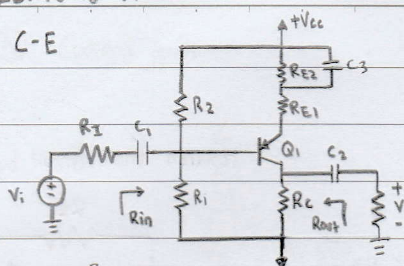
No.: .... Date: ....

Esmund Lim — A·E Tutorial 6

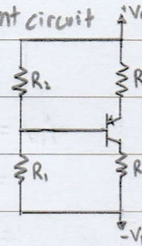
1) C-E



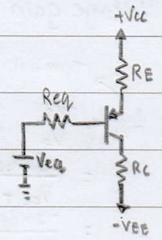
$\beta = 100$

DC Equivalent circuit

→ DC Analysis



→ simplify by thevenin

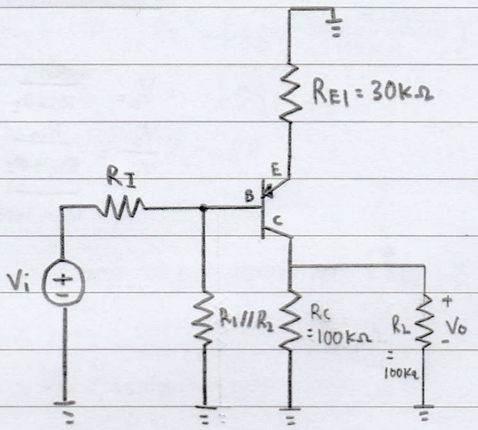


$Req = R_1 // R_2 = 100k\Omega$

$Veq = 0V$

Q-pt (I_C, V_{EC}) $(50\mu A, 10.86V)$ — solve by KVL in T5 Q3

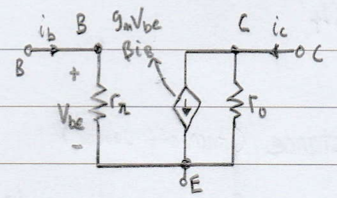
ac Equivalent circuit (cap S/C, DC voltage GND)



$\beta = 100$

$V_A = 75V$

Early voltage



① input : left common
output : Right : down

② r_π : between B & E

③ ⬦ & r_o parallel : between C & E
Flow from C to E
r_o very huge
i_{ro} very small

$V_{be} = i_b r_\pi$

$g_m V_{be} = g_m i_b r_\pi$ ($g_m r_\pi \to \beta$)

$= \beta i_b$

$i_c = \beta i_b + \frac{V_{ce}}{r_o}$ (V_{ce} very small, r_o very large)

$\approx \beta i_b$

$r_\pi = \frac{\beta}{g_m}$ $g_m r_\pi = \beta$

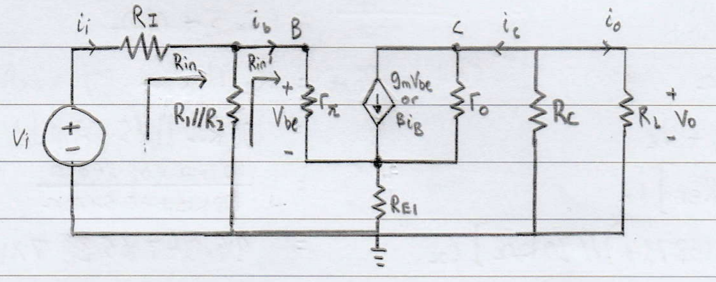
$g_m = \frac{I_C}{V_T} \approx 40 I_C$

$V_T = \frac{kT}{q} \approx 25mV$

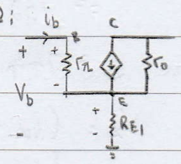
$r_\pi = \frac{\beta}{g_m}$ (r_π = input) (r_o = output)

$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$

Small signal ac equivalent circuit



Method 2:



