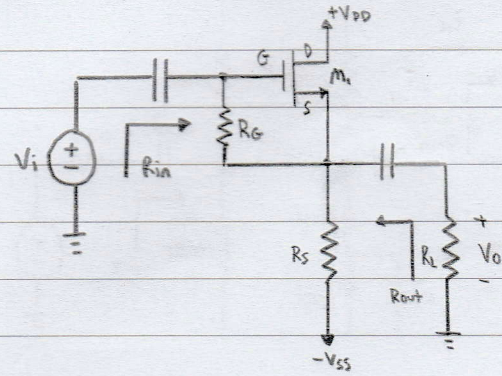
No.: .... Date: ....

Esmund Lim A.E Tutorial 7

1) Common-drain (Voltage follower)



$g_m = 3.54mS$

r_o can be neglected $\therefore r_o = \infty$ (O/C)

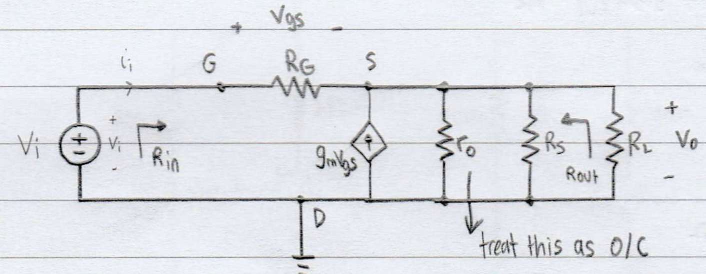
$R_G = 1M\Omega$

$R_S = 2k\Omega$

$R_L = 100k\Omega$

$V_{DD} = V_{SS} = 10V$

Small signal ac equivalent circuit



KCL

$$\frac{V_i - V_o}{1\times10^6} + g_m V_{gs} = \frac{V_o}{R_S // R_L}$$

$$V_{gs} = V_i - V_o$$

$$\frac{V_i}{1\times10^6} - \frac{V_o}{1\times10^6} + g_m(V_i - V_o) = \frac{V_o}{2k\Omega // 100k\Omega}$$

$$\frac{1}{1\times10^6}V_i - \frac{1}{1\times10^6}V_o + 3.54\times10^{-3}V_i - 3.54\times10^{-3}V_o = \frac{1}{2k\Omega // 100k\Omega}V_o$$

$$\frac{1}{1\times10^6}V_i - \frac{1}{1\times10^6}V_o + 3.54\times10^{-3}V_i - 3.54\times10^{-3}V_o = 0.51\times10^{-3}V_o$$

$$-\frac{1}{1\times10^6}V_o - 3.54\times10^{-3}V_o - 0.51\times10^{-3}V_o = -\frac{1}{1\times10^6}V_i - 3.54\times10^{-3}V_i$$

$$-4.051\times10^{-3}V_o = -3.541\times10^{-3}V_i$$

$$\frac{V_o}{V_i} = \frac{-3.541\times10^{-3}}{-4.051\times10^{-3}} \qquad \therefore V_o = 0.874V_i$$

$$A_v = 0.8741051592$$

$$\approx 0.874$$

$$R_{in} = \frac{v_i}{i_i}$$

$$= \frac{V_i}{\frac{V_i - V_o}{R_G}}$$

$$= V_i \times \frac{R_G}{V_i - V_o}$$

$$= \frac{1\times10^6 V_i}{V_i - V_o}$$

$$= \frac{1\times10^6 V_i}{V_i - 0.874V_i}$$

$$= \frac{1\times10^6 \cancel{V_i}}{0.126\cancel{V_i}}$$

$$= 7.936507937\times10^6\Omega$$

$$\approx 7.94M\Omega$$

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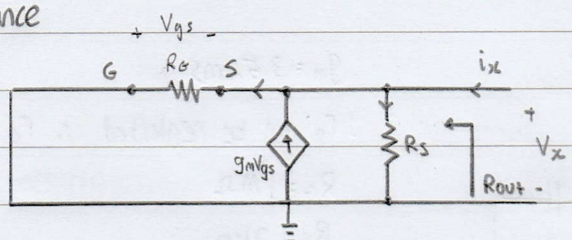
Output Resistance



