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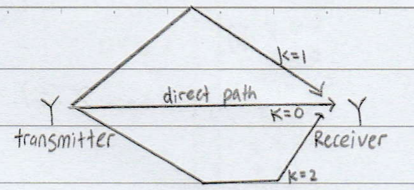
Esmund Lim Signal Tutorial 4

1) $h(t) = \sum_{k=0}^{2} (\frac{1}{2})^k \delta(t-3k)$

$= (\frac{1}{2})^0 \delta(t-3(0)) + (\frac{1}{2})^1 \delta(t-3(1)) + (\frac{1}{2})^2 \delta(t-3(2))$

$= \delta(t) + \frac{1}{2}\delta(t-3) + \frac{1}{4}\delta(t-6)$



- There are many possible path from transmitter to receiver
- Time it travel is proportional to the distance
  Shortest distance = shortest delay
  Longest distance = Longest delay
  attenuation

a) Determine whether the system is memoryless, causal, and stable

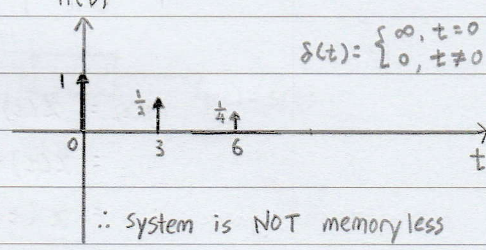
(i) memoryless

→ A system is memoryless if its output signal depends only on the present value of the input signal

→ A LTI system is memoryless if and only if its impulse response is given by:

DT system : $h[n] = c\,\delta[n]$

CT system : $h(t) = c\,\delta(t)$



$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases}$

∴ System is NOT memoryless

$h(t) \neq c\delta(t)$

* memoryless system only allow one impulse at location 0

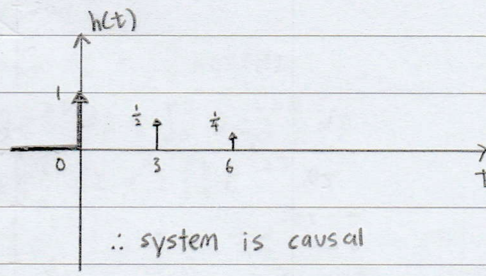
(ii) Causal

→ A system is causal if the present value of the output signal depends only on the present or past values of the input signal

→ A LTI system is causal if and only if its impulse response satifies the following condition:

DT system $h[n] = 0$, for $n < 0$

CT system $h(t) = 0$, for $t < 0$



∴ system is causal

$h(t) = 0$, for $t < 0$

→ anything to the left of $t=0$, $h(t) = 0$

$h(t) = \delta(t) + \frac{1}{2}\delta(t-3) + \frac{1}{4}\delta(t-6)$

let $t = -1$

$h(-1) = \delta(-1) + \frac{1}{2}\delta(-1-4) + \frac{1}{4}\delta(-1-6)$

$= \delta(-1) + \frac{1}{2}\delta(-5) + \frac{1}{4}\delta(-7)$ * $\delta(t) = 0,\ t \neq 0$

$= 0 + \frac{1}{2}(0) + \frac{1}{4}(0)$

$= 0$

iii) Stable

* BIBO
Bounded input
Bounded Output

→ finite input into the system = output is finite

System is BIBO stable

→ A LTI system is BIBO stable if its impulse response

DT system $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

CT system $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$

$\int_{-\infty}^{\infty} |h(t)|\,dt = 1 + \frac{1}{2} + \frac{1}{4}$

$= \frac{7}{4}$

$\frac{7}{4} < \infty$

∴ System is stable

* ∴ This system is NOT memoryless, it is causal and stable

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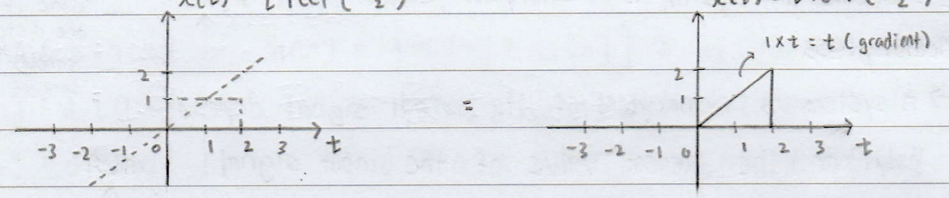
b)

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

$$t \times \mathrm{rect}\left(\frac{t-1}{2}\right) \longrightarrow \boxed{\sum_{k=0}^{2} \left(\tfrac{1}{2}\right)^k \delta(t-3k)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t) = t\,\mathrm{rect}\left(\frac{t-1}{2}\right)$ $\qquad$ $x(t) = t\,\mathrm{rect}\left(\frac{t-1}{2}\right)$

