^{1-\rho}}{\nu_H + \nu_F\Big((1+\bar{\tau}^T_F)\,\bar{\mathcal{P}}_1^{-1}\Big)^{1-\rho} + \nu_W\Big((1+\bar{\tau}^T_{RoW})\,\bar{\mathcal{P}}_2^{-1}\Big)^{1-\rho}}\Big(\ln(\widehat{1+\tau^T_{RoW,t}}) - \hat{p}_{2,t}\Big)\end{aligned}$$

$$\begin{aligned}\hat{L}^{N*}_t =& \hat{c}^{T*}_t - \hat{a}^*_t + \frac{\nu^*_H\big((1+\bar{\tau}^{T*}_H)\,\bar{\mathcal{P}}_1\big)^{1-\rho}}{\nu^*_H\Big((1+\bar{\tau}^{T*}_H)\,\bar{\mathcal{P}}_1\Big)^{1-\rho} + \nu^*_F + \nu^*_W\Big((1+\bar{\tau}^{T*}_{RoW})\,\bar{\mathcal{P}}_3\Big)^{1-\rho}}\Big(\ln(\widehat{1+\tau^{T*}_{H,t}}) + \hat{p}_{1,t}\Big) \\ &+ \frac{\nu^*_W\big((1+\bar{\tau}^{T*}_{RoW})\,\bar{\mathcal{P}}_3\big)^{1-\rho}}{\nu^*_H\Big((1+\bar{\tau}^{T*}_H)\,\bar{\mathcal{P}}_1\Big)^{1-\rho} + \nu^*_F + \nu^*_W\Big((1+\bar{\tau}^{T*}_{RoW})\,\bar{\mathcal{P}}_3\Big)^{1-\rho}}\Big(\ln(\widehat{1+\tau^{T*}_{RoW,t}}) + \hat{p}_{3,t}\Big)\end{aligned}$$

$$
\begin{aligned}
\hat{L}_t^{N**} = \hat{c}_t^{T**} & \\
& + \frac{\nu_H^{**}\left((1+\bar{\tau}_H^{T**})\bar{\mathcal{P}}_2\right)^{1-\rho}}{\nu_H^{**}\left((1+\bar{\tau}_H^{T**})\bar{\mathcal{P}}_2\right)^{1-\rho} + \nu_F^{**}\left((1+\bar{\tau}_F^{T**})\bar{\mathcal{P}}_3^{-1}\right)^{1-\rho} + \nu_W^{**}} \left(\ln(\widehat{1+\tau_{H,t}^{T**}}) + \hat{p}_{2,t}\right) \\
& + \frac{\nu_F^{**}\Big((1+\bar{\tau}_F^{T**})\bar{\mathcal{P}}_3^{-1}\Big)^{1-\rho}}{\nu_H^{**}\left((1+\bar{\tau}_H^{T**})\bar{\mathcal{P}}_2\right)^{1-\rho} + \nu_F^{**}\left((1+\bar{\tau}_F^{T**})\bar{\mathcal{P}}_3^{-1}\right)^{1-\rho} + \nu_W^{**}} \left(\ln(\widehat{1+\tau_{F,t}^{T**}}) - \hat{p}_{3,t}\right) \\
& - \frac{(\bar{a}^{**})^{1-\eta}}{(\bar{a}^{**})^{1-\eta} + (1-\gamma)\big(\overline{y_F^{T,Re}}\big)^{1-\eta}}\,\hat{a}_t^{**} \; - \; \frac{(1-\gamma)\big(\overline{y_F^{T,Re}}\big)^{1-\eta}}{(\bar{a}^{**})^{1-\eta} + (1-\gamma)\big(\overline{y_F^{T,Re}}\big)^{1-\eta}}\,\widehat{y_{F,t}^{T,Re}}
\end{aligned}
$$

$$
\frac{\bar{L}^N}{\bar{L}}\,\hat{L}_t^N + \frac{\bar{L}^T}{\bar{L}}\,\hat{L}_t^T + \frac{\bar{L}^R}{\bar{L}}\,\hat{L}_t^R = 0
$$

$$
\frac{\bar{L}^{N*}}{\bar{L}^*}\,\hat{L}_t^{N*} + \frac{\bar{L}^{T*}}{\bar{L}^*}\,\hat{L}_t^{T*} + \frac{\bar{L}^{R*}}{\bar{L}^*}\,\hat{L}_t^{R*} = 0
$$

$$
\frac{\bar{L}^{N**}}{\bar{L}^{**}}\,\hat{L}_t^{N**} + \frac{\bar{L}^{T**}}{\bar{L}^{**}}\,\hat{L}_t^{T**} + \frac{\bar{L}^{R**}}{\bar{L}^{**}}\,\hat{L}_t^{R**} = 0
$$