By KCL

$i_x + g_m V_{gs} = \frac{V_x}{R_S} + \frac{V_x}{R_G}$

$V_{gs} = -V_x$

$i_x + g_m(-V_x) = \frac{V_x}{R_S} + \frac{V_x}{R_G}$

$i_x = \frac{V_x}{R_S} + \frac{V_x}{R_G} + g_m V_x$

$i_x = \frac{1}{2\times10^3} V_x + \frac{1}{1\times10^6} V_x + 3.54\times10^{-3} V_x$

$i_x = 4.041\times10^{-3} V_x$

$R_{out} = \frac{V_x}{i_x}$

$= \frac{1}{4.041\times10^{-3}}$

$= 247.4634991\,\Omega$

$\approx 247.46\,\Omega$

b) To acheive $A_v = 1$

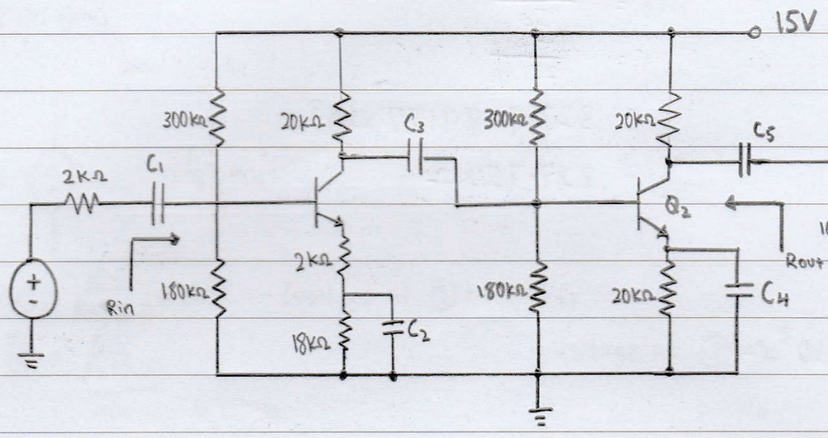
$V_o = V_i$

$R_{in} = \frac{V_i}{\frac{V_i - V_o}{R_G}}$

$= \frac{V_i}{0}$

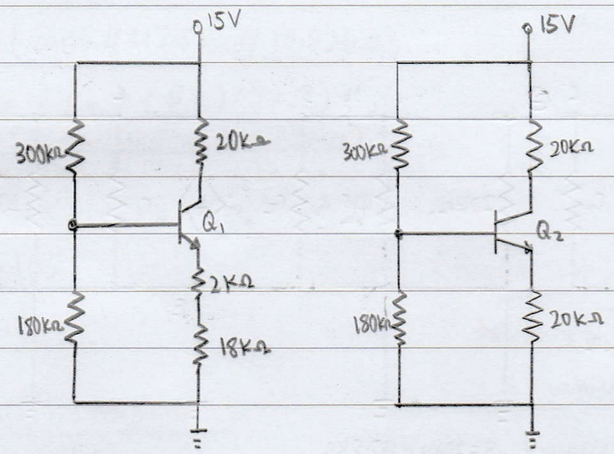
$= \infty$

2)



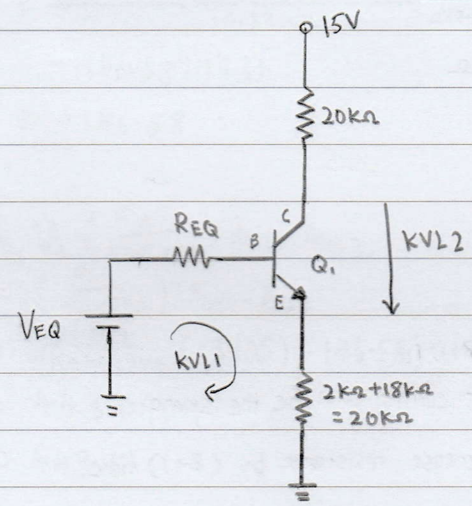
$\beta = 100 \quad V_A = 70V$

DC equivalent circuit



Q_1 and Q_2 will have the same Q-point

Q-point
(I_C, V_{CE})
$(231\mu A, 5.71V)$



$$R_{EQ} = 300k\Omega \,//\, 180k\Omega$$
$$= \frac{300k\Omega \times 180k\Omega}{300k\Omega + 180k\Omega}$$
$$= 112.5\ k\Omega$$