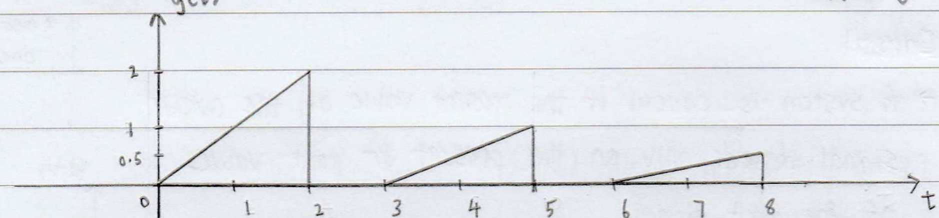


$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= x(t) * \left[\delta(t) + \tfrac{1}{2}\delta(t-3) + \tfrac{1}{4}\delta(t-6)\right] \\
&= x(t) + \tfrac{1}{2}x(t-3) + \tfrac{1}{4}x(t-6)
\end{aligned}
$$

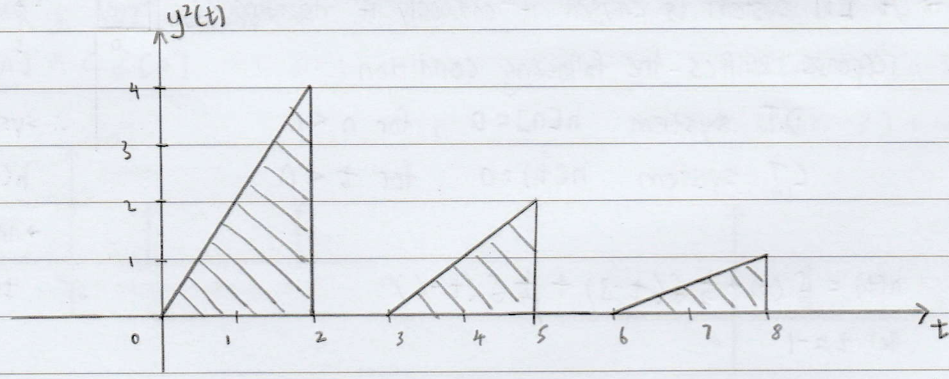
* Properties

Convolution with Delta function

- $x[n] * \delta[n-k_0] = x[n-k_0]$
- $x(t) * \delta(t-T_0) = x(t-T_0)$



c)



E_x $\qquad$ $\left(\frac{1}{2}\right)^2 E_x$ $\qquad$ $\left(\frac{1}{4}\right)^2 E_x$

$$
\begin{aligned}
E_y &= \left(1 + \tfrac{1}{4} + \tfrac{1}{16}\right) E_x \\
&= \tfrac{21}{16} E_x
\end{aligned}
$$

or

$$
\begin{aligned}
E_y &= \int_{-\infty}^{\infty} |y(t)|^2\, dt \\
&= \int_{-\infty}^{\infty} \sum_{k=0}^{2} \left(\tfrac{1}{2}\right)^{2k} |x(t-3k)|^2\, dt \\
&= \sum_{k=0}^{2} \left(\tfrac{1}{2}\right)^{2k} \int_{-\infty}^{\infty} |x(t-3k)|^2\, dt \\
&= \sum_{k=0}^{2} \left(\tfrac{1}{2}\right)^{2k} E_x \\
&= \left[\left(\tfrac{1}{2}\right)^{2(0)} + \left(\tfrac{1}{2}\right)^{2(1)} + \left(\tfrac{1}{2}\right)^{2(2)}\right] E_x \\
&= \tfrac{21}{16} E_x
\end{aligned}
$$

* Area is independent of the absolute location, it is always the same wherever you shift it

1. Area under graph = E_x
2. Area under graph $(\text{half the amplitude})^2 = \frac{1}{4} E_x$
3. Area under graph $\left(\frac{1}{4} \text{ the amplitude}\right)^2 = \frac{1}{16} E_x$

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2) $h(t) = 2e^{-2|t|}$

$$= \begin{cases} 2e^{-2t}, & t \geq 0 \\ 2e^{2t}, & t < 0 \end{cases}$$

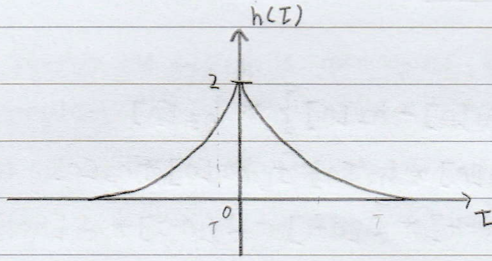
step response

$$s(t) = u(t) * h(t)$$
$$= h(t) * u(t)$$
$$= \int_{-\infty}^{\infty} h(\tau) u(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{t} h(\tau)\, d\tau$$

(There are 2 values of t due to the absolute sign)