$Rin' = \frac{V_b}{i_b}$

$= \frac{i_b(r_\pi) + (\beta+1) i_b R_{E1}}{i_b}$

$= r_\pi + (\beta+1) R_{E1}$

$Rin = Rin' // R_1 // R_2$

$g_m = 40 I_C$ or $= \frac{I_C}{V_T}$

$= 40(50\mu A)$ $= \frac{50\mu A}{25mV}$

$= 2mS$ $= 2mS$

$r_\pi = \frac{\beta}{g_m}$

$= \frac{100}{2mS}$

$= 50k\Omega$

$* r_o \approx \frac{V_A}{I_C}$ if $V_{CE} << V_A$

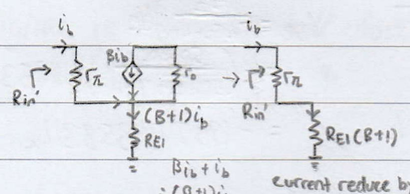
$r_o = \frac{V_A + |V_{CE}|}{I_C}$

$= \frac{75V + 10.86V}{50\mu A}$

$= 1.7172 M\Omega$

① Input Resistance (2 methods)

Method 1 : By inspection



current reduce by $(\beta+1)$ time
= voltage smaller by $(\beta+1)$ time
∴ in order to keep the structure increase R_{E1} by $(\beta+1)$ time

$Rin' = r_\pi + (\beta+1) R_{E1}$

$= 50k\Omega + (101) 30k\Omega$

$= 3.08 M\Omega$

$Rin = R_1 // R_2 // Rin'$

$= \frac{100k\Omega \times 3.08M\Omega}{100k\Omega + 3.08M\Omega}$

$= 96.85534591 \times 10^3 \Omega$

$\approx 96.86 k\Omega$

② Voltage gain

Terminal voltage gain

$$A_{vt} = \frac{V_o}{V_b}$$

$$= \frac{-g_m V_{be}(R_C \| R_L)}{V_{be} + g_m V_{be}(R_{E1})}$$

$$= \frac{-g_m \cancel{V_{be}}(R_C \| R_L)}{\cancel{V_{be}}(1 + g_m R_{E1})}$$

$$= \frac{-g_m(R_C \| R_L)}{1 + g_m R_{E1}}$$

$R_C \| R_L = \frac{100 \times 100}{100 + 100} = 50k\Omega$

$$= \frac{-2 \times 10^{-3}(50k\Omega)}{1 + (2 \times 10^{-3} \times 30 \times 10^3)}$$

$$= -1.639344262$$

$$\approx -1.64$$

$$A_v = \frac{V_b}{V_i} \times \frac{V_o}{V_b}$$

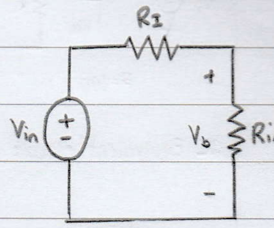
$$= \left(\frac{R_{in}}{R_{in} + R_I}\right) A_{vt}$$

$$= \frac{96.86k\Omega}{96.86k\Omega + 750\Omega} \times (-1.64)$$

$$= -1.627398832$$

$$\approx -1.63$$

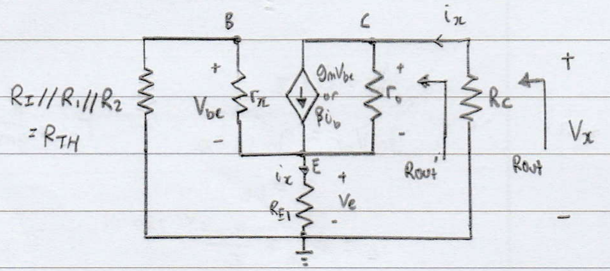
replaced whole circuit by Rin

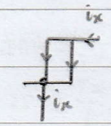


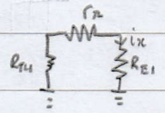
$$V_b = \frac{R_{in}}{R_{in} + R_I} V_{in}$$

$$\frac{V_b}{V_i} = \underbrace{\frac{R_{in}}{R_{in} + R_I}}_{\text{input loading factor}}$$