$$V_{EQ} = \frac{180\ k\Omega}{180k\Omega + 300k\Omega} \times 15V$$
$$= 5.625V$$

$$I_C = \beta I_B \qquad I_E = (\beta+1) I_B$$
$$= 100(2.31\mu A) \qquad = 101(2.31\mu A)$$
$$= 231\mu A \qquad = 233.31\mu A$$

By KVL 1

$$-V_{EQ} + I_B R_{EQ} + V_{BE} + I_E(20k\Omega) = 0$$
$$I_B(112.5k\Omega) + I_E(20k\Omega) = V_{EQ} - V_{BE}$$
$$I_B(112.5k\Omega) + (\beta+1)I_B(20k\Omega) = 5.625 - 0.7$$
$$2132500 I_B = 4.925$$
$$I_B = 2.309495897 \times 10^{-6}\ A$$
$$\approx 2.31\mu A$$

$I_C = \beta I_B$
$= \alpha I_E$

$I_E = (\beta+1) I_B$

$\alpha = \frac{\beta}{\beta+1}$

By KVL2

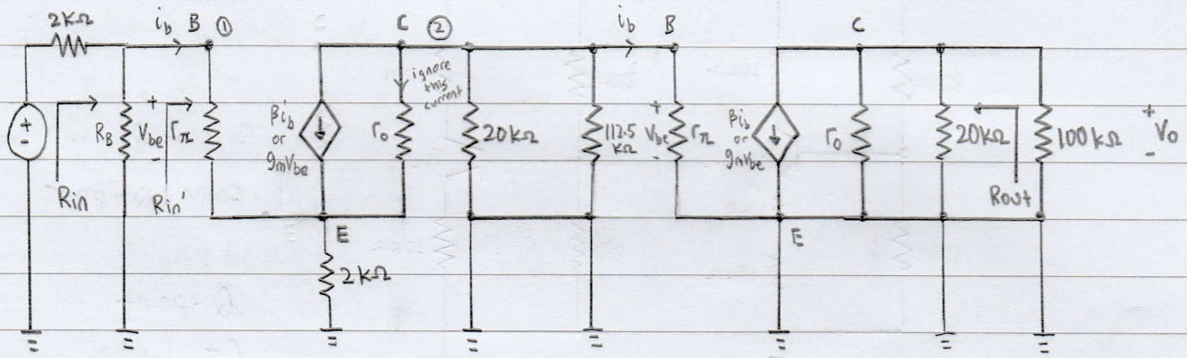
$$-15 + I_C R_C + V_{CE} + I_E(20k\Omega) = 0$$
$$V_{CE} = 15 - 231\mu A(20k\Omega) - 233.31\mu A(20k\Omega)$$
$$= 5.7138V$$
$$\approx 5.71V$$

$g_m = 40 I_c$
$= 40\,(231\,\mu A)$
$= 9.24\,mS$

$r_o = \dfrac{V_A + V_{CE}}{I_c}$
$= \dfrac{70 + 5.71}{231\,\mu A}$
$= 327.7489177 \times 10^3\,\Omega$
$\approx 327.75\,k\Omega$

