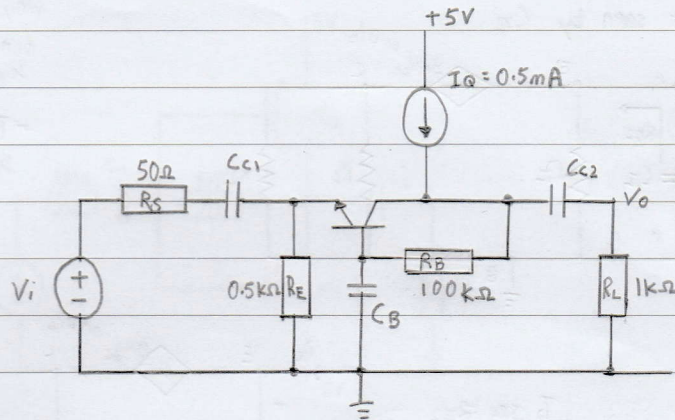


Esmund Lim

## A.E Tutorial 12

1)



$$\beta = 100$$

$$V_A = \infty$$

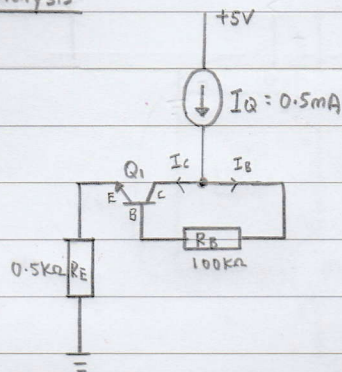
$$C_u = 1\text{pF}$$

$$f_T = 285\text{MHz} \quad (\text{unity gain frequency}) \text{ GBW}$$

$$\text{Assume } V_T = 25\text{mV}$$

Gain  
BW  
product

## DC Analysis



$$I_Q = I_C + I_B \quad \text{--- (1)}$$

$$I_C = \beta I_B$$

$$I_B = \frac{I_C}{\beta} \quad \text{--- (2)}$$

sub (2) into (1)

$$I_Q = I_C + \frac{I_C}{\beta}$$

$$0.5\text{mA} = I_C + \frac{1}{100} I_C$$

$$I_C = 0.495049505\text{mA}$$

$$\approx 0.5\text{mA}$$

$$g_m = \frac{I_C}{V_T}$$

$$= \frac{0.5\text{mA}}{25\text{mV}}$$

$$= 0.02\text{S}$$

$$r_o = \frac{V_A + V_{CE}}{I_C}$$

$$= \frac{\infty + V_{CE}}{I_C}$$

$$= \infty$$

$$g_m r_{\pi} = \beta$$

$$r_{\pi} = \frac{\beta}{g_m}$$

$$= \frac{100}{0.02}$$

$$= 5000\Omega$$

$$= 5\text{k}\Omega$$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_u)}$$

$$= \frac{g_m}{2\pi(C_{\pi} + C_u)}$$

$$f_T \div \frac{g_m}{2\pi} = \frac{1}{C_{\pi} + C_u}$$

$$\frac{f_T 2\pi}{g_m} = \frac{1}{C_{\pi} + C_u}$$

$$\frac{g_m}{f_T 2\pi} = C_{\pi} + C_u$$

$$C_{\pi} = \frac{g_m}{f_T 2\pi} - C_u$$

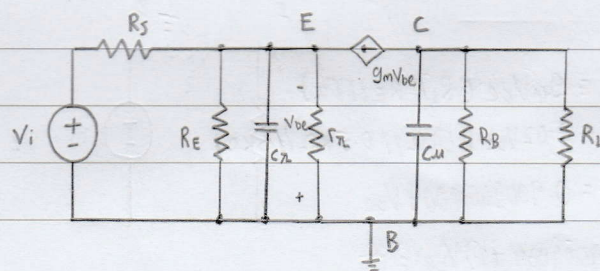
$$= \frac{0.02}{285 \times 10^6 \times 2\pi} - 1 \times 10^{-12}$$

