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Esmund Lim Signals tutorial 7

1) $x(t) = \frac{\pi}{3} + \sin \omega_0 t + 2\cos \omega_0 t + \frac{\sqrt{5}}{2}\cos\left(2\omega_0 t + \frac{\pi}{4}\right)$

$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$ C.E Form

convert to complex exponential form

Euler identities

$\cos\phi = \frac{e^{j\phi}+e^{-j\phi}}{2}$

$\sin\phi = \frac{e^{j\phi}-e^{-j\phi}}{j2}$

$x(t) = \frac{\pi}{3} + \frac{1}{j2}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \cancel{2}\left[\frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)\right] + \frac{\sqrt{5}}{2}\left[\frac{1}{2}\left(e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right)\right]$

$= \frac{\pi}{3} + \frac{1}{j2}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \frac{\sqrt{5}}{4}\left(e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right)$

$= \frac{\pi}{3} + \frac{1}{j2}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) + \frac{\sqrt{5}}{4}\left(e^{j(2\omega_0 t)} \cdot e^{j(\pi/4)} + e^{-j(2\omega_0 t)} \cdot e^{-j(\pi/4)}\right)$

$= \underbrace{\frac{\pi}{3}}_{C_0} + \underbrace{\left(1+\frac{1}{j2}\right)}_{C_1} e^{j\omega_0 t} + \underbrace{\left(1-\frac{1}{j2}\right)}_{C_{-1}} e^{-j\omega_0 t} + \underbrace{\left(\frac{\sqrt{5}}{4} e^{j(\pi/4)}\right)}_{C_2} e^{j(2\omega_0 t)} + \underbrace{\left(\frac{\sqrt{5}}{4} e^{-j(\pi/4)}\right)}_{C_{-2}} e^{-j(2\omega_0 t)}$

a) $C_0 = \frac{\pi}{3} + j0 \quad \left(\frac{\pi}{3}, 0\right)$

$|C_0| = \sqrt{\left(\frac{\pi}{3}\right)^2 + 0^2} = \frac{\pi}{3}$ $\quad \measuredangle C_0 = \tan^{-1}\left(\frac{0}{\pi/3}\right) = 0°$

$\boxed{\text{if } C_0 = -\frac{\pi}{3} + j0 \; \left(-\frac{\pi}{3}, 0\right) \quad \measuredangle C_0 = 180°}$

DC component $= \frac{\pi}{3}$

Complex Number

[Rect]: $z = x + jy = r\cos\theta + jr\sin\theta$

[Polar]: $z = r\angle\theta$

[Exp]: $z = r e^{j\theta}$ (r: magnitude, θ: phase) $= r[\cos\theta + j\sin\theta]$

$|z| = r = \sqrt{x^2+y^2}$ $\quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta \quad y = r\sin\theta$

$j = \sqrt{-1} \quad j^2 = j \times j = -1 \quad j^3 = j \times j^2 = -j$

$j^4 = j^2 \times j^2 = 1$

$C_1 = 1 + \frac{1}{j2}$

$= 1 + \frac{1 \times j}{j2 \times j}$ (To get rid of the bottom j)

$= 1 + \frac{j}{2(-1)}$

$= 1 + \left(-\frac{1}{2}\right)j$

$|C_1| = \sqrt{(1)^2 + \left(-\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{2}$ $\quad \measuredangle C_1 = \tan^{-1}\left(\frac{-0.5}{1}\right) = -26.56505118°$

$C_{-1} = 1 - \frac{1}{j2}$

$= 1 - \frac{1 \times j}{j2 \times j}$

$= 1 - \frac{j}{2(-1)}$

$= 1 + \left(\frac{1}{2}\right)j$

$|C_{-1}| = \sqrt{(1)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{2}$ $\quad \measuredangle C_1 = \tan^{-1}\left(\frac{0.5}{1}\right) = 26.56505118°$