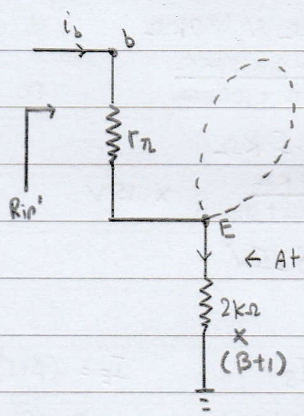
$r_\pi = \dfrac{\beta}{g_m}$
$= \dfrac{100}{9.24\,mS}$
$= 10.82251082 \times 10^3\,\Omega$
$\approx 10.82\,k\Omega$

small signal ac equivalent circuit



$R_B = 300k\Omega // 180k\Omega$
$= \dfrac{300k\Omega \times 180k\Omega}{300k\Omega + 180k\Omega}$
$= 112.5\,k\Omega$

$R = 20k\Omega // 112.5\,k\Omega$
$= \dfrac{20k\Omega \times 112.5k\Omega}{20k\Omega + 112.5k\Omega}$
$= 16.98\,k\Omega$



← At first this current is $(\beta+1)\, i_b$

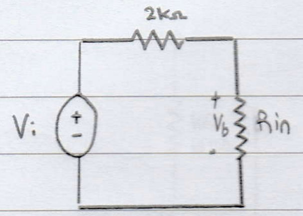
∴ after removing it current will be the same

∴ have to increase resistance by $(\beta+1)$ time

$R_{in}' = r_\pi + (\beta+1)(2k\Omega)$
$= 10.82k\Omega + (100+1)(2k\Omega)$
$= 212.82\,k\Omega$

$R_{in} = R_{in}' // R_B$
$= 212.82k\Omega // 112.5\,k\Omega$
$= 73.59599779 \times 10^3\,\Omega$
$\approx 73.6\,k\Omega$

voltage gain



$V_b = \frac{R_{in}}{R_{in} + 2k\Omega} \times V_i$ (voltage at ①) $\times A_{vt1}$

$\frac{V_b}{V_i} = \underbrace{\frac{R_{in}}{R_{in} + 2k\Omega}}_{\text{input loading factor}}$ $= \text{voltage at ②} \times A_{vt2}$

$= V_o$

$$A_{vt1} = \frac{V_c}{V_b}$$

$$= \frac{-\beta i_b (20k\Omega // 112.5k\Omega // 10.82k\Omega)}{i_b(r_\pi + (\beta+1)(2k\Omega))}$$

$$= \frac{-100(20k\Omega // 112.5k\Omega // 10.82k\Omega)}{10.82k\Omega + 101(2k\Omega)}$$

$$= -3.10541024$$

$$\approx -3.105$$

(Reason why this gain is so small as compared to A_{vt2} is because there is a resistor at E while the other circuit have that resistor bypassed)

$$A_{vt2} = \frac{V_c}{V_b}$$

$$= \frac{-\beta i_b (r_o // 20k\Omega // 100k\Omega)}{i_b r_\pi}$$

$$= \frac{-100(327.75k\Omega // 20k\Omega // 100k\Omega)}{10.82k\Omega}$$

$$= -146.5817931$$

$$\approx -146.58$$

$$V_o = V_i \left(\frac{R_{in}}{R_{in} + 2k\Omega}\right) A_{vt1} \cdot A_{vt2}$$

$$\frac{V_o}{V_i} = \left(\frac{R_{in}}{R_{in} + 2k\Omega}\right) A_{vt1} \cdot A_{vt2}$$

$$A_v = \frac{73.6k\Omega}{73.6k\Omega + 2k\Omega}(-3.105)(-146.58)$$

$$= 443.0904$$

$$\approx 443.1$$

Output resistance

$$R_{out} = 20k\Omega // r_o$$

$$= 20k\Omega // 327.75k\Omega$$

$$= \frac{20k\Omega \times 327.75k\Omega}{20k\Omega + 327.75k\Omega}$$

$$= 18.84974838 \times 10^3 \ \Omega$$

$$\approx 18.8k\Omega$$

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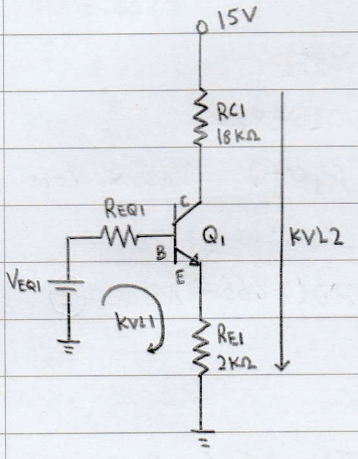
3) DC analysis