Resources Constraints

$$
-\hat{p}_{3,t} = \eta \frac{\gamma\left(\dfrac{\bar{a}^{**}}{(1-\delta)\,\bar{y}_F^{T,Re}}\right)^{1-\eta}}{\gamma\left(\dfrac{\bar{a}^{**}}{(1-\delta)\,\bar{y}_F^{T,Re}}\right)^{1-\eta} + (1-\gamma)} \left(\hat{a}_t^{**} - \widehat{y_F^{T,Re}}\right) + \hat{L}_t^{T**}
$$

$$
\hat{L}_t^T = \frac{\bar{c}_H^T}{\mathcal{Z}\overline{L^T}}\hat{c}_{H,t}^T + \frac{\bar{c}_H^{T*}}{\mathcal{Z}\overline{L^T}}\hat{c}_{H,t}^{T*} + \frac{\bar{c}_H^{T**}}{\mathcal{Z}\overline{L^T}}\hat{c}_{H,t}^{T**}
$$

$$
\hat{a}_t^* + \widehat{L_t^{T*}} = \frac{\bar{c}_F^T}{\mathcal{Z}a^*L^{T*}}\,\hat{c}_{F,t}^T + \frac{\bar{c}_F^{T*}}{\mathcal{Z}a^*L^{T*}}\,\hat{c}_{F,t}^{T*} + \frac{\bar{c}_F^{T**}}{\mathcal{Z}a^*L^{T*}}\,\hat{c}_{F,t}^{T**} + \frac{\bar{y}_F^{T,Re}}{\mathcal{Z}a^*L^{T*}}\,\widehat{y_{F,t}^{T,Re}}
$$

$$
\begin{aligned}
\frac{(\bar{a}^{**})^{1-\eta}}{(\bar{a}^{**})^{1-\eta} + (1-\gamma)(\bar{y}_F^{T,Re})^{1-\eta}}\hat{a}_t^{**} + \frac{(1-\gamma)(\bar{y}_F^{T,Re})^{1-\eta}}{(\bar{a}^{**})^{1-\eta} + (1-\gamma)(\bar{y}_F^{T,Re})^{1-\eta}}\widehat{y_F^{T,Re}} + \widehat{L_{RoW,t}^{T**}} & \\
= \frac{\bar{c}_{RoW}^T}{\mathcal{Z}\left((\bar{a}^{**})^{1-\eta} + (1-\gamma)(\bar{y}_F^{T,Re})^{1-\eta}\right)^{\frac{1}{1-\eta}} L_{RoW}^{T**}}\,\hat{c}_{RoW,t}^T & \\
+ \frac{\bar{c}_{RoW}^{T*}}{\mathcal{Z}\left((\bar{a}^{**})^{1-\eta} + (1-\gamma)(\bar{y}_F^{T,Re})^{1-\eta}\right)^{\frac{1}{1-\eta}} L_{RoW}^{T**}}\,\hat{c}_{RoW,t}^{T*} & \\
+ \frac{\bar{c}_{RoW}^{T**}}{\mathcal{Z}\left((\bar{a}^{**})^{1-\eta} + (1-\gamma)(\bar{y}_F^{T,Re})^{1-\eta}\right)^{\frac{1}{1-\eta}} L_{RoW}^{T**}}\,\hat{c}_{RoW,t}^{T**} &
\end{aligned}
$$