$$= 10.16876794 \times 10^{-12}\text{F}$$

$$\approx 10\text{pF}$$

$$W_{H-3dB} = \frac{1}{\frac{1}{C_{\pi} R_{\pi}} + \frac{1}{C_u R_{u0}}}$$

## AC analysis

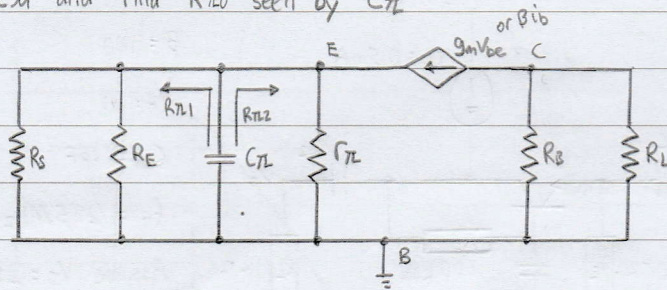
\*  $r_o = \infty$   
open-circuit $C_{\pi}$  is always between  
base and emitter $C_u$  is always between  
base and collector

high frequency small signal equivalent circuit by replacing  
external capacitor  $C_{c1}$ ,  $C_B$ ,  $C_{c2}$  by short circuit (ideal case)

Add in internal capacitor  $C_{\pi}$  and  $C_u$



Open circuit  $C_M$  and find  $R_{\pi 0}$  seen by  $C_{\pi}$



Why is it called open circuit time constant when you find higher cutoff frequency?

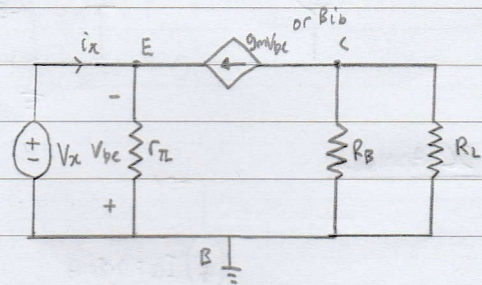
- To see resistance seen by one capacitor without the other capacitor  
 $\hookrightarrow$  O/C "killed" it

$$R_{\pi 1} = R_s // R_E$$

$$= \frac{50 \times 0.5 \times 10^3}{50 + 0.5 \times 10^3}$$

$$= 45.45454545 \Omega$$

To see  $R_{\pi 2}$



$$R_{\pi 2} = \frac{V_x}{i_x}$$

$$= \frac{-V_{be}}{-g_m V_{be}}$$

$$= \frac{1}{g_m}$$

$$= \frac{1}{0.02}$$

$$= 50 \Omega$$

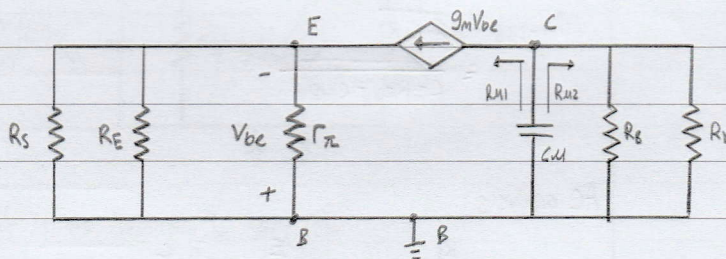
$$\therefore R_{\pi 0} = R_{\pi 1} // R_{\pi 2}$$

$$= \frac{45.45 \times 50}{45.45 + 50}$$

$$= 23.80952381 \Omega$$

$$\approx 24 \Omega$$

Open circuit  $C_{\pi}$  and find  $R_{u0}$



$$-V_{be} = g_m V_{be} (R_s // R_E // R_{\pi})$$

$$= 0.02 V_{be} (50 \Omega // 0.5 \text{ k}\Omega // 15 \text{ k}\Omega)$$

$$= 0.9009009009 V_{be}$$

$$(0.9009009009 + 1) V_{be} = 0$$

$$V_{be} = 0$$

Since  $V_{be} = 0$

$$R_{u1} = \infty$$

$$R_{u0} = R_{u1} // R_{u2}$$

$$R_{u2} = R_B // R_L$$

$$= \infty // R_{u2}$$

$$= 100 \text{ k}\Omega // 1 \text{ k}\Omega$$

$$= 990 \Omega$$

$$= 990.0990099 \Omega$$

$$\approx 990 \Omega$$

$\therefore$  By OCTC method

$$\omega_{H-3dB} = \frac{1}{R_{u0} C_{\pi} + R_{\pi 0} C_{\pi}}$$

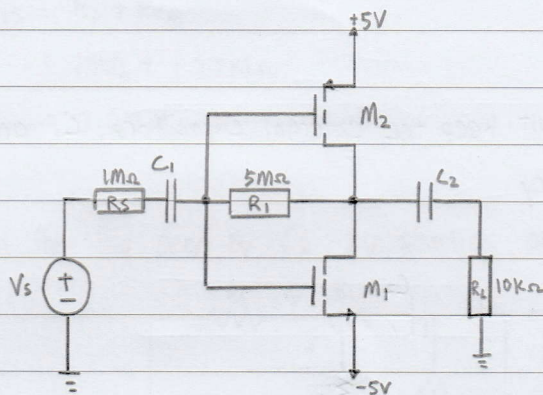
$$= \frac{1}{990(1 \times 10^{-12}) + 24(10 \times 10^{-12})}$$

