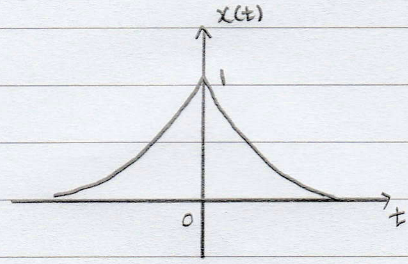
No.: .... Date: ....

Esmund Lim Signals Tutorial 9

1) $x(t) = e^{-a|t|}$, $a > 0$



$$X(\omega) = \mathcal{F}[x(t)]$$
$$= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$ Fourier Transform

$$x(t) = \mathcal{F}^{-1}[X(\omega)]$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$$ Inverse Fourier Transform

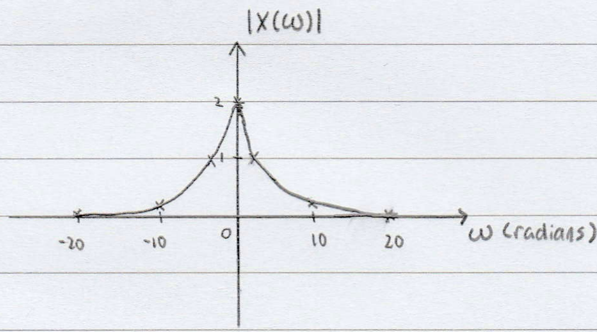
$$X(\omega) = \underbrace{|X(\omega)|}_{\text{magnitude}}\, e^{j\underbrace{\theta(\omega)}_{\text{phase}}}.$$

↳ complex-valued continuous

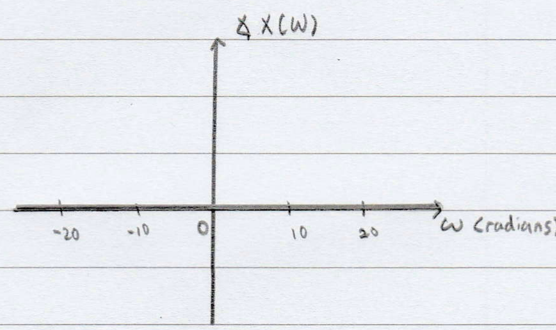
$$X(\omega) = \mathcal{F}[x(t)]$$
$$= \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t}\, dt$$
$$= \int_{-\infty}^{0} e^{at}\cdot e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\cdot e^{-j\omega t}\, dt$$
$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{(-a-j\omega)t}\, dt$$
$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$
$$= \left[\frac{1}{a-j\omega}e^{(a-j\omega)t}\right]_{-\infty}^{0} + \left[-\frac{1}{a+j\omega}e^{-(a+j\omega)t}\right]_{0}^{\infty}$$
$$= \left[\frac{1}{a-j\omega}e^{(a-j\omega)(0)} - \frac{1}{a-j\omega}e^{(a-j\omega)(-\infty)}\right] + \left[-\frac{1}{a+j\omega}e^{-(a+j\omega)(\infty)} - \left(-\frac{1}{a+j\omega}e^{-(a+j\omega)0}\right)\right]$$
$$= \left[\frac{1}{a-j\omega} - 0\right] + \left[-0 + \frac{1}{a+j\omega}\right]$$
$$= \left[\frac{1}{a-j\omega}\right] + \left[\frac{1}{a+j\omega}\right]$$
$$= \frac{a+j\omega}{(a-j\omega)(a+j\omega)} + \frac{a-j\omega}{(a-j\omega)(a+j\omega)}$$
$$= \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2+aj\omega-aj\omega-j^2\omega^2}$$
$$= \frac{2a}{a^2+\omega^2}$$

$$|X(\omega)| = \frac{2a}{a^2+\omega^2}$$
$$\measuredangle X(\omega) = 0^\circ$$

magnitude with a=1



phase with a = 1



$$|X(10)| = \frac{2(1)}{1^2+10^2}$$
$$= 0.19$$

$$|X(1)| = \frac{2(1)}{1^2+1^2}$$
$$= 1$$