Price

$$
\hat{p}_{3,t} \;=\; \hat{p}_{1,t} \;-\; \hat{p}_{2,t}.
$$

Budget Constraint

$$\mathcal{Z}\,\bar{L}^T\,\hat{L}_t^T \;+\; \Big[\nu_H + \nu_F\left((1+\tau_F^T)\bar{\mathcal{P}}_1^{-1}\right)^{1-\rho} + \nu_W\left((1+\tau_{RoW}^T)\bar{\mathcal{P}}_2^{-1}\right)^{1-\rho}\Big]^{\frac{1}{1-\rho}}\,\kappa_H\,(\hat{g}_{t+1} - \hat{R}_t^n)$$

$$=\; \bar{c}_H^T\,\hat{c}_{H,t}^T \;+\; \bar{\mathcal{P}}_1^{-1}\bar{c}_F^T\,(-\hat{p}_{1,t} + \hat{c}_{F,t}^T) \;+\; \bar{\mathcal{P}}_2^{-1}\bar{c}_{RoW}^T\,(-\hat{p}_{2,t} + \hat{c}_{RoW,t}^T)$$

$$\left[\left(\bar{a}^{**\,1-\eta} + (1-\gamma)\,\bar{y}_F^{T,Re\,1-\eta}\right)^{\frac{1}{1-\eta}}\,\mathcal{Z}\,\bar{L}^{T**}\right]\hat{L}_t^{T**} \;+\; \kappa_{RoW}\,\Big[\nu_H^{**}\left((1+\tau_{H,t}^{T**})\bar{\mathcal{P}}_{2,t}\right)^{1-\rho} + \nu_F^{**}\left((1+\tau_F^{T**})\bar{\mathcal{P}}_{3,t}^{-1}\right)^{1-\rho} + \nu_W^{**}\Big]^{\frac{1}{1-\rho}}$$

$$=\; \bar{\mathcal{P}}_2\,\bar{c}_H^{T**}\,(\hat{p}_{2,t} + \hat{c}_{H,t}^{T**}) \;+\; \bar{\mathcal{P}}_3^{-1}\bar{c}_F^{T**}\,(-\hat{p}_{3,t} + \hat{c}_{F,t}^{T**}) \;+\; \bar{\mathcal{P}}_3^{-1}\bar{y}_F^{T,Re}\,(-\hat{p}_{3,t} + \hat{y}_{F,t}^{T,Re}) \;+\; \bar{c}_{RoW}^{T**}\,\hat{c}_{RoW,t}^{T**}.$$

Remark on exponents. All nine linearizations above use the rule: if $c = \nu\,[S]^\alpha c^T$ with $S = \sum_k a_k x_k^{1-\rho}$, then

$$\hat{c} = \hat{c}^T + \underbrace{\alpha(1-\rho)}_{\text{multiplier}}\sum_k \left(\frac{a_k \bar{x}_k^{1-\rho}}{\bar{S}}\right)\hat{x}_k.$$

In my system, $\alpha = \frac{\rho}{1-\rho}$ for all equations except any line where the level equation would use $\alpha = \frac{1}{1-\rho}$ (then the multiplier becomes 1 instead of ρ). The equations above reflect the exponents exactly as you wrote them.

Definitions. $\hat{p}_{j,t} \equiv \ln \mathcal{P}_{j,t} - \ln \bar{\mathcal{P}}_j$. $\ln\widehat{(1+\tau_{i,t})} \equiv \ln(1+\tau_{i,t}) - \ln(1+\bar{\tau}_i)$. Overbars denote steady-state values.