$Ae^{j\theta}$

$C_2 = \frac{\sqrt{5}}{4} e^{j(\pi/4)}$ $\quad |C_2| = \frac{\sqrt{5}}{4}$ $\quad \measuredangle C_2 = \frac{\pi}{4} = 45°$

$C_{-2} = \frac{\sqrt{5}}{4} e^{-j(\pi/4)}$ $\quad |C_{-2}| = \frac{\sqrt{5}}{4}$ $\quad \measuredangle C_{-2} = -\frac{\pi}{4} = -45°$

b) ac component of $x(t)$

→ all the remaining terms that involves ω_0

c) fundamental angular frequency of 2nd harmonics $= 2\omega_0$

With a magnitude contribution of $\frac{\sqrt{5}}{4}$ at $2\omega_0$ and $\frac{\sqrt{5}}{4}$ at $-2\omega_0$ (2 sided spectrum complex Ex FS)

d) fundamental angular frequency of 3rd harmonics $= 3\omega_0$

With zero contribution

e) fundamental angular frequency $= \omega_0$

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f) Fourier series coefficients of the signal $x(t)$

$C_0 = \frac{\pi}{3} + j0$
$= \frac{\pi}{3} e^{j(0^\circ)}$
$= \frac{\pi}{3} \angle 0^\circ$
$= 1.047 \angle 0^\circ$

$C_1 = 1 + \frac{1}{2j}$
$= \frac{\sqrt{5}}{2} e^{j(-26.57^\circ)}$
$= 1.118033989 \angle -26.57^\circ$
$\approx 1.12 \angle -26.57^\circ$

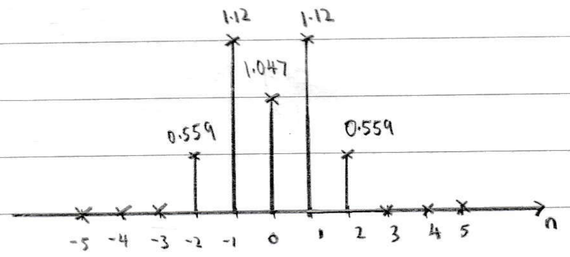
$C_{-1} = 1 - \frac{1}{2j}$
$= \frac{\sqrt{5}}{2} e^{j(26.57^\circ)}$
$= 1.12 \angle 26.57^\circ$

$C_2 = \frac{\sqrt{5}}{4} e^{j(\pi/4)}$
$= \frac{\sqrt{5}}{4} \angle 45^\circ$
$= 0.5590169944 \angle 45^\circ$
$\approx 0.559 \angle 45^\circ$

$C_{-2} = \frac{\sqrt{5}}{4} e^{-j(\pi/4)}$
$= \frac{\sqrt{5}}{4} \angle -45^\circ$
$= 0.559 \angle -45^\circ$

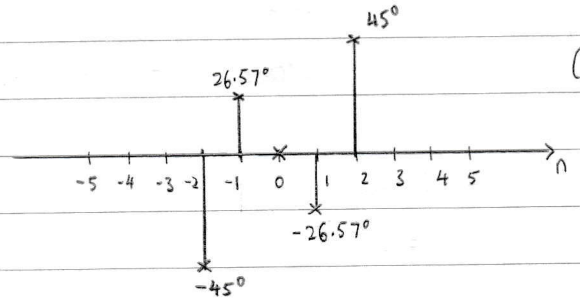
$C_n = 0 \quad , |n| > 2$

g) Amplitude spectrum $|C_n|$ of $x(t)$



(Note: Even Function)

Phase spectrum $\measuredangle C_n$ of $x(t)$



(Note: odd Function)

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2a) $x(t) = \cos t + \sin \sqrt{2}\, t$

