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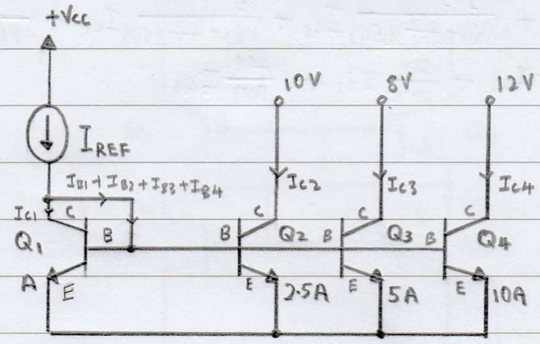
Esmund Lim A·E Tutorial 10

1)



$I_{REF} = 100\mu A$

$\beta = 50$

$V_A = 50V$

$$I_C = I_S e^{\left(\frac{V_{BE}}{V_T}\right)}\left(1 + \frac{V_{CE}}{V_A}\right)$$ if $V_A = \infty$ (ideal)

→ I_C will not be dependent on V_{CE}

↳ $I_S \propto$ $\underbrace{\text{Area of emitter}}$

able to control I_S directly → affect the I_C

∴ only I_S and V_{BE} will affect I_C

since all 4 BJT have the same V_{BE}, I_C will only be controlled by I_S which is also proportional to Area of emitter (ideally)

also if all the transistor has infinite β

$I_B = 0$ ($I_C = \beta I_B$, $I_B = I_C/\beta$), $I_C = I_{REF}$

→ If the outcome is not the predicted value it means that ① $V_A \neq \infty$

② $I_C \neq I_{REF}$ because there is I_B

* β of transistor, in small signal amplifier, assume β is fixed but in actual fact the transistor β do change with V_{CE}

$$\beta_i = \beta\left(1 + \frac{V_{CEi}}{V_A}\right)$$

$$I_C = I_S e^{\left(\frac{V_{BE}}{V_T}\right)}\left(1 + \frac{V_{CE}}{V_A}\right) \quad \text{and} \quad I_S \propto A$$

all the transistor will be in the forward active region

$$V_{BE1} = V_{BE2} = V_{BE3} = V_{BE4}$$

$$\frac{I_{C2}}{I_{C1}} = \frac{A_2 \cancel{e^{\left(\frac{V_{BE2}}{V_T}\right)}}\left(1 + \frac{V_{CE2}}{V_A}\right)}{A_1 \cancel{e^{\left(\frac{V_{BE1}}{V_T}\right)}}\left(1 + \frac{V_{CE1}}{V_A}\right)}$$

$$= \frac{A_2\left(1 + \frac{V_{CE2}}{V_A}\right)}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)}$$

$$I_{C2} = \frac{A_2\left(1 + \frac{V_{CE2}}{V_A}\right)}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)} I_{C1}$$

However $I_{REF} \neq I_{C1}$ unlike in the case of MOSFET current mirrors because of base currents

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} + I_{B3} + I_{B4}$$ ($I_C = \beta I_B$, $I_B = \frac{I_C}{\beta}$)

$$= I_{C1} + \frac{I_{C1}}{\beta_1} + \frac{I_{C2}}{\beta_2} + \frac{I_{C3}}{\beta_3} + \frac{I_{C4}}{\beta_4}$$

$$= I_{C1} + \frac{I_{C1}}{\beta\left(1 + \frac{V_{CE1}}{V_A}\right)} + \frac{A_2\cancel{\left(1 + \frac{V_{CE2}}{V_A}\right)} I_{C1}}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)} \cdot \frac{1}{\beta\cancel{\left(1 + \frac{V_{CE2}}{V_A}\right)}} + \frac{A_3\cancel{\left(1 + \frac{V_{CE3}}{V_A}\right)} I_{C1}}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)} \cdot \frac{1}{\beta\cancel{\left(1 + \frac{V_{CE3}}{V_A}\right)}}$$

$$+ \frac{A_4\cancel{\left(1 + \frac{V_{CE4}}{V_A}\right)} I_{C1}}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)} \cdot \frac{1}{\beta\cancel{\left(1 + \frac{V_{CE4}}{V_A}\right)}}$$

$$= I_{C1} + \frac{I_{C1}}{\beta\left(1 + \frac{V_{CE1}}{V_A}\right)} + \frac{A_2 I_{C1}}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)\beta} + \frac{A_3 I_{C1}}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)\beta} + \frac{A_4 I_{C1}}{A_1\left(1 + \frac{V_{CE1}}{V_A}\right)\beta}$$

