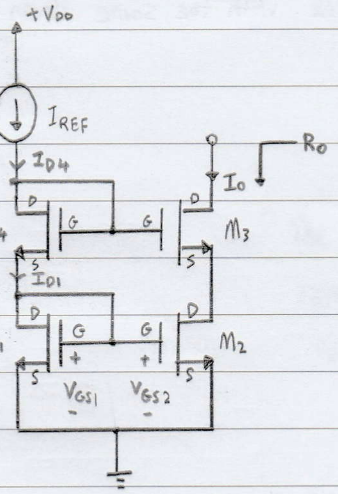
No.: Date:

Esmund Lim A·E Tutorial 11

1)



All the transistor have the same $\frac{W}{L}$, no gate current

$\therefore I_{REF} = I_{D4} = I_{D1}$

Since $V_{GS1} = V_{GS2}$

$\rightarrow I_D = \frac{1}{2} \underbrace{k_n'}_{\text{same fabrication}} \underbrace{\left(\frac{W}{L}\right)}_{\text{same size}} \underbrace{(V_{GS} - V_{TN})^2}_{\text{same}} \underbrace{(1 + \lambda V_{DS})}_{\text{ignore early effect}}$

$\therefore I_{D2} = I_{D1}$

$= I_o$ $\therefore$ Since all of them have same I_D

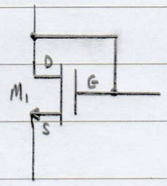
They must have same V_{GS}

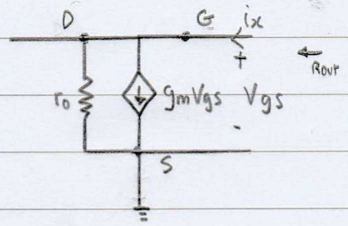
M_4 and M_1 will always be in the saturation region due to diode connected

M_2, $V_{DS} = V_{GS}$ due to the connection

The only problem is M_3, it depend on V_o, if V_o is very low, V_{DS} could be lower than $V_{GS} - V_{TN}$ $\therefore$ might not be in saturation region $\rightarrow$ current mirror fail

①





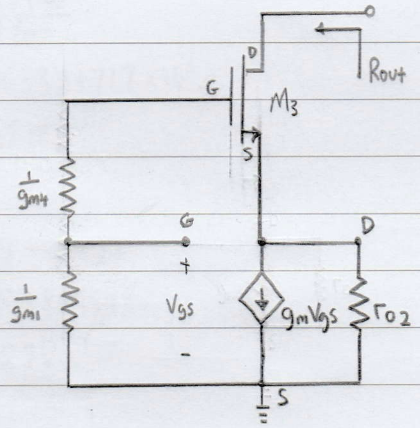
$R_{out} = \frac{V_{gs}}{i_x}$

$= \frac{V_{gs}}{g_m V_{gs}}$

$= \frac{1}{g_m}$ $\rightarrow$ for all diode connected MOSFET

②

replace M_2 with small signal

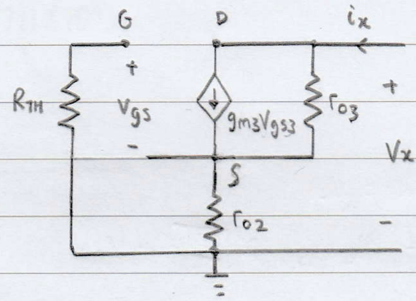


No gate current

$V_{gs} = 0$

$\therefore$ current source no value : o/c

③



$R_{TH} = \frac{1}{g_{m4}} + \frac{1}{g_{m1}}$

$R_{out} = \frac{V_x}{i_x}$

$= \frac{(i_x - g_{m3} V_{gs3}) r_{o3} + i_x r_{o2}}{i_x}$

$V_{gs3} = -i_x r_{o2}$

$= \frac{(i_x - g_{m3}(-i_x r_{o2})) r_{o3} + i_x r_{o2}}{i_x}$

$= r_{o3} + g_{m3} r_{o2} r_{o3} + r_{o2}$

$= r_{o3}(1 + g_{m3} r_{o2}) + r_{o2}$

$\approx g_{m3} r_{o2} r_{o3}$

No.: Date:

b) Assuming all the transistors are of the same W/L. with the same drain current

$$V_{GS1} = V_{GS2} = V_{GS}$$

$$I_{REF} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_4 (V_{GS} - V_{TN})^2$$

$$= \frac{1}{2} k_n (V_{GS} - V_{TN})^2$$

$$\frac{I_{REF}}{\frac{1}{2}k_n} = (V_{GS} - V_{TN})^2$$

$$\sqrt{\frac{I_{REF}}{\frac{1}{2}k_n}} = V_{GS} - V_{TN}$$

$$V_{GS} = \sqrt{\frac{I_{REF}}{\frac{1}{2}k_n}} + V_{TN}$$

$$= \sqrt{\frac{2I_{REF}}{k_n}} + V_{TN}$$

The gate voltage of M_3 and M_4:

$$V_{G3} = V_{G4}$$

$$= V_{GS1} + V_{GS4}$$

$$= 2V_{GS}$$

The drain source voltage of M_2:

$$V_{DS2} = V_{G3} - V_{GS3}$$

$$= 2V_{GS} - V_{GS}$$

$$= V_{GS}$$

in the saturation region

$$= \sqrt{\frac{2I_{REF}}{k_n}} + V_{TN}$$

For M_3 to remain in the saturation region

$$V_{DS3} \geqslant V_{GS3} - V_{TN}$$

$$V_{DS3} \geqslant \sqrt{\frac{2I_{REF}}{k_n}} + V_{TN} - V_{TN}$$

$$V_{DS3} \geqslant \sqrt{\frac{2I_{REF}}{k_n}}$$

The minimum voltage for the whole current mirror to work

$$V_{D3} = V_{DS2} + V_{DS3}$$

$$V_{D3} \geqslant \sqrt{\frac{2I_{REF}}{k_n}} + V_{TN} + \sqrt{\frac{2I_{REF}}{k_n}}$$

$$V_{D3} \geqslant 2\sqrt{\frac{2I_{REF}}{k_n}} + V_{TN}$$

The minimum voltage at the drain of M_3 is $2\sqrt{\frac{2I_{REF}}{k_n}} + V_{TN}$

No.: Date:

c) Given

$I_{REF} = 17.5\mu A$

$V_{DD} = 5V$

$k_n = 75\mu A / V^2$

$V_{TN} = 0.75V$

$\lambda = 0.0125 V^{-1}$

$R_o \approx g_{m3} r_{o2} r_{o3}$ — ①

$V_{DS2} = \sqrt{\frac{2 I_{REF}}{k_n}} + V_{TN}$

$= \sqrt{\frac{2(17.5\times10^{-6})}{75\times10^{-6}}} + 0.75$

$= 1.433130051V$

$\approx 1.43V$

$r_{o2} = \frac{\frac{1}{\lambda} + V_{DS2}}{I_D}$

$= \frac{\frac{1}{0.0125} + 1.43}{17.5\mu A}$

$= 4.653321717 \times 10^6 \Omega$

$\approx 4.65 M\Omega$

$V_{DS3} = 5V - 1.43V$

$= 3.566869949V$

$\approx 3.57V$

$r_{o3} = \frac{\frac{1}{0.0125} + 3.57}{17.5\mu A}$

$= 4.775249711 \times 10^6 \Omega$

$\approx 4.78 M\Omega$

$g_{m3} = \sqrt{2 k_n I_D (1 + \lambda V_{DS3})}$

$= \sqrt{2(75\times10^{-6})(17.5\times10^{-6})(1 + 0.0125(3.57))}$

$= 5.236447193 \times 10^{-5} S$

$\approx 5.24 \times 10^{-5} S$

$R_o \approx 5.24\times10^{-5} \times 4.78\times10^6 \times 4.65\times10^6$

$= 1.163579054 \times 10^9 \Omega$

$\approx 1.16 G\Omega$

The minimum voltage at drain M_3 for all transistors to remain in saturation region:

$V_{DS} \geq 2\sqrt{\frac{2 I_{REF}}{k_n}} + V_{TN}$

$= 2(0.683130051) + 0.75$

$= 2.116260102V$

$\approx 2.12V$

No.: .... Date: ....