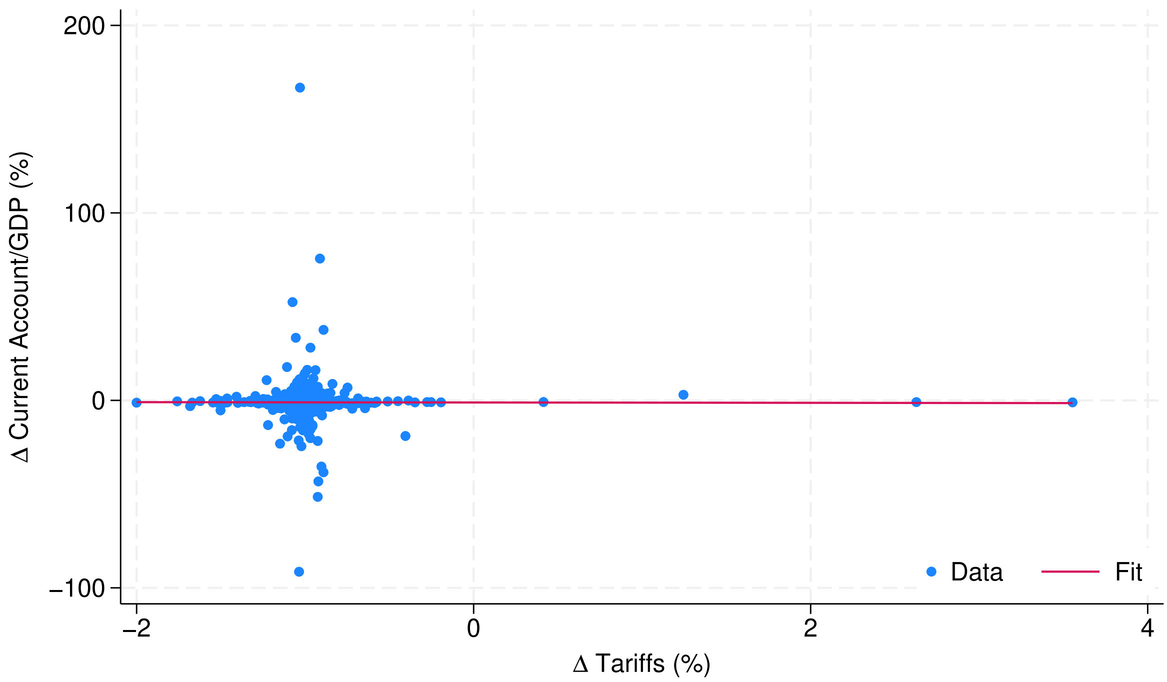

Figure 20: Scatter Plots Between the Change of Tariffs to The CA-to-GDP Ratio
This figure illustrates the correlation between change of tariffs to a country's current account share, which shows no evidence regarding the correlations.

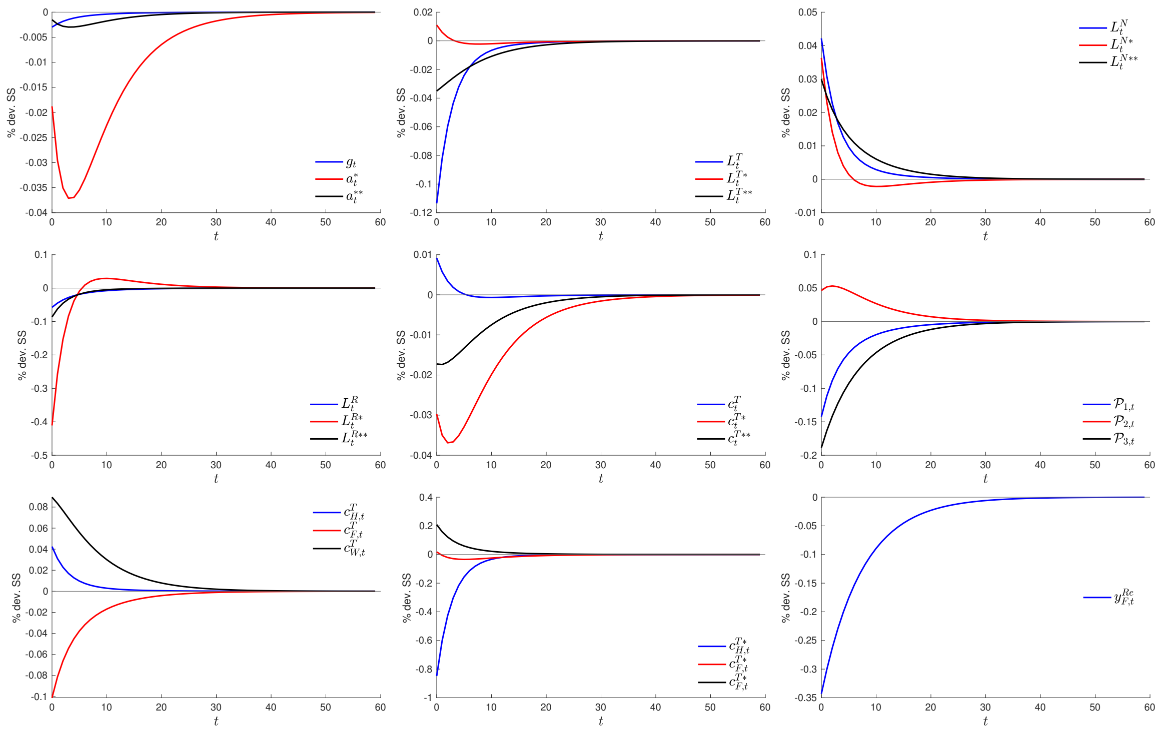

Figure 21: Dynamics under Unilateral Tariff Regime: Foreign Country against the Home Country, Three Country Cases.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{FH,t} = 1\%$, I hereby assume no retaliations from the foreign country and $\eta = 0.67$. **Assumption 1, 2** holds.