③ output resistance (turn off source)







$$R_{TH} = R_I \| R_1 \| R_2$$

$$= R_I \| 100k\Omega$$

$$= 750 \| 100k\Omega$$

$$= 744.4168734\Omega$$

$$V_x = (i_x - g_m V_{be}) r_o + V_e$$

$$V_e = \left[(r_\pi + R_{TH}) \| R_{E1}\right] i_x$$

$$= \left[(50k\Omega + 744.4168734) \| 30k\Omega\right] i_x$$

$$= 18853.7185 i_x$$

$$\therefore R_{out}' = \frac{V_x}{i_x}$$

$$= 65.53736901 \times 10^6$$

$$\approx 65.54 M\Omega$$

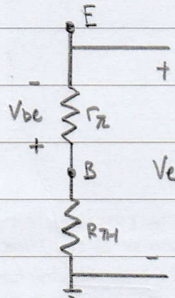
$$R_{out} = R_C \| R'_{out}$$

$$= 100k\Omega \| 65.54M\Omega$$

$$= \frac{100k\Omega \times 65.54M\Omega}{100k\Omega + 65.54M\Omega}$$

$$= 99.84765387 \times 10^3$$

$$\approx 100k\Omega$$



$$V_{be} = -\left(\frac{r_\pi}{r_\pi + R_{TH}}\right) V_e$$

$$= -\left(\frac{50k\Omega}{50k\Omega + 744}\right)(18853.7185 i_x)$$

$$= -18577.13583 i_x$$

$$V_x = (i_x - 2 \times 10^{-3}(-18577.13583 i_x))\, 1.7172M\Omega + 18853.7185 i_x$$

$$= (i_x + 37.15427166 i_x)\, 1.7172M\Omega + 18853.7185 i_x$$

$$= 65.53736901 \times 10^6 \, i_x$$

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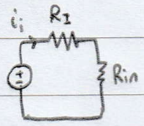
④ Current gain



$$A_i = \frac{i_o}{i_i} = \frac{\frac{V_o}{R_L}}{\frac{V_i}{R_I + R_{in}}}$$

$$= \frac{V_o}{R_L} \times \frac{R_I + R_{in}}{V_i}$$

$$= \frac{V_o}{V_i} \times \frac{R_I + R_{in}}{R_L}$$

$$= A_v \times \frac{R_I + R_{in}}{R_L}$$

$$= -1.63\left(\frac{750 + 96.86\,k\Omega}{100\,k\Omega}\right)$$

$$= -1.591043$$

$$\approx -1.59$$

For small-signal operation $|V_{be}| \leq 5mV$

$$V_{be} = i_b r_\pi$$

$$= \left(\frac{V_b}{R'_{in}}\right) r_\pi$$

$$= \frac{50\,k\Omega}{3.08\,M\Omega} V_b$$

$$0.01623376623\,|V_b| \leq 5mV$$

$$|V_b| \leq 0.308V$$

$$\leq 308mV$$

$$A_{vt} = \frac{V_o}{V_b}$$

$$|V_o| = |A_{vt} V_b|$$

$$= 1.64\,|V_b|$$

$$= 1.64 \times 308mV$$

$$= 505.12mV$$

$\therefore$ The maximum ac output signal is $505.12\,mV$ for small-signal operation

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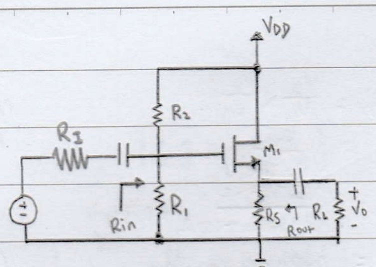
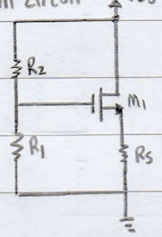
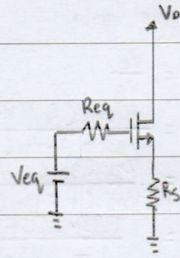
2)