2) Given transfer function

$$T(s) = \frac{10^{12} s^2}{(s+10)(s+10^3)(s+10^6)}$$

$s = j\omega = j1 \quad \sqrt{1^2} = 1$

$j\omega + 10 = \sqrt{1^2+10^2} \approx 10$

Note: $|j\omega + a| = \sqrt{\omega^2 + a^2}$ if $\omega << a$, then $|j\omega+a| \approx a$

if $\omega = a$: $= \sqrt{2a^2} = \sqrt{2}a = 1.414a \approx a$

if $\omega >> a$ or $a >> \omega$: $\approx \sqrt{\omega^2} = \omega$ $\quad \approx \sqrt{a^2} = a$

cannot start from 0 rad/s → impossible to use decades of 0

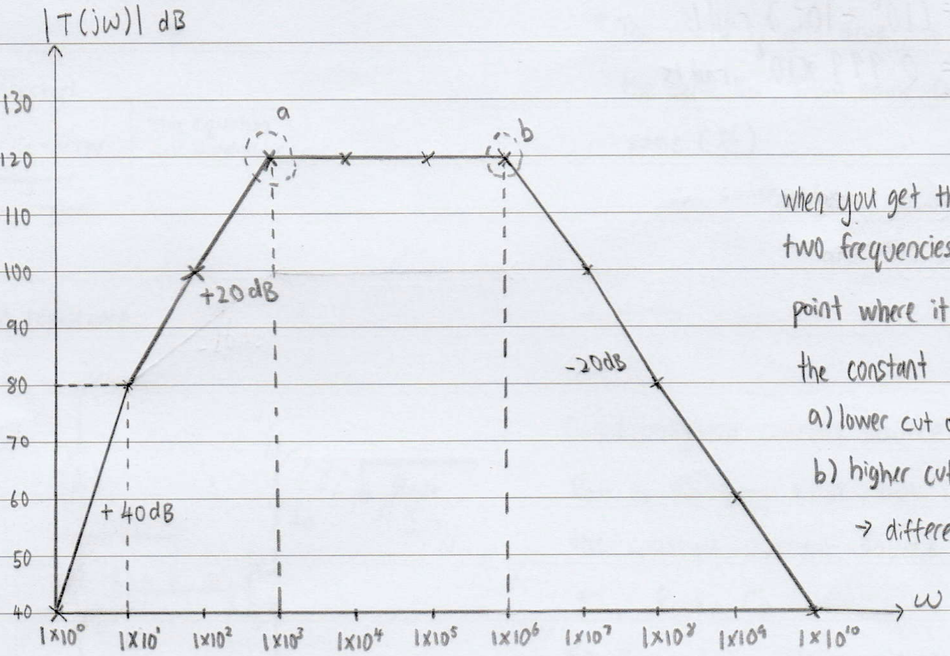
At $\omega = 1\,rad/s$, $|T(j1)| \approx \frac{10^{12}(1)}{10 \times 10^3 \times 10^6}$

≈ 100

$= 20\log 100\ dB$

$= 40\,dB$

Voltage gain = 20log
power gain = 10log



When you get the Bode plot, there are two frequencies point of interest, point where it start to deviate from the constant

a) lower cut off frequency

b) higher cut off frequency

→ difference between a and b = bandwidth

↳ How much frequency band you can work with the constant gain

2 zeros

$\underbrace{(s+0)}_{+20dB}\underbrace{(s+0)}_{+20dB}$ → hit $(s+10)$ = -20dB → hit $(s+10^3)$ = -20dB → hit $(s+10^6)$ = -20dB

poles ↳ 10 rad/s; poles ↳ 10^3 rad/s; poles ↳ 10^6 rad/s

Two zeros at $s = 0$

Three poles at $s = -10$, $s = -10^3$, $s = -10^6$

Av estimation from Bode plot:

$A_v|_{10^3} = 120\,dB \Rightarrow \omega_L = 10^3\ rad/s$

$A_v|_{10^6} = 120\,dB \Rightarrow \omega_H = 10^6\ rad/s$

Av estimation from calculation

(Note: If $\omega \leq a$, then $|j\omega + a| \approx a$, else $|j\omega + a| \approx \omega$

$A_v|_{10^3} \approx \frac{10^{12}(10^3)^2}{10^3 10^3 10^6} = 10^6 = 120\,dB \Rightarrow \omega_L = 10^3\ rad/s$

$A_v|_{10^6} \approx \frac{10^{12}(10^6)^2}{10^6 10^6 10^6} = 10^6 = 120\,dB \Rightarrow \omega_H = 10^6\ rad/s$

$\frac{10^{12}(10^3)^2}{(10^3 > 10)(10^3 = 10^3)(10^3 < 10^6)}$

No.: .... Date: ....