$$= 813.0081301 \times 10^6 \text{ rad/s}$$

$$\approx 813 \text{ M rad/s}$$



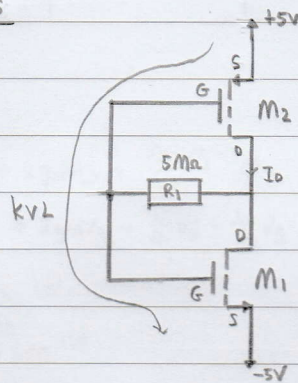
2)



$$\mu_n C_{ox1} \left( \frac{W_1}{L_1} \right) = \mu_p C_{ox2} \left( \frac{W_2}{L_2} \right) = 50 \mu A/V^2$$

$$|V_{TP}| = V_{TN} = 2V$$

$$\lambda = 0.005 V^{-1}$$

DC analysis

By KVL

$$-5 + V_{SG2} + V_{GS1} - 5 = 0$$

$$V_{SG2} + V_{GS1} = 10$$

$$V_{GS} = 5V$$

$$V_{SG2} = 5V, V_{GS1} = 5V$$

$$I_D = \frac{K_n}{2} (V_{GS} - V_{TN})^2$$

$$= \frac{50 \mu A}{2} (5 - 2)^2$$

$$= 225 \mu A$$

$$g_{m1} = g_{m2} = g_m$$

$$= \sqrt{2K_n I_D}$$

$$= \sqrt{2(50 \mu A)(225 \mu A)}$$

$$= 150 \times 10^{-6} S$$

$$r_{o1} = r_{o2} = r_o$$

$$= \frac{1}{\lambda I_D}$$

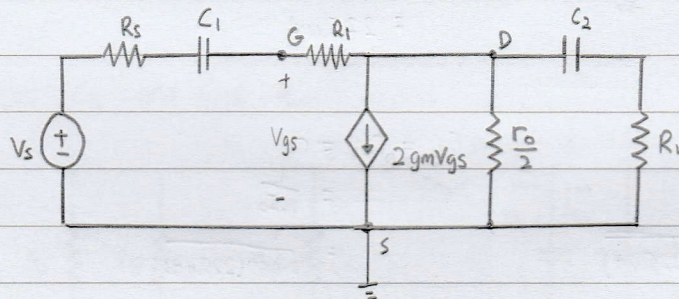
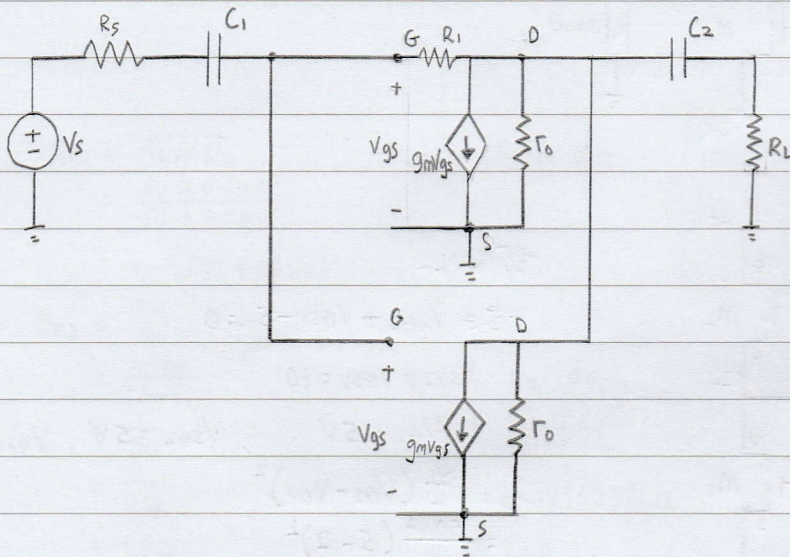
$$= \frac{1}{0.005(225 \mu A)}$$