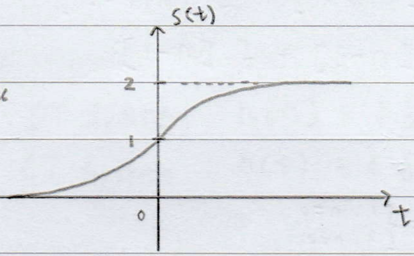
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$u(t) = \begin{cases} 1, & t-\tau \geq 0 \\ 0, & t-\tau < 0 \end{cases} = \begin{cases} 1, & t \geq \tau \\ 0, & t < \tau \end{cases}$$



$x(t) = u(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = s(t)$

* step response is the integration of impulse response to current time, area keep increasing if t is increasing



When $t < 0$

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$
$$= \int_{-\infty}^{t} 2e^{2\tau}\, d\tau$$
$$= 2\int_{-\infty}^{t} e^{2\tau}\, d\tau$$
$$= 2\left[\tfrac{1}{2}e^{2\tau}\right]_{-\infty}^{t}$$
$$= 2\left[\tfrac{1}{2}e^{2t} - \tfrac{1}{2}e^{2(-\infty)}\right]$$
$$= e^{2t}$$

when $t \geq 0$

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$
$$= \int_{-\infty}^{0} h(\tau)\, d\tau + \int_{0}^{t} h(\tau)\, d\tau$$
$$= \int_{-\infty}^{0} 2e^{2\tau}\, d\tau + \int_{0}^{t} 2e^{-2\tau}\, d\tau$$
$$= 2\int_{-\infty}^{0} e^{2\tau}\, d\tau + 2\int_{0}^{t} e^{-2\tau}\, d\tau$$
$$= 2\left[\tfrac{1}{2}e^{2\tau}\right]_{-\infty}^{0} - 2\left[\tfrac{1}{2}e^{-2\tau}\right]_{0}^{t}$$
$$= 2\left[\tfrac{1}{2}e^{2(0)} - \tfrac{1}{2}e^{2(-\infty)}\right] - 2\left[\tfrac{1}{2}e^{-2t} - \tfrac{1}{2}e^{-2(0)}\right]$$
$$= 2\left[\tfrac{1}{2} - 0\right] - 2\left[\tfrac{1}{2}e^{-2t} - \tfrac{1}{2}\right]$$
$$= 1 - e^{-2t} + 1$$
$$= 2 - e^{-2t}$$

Hence, the overall step response is given by

$$s(t) = \begin{cases} 2 - e^{-2t}, & t \geq 0 \\ e^{2t}, & t < 0 \end{cases}$$

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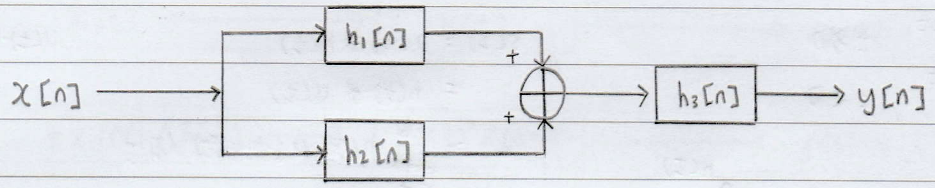
3)



$h_1[n] = \delta[n-4]$

$h_2[n] = \delta[n-2]$

$h_3[n] = \delta[n+1]$

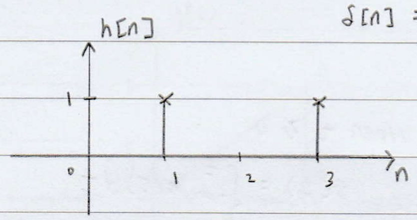
$$h[n] = \{h_1[n] + h_2[n]\} * h_3[n]$$
$$= h_1[n] * h_3[n] + h_2[n] * h_3[n]$$
$$= \delta[n-4] * \delta[n+1] + \delta[n-2] * \delta[n+1]$$
$$= \delta[n+1-4] + \delta[n+1-2]$$
$$= \delta[n-3] + \delta[n-1]$$

* or we can sum first and then convolute, This method, we only have to do convolution once

* $x[n] * \delta[n+n_0] = x[n+n_0]$ replace n with $n+n_0$

b) i) memoryless

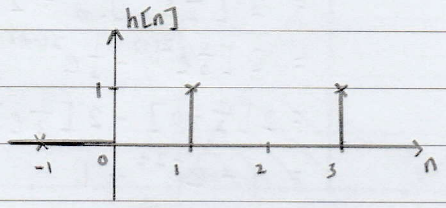
$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$



∴ System is NOT memoryless

$h[n] \neq c\,\delta[n]$

ii)



∴ system is causal

$h[n] = 0$, for $n < 0$

iii) $\sum_{n=-\infty}^{\infty} |h[n]| = 1 + 1$

$= 2$

∴ system is stable

* The system is NOT memoryless, it is causal and stable

c) Since, $h[n] = \delta[n-3] + \delta[n-1]$

$$s[n] = u[n] * h[n]$$
$$= u[n] * \{\delta[n-3] + \delta[n-1]\}$$
$$= u[n] * \delta[n-3] + u[n] * \delta[n-1]$$
$$= u[n-3] + u[n-1]$$