No.: .... Date: ....

Esmund Lim Signals Tutorial 6

Fundamental period T_0 → minimum time period

$$f(t) = f(t + \underbrace{n}_{\text{arbitrary integer}} T_0)$$

Signals

1. REAL Sinusoid : $x(t) = A\cos(\omega t + \underbrace{\theta}_{\text{phase change}})$ or $A\sin(\omega t + \theta)$

$\sin(\omega t + \pi/2) = \cos(\omega t)$
$\cos(\omega t + \pi/2) = -\sin(\omega t)$

2. COMPLEX Sinusoid : $x(t) = Ae^{j(\omega t + \theta)}$ or $Ae^{-j(\omega t + \theta)}$

$$= \underbrace{A\cos(\omega t + \theta)}_{\text{real}} + j\underbrace{A\sin(\omega t + \theta)}_{\text{imaginary}} \qquad = A\cos(\omega t + \theta) - jA\sin(\omega t + \theta)$$

$z = a + jb$

By Euler identities : $A\cos(\omega t + \theta) = A\left[\frac{e^{j(\omega t+\theta)} + e^{-j(\omega t + \theta)}}{2}\right]$

$e^{j\omega t} \cdot e^{j\theta}$

$$\underbrace{\left(\frac{A}{2}e^{j\theta}\right)}\underbrace{e^{j\omega t}}$$

$= \cos\omega t + j\sin\omega t$

$\hookrightarrow = \frac{A}{2}\cos\theta + j\frac{A}{2}\sin\theta$

$$A\sin(\omega t + \theta) = A\left[\frac{e^{j(\omega t+\theta)} - e^{-j(\omega t+\theta)}}{j2}\right]$$

Euler identities

$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$

$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{j2}$

1) a) $x(t) = \cos\left(t + \frac{\pi}{4}\right)$

$\omega_0 = 1 \qquad \omega_0 = 2\pi f_0$

$f_0 = \frac{\omega_0}{2\pi} \qquad T_0 = \frac{1}{f_0}$

$= \frac{1}{2\pi} \qquad = 2\pi\ s$

b) $x(t) = \pi\sin\frac{2\pi t}{3}$

$\omega_0 = \frac{2\pi}{3}$

$f_0 = \frac{2\pi}{3} \div 2\pi$

$= \frac{2\pi}{3} \times \frac{1}{2\pi}$

$= \frac{1}{3}$

$T_0 = \frac{1}{f_0}$

$= 3\ s$

c) $x(t) = 2\sin^2 t$

$= 2\left[\frac{1}{2}(1 - \cos 2t)\right]$

$= 2\left[\frac{1}{2} - \frac{1}{2}\cos 2t\right]$

$= 1 - \cos(2t)$

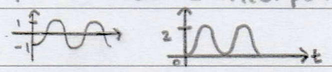
$\omega_0 = 2 \qquad T_0 = \frac{1}{f_0}$

$f_0 = \frac{2}{2\pi} \qquad = \pi\ s$

$= \frac{1}{\pi}$

* $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

1 = dc offset [does not affect period]



* When you square the sinusoid signal
→ double the frequency
↳ half the period

d) $x(t) = 5e^{j\left[\left(\frac{\pi t}{2}\right) - 1\right]}$

$= \underbrace{5e^{-j}}_{\text{constant (complex number)}} \cdot e^{j\frac{\pi}{2}t}$

$\omega_0 = \frac{\pi}{2}$

$f_0 = \frac{\pi}{2} \div 2\pi \qquad T_0 = \frac{1}{f_0}$

$= \frac{\pi}{2} \times \frac{1}{2\pi} \qquad = 4\ s$

$= \frac{1}{4}$

2) $x_1(t)\cos 5t$, $x_2(t) = 2\sin 5t$

$x_1(t) + x_2(t)$

$\cos 5t + 2\sin 5t$

$\underbrace{}$ $A\cos(\omega t + \theta)$

$= A\cos\omega t\cos\theta - A\sin\omega t\sin\theta$

$= A\cos\theta\cos\omega t - A\sin\theta\sin\omega t$

* $\underbrace{P\sin\omega t + Q\cos\omega t}_{\text{without phase}} = \underbrace{A\sin(\omega t + \theta)}_{\text{with phase}}$

$\underbrace{P\cos\omega t + Q\sin\omega t}_{\text{without phase}} = \underbrace{A\cos(\omega t + \theta)}_{\text{with phase}}$

same frequency ω_0

$\therefore A\cos\theta = 1$ — ①

$-A\sin\theta = 2$

$A\sin\theta = -2$ — ②

$\omega_0 = 5$ rad/s

$①^2 + ②^2$:

$A^2\cos^2\theta + A^2\sin^2\theta = 1^2 + (-2)^2$

$A^2 = 5$

$A = \sqrt{5}$

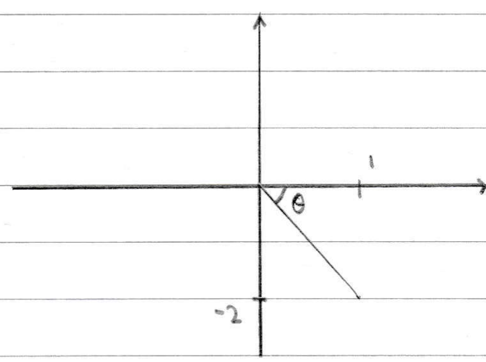
$\frac{②}{①}$:

$\frac{A\sin\theta}{A\cos\theta} = \frac{-2}{1}$

$\tan\theta = \frac{-2}{1}$

$= -63.43494882°$

$\approx -63.43°$



CW : negative angle

CCW : positive angle