DC Equivalent circuit



$\rightarrow$ simplify



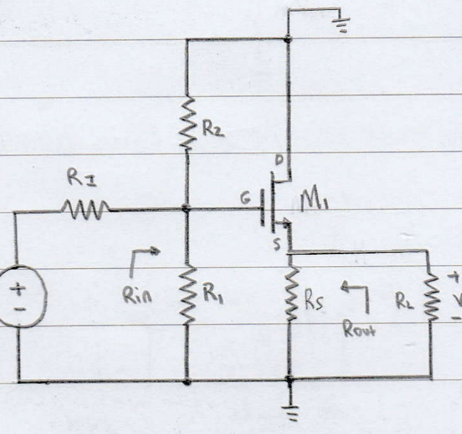
$R_{eq} = 518k\Omega$

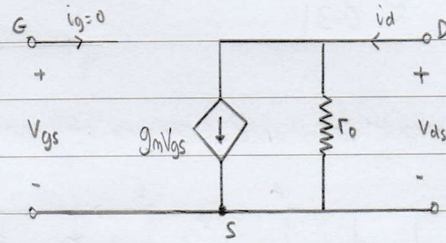
$V_{eq} = 8.53V$

Q-pt (I_D, V_{DS}) solved by KVL in T5Q4

$(1.87mA, 9.39V)$

ac equivalent circuit (cap S/C, DC voltage GND)





* no r_π

& r_o parallel between D&S

Flow from D to S

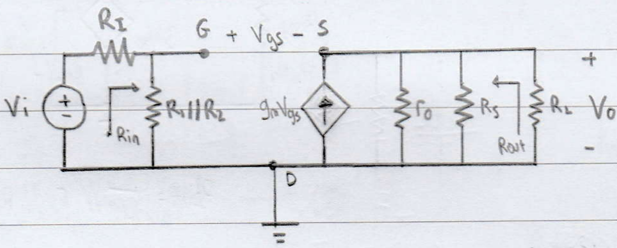
r_o very huge i_{r_o} very small

$$i_D = \frac{k_n}{2}(V_{GS}-V_{TN})^2(1+\lambda V_{DS})$$

$$g_m = k_n(V_{GS}-V_{TN})(1+\lambda V_{DS}) \qquad k_n = \mu_n C_{ox}\left(\frac{W}{L}\right)$$

$$\left.\begin{aligned} g_m &= \sqrt{2k_n I_D} \\ r_o &= \frac{\frac{1}{\lambda}+V_{DS}}{I_D} \approx \frac{1}{\lambda I_D} \end{aligned}\right\} \text{important}$$

Small signal ac equivalent circuit



$k_n = 1mA/V^2$ $\quad R_I = 100\Omega$ $\quad R_L = 250\Omega$

$V_{TN} = 1V$ $\quad R_1 = 1.2M\Omega$ $\quad V_{DD} = 15V$

$\lambda = 0.02V^{-1}$ $\quad R_2 = 910k\Omega$

$R_S = 3k\Omega$

$$\begin{aligned} g_m &= \sqrt{2k_n I_D} \\ &= \sqrt{2(1\times10^{-3})(1.87\times10^{-3})} \\ &= 1.933907961\times10^{-3} \\ &\approx 1.93mS \end{aligned}$$

$$\begin{aligned} r_o &= \frac{1}{\lambda I_D} \\ &= \frac{1}{0.02(1.87\times10^{-3})} \\ &= 26.73796791\times10^{3}\ \Omega \\ &\approx 26.74k\Omega \end{aligned}$$