$R_{EQ1} = R_1 // R_2$

$= 820k\Omega // 100k\Omega$

$= \frac{820k\Omega \times 100k\Omega}{820k\Omega + 100k\Omega}$

$= 89.13043478 \times 10^3 \Omega$

$\approx 89.13k\Omega$

$V_{EQ1} = \frac{R_2}{R_1+R_2} \times V_{CC}$

$= \frac{100k\Omega}{100k\Omega + 820k\Omega} \times 15$

$= 1.630434783V$

$\approx 1.63V$

By KVL1

$-V_{EQ1} + I_B R_{EQ1} + V_{BE} + I_E R_{E1} = 0$

$I_B R_{EQ1} + (\beta+1) I_B R_{E1} = V_{EQ1} - V_{BE}$

$I_B(89.13k\Omega) + (100+1)(2000) I_B = 1.63 - 0.7$

$I_B = 3.194449215 \times 10^{-6} A$

$\approx 3.19 \mu A$

$I_C = \beta I_B$

$= 100 \times 3.19 \mu A$

$= 319 \mu A$

$I_E = (\beta+1) I_B$

$= 101 \times 3.19 \mu A$

$= 322.19 \mu A$

By KVL 2

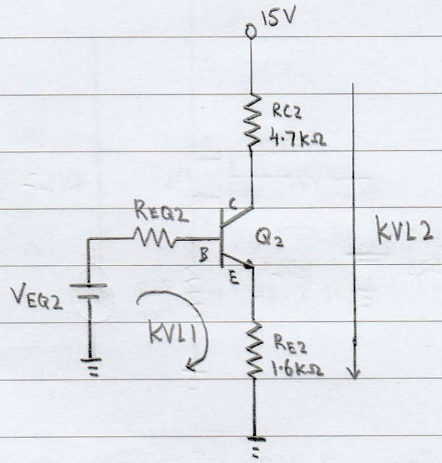
$-15 + 18k\Omega(319\mu A) + V_{CE} + 322.19\mu A(2k\Omega) = 0$