Since $V_{CE1} = V_{BE1} = 0.7V$ (forward active region)

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$$I_{REF} = I_{C1} + \frac{1}{50(1+\frac{0.7}{50})} I_{C1} + \frac{2.5}{1(1+\frac{0.7}{50})(50)} I_{C1} + \frac{5}{1(1+\frac{0.7}{50})(50)} I_{C1} + \frac{10}{1(1+\frac{0.7}{50})(50)} I_{C1}$$

$$= I_{C1} + \frac{10}{507} I_{C1} + \frac{25}{507} I_{C1} + \frac{50}{507} I_{C1} + \frac{100}{507} I_{C1}$$

$$= 1.364891519\, I_{C1}$$

$$I_{C1} = \frac{I_{REF}}{1.364891519}$$

$$= \frac{100\mu A}{1.364891519}$$

$$= 73.26589595 \times 10^{-6} A$$

$$\approx 73.3 \mu A$$

$$I_{C2} = \frac{A_2(1+\frac{V_{CE2}}{V_A})}{A_1(1+\frac{V_{CE1}}{V_A})} I_{C1} \qquad r_{o2} = \frac{V_A + V_{CE2}}{I_{C2}}$$

$$= \frac{2.5(1+\frac{10}{50})}{1(1+\frac{0.7}{50})} . 73.3 \mu A \qquad = \frac{50+10}{216.76 \mu A}$$

$$= 216.7630058 \times 10^{-6} A \qquad = 276.8 k\Omega$$

$$\approx 216.76 \mu A$$

$$I_{C3} = \frac{A_3(1+\frac{V_{CE3}}{V_A})}{A_1(1+\frac{V_{CE1}}{V_A})} I_{C1} \qquad r_{o3} = \frac{V_A + V_{CE3}}{I_{C3}}$$

$$= \frac{5(1+\frac{8}{50})}{1(1+\frac{0.7}{50})} . 73.3 \mu A \qquad = \frac{50+8}{419.08 \mu A}$$

$$= 419.0751445 \times 10^{-6} A \qquad = 138.4 k\Omega$$

$$\approx 419.08 \mu A$$

$$I_{C4} = \frac{A_4(1+\frac{V_{CE4}}{V_A})}{A_1(1+\frac{V_{CE1}}{V_A})} I_{C1} \qquad r_{o4} = \frac{V_A + V_{CE4}}{I_{C4}}$$

$$= \frac{10(1+\frac{12}{50})}{1(1+\frac{0.7}{50})} 73.3 \mu A \qquad = \frac{50+12}{895.95 \mu A}$$

$$= 895.9537572 \times 10^{-6} A \qquad = 69.2 k\Omega$$

$$\approx 895.95 \mu A$$

Note that the accuracy of the current mirror is dependent on β, V_A and the number of mirror transistors connected to the same diode-connected transistor

→ more base current, less accurate

↳ MOS don't have this problem → no gate current

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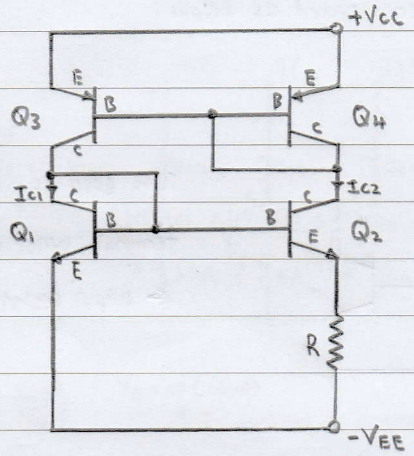
2)