① Input Resistance

$$\begin{aligned} R_{in} &= R_1 // R_2 \\ &= \frac{1.2M\Omega\times910k\Omega}{1.2M\Omega+910k\Omega} \\ &= 517.535545\times10^{3}\ \Omega \\ &\approx 517.54k\Omega \end{aligned}$$

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② Voltage gain

$$A_{vt} = \frac{V_o}{V_g}$$

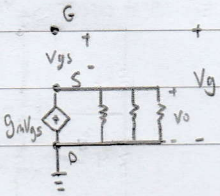
$$= \frac{g_m V_{gs}(r_o // R_S // R_L)}{V_{gs} + g_m V_{gs}(r_o // R_S // R_L)}$$

$$= \frac{\cancel{V_{gs}}\,(g_m(r_o // R_S // R_L))}{\cancel{V_{gs}}\,(1 + g_m(r_o // R_S // R_L))}$$

$$= \frac{1.93\text{mS}(26.74\text{k}\Omega // 3\text{k}\Omega // 250\Omega)}{1 + 1.93\text{mS}(26.74\text{k}\Omega // 3\text{k}\Omega // 250\Omega)}$$

$$= 0.306313689$$

$$\approx 0.31$$



$$A_v = \frac{V_g}{V_i} \times \frac{V_o}{V_g}$$

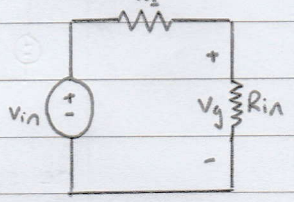
$$= \left(\frac{R_{in}}{R_{in} + R_I}\right) \times 0.31$$

$$= \frac{517.54\text{k}\Omega}{517.54\text{k}\Omega + 100\Omega} \times 0.31$$

$$= 0.3099401128$$

$$\approx 0.31$$

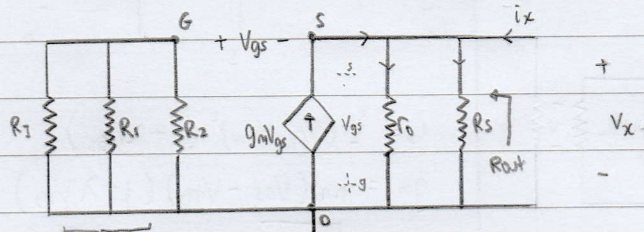
replaced whole circuit by Rin



$$V_g = \frac{R_{in}}{R_{in} + R_I} V_{in}$$

$$\frac{V_g}{V_{in}} = \frac{R_{in}}{R_{in} + R_I}$$

③ Output Resistance (switch off source)



no current flowing

$$\text{KCL:} \quad g_m V_{gs} + i_x = \frac{V_x}{r_o // R_S}$$

$$\therefore V_{gs} = -V_x$$

$$g_m(-V_x) + i_x = \frac{V_x}{r_o // R_S}$$

$$i_x = \frac{V_x}{r_o // R_S} + g_m V_x$$

$$= V_x\left(\frac{1}{r_o // R_S} + g_m\right)$$

$$R_{out} = \frac{V_x}{i_x}$$

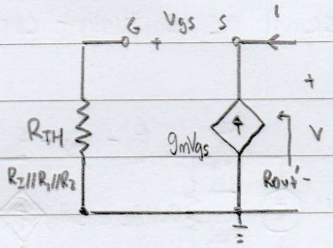
$$= \frac{1}{\frac{1}{r_o // r_s} + g_m}$$

$$= 1 \div \left(\frac{1}{26.74\text{k}\Omega // 3\text{k}\Omega} + 1.93\text{mS}\right)$$

$$= 434.6445635\,\Omega$$

$$\approx 434.64\,\Omega$$

by inspection



$$R_{out}' = \frac{v}{i}$$

$$= \frac{-\cancel{V_{gs}}}{\cancel{-}\,g_m \cancel{V_{gs}}}$$

$$= \frac{1}{g_m}$$

$$R_{out} = \frac{1}{g_m} // r_o // R_S$$

For small signal operation $V_{gs} \leq 0.2(V_{GS} - V_{TN})$

Start with V_{gs}
express in V_g
express in V_i

$V_{GS} = 2.92\text{V}$ (calculated in TSQ4)