$\omega_1 = 1 \qquad \dfrac{\omega_1}{\omega_2} = \dfrac{1}{\sqrt{2}}$

$\omega_2 = \sqrt{2}$

$\therefore$ $x(t)$ is not periodic

↳ No F.S representation

b) $x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t$

$\omega_1 = \frac{\pi}{3} \qquad \dfrac{\omega_1}{\omega_2} = \dfrac{\frac{\pi}{3}}{\frac{\pi}{4}} = \dfrac{4}{3}$

$\omega_2 = \frac{\pi}{4}$

$\therefore$ $x(t)$ is periodic

$\omega_0 = 2\pi f_0 = 2\pi(\frac{1}{T_0})$

$T_0 = \frac{2\pi}{\omega_0}$

$T_1 = \frac{2\pi}{\pi/3} = 6 \qquad T_2 = \frac{2\pi}{\pi/4} = 8 \qquad$ or $\omega_0 = GCF(\frac{\pi}{3}, \frac{\pi}{4}) = ?$

$T_0 = LCM(T_1, T_2) = LCM(6, 8) = 24$

$2 \underline{|6, 8}$
$\quad 3, 4$

$LCM = 2\times3\times4 = 24$

c) $x(t) = \sin^2(\frac{5\pi}{4} t) + \cos(3\pi t + \frac{\pi}{3})$

$= \frac{1}{2}(1 - \cos[2\times\frac{5\pi}{4}t]) + \cos(3\pi t + \frac{\pi}{3})$

$= \frac{1}{2} - \frac{1}{2}\cos(\frac{5\pi}{2}t) + \cos(3\pi t + \frac{\pi}{3})$

$\omega_1 = \frac{5\pi}{2} \qquad \dfrac{\omega_1}{\omega_2} = \dfrac{\frac{5\pi}{2}}{3\pi} = \dfrac{5}{6}$

$\omega_2 = 3\pi$

$\therefore$ $x(t)$ is periodic

$T_1 = \frac{2\pi}{5\pi/2} = \frac{4}{5} \qquad T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$

$T_0 = LCM(\frac{4}{5}, \frac{2}{3}) = 4$

$\frac{4}{5}\times 15 \quad \frac{2}{3}\times 15$
$= 12 \quad 10$

$LCM(12, 10)$

$2 \underline{|12, 10}$
$\quad 6, 5$

$2\times6\times5 = 60$

$\therefore LCM = \frac{60}{15} = 4$

or

$\omega_0 = GCF(\frac{5\pi}{2}, 3\pi) = ?$

Summation of two sinusoids

→ $x_1(t)$, $x_2(t)$ periodic C-T signal with T_1, T_2 fundamental period

→ $x(t) = x_1(t) + x_2(t)$

$\underbrace{\text{periodic + periodic}}_{\text{not always periodic}}$

check

$\frac{T_1}{T_2} = \frac{k}{m} = \frac{\text{Integer}}{\text{Integer}}$ = rational number

or

$\frac{\omega_2}{\omega_1} = \frac{f_2}{f_1} = \frac{\text{Integer}}{\text{Integer}}$

if it is periodic

$T = LCM(T_1, T_2)$

$f = GCF(f_1, f_2)$

① get rid of denominator by multiplying product of 2 denominator

② get LCM

③ divide it back

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3) For Qns 2(a)

→ No Fourier series representation

For Qns 2(b)

$w_0 = \frac{2\pi}{T_0}$

$= \frac{2\pi}{24}$

$= \frac{\pi}{12}$

* Find Wo first, from Wo then we know what is the corresponding n

$x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t$

$= \cos 4 w_0 t + \sin 3 w_0 t$

$a_4 = 1$, $b_3 = 1$ (Trigonometric form)

$n \cdot w_0 = \frac{\pi}{3}$

$n = \frac{\pi}{3} \div w_0$

$= \frac{\pi}{3} \div \frac{\pi}{12}$

$= 4$

$n \cdot w_0 = \frac{\pi}{4}$

$n = \frac{\pi}{4} \div \frac{\pi}{12}$

$= 3$

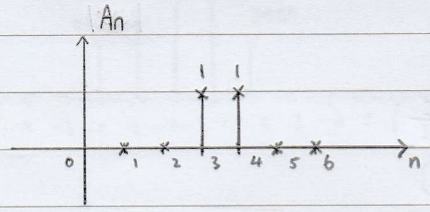
$x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t$