$V_{CE} = 15 - 18k\Omega(319\mu A) - 2k\Omega(322.19\mu A)$

$= 8.61362V$

$\approx 8.61V$

Q-point $(319\mu A, 8.61V)$



$R_{EQ2} = R_3 // R_4$

$= 160k\Omega // 43k\Omega$

$= \frac{160k\Omega \times 43k\Omega}{160k\Omega + 43k\Omega}$

$= 33.89162562 \times 10^3 \Omega$

$\approx 33.89k\Omega$

$V_{EQ2} = \frac{R_4}{R_4+R_3} \times 15V$

$= \frac{43k\Omega}{43k\Omega + 160k\Omega} \times 15V$

$= 3.177339901V$

$\approx 3.18V$

By KVL1

$-V_{EQ2} + I_B R_{EQ2} + V_{BE} + I_E R_{E2} = 0$

$I_B R_{EQ2} + (\beta+1) I_B R_{E2} = V_{EQ} - V_{BE}$

$I_B(33.89k\Omega) + (100+1)(1600) I_B = 3.18 - 0.7$

$I_B = 12.6860709 \times 10^{-6} A$

$\approx 12.69 \mu A$

$I_C = \beta I_B$

$= 100 \times 12.69 \mu A$

$= 1.269 mA$

$I_E = (\beta+1) I_B$

$= 101 \times 12.69 \mu A$

$= 1.28 mA$

By KVL 2

$-15 + 4.7k\Omega(1.269mA) + V_{CE} + 1.6k\Omega(1.28mA) = 0$

$V_{CE} = 15 - 4.7k\Omega(1.269mA) - 1.6k\Omega(1.28mA)$

$= 6.9877V$

$\approx 6.99V$

Q-point $(1.269mA, 6.99V)$

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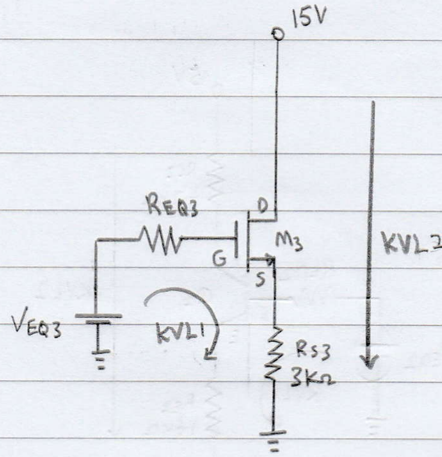

* $I_G = 0$

$R_{EQ3} = R_5 // R_6$
$= 910k\Omega // 1.2M\Omega$
$= \frac{910k\Omega \times 1.2M\Omega}{910k\Omega + 1.2M\Omega}$
$= 517.535545 \times 10^3 \Omega$
$\approx 517.54k\Omega$

$V_{EQ3} = \frac{R_6}{R_5+R_6} \times 15V$
$= \frac{1.2M\Omega}{1.2M\Omega + 910k\Omega} \times 15V$
$= 8.530805687V$
$\approx 8.53V$

By KVL1

$-V_{EQ3} + V_{GS} + I_D R_{S3} = 0$
$V_{GS} = V_{EQ3} - I_D R_{S3}$
$= 8.53 - 3000 I_D$ — ①

$I_D = \frac{k_n}{2}(V_{GS} - V_{TN})^2$ — ②

Sub ① into ②

$I_D = \frac{k_n}{2}(8.53 - 3000I_D - 1)^2$
$= \frac{1\times10^{-3}}{2}(7.53 - 3000I_D)^2$
$= 5\times10^{-4}[(7.53-3000I_D)(7.53-3000I_D)]$
$= 5\times10^{-4}[56.7009 - 45180I_D + 9\times10^6 I_D^2]$
$= 0.02835045 - 22.59I_D + 4500I_D^2$

$4500I_D^2 - 23.59I_D + 0.02835045 = 0$

$I_D = 3.376176311\times10^{-3}A$ or $1.866045911\times10^{-3}A$
$\approx 3.38mA$ or $\approx 1.866mA$

if $I_D = 3.38mA$
$V_{GS} = 8.53 - 3000I_D$
$= 8.53 - 3000(3.38mA)$
$= -1.61V$ (reject) $V_{GS} < V_{TN}$