$$= 0.888 M\Omega$$



AC Analysis

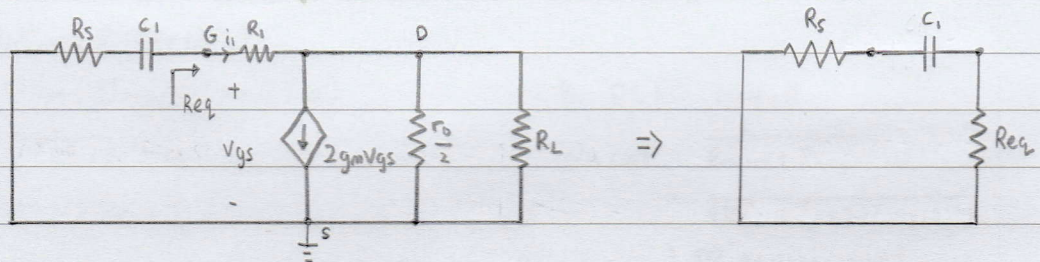
Low frequency small signal equivalent circuit. keep the external capacitors,  $C_1$  and  $C_2$ , that contribute to the lower cut-off frequency



$$\omega_L - 3dB = \sum \frac{1}{C_i R_{is}} \rightarrow \text{short circuit}$$

$$= \frac{1}{C_1 R_{is} + C_2 R_{is}}$$

Find  $R_{is}$  seen by  $C_1$  by shorting other capacitor



$$R_{eq} = \frac{V_{gs}}{I_1}$$

By KCL

$$\frac{V_g - V_d}{R_i} = 2g_m V_{gs} + \frac{V_d}{r_o/2} + \frac{V_d}{R_L}$$

$$\frac{V_g - V_d}{R_i} = 2g_m V_g + \frac{V_d}{r_o/2} + \frac{V_d}{R_L}$$

$$\frac{1}{R_i} V_g - \frac{1}{R_i} V_d = 2g_m V_g + \frac{1}{r_o/2} V_d + \frac{1}{R_L} V_d$$

$$\left(\frac{1}{R_i} - 2g_m\right) V_g = \left(\frac{2}{r_o} + \frac{1}{R_L} + \frac{1}{R_i}\right) V_d$$

$$V_{gs} = V_g - V_s$$

$$= V_g - 0$$

$$= V_g$$

$$\left[\frac{1}{5 \times 10^6} - 2(180 \mu)\right] V_g = \left(\frac{2}{0.888 M} + \frac{1}{10 k\Omega} + \frac{1}{5 M}\right) V_d$$

$$-299.8 \times 10^{-6} V_g = 1.024522523 \times 10^{-4} V_d$$

$$V_d = -2.926241183 V_g$$

$$R_{eq} = \frac{V_g}{(V_g - V_d)/R_i}$$

$$= \frac{V_g}{(V_g - (-2.926)V_g)/5 M}$$

$$= 1.273482643 \times 10^6 \Omega$$

$$\approx 1.273 M\Omega$$

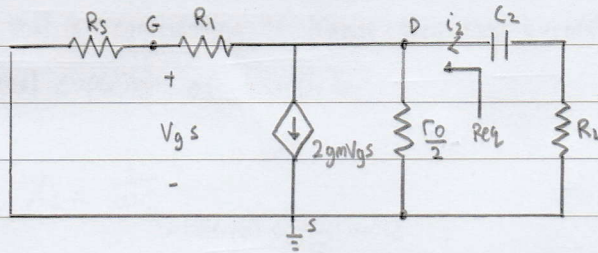


$$R_{1s} = R_s + R_{eq}$$

$$= 1\text{M}\Omega + 1.27\text{M}\Omega$$

$$= 2.27\text{M}\Omega$$

Find the  $R_{2s}$  seen by  $C_2$  by shorting other capacitor



By KCL

$$i_2 = \frac{V_d}{r_{o/2}} + 2gmV_{gs} + \frac{V_d - V_g}{R_1}$$

$$= \frac{2}{r_o} V_d + 2gmV_g + \frac{1}{R_1} V_d - \frac{1}{R_1} V_g$$

$$V_g = \frac{R_s}{R_s + R_1} V_d$$

$$= \frac{1\text{M}\Omega}{1\text{M}\Omega + 5\text{M}\Omega} V_d$$

$$= \frac{1}{6} V_d$$

$$V_d = 6 V_g$$

$$R_{eq} = \frac{V_d}{i_2}$$

$$= \frac{V_d}{\frac{2}{r_o} V_d + 2gmV_g + \frac{1}{R_1} V_d - \frac{1}{R_1} V_g}$$

$$= \frac{6 V_g}{\frac{2}{0.833\text{M}} (6 V_g) + 2(150 \times 10^{-6}) V_g + \frac{1}{5 \times 10^6} (6 V_g) + \frac{1}{5 \times 10^6} V_g}$$

