EE2010/IM2004 Signals and Systems

Tutorial #6

Note: In this tutorial, the Quiz #1 will be conducted in your tutorial session! Your HW#1 will be due, too.

(1). Determine the fundamental period of each sinusoidal signal.

$(a).\quad x(t) = \cos\left(t + \frac{\pi}{4}\right)$

$(c).\quad x(t) = 2\sin^2 t$

$(b).\quad x(t) = \pi \sin\frac{2\pi t}{3}$

$(d).\quad x(t) = 5e^{j\left[\left(\frac{\pi t}{2}\right) - 1\right]}$

(2). Find the amplitude, the radian frequency, and the phase of the sum of the following two sinusoidal signals: $x_1(t) = \cos 5t$ and $x_2(t) = 2\sin 5t$.

Partial Answers:

1. $(a).\ T_0 = 2\pi;\quad (d).\ T_0 = 4.$
2. $A = \sqrt{5},\ \omega_0 = 5$ rad/s, and $\theta = \tan^{-1}(-2/1) = -63.4^\circ$

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Tutorial #7

Get your <u>HW#2</u> assignment in this tutorial session

1. Let $x(t) = \frac{\pi}{3} + \sin\omega_0 t + 2\cos\omega_0 t + \frac{\sqrt{5}}{2}\cos\left(2\omega_0 t + \frac{\pi}{4}\right)$

(a). What is the dc component of $x(t)$?

(b). What is the ac component of $x(t)$?

(c). What is the fundamental angular frequency of the 2nd harmonics and its contribution in $x(t)$?

(d). What is the fundamental angular frequency of the 3rd harmonics and its contribution in $x(t)$?

(e). What is the fundamental angular frequency of $x(t)$?

(f). Compute the Fourier series coefficients of the signal $x(t)$, using the complex exponential form.

(g). Graph the two-sided plot of the amplitude and phase spectra of $x(t)$.

2. Determine whether each of the following signals is periodic. If it is periodic, find its fundamental period.

(a). $x(t) = \cos t + \sin\sqrt{2}\, t$ (c). $x(t) = \sin^2\left(\frac{5\pi}{4}t\right) + \cos\left(3\pi t + \frac{\pi}{3}\right)$

(b). $x(t) = \cos\frac{\pi t}{3} + \sin\frac{\pi t}{4}$

3. Find the Fourier series representation for each signal $x(t)$ of the previous two questions given in this tutorial using (i) **trigonometric** form and (ii) **amplitude-phase** form. Plot the **one**-sided plot of the amplitude and phase spectra. For Q.1, let $\omega_0 = 5$.

Partial Answers:

Q.1-(f) $c_0 = \frac{\pi}{3} = \frac{\pi}{3}e^{j(0^0)} = \frac{\pi}{3}\angle 0^0 = 1.047\angle 0^0$; $c_1 = 1 + \frac{1}{2j} = 1.12\,e^{j(-26.57^0)} = 1.12\angle -26.57^0$;

$c_2 = \frac{\sqrt{5}}{4}e^{j\left(\frac{\pi}{4}\right)} = 0.559\angle 45^0$; ...

Q.2-(c) 4 seconds;

Q.3-[based on the $x(t)$ of Q.1]: $x(t) = \frac{\pi}{3} + \sin 5t + 2\cos 5t + \frac{\sqrt{10}}{4}\cos 10t - \frac{\sqrt{10}}{4}\sin 10t$

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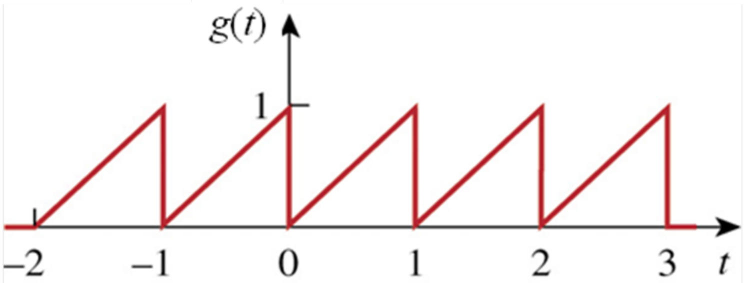
Tutorial #8

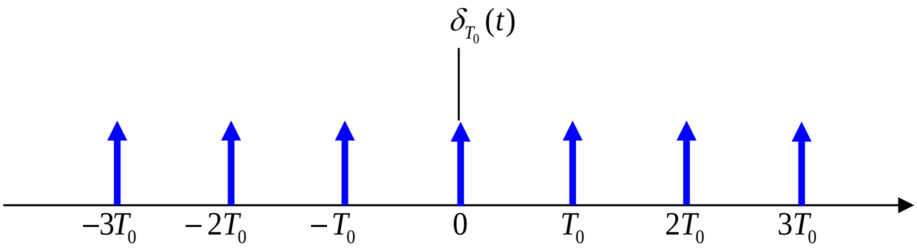
*** * * e-learning Week * * ***

[Full solution will be uploaded later for your e-learning]

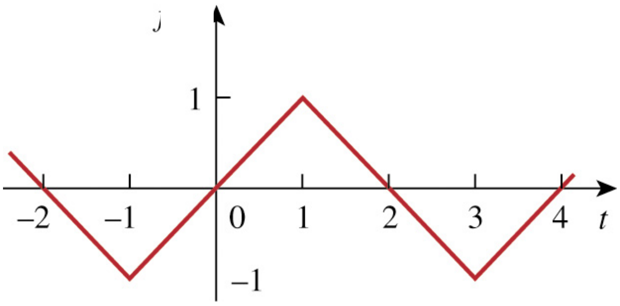
1. Determine the Fourier series of a sawtooth waveform using the **trigonometric form**, and plot its line spectra. Further plot the truncated (approximated) Fourier series representation of $g(t)$ using only the dc term and the first 4 ac harmonics terms.