$$V_{gs} = V_g - V_s \quad , A_{vt} = \frac{V_s}{V_g} \rightarrow V_s = V_g A_{vt}$$

$$= V_g - V_g A_{vt}$$

$$= V_g(1 - A_{vt})$$

$$= \left(\frac{R_{in}}{R_{in} + R_I} V_{in}\right)(1 - A_{vt})$$

$$= \frac{517.54\text{k}\Omega}{517.54\text{k}\Omega + 100\Omega} V_i (1 - 0.31)$$

$$= 0.6898667027 V_i$$

$$V_{gs} \leq 0.2(2.92 - 1)$$

$$\leq 0.384$$

$$0.6898667027 V_i \leq 0.384$$

$$V_i \leq \frac{0.384}{0.6898667027}$$

$$\leq 556.6292713\text{mV}$$

$$\leq 556.63\text{mV}$$

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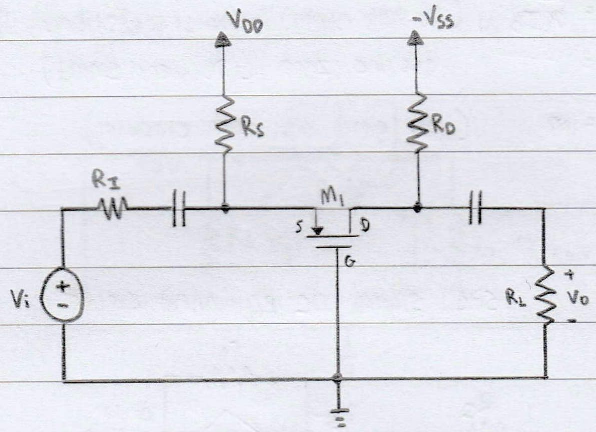
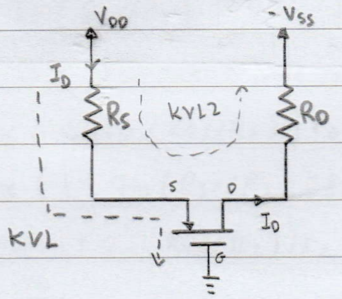
3)



* DC analysis for MOSFET solve quadratic equation

DC equivalent circuit (cap O/C, AC source off)



$k_p = 200\mu A / V^2$ $\quad R_I = 250\Omega$ $\quad V_{DD} = V_{SS} = 15V$

$V_{TP} = -1V$ $\quad R_S = 68k\Omega$

$R_L = 200k\Omega$

$R_D = 43k\Omega$

NMOS Normal PMOS use absolute but PMOS will have a negative V_{GS}

$I_D = \frac{k_p}{2}(|V_{GS}| - |V_{TP}|)^2$

$= \frac{200\mu A}{2}(|V_{GS}| - 1)^2$ — ①

By KVL

$-V_{DD} + I_D R_S + |V_{GS}| = 0$

$I_D = \frac{V_{DD} - |V_{GS}|}{R_S}$

$= \frac{15 - |V_{GS}|}{68\times10^3}$ — ②

sub ② into ①

$\frac{15 - |V_{GS}|}{68\times10^3} = \frac{200\mu A}{2}(|V_{GS}| - 1)^2$

$= 100\mu A\,(|V_{GS}| - 1)(|V_{GS}| - 1)$

$= 100\mu A\,[\,|V_{GS}|^2 - 2|V_{GS}| + 1\,]$

$= 100\mu A\,|V_{GS}|^2 - 200\mu A\,|V_{GS}| + 100\mu A$

$15 - |V_{GS}| = [100\mu A\,|V_{GS}|^2 - 200\mu A\,|V_{GS}| + 100\mu A\,] \times 68\times10^3$