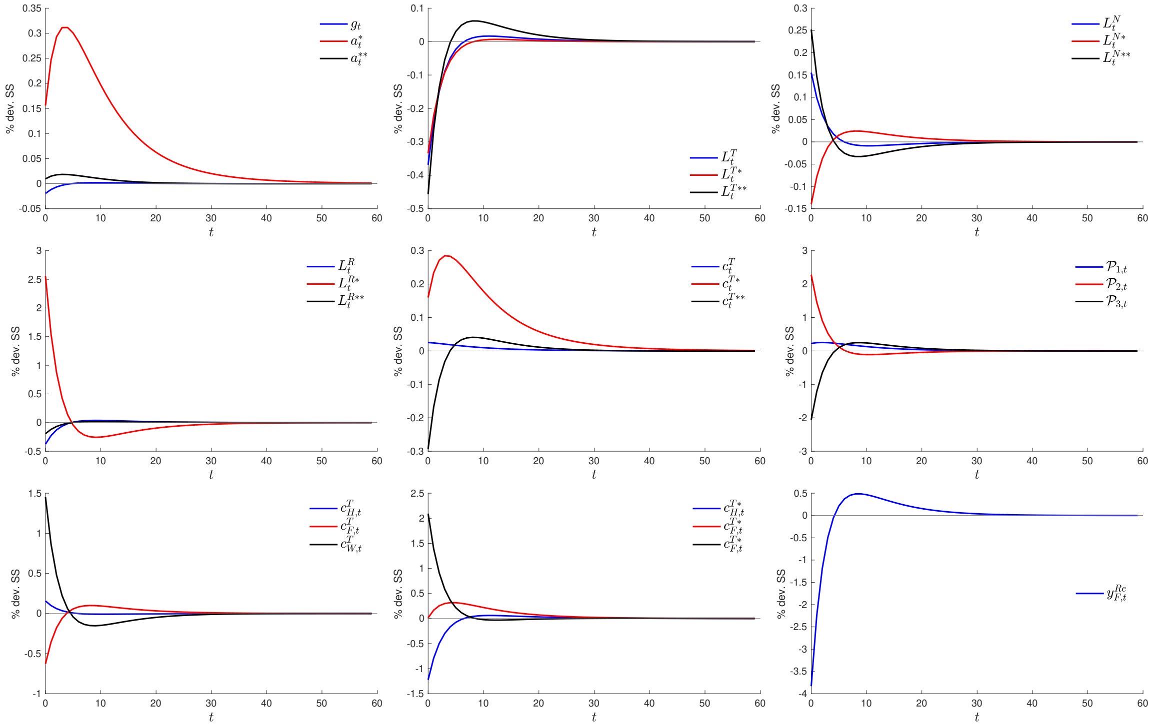

Figure 22: Dynamics under The Global Trade War, Three Country Cases.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country, and a simultaneous retaliation from the foreign country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{HF,t} = \varepsilon_{HW,t} = \varepsilon_{FH,t} = 1\%$, I hereby assume no retaliations from the foreign country and $\eta = 0.67$. **Assumption 1, 2** holds.