if $I_D = 1.866mA$
$V_{GS} = 8.53 - 3000(1.866mA)$
$= 2.932V$
$V_{GS} > V_{TN}$

$\therefore$ Hence, $I_D = 1.866mA$

By KVL 2

$-15 + V_{DS} + I_D(3000) = 0$
$V_{DS} = 15 - 3000(1.866mA)$
$= 9.402V$

Q-point $(1.866mA, 9.402V)$

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$g_{m1} = 40 I_c$

$= 40(319\mu A)$

$= 12.76\,mS$

$r_{\pi 1} = \frac{B}{g_m}$

$= \frac{100}{12.76ms}$

$= 7.836990596 \times 10^3 \Omega$

$\approx 7.84\,k\Omega$

$r_{o1} = \frac{V_A + V_{CE}}{I_c}$

$= \frac{70 + 8.61}{319\mu A}$

$= 246.4263323 \times 10^3 \Omega$

$\approx 246.43\,k\Omega$

$g_{m2} = 40 I_c$

$= 40(1.269mA)$

$= 50.76\,mS$

$r_{\pi 2} = \frac{B}{g_m}$

$= \frac{100}{50.76ms}$

$= 1.970055162 \times 10^3 \Omega$

$\approx 1.97\,k\Omega$

$r_{o2} = \frac{V_A + V_{CE}}{I_c}$

$= \frac{70 + 6.99}{1.269mA}$

$= 60.66981875 \times 10^3 \Omega$

$\approx 60.67\,k\Omega$

$g_{m3} = \sqrt{2 k_n I_D}$

$= \sqrt{2(1 \times 10^{-3})(1.866mA)}$

$= 1.93\,mS$

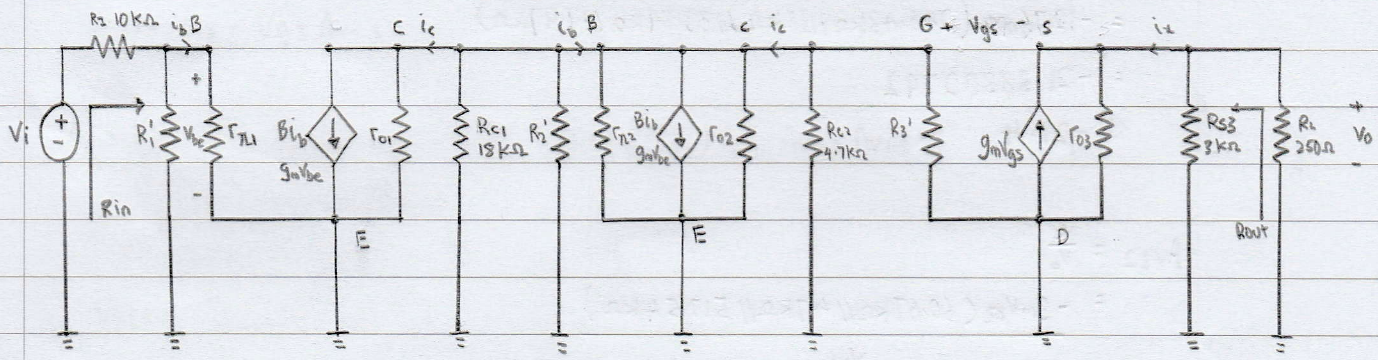
$r_{o3} = \frac{1}{\lambda I_D}$

$= \frac{1}{0.02(1.866mA)}$

$= 26.79528403 \times 10^3$

$\approx 26.8\,k\Omega$

small signal ac equivalent circuit



$R_1' = R_1 // R_2$

$= 820k\Omega // 100k\Omega$

$= \frac{820k\Omega \times 100k\Omega}{820k\Omega + 100k\Omega}$

$= 89.13043478 \times 10^3 \Omega$

$\approx 89.13\,k\Omega$

$R_2' = R_3 // R_4$

$= 160k\Omega // 43k\Omega$

$= \frac{160k\Omega \times 43k\Omega}{160k\Omega + 43k\Omega}$

$= 33.89162562 \times 10^3 \Omega$

$\approx 33.89\,k\Omega$

$R_3' = R_5 // R_6$

$= 910k\Omega // 1.2M\Omega$

$= \frac{910k\Omega \times 1.2M\Omega}{910k\Omega + 1.2M\Omega}$

$= 517.535545 \times 10^3 \Omega$

$\approx 517.54\,k\Omega$

① Input Resistance

$R_{in} = R_1' // r_{\pi 1}$

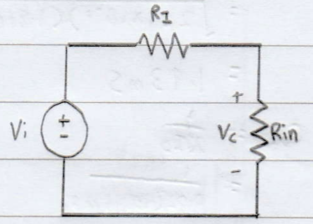
$= 89.13k\Omega // 7.84k\Omega$

$= \frac{89.13k\Omega \times 7.84k\Omega}{89.13k\Omega + 7.84k\Omega}$

$= 7.206137981 \times 10^3 \Omega$

$\approx 7.21\,k\Omega$

② Voltage gain



$$V_c = \frac{R_{in}}{R_{in}+R_I} \times V_i$$

$$\hookrightarrow \times A_{vt1} \times A_{vt2} \times A_{vt3} = V_o$$

$$\therefore A_v = \frac{V_o}{V_i}$$

$$= \left(\frac{R_{in}}{R_{in}+R_I}\right) A_{vt1} A_{vt2} A_{vt3}$$

$$A_{vt1} = \frac{V_c}{V_b}$$

$$= \frac{-g_m \cancel{V_{be}} (246.43k\Omega // 18k\Omega // 33.89k\Omega // 1.97k\Omega)}{\cancel{V_{be}}}$$