$= 6.8|V_{GS}|^2 - 13.6|V_{GS}| + 6.8$

$6.8|V_{GS}|^2 - 12.6|V_{GS}| - 8.2 = 0$

$|V_{GS}| = 2.363213468V$ or $-0.510272292V$ (infeasible because $|V_{GS}| \geq 0$)

$\approx 2.363V$

$V_{GS} = -2.363V$ (without absolute)

$I_D = \frac{15 - 2.363}{68\times10^3}$

$= 185.8382353\times10^{-6}$ A

$\approx 186\mu A$

By KVL 2

$-15 + I_D R_S + V_{SD} + I_D R_D + (-15) = 0$

$V_{SD} = 30 - I_D R_S - I_D R_D$

$= 30 - 186\mu A(68k\Omega) - 186\mu A(43k\Omega)$

$= 9.354V$

$\therefore V_{DS} = -9.354V$ $\left(\begin{array}{c}\text{PMOS in saturation mode} \\ |V_{DS}| \geq |V_{GS}| - |V_{TP}|\end{array}\right)$

$9.354 \geq 2.363 - 1$

Q-pt

$(186\mu A, -9.354V)$

$g_m = \sqrt{2K_p I_D}$

$= \sqrt{2(200\mu A)(186\mu A)}$

$= 272.7636339 \times 10^{-6}\ s$

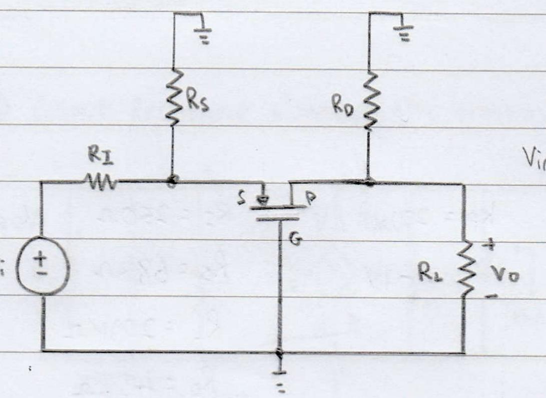
$\approx 273\ \mu S$

$r_o = \frac{1}{\lambda I_D}$ (λ not given in this question)

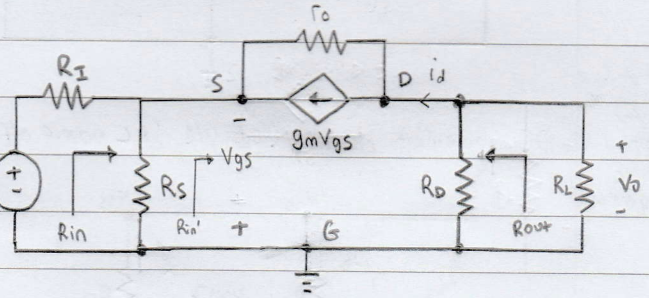
assume zero (λ very small)

$= \infty$ (as good as open circuit)

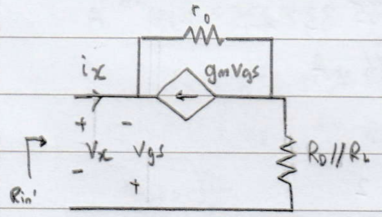
ac equivalent circuit



Small signal ac equivalent circuit



① Input Resistance



$R_{in}' = \frac{V_x}{i_x}$

$= \frac{-V_{gs}}{-g_m V_{gs}}$

$= \frac{1}{g_m}$

$R_{in} = R_S // R_{in}'$

$= 68\ k\Omega // \frac{1}{273\ \mu S}$

$= 3.475771826 \times 10^3\ \Omega$

$\approx 3.476\ k\Omega$

② Voltage gain

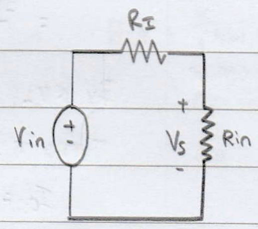
$A_{vt} = \frac{V_o}{V_s}$

$= \frac{-g_m V_{gs}(R_D // R_L)}{-V_{gs}}$

$= g_m (R_D // R_L)$

$= 273\mu S \left(\frac{43k\Omega \times 200k\Omega}{43k\Omega + 200k\Omega}\right)$