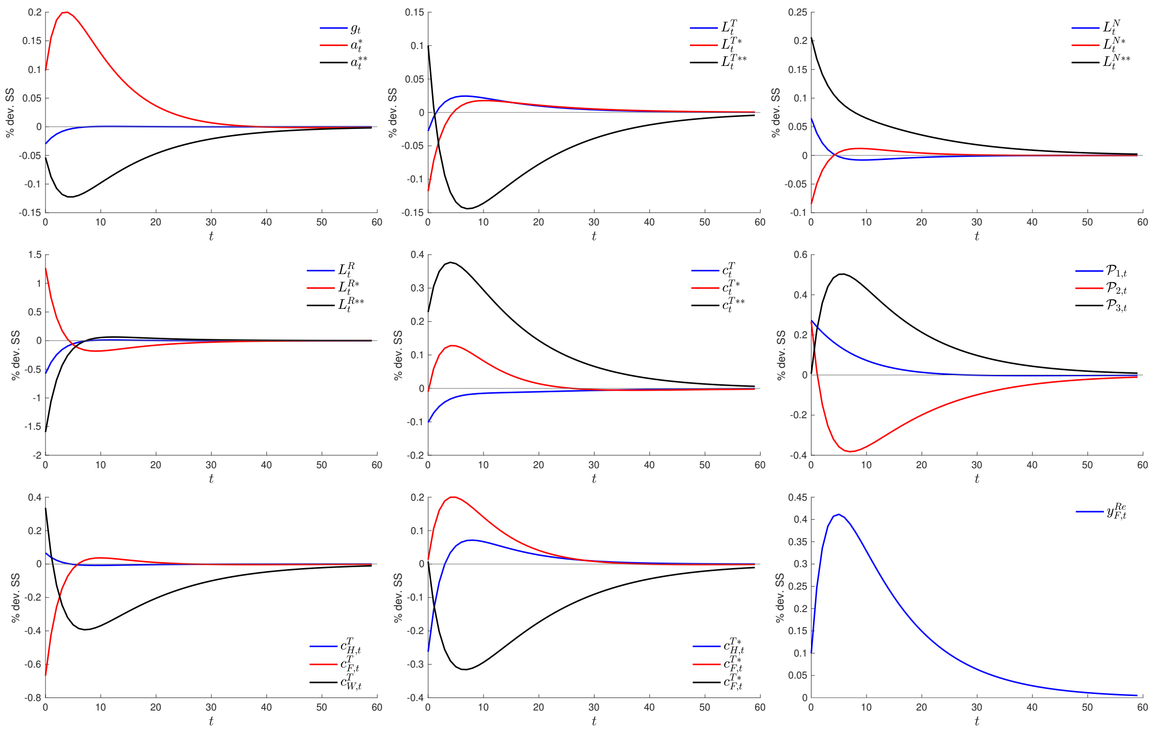

Figure 23: Dynamics under Unilateral Tariff Regime: Home Country against the Foreign Country, Three Country Cases, $\eta > 1$.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{HF,t} = 1\%$, I hereby assume no retaliations from the foreign country and $\eta = 1.5$. **Assumption 1, 2** holds.