Actual Gain at 10^3 rad/s or 10^6 rad/s = 117dB (Hence, the name -3dB frequency)

$$|T(j10^3)| = 10^{12}\left(\frac{(10^3)^2}{\sqrt{[(10^3)^2+(10)^2][(10^3)^2+(10^3)^2][(10^3)^2+(10^6)^2]}}\right)$$

$= 707071.075$

$= 20\log(707071.075)$ dB

$= 116.9892614$ dB

≈ 117 dB

↳ 3dB below from the approximate bode plot

$BW = \omega_H - \omega_L$

$= (10^6 - 10^3)$ rad/s

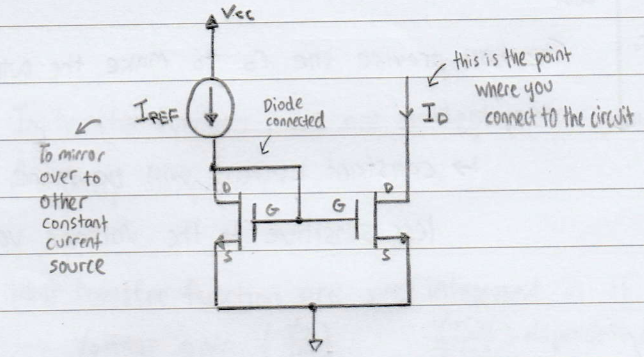
$= 0.999 \times 10^6$ rad/s

$1.414284978 \times 10^{12}$

Concept for this tutorial

$$I_D = \frac{1}{2} k_n' \left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$$

Simple current mirror circuit



if the two transistor had the same $\left(\frac{W}{L}\right)$, fabricated under the same process same k_n'

V_{GS} same, ignore early effect

$\therefore \underbrace{I_D = I_{REF}}_{\text{mirror over}}$

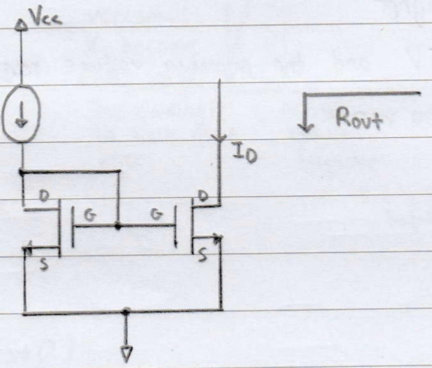
* The reverse is also true, if two transistor had the same I_D, from same fabrication process same $\left(\frac{W}{L}\right)$

$\therefore$ same V_{gs}

Diode connected

$\left.\underbrace{V_{DS} \geq V_{GS} - V_{TN}}_{\text{Saturation region}}\right\}$ This equation will always be true

about output resistance

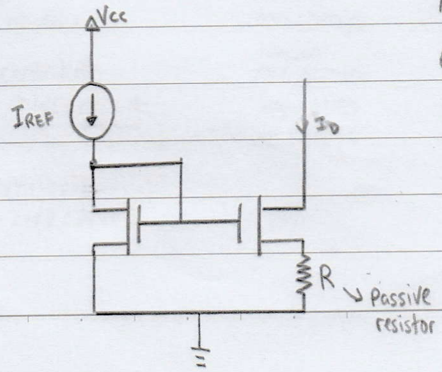


Good constant current source will require Rout to be very high, but if you implement the constant current source like the circuit ←, $R_{out} = r_o$

so I need to increase it to make it a better constant source, if r_o is higher it means that the current I_D is more constant with respect to the change of voltage