$= 9.661728395$



$V_s = \frac{R_{in}}{R_{in}+R_I} V_{in}$

$\frac{V_s}{V_{in}} = \frac{R_{in}}{R_{in}+R_I}$

$A_v = \frac{V_s}{V_i} \times \frac{V_o}{V_s}$

$= \left(\frac{R_{in}}{R_{in}+R_I}\right)(9.661728395)$

$= \left(\frac{3.476k\Omega}{3.476k\Omega + 250}\right)(9.661728395)$

$= 9.013464278$

≈ 9.01

Current gain

$A_i = \frac{i_o}{i_i}$

$= \frac{\frac{V_o}{R_L}}{\frac{V_i}{R_I + R_{in}}}$

$= \left(\frac{V_o}{R_L} \times \frac{R_I + R_{in}}{V_i}\right)$

$= \frac{V_o}{V_i}\left(\frac{R_I + R_{in}}{R_L}\right)$

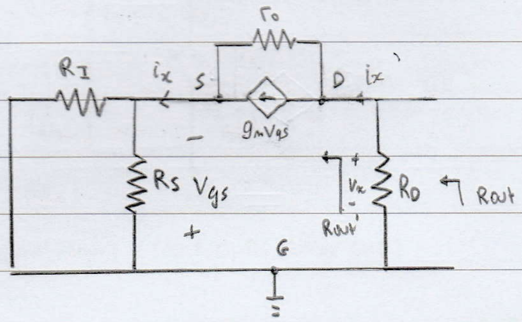
$= 9.01\left(\frac{250 + 3.476k\Omega}{200k\Omega}\right)$

$= 0.1678563$

≈ 0.168

No.: .... Date: ....

③ Output Resistance (turn-off source)



$$V_x = (i_x - g_m V_{gs}) r_o + i_x (R_S // R_I)$$

express in i_x $\quad V_{gs} = -i_x (R_S // R_I)$

$$V_x = (i_x - g_m(-i_x(R_S//R_I)))r_o + i_x(R_S//R_I)$$

$$= [i_x + g_m i_x (R_S//R_I)] r_o + \underbrace{i_x(R_S//R_I)}_{\text{ignore this}}$$

$$= i_x(1 + g_m(R_S//R_I)) r_o$$

$$V_x = i_x(1 + g_m(R_S//R_I)) r_o$$

$$R_{out}' = \frac{V_x}{i_x}$$

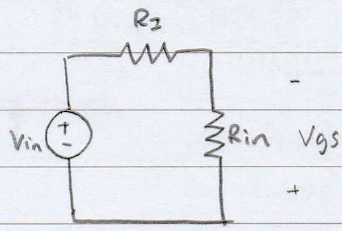
$$= 1 + g_m(R_S//R_I) r_o \rightarrow \infty$$

$$R_{out}' = \infty$$

$$R_{out} = R_{out}' // R_D$$

$$= \infty // 43k\Omega$$

$$= 43k\Omega$$



$$|V_{gs}| = \frac{R_{in}}{R_I + R_{in}} V_{in}$$

For small signal operation $|V_{gs}| \le 0.2(|V_{GS}| - |V_{TP}|)$

$$V_{GS} = 2.363V$$

$$\left(\frac{R_{in}}{R_I + R_{in}}\right) V_i \le 0.2(2.363 - 1)$$

$$\frac{3.476k\Omega}{3.476k\Omega + 250} V_i \le 0.2(2.363 - 1)$$

$$V_i \le 292.2058688 \times 10^{-3}\ V$$

$$\le 292.21\ mV$$