No.: Date:

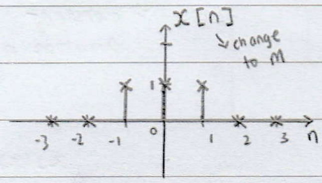
Esmund Lim

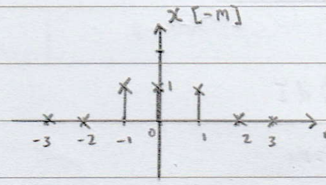
Signal Tutorial 3

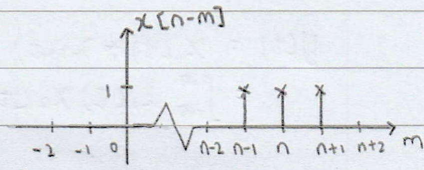
1) $y[n] = x[n] * x[n]$

$= \sum_{m=-\infty}^{\infty} x[m]\,x[n-m]$

$x[n] = \begin{cases} 1, & \text{for } n=-1,0,1 \\ 0, & \text{otherwise} \end{cases}$







$x[m] \rightarrow x[-m]$

time scaling, mirror image

$x[-m] \longrightarrow x[n-m]$

time shifting

$x[-(m-n)]$

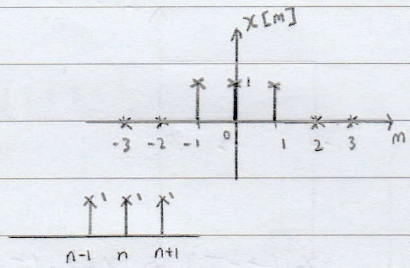
$x[n-m]$

shift by n

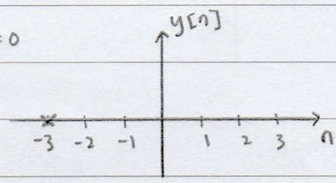
(i) n = -3 (first one with no overlap)

n+1 = -2

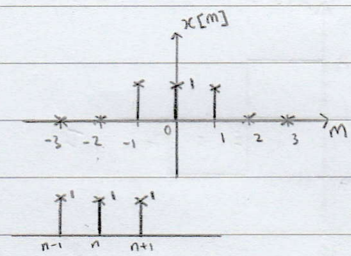
n = -3



$\therefore y[-3] = 0$

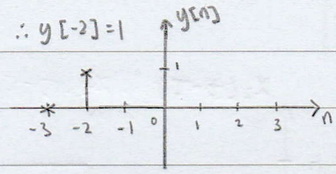


ii) n = -2 (shift right by 1)

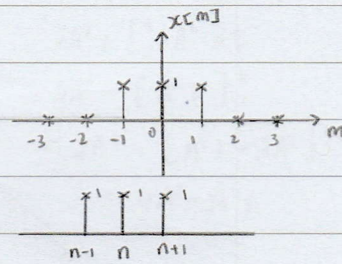


$|\times 1 = 1$

$\therefore y[-2] = 1$

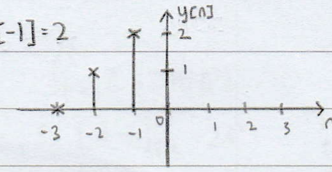


iii) n = -1 (shift right by 1)

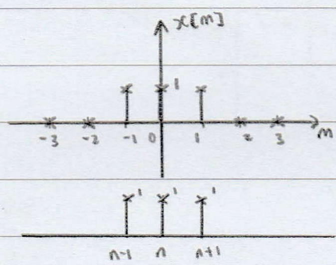


$1\times1=1,\ 1\times1=1 \quad 1+1=2$

$\therefore y[-1] = 2$

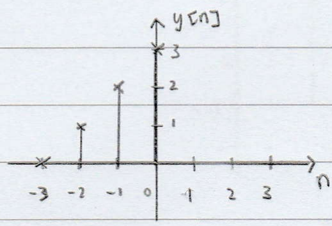


iv) n = 0

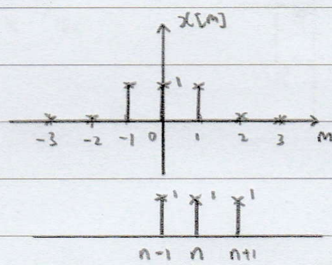


$1\times1=1\ 1\times1=1\ 1\times1=1 \quad 1+1+1=3$

$\therefore y[0] = 3$

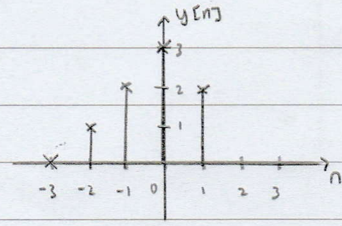


v) n = 1

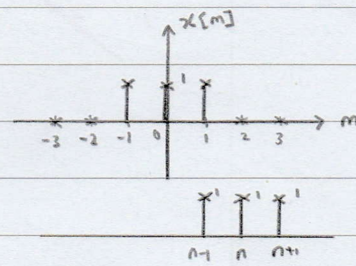


$1\times1=1\ 1\times1=1 \quad 1+1=2$

$\therefore y[1] = 2$

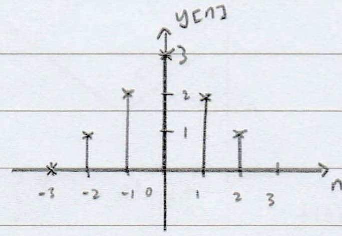


vi) n = 2



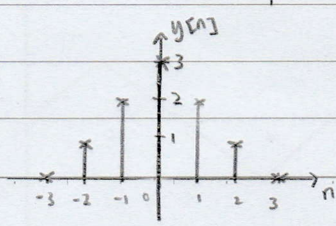
$1\times1=1$

$\therefore y[2] = 1$



vii) n = 3

y[3] = 0



* multiply first and sum it

2) $x_1(t) = A\,rect\left(\frac{t-1}{2}\right) - A\,rect\left(\frac{t-3}{2}\right)$ $\qquad x_2(t) = A\,rect\left(\frac{t-2}{4}\right)$

$x_1(t) \longrightarrow \boxed{x_2(t)} \longrightarrow y(t)$

$-\frac{T}{2} \le t \le \frac{T}{2}$

variable

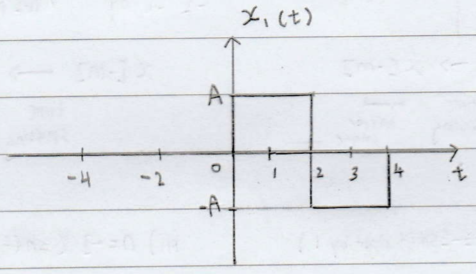
$rect\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \le T/2 \\ 0, & \text{otherwise} \end{cases}$

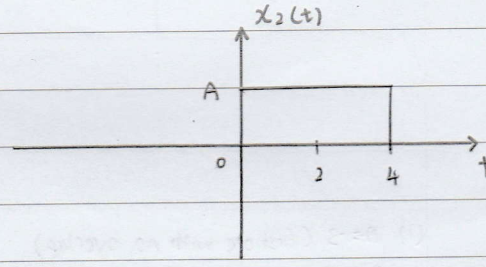
constant