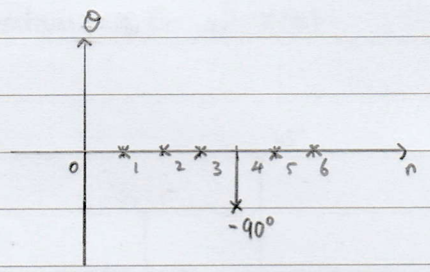
$= \cos \frac{\pi}{3} t + \cos(\frac{\pi}{4} t - 90°)$

$= \cos(4 w_0 t + 0°) + \cos(3 w_0 t - 90°)$

$A_4 = 1$ $\theta_4 = 0$

$A_3 = 1$ $\theta_3 = -90°$





1 sided plot : n is positive

2 sided plot : n is both positive and negative (complex)

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For Qns 2(c)

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{5\pi}{2}t\right) + \cos\left(3\pi t + \frac{\pi}{3}\right)$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$n\omega_0 = \frac{5\pi}{2}$, $n = \frac{5\pi}{2} \div \frac{\pi}{2} = 5$

$n\omega_0 = 3\pi$, $n = 3\pi \div \frac{\pi}{2} = 6$

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos(5\omega_0 t) + \cos\left(6\omega_0 t + \frac{\pi}{3}\right)$$

$+\frac{1}{2}\cos(5\omega_0 t - \pi)$
$+\frac{1}{2}\cos(5\omega_0 t + \pi)$

$$= \frac{1}{2} + \frac{1}{2}\cos(5\omega_0 t + \pi) + \cos\left(6\omega_0 t + \frac{\pi}{3}\right)$$

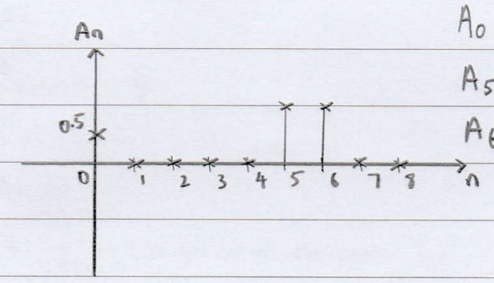
(Amplitude phase form)

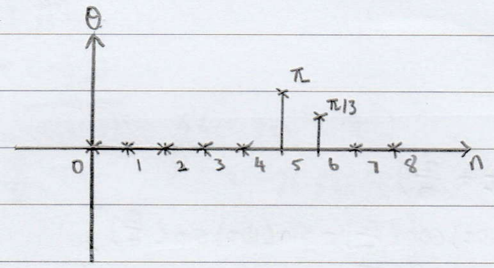
amplitude/Magnitude always positive
take care of the '−' sign by adding/subtracting 180° ($\cos 180° = -1$)

$A_0 = \frac{1}{2}$ $\theta_0 = 0°$

$A_5 = \frac{1}{2}$ $\theta_5 = \pi$ or $180°$

$A_6 = 1$ $\theta_6 = \frac{\pi}{3}$ or $60°$





$$x(t) = \frac{1}{2} - \frac{1}{2}\cos(5\omega_0 t) + \cos\left(6\omega_0 t + \frac{\pi}{3}\right)$$

$$= \frac{1}{2} - \frac{1}{2}\cos(5\omega_0 t) + \cos(6\omega_0 t)\cos\left(\frac{\pi}{3}\right) - \sin(6\omega_0 t)\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2} - \frac{1}{2}\cos(5\omega_0 t) + \frac{1}{2}\cos(6\omega_0 t) - \frac{\sqrt{3}}{2}\sin(6\omega_0 t)$$ [Trigonometric form]