How to increase output resistance

→ Widlar Current mirror



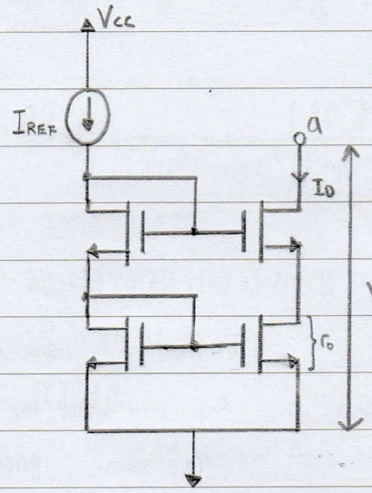
Make circuit in Ic chips, the area is very costly, for the same amount of area, if I want a very high resistance, I have to make very high R, but that R will take up a lot of area and very costly

$\therefore$ Better way

=> use transistor (Cascode)

By making it into a Passive resistor

No.: .... Date: ....



cascode current mirror

This will provide the r_o to make the output resistance very high

↳ constant current will be more constant less sensitive to the voltage variation across it

But is there a price to pay?

→ there must be a minimum voltage at point "a" so that all the transistor are in the saturation region, if any of the transistor not in the saturation region, the whole current mirror fail to function

The higher you stack, the higher is the output resistance and so the minimum voltage required is also higher

→ if a battery is 1.5 V and the minimum voltage required is 2V

→ fail to work

No.: Date:

Concept for frequency response

I/P → **Transfer Function** → O/P

↳ Ratio of the quantity of interest $\frac{O/P}{I/P}$

↳ Can be voltage / current

In transfer function, we are dealing with signal that have frequency and the circuit might have frequency component

What transfer function are we interested in if it is a amplifier?

→ Voltage gain $\left(\frac{V_o}{V_i}\right)$ $\frac{V_o(s)}{V_i(s)}$ → dependent on frequency

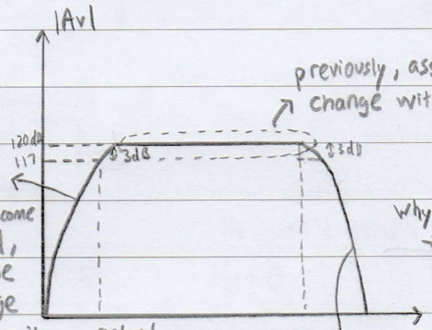
$s \to j\omega$, $\omega \to 2\pi f$

$X_c = \frac{1}{\omega C}$ → Fixed value

as frequency decreases, gain start to drop due to capacitor not short circuit any more

when ω become very small, X_c become very large

Thus causing it to have more error



actual cutoff frequency → 3 dB cutoff frequency

previously, assume gain doesn't change with frequency

Why 3dB? → more than half of the power

predict very well but after certain frequency, it will drop

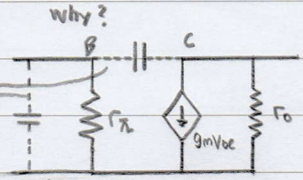
in AC capacitor } assume s/c

$X_c = \frac{1}{\omega C} = 0$, $\omega \to \infty$

assumption is incorrect if $C \neq 0$

Why?

There will be some capacitive effect



↳ but it will be very small $< 10^{-15}$

There will be some signal, if their frequency is high enough, they will be able to pass through because of the capacitor

↳ not completely open

$X_c = \frac{1}{\omega C}$ ↑ reactance very high; ωC → very small; → very big

Bode plot

$s^2 = (s+0)(s+0)$ → $j\omega$ → frequency = 0 rad/s

$(s+10)$ $(s+10^3)$

$(j\omega+10)$ $(j\omega+10^3)$ → 10^3 rad/s

→ frequency = 10 rad/s

→ turning point of frequency: when you hit that frequency the slope will change transfer characteristic slope going to change

$\frac{\text{numerator} \to \text{zeros}}{\text{denominator} \to \text{poles}}$

each zeros contribute a positive + 20dB/$\underbrace{\text{decade}}_{10\text{ time}}$

each poles contribute a negative − 20dB / decade

decade: 1 rad/s → 10 rad/s → 100 rad/s

every decade it increase 20dB

so given a transfer function

→ draw the $\underbrace{\text{frequency plot}}_{\text{bode plot}}$