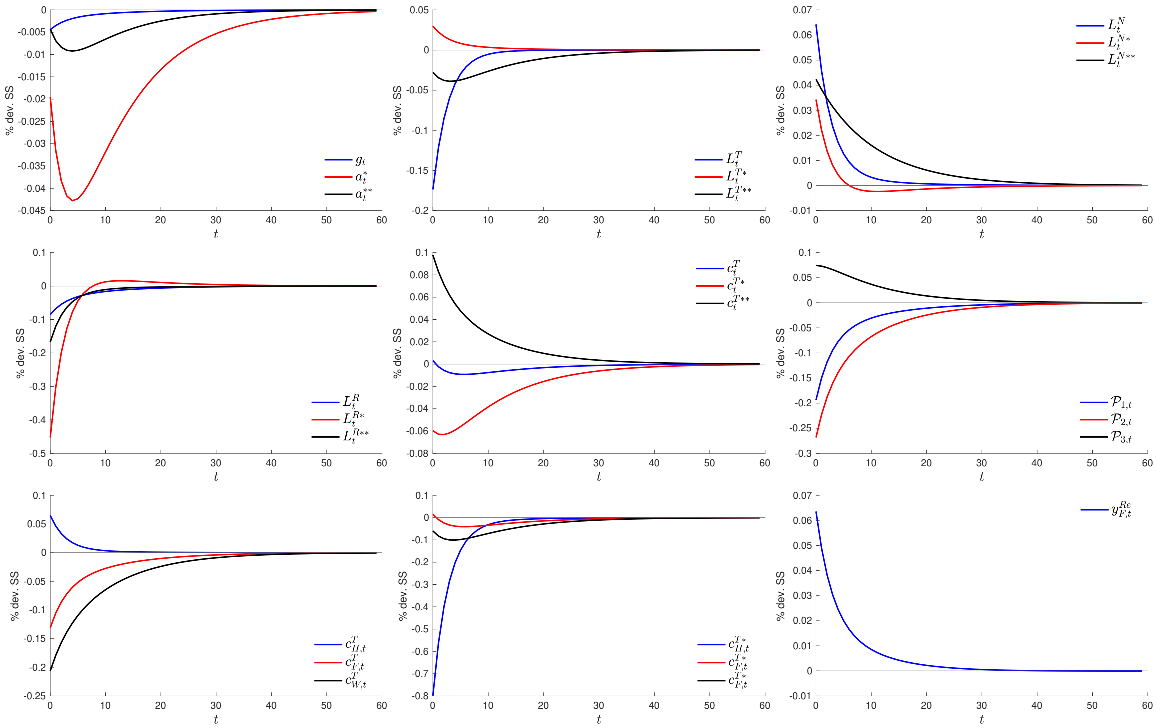

Figure 24: Dynamics under Unilateral Tariff Regime: Foreign Country against the Home Country, Three Country Cases, $\eta > 1$.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{FH,t} = 1\%$, I hereby assume no retaliations from the foreign country and $\eta = 1.5$. **Assumption 1, 2** holds.

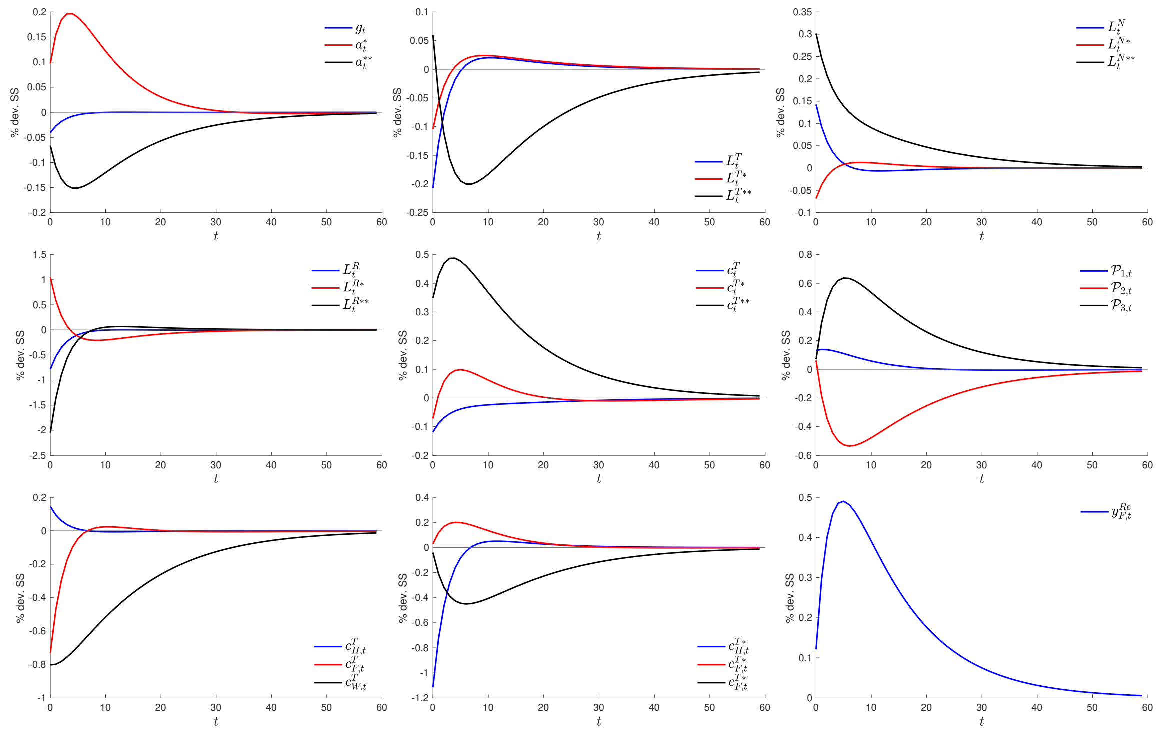

Figure 25: Dynamics under The Global Trade War, Three Country Cases, $\eta > 1$.
This figure illustrates the adjustment dynamics of key macroeconomic variables following the imposition of a unilateral tariff by the home country, and a simultaneous retaliation from the foreign country. I hereby assume $\rho_\tau = 0.7$, $\varepsilon_{HF,t} = \varepsilon_{HW,t} = \varepsilon_{FH,t} = 1\%$, I hereby assume no retaliations from the foreign country and $\eta = 0.67$. **Assumption 1, 2** holds.