2. Find the complex-exponential form of the Fourier series representation of the impulse train with the fundamental period T_0.



3. (a). Find the trigonometric-form Fourier series of the periodic triangle waveform $x(t)$.



(b). Let $y(t) = dx(t)/dt$. Plot $y(t)$ and find its **trigonometric**-form Fourier series.
(c). Further find its **complex-exponential** form Fourier series based on the results obtained in (b).
(d). Show that the Fourier series result of $y(t)$ can be also obtained by properly shifting and scaling of the example $f(t)$ as studied in the duty cycle topic in the notes.

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Tutorial #9

Note: In this tutorial, the Quiz #2 will be conducted in your tutorial session! HW#2 will be due, too.

1. Find the Fourier Transform of a double-sided exponential signal and plot its amplitude response and phase response, for $a = 1$.

$$x(t) = e^{-a|t|}\ ,\ \ a > 0$$

Partial Answers: $X(\omega) = \dfrac{2a}{a^2 + \omega^2}$

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Tutorial #10

1. Find the Fourier transform of the following signals.

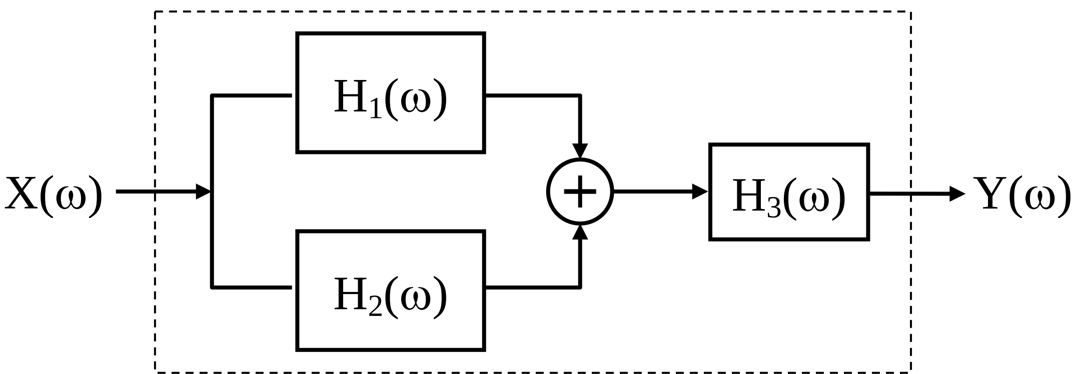
   (a). $x(t) = \begin{cases} \exp(-at), & t > 0; \\ 0, & t = 0; \\ -\exp(at), & t < 0. \end{cases}$

   (b). the signum function, $\operatorname{sgn}(t)$;

2. Find the Fourier transform of the following periodic signals:

   (a). $\cos\omega_0 t$ (b). $\sin\omega_0 t$

3. Find the equivalent frequency response in terms of $H_1(\omega)$, $H_2(\omega)$, and $H_3(\omega)$.



4. Find the impulse response of the following LTI system:

$$\frac{d^2 y(t)}{dt^2} - 9y(t) = \frac{d^2 x(t)}{dt^2} - 21x(t)$$

Partial Answers:

1. (a). $\frac{-j2\omega}{a^2+\omega^2}$; (b). $\frac{2}{j\omega}$
2. $\mathscr{F}\{\cos\omega_0 t\} \quad \leftrightarrow \quad \pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
3. Based on the previous module's material and the FT properties to derive.
4. (Hint): Use existing FT pair, rather than direct evaluation via integration.

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Tutorial #11

1. A signal $x(t) = 5\cos(500t)$ is sampled with the sampling frequency $\omega_s = 1200$. Sketch the resulted Fourier transform magnitude spectrum.

2. Consider the signal $x(t) = 10\cos(426\pi t - 60^\circ) - 2.5\sin(1200\pi t - 20^\circ)$.
   (a). What is the Nyquist rate of $x(t)$?
   (b). If $x(t)$ is sampled with the sampling frequency f_s =1000 Hz, find the resulted discrete-time signal $x[n]$.
   (c). If the same f_s =1000 Hz is used to covert $x[n]$ back to a continuous-time signal $\tilde{x}(t)$, is $\tilde{x}(t) = x(t)$?

3. Consider the analog periodic signal $x(t) = A\cos\left(2\pi f_0 t + \theta\right)$, which is under uniform sampling with the sampling frequency f_s to produce $x[n]$. Find the condition on the value of the normalized (digital) frequency f_d so that $x[n]$ is periodic.

Partial Answers:

1. (a). $\mathscr{F}\left\{x\left(nT_s\right)\right\} = 3000\sum_{n=-\infty}^{\infty}\left[\delta(\omega + 500 - 1200n) + \delta(\omega - 500 - 1200n)\right]$

2. (b). $x[n] = 10\cos\left(\frac{213}{500}n\pi - 60^\circ\right) + 2.5\sin\left(\frac{4}{5}n\pi + 20^\circ\right)$; (c). $\tilde{x}(t) \neq x(t)$ (Why?)

3. $f_d = \frac{f_a}{f_s}$ must be a rational number!