$$= 19.0528502 \times 10^3 \Omega$$

$$\approx 19.05\text{k}\Omega$$

$$R_{2s} = R_L + R_{eq}$$

$$= 10\text{k}\Omega + 19.05\text{k}\Omega$$

$$= 29.05\text{k}\Omega$$

By SCTC method

$$\omega_{L-3dB} = \frac{\sum \frac{1}{C_i R_{is}}}{1}$$

$$= \frac{1}{C_1 R_{1s}} + \frac{1}{C_2 R_{2s}}$$

$$= \frac{1}{1 \times 10^{-6} \times 2.27 \times 10^6} + \frac{1}{1 \times 10^{-6} \times 29.05 \times 10^3}$$

$$= 34.86393655 \text{ rad/s}$$

$$\approx 34.9 \text{ rad/s}$$



Concept for this tutorial

① need to know which capacitor contribute to higher frequency cutoff and which capacitor contributed to lower frequency cutoff

→ Capacitor that is contributing to higher frequency cutoff is very small capacitance which is the internal capacitor of transistor

$$X_c = \frac{1}{\omega C}$$

→ internal capacitance

$C_{\pi}, C_{\mu}$

value of internal capacitance very small

\*

$C_{\pi}$   $C_{\mu}$   
B to E B to C  
G to S G to D

→ no matter what frequency  $\omega$  you have  $X_c \approx$  large (o/c)

if we are interested in very high  $\omega$

$$\text{eg } \frac{(1 \times 10^{-12})}{\omega} = 1 \Omega$$

→ very small resistance

cannot ignore this resistance

$$\begin{aligned} \omega_{H-3dB} &= \frac{1}{\sum C_i R_i} \\ &= \frac{1}{C_{\pi} R_{\pi 0} + C_{\mu} R_{\mu 0}} \end{aligned}$$

→ Capacitor that is contributing to lower frequency cut-off is very large capacitance which is the external capacitor of transistor

$$X_c = \frac{1}{\omega C} \rightarrow \infty$$

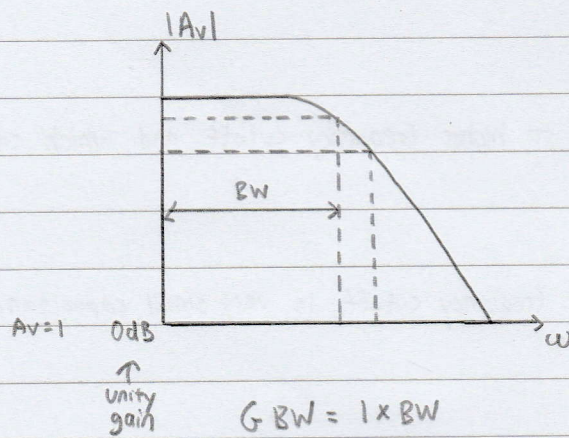
= 0 no matter what  $\omega$  input

$$X_c = \frac{1}{\omega C} \rightarrow \text{finite}$$

if  $\omega$  get lower, it contribute to lower cut-off frequency

$$\omega_{L-3dB} = \frac{1}{\sum C_i R_i}$$





$\leftarrow A_v$   
 $\leftarrow GBW$   
 Bandwidth

lower gain = larger bandwidth

If gain = 1

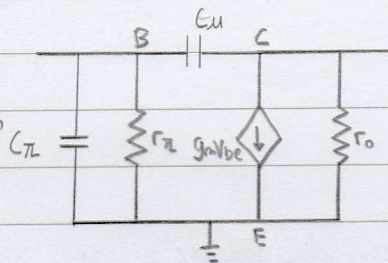
$$20 \log(1) = 0 \text{ dB}$$

$f_T \rightarrow$  if you have a transistor that can have this  $f_T$  for building amplifier, that is the limit of your frequency operation, because you can't build an amplifier which have a gain lesser than 1  
 $\rightarrow$  how high the frequency can operate this

$$\omega_T = \frac{g_m}{C_\pi + C_\mu}, \quad f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

gain is affected  $g_m$

larger  $g_m$  = larger gain  
 as gain get larger  
 bw get smaller



gain equation  
 $\# g_m$  always at  
 numerator

ideally if the frequency is very low

$$C_\pi, C_\mu = 0/c$$

but when frequency is very high

$X_c$  become small enough that some frequency can pass by  $C_\pi, C_\mu$  without being amplify by  $g_m v_{be}$