No.: Date:

Esmund Lim Signals Tutorial 11

1) $x(t) = 5\cos(500t)$

$x(t) \xrightarrow{t = nT_s} x[n]$, $T_s = \frac{1}{F_s}$

Original signal — Sampled signal — reconstruction

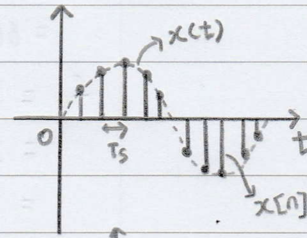
$\omega_0 = 500$ rad/s

$f_0 = \frac{500}{2\pi} = \frac{250}{\pi}$ Hz

$\omega_N = 2 \times \omega_m$ Nyquist rate = 2 x maximum frequency

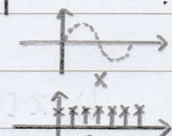
$= 2 \times 500$ rad/s

$= 1000$ rad/s $\omega_s = 1200$ rad/s $T_s = \frac{2\pi}{1200} = \frac{\pi}{600}$ Hz



$y(t) = x(t) \cdot \delta_{T_s}(t)$

$= x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$



$\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$

$y(t) = \sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT_s)$

$\mathcal{F}\{y(t)\} = \sum_{n=-\infty}^{\infty} \mathcal{F}\{x(t) \cdot \delta(t - nT_s)\}$

$Y(\omega) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}[\mathcal{F}\{x(t)\} * \mathcal{F}\{\delta(t-nT_s)\}]$

$x_1(t)\, x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$

Multiplication in time $\Leftrightarrow$ convolution in freq

$\mathcal{F}\{x(t)\} = X(\omega)$

$\mathcal{F}\{\delta(t - nT_s)\} = \omega_s \delta(\omega - n\omega_s)$

$= \frac{2\pi}{T_s} \delta(\omega - n\omega_s)$

$Y(\omega) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T_s}\delta(\omega - n\omega_s)\right]$

$= \sum_{n=-\infty}^{\infty} \frac{1}{T_s}[X(\omega) * \delta(\omega - n\omega_s)]$

$= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

$= \frac{600}{\pi} \sum_{n=-\infty}^{\infty} X(\omega - 1200n)$

$\omega_0 = 500$ rad/s

$\omega_N = 1000$ rad/s

$\omega_s = 1200$ rad/s greater by 20%

1. Oversampling $f_s > 2f_m$ (no overlap)
   more samples are generated
2. Critical or Nyquist sampling $f_s = 2f_m$
   minimum sampling rate
3. Undersampling $f_s < 2f_m$
   less samples generated, aliasing occurred

$x(t) = 5\cos(500t)$

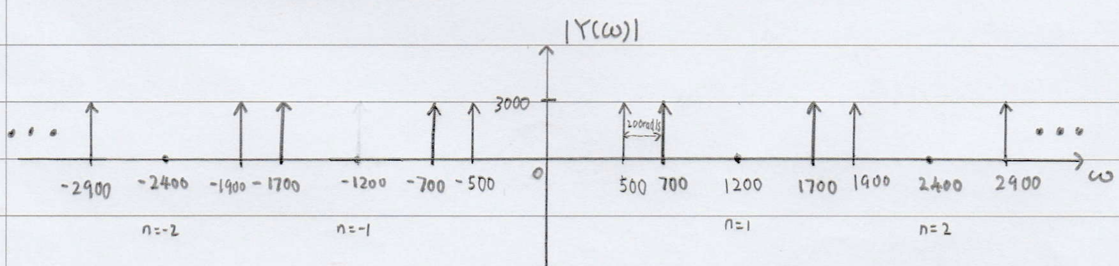
$X(\omega) = \mathcal{F}\{5\cos(500t)\}$

$= 5\,\mathcal{F}\{\cos(500t)\}$

$= 5\pi[\delta(\omega - 500) + \delta(\omega + 500)]$

$\therefore Y(\omega) = \frac{600}{\pi} \times 5\pi \sum_{n=-\infty}^{\infty} [\delta(\omega - 500 - 1200n) + \delta(\omega + 500 - 1200n)]$

$= 3000 \sum_{n=-\infty}^{\infty} [\delta(\omega - 500 - 1200n) + \delta(\omega + 500 - 1200n)]$



No.: .... Date: ....