$a_0 = \frac{1}{2}$, $a_5 = -\frac{1}{2}$, $a_6 = \frac{1}{2}$, $b_6 = -\frac{\sqrt{3}}{2}$

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For Qns 1

Let $\omega_0 = 5$

$$x(t) = \frac{\pi}{3} + \sin \omega_0 t + 2\cos \omega_0 t + \frac{\sqrt{5}}{2}\cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$

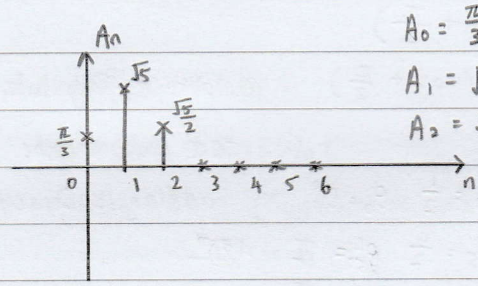
$$= \frac{\pi}{3} + \sin 5t + 2\cos 5t + \frac{\sqrt{5}}{2}\cos\left(10t + \frac{\pi}{4}\right)$$

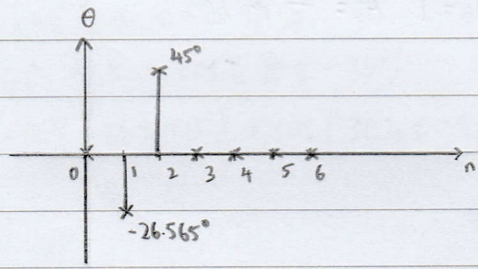
$$= \frac{\pi}{3} + \sqrt{5}\cos(5t - 26.565^\circ) + \frac{\sqrt{5}}{2}\cos\left(10t + \frac{\pi}{4}\right) \quad \text{(Amplitude-phase)}$$

$A_0 = \frac{\pi}{3}$ $\quad \theta_0 = 0^\circ$

$A_1 = \sqrt{5}$ $\quad \theta_1 = -26.565^\circ$

$A_2 = \frac{\sqrt{5}}{2}$ $\quad \theta_2 = 45^\circ$





$2\cos 5t + \sin 5t$

$= A\cos(\omega t + \theta)$ (A.P form that's why change to cos)

$= A\cos\omega t\cos\theta - A\sin\theta\sin\omega t$

$A\cos\theta = 2$ — ①

$-A\sin\theta = 1$

$A\sin\theta = -1$ — ②

①^2 + ②^2

$A^2\cos^2\theta + A^2\sin^2\theta = 2^2 + (-1)^2$

$A^2 = 5$

$A = \sqrt{5}$

$\frac{②}{①}: \frac{A\sin\theta}{A\cos\theta} = \frac{-1}{2}$

$\tan\theta = -\frac{1}{2}$

$\theta = -26.565^\circ$

$$x(t) = \frac{\pi}{3} + \sin 5t + 2\cos 5t + \frac{\sqrt{5}}{2}\cos\left(10t + \frac{\pi}{4}\right)$$

$$= \frac{\pi}{3} + \sin 5t + 2\cos 5t + \frac{\sqrt{5}}{2}\left[\cos(10t)\cos\left(\frac{\pi}{4}\right) - \sin(10t)\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \frac{\pi}{3} + \sin 5t + 2\cos 5t + \frac{\sqrt{10}}{4}\cos(10t) - \frac{\sqrt{10}}{4}\sin(10t) \quad \text{(Trigonometric form)}$$

$a_0 = \frac{\pi}{3}$ $\quad a_1 = 2$ $\quad a_2 = \frac{\sqrt{10}}{4}$ $\qquad a_n = 0$ for $n \geq 3$

$b_1 = 1$, $b_2 = -\frac{\sqrt{10}}{4}$ $\qquad b_n = 0$ for $n \geq 3$