$$= -12.76mS\,(246.43k\Omega // 18k\Omega // 33.89k\Omega // 1.97k\Omega)$$

$$= -21.38302792$$

$$\approx -21.4$$

$$A_{vt2} = \frac{V_c}{V_b}$$

$$= \frac{-g_m \cancel{V_{be}} (60.67k\Omega // 4.7k\Omega // 517.54k\Omega)}{\cancel{V_{be}}}$$

$$= -50.76mS\,(60.67k\Omega // 4.7k\Omega // 517.54k\Omega)$$

$$= -219.5684203$$

$$\approx -220$$

$$A_{vt3} = \frac{V_o}{V_g}$$

$$= \frac{g_m V_{gs} (26.8k\Omega // 3k\Omega // 250\Omega)}{V_{gs} + g_m V_{gs} (26.8k\Omega // 3k\Omega // 250\Omega)}$$

$$= \frac{g_m (26.8k\Omega // 3k\Omega // 250\Omega)}{1 + g_m (26.8k\Omega // 3k\Omega // 250\Omega)}$$

$$= \frac{1.93mS\,(26.8k\Omega // 3k\Omega // 250\Omega)}{1 + 1.93mS\,(26.8k\Omega // 3k\Omega // 250\Omega)}$$

$$= 0.3063177594$$

$$\approx 0.306$$

$$\therefore A_v = \left(\frac{R_{in}}{R_{in}+R_I}\right) A_{vt1} A_{vt2} A_{vt3}$$

$$= \frac{7.21k\Omega}{7.21k\Omega + 10k\Omega}(-21.4)(-220)(0.306)$$

$$= 603.5486392$$

$$\approx 603.55$$

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③ output Resistance

By KCL

$i_x + g_m V_{gs} = \frac{V_x}{3k\Omega \parallel 26.8k\Omega}$ $V_{gs} = -V_x$

$i_x + g_m(-V_x) = \frac{V_x}{3k\Omega \parallel 26.8k\Omega}$

$i_x - g_m V_x = \frac{V_x}{3k\Omega \parallel 26.8k\Omega}$

$i_x = 3.706467662 \times 10^{-4} V_x + 1.93mS (V_x)$

$R_{out} = \frac{V_x}{i_x}$

$= 434.6\Omega$

For small signal operation $V_{gs3} \leq 0.2(V_{GS3} - V_{TN3})$

$A_{vt3} = \frac{V_o}{V_{g3}}$, $V_o = g_m V_{gs3} (26.8k\Omega \parallel 3k\Omega \parallel 250\Omega)$

$V_{g3} = V_{gs3} + V_o$

$V_{gs3} = V_{g3} - V_o$

$= V_{g3} - V_{g3} A_{vt3}$

$A_v = \frac{V_o}{V_i}$ $V_o = \left(\frac{R_{in}}{R_{in}+R_I}\right) A_{vt1} A_{vt2} A_{vt3} V_i$

$= \left(\frac{R_{in}}{R_{in}+R_I}\right) A_{vt1} A_{vt2} A_{vt3}$

$V_{gs3} = V_{g3}(1 - A_{vt3})$

$= \frac{V_o}{A_{vt3}}(1 - A_{vt3})$

$= \left(\frac{R_{in}}{R_{in}+R_I}\right) A_{vt1} A_{vt2} V_i (1 - A_{vt3})$

$V_{gs3} \leq 0.2(2.932V - 1)$

≤ 0.3864

$V_i \leq 0.3864 \div \left(\frac{R_{in}}{R_{in}+R_I}\right)(A_{vt1})(A_{vt2})(1 - A_{vt3})$

$V_i \leq 0.3864 \div \left(\frac{7.21k\Omega}{7.21k\Omega + 10k\Omega}\right)(-21.4)(-220)(1 - 0.306)$

$V_i \leq 282.2843483 \mu V$

$\leq 282.3 \mu V$