$A_{E3} = 10 A_{E4}$

$A_{E2} = 10 A_{E1}$

$R = 1 K\Omega$

$\beta \to \infty$

ignore Early effect

since $\beta \to \infty$, no I_B

$I_{c1} = I_{c3}$

$I_{c2} = I_{c4}$

ignore Early effect

$I_c = I_s e^{\left(\frac{V_{BE}}{V_T}\right)}$

since $A_{E3} = 10 A_{E4}$, $I_{c3} = 10 I_{c4}$

since $I_{c3} = I_{c1}$, $I_{c1} = 10 I_2$

$A_{E2} = 10 A_{E1}$, $I_{s2} = 10 I_{s1}$

The voltage across R is $V_{BE1} - V_{BE2}$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$

$$\begin{aligned}
V_{BE1} - V_{BE2} &= V_T \ln\left(\frac{I_{c1}}{I_{s1}}\right) - V_T \ln\left(\frac{I_{c2}}{I_{s2}}\right) \\
&= V_T \left[\ln\left(\frac{I_{c1}}{I_{s1}}\right) - \ln\left(\frac{I_{c2}}{I_{s2}}\right)\right] \\
&= V_T \ln\left(\frac{I_{c1}}{I_{s1}} \times \frac{I_{s2}}{I_{c2}}\right) \\
&= V_T \ln\left(\frac{I_{s2}}{I_{s1}} \times \frac{I_{c1}}{I_{c2}}\right) \\
&= V_T \ln(10 \times 10) \\
&= 0.025 \ln(100) \\
&= 0.1151292546 \\
&\approx 0.115
\end{aligned}$$

$$\begin{aligned}
I_{c2} \approx I_{E2} &= \frac{V_{BE1} - V_{BE2}}{R} \\
&= \frac{0.115}{1000} \\
&= 115\,\mu A
\end{aligned}$$

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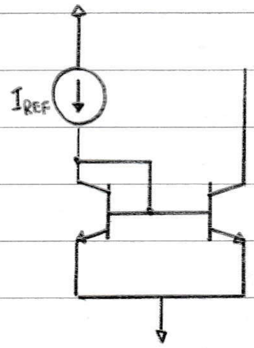
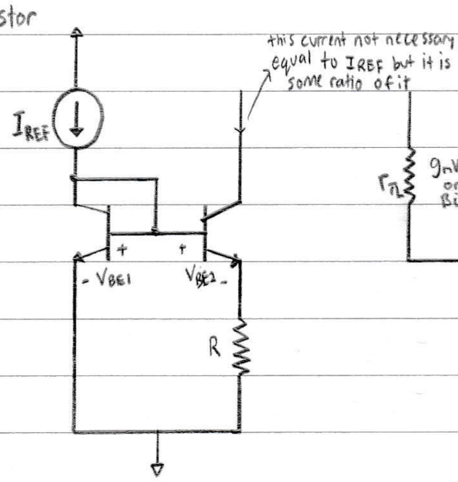
a simple current mirror can be constructed as below :

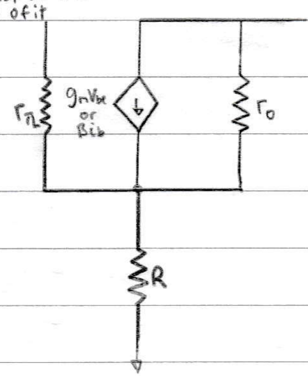


How good is this current mirror depends on across the terminal , what is output resistance

← output resistance = r_o

By adding a resistor





output resistance : $(1+g_m V_{be}) r_o$

$$V_{BE1} = V_{BE2} + V_R$$

$$V_R = V_{BE1} - V_{BE2}$$

$$I_c = I_s e^{\left(\frac{V_{BE}}{V_T}\right)} \left(1+\frac{V_{CE}}{V_A}\right)$$

$$I_c / \left(1+\frac{V_{CE}}{V_A}\right) = I_s e^{\left(\frac{V_{BE}}{V_T}\right)}$$

$$I_c / \left(1+\frac{V_{CE}}{V_A}\right) . I_s = e^{\left(\frac{V_{BE}}{V_T}\right)}$$

$$\ln\left(\frac{I_c}{I_s} \cdot \frac{1}{1+\frac{V_{CE}}{V_A}}\right) = \frac{V_{BE}}{V_T}$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s} \cdot \frac{1}{1+\frac{V_{CE}}{V_A}}\right)$$

$$V_T = \frac{kT}{q}$$