2) $x(t) = 10\cos(426\pi t - 60^\circ) - 2.5\sin(1200\pi t - 20^\circ)$

$f_1 = 213\text{ Hz}$

$f_2 = 600\text{Hz}$

$f_m = \max\{213\text{Hz}, 600\text{Hz}\}$

$= 600\text{ Hz}$

$f_N = 2 \times f_m$

$= 2 \times 600\text{Hz}$

$= 1200\text{Hz}$

b) $x[n] = x(t)|_{t=nT_s}$

$T_s = \frac{1}{f_s} = \frac{1}{1000}$

$f_s = 2f_m$

$f_m = 0.5 f_s$

$= 10\cos(2\pi\, 213\, nT_s - 60^\circ) - 2.5\sin(2\pi\, 600\, nT_s - 20^\circ)$

$= 10\cos(\cancel{2}\pi n \frac{213}{1000} - 60^\circ) - 2.5\sin(2\pi n \frac{600}{1000} - 20^\circ)$

$= 10\cos(\frac{213}{500} n\pi - 60^\circ) - 2.5\sin(2\pi n(1 - \frac{2}{5}) - 20^\circ)$

$= 10\cos(\frac{213}{500} n\pi - 60^\circ) - 2.5\sin(2n\pi(-\frac{2}{5}) - 20^\circ)$

$= 10\cos(\frac{213}{500} n\pi - 60^\circ) + 2.5\sin(\frac{4}{5} n\pi + 20^\circ)$

c) First term : $10\cos(2\pi \times \frac{213}{1000} \times n - 60^\circ)$

$f_{d1} = \frac{213}{1000} = \frac{f_{a1}}{f_s}$

normalized digitial frequency

$f_{a1} = 213\text{Hz}$

$f_d = \frac{f_a}{f_s}$

Second term : $2.5\cos(2\pi \times \frac{2}{5} \times n + 20^\circ)$

$f_{a2} : \frac{2}{5} = \frac{f_{a2}}{f_s}$

$= \frac{400}{1000}$

$f_{a2} = 400\text{Hz}$

Thus $\tilde{x}(t) = 10\cos(426\pi t - 60^\circ) + 2.5\sin(800\pi t + 20^\circ)$

$\neq x(t)$

due to the aliasing incurred in the second term

No.: .... Date: ....

3) $x(t) = A\cos(2\pi f_0 t + \theta)$

$x[n] = x(t)\big|_{t=nT_s}$
$t = n(\frac{1}{f_s})$

$= A\cos(2\pi f_0 (\frac{n}{f_s}) + \theta)$
$= A\cos(2\pi n (\frac{f_0}{f_s}) + \theta)$
$= A\cos(2\pi f_d n + \theta)$ — ①

$x[n+N_0] = A\cos[2\pi f_d (n+N_0) + \theta]$
$= A\cos[2\pi f_d n + 2\pi f_d N_0 + \theta]$ — ②

if $x[n]$ is periodic $x[n] = x[n+N_0]$

Equate ① = ②

extra term

$\cancel{A}\cos[2\pi f_d n + \theta] = \cancel{A}\cos[2\pi f_d n + 2\pi f_d N_0 + \theta]$

$\cancel{2\pi} f_d N_0 = \cancel{2\pi} \times m$ where m is an integer

$f_d = \frac{m}{N_0}$ = rational number

$\therefore x[n]$ is periodic

$2\pi f_d N_0$ should be a multiple of 2π
↳ go back to original signal

$\cos\theta$
$= \cos(\theta + 2\pi n)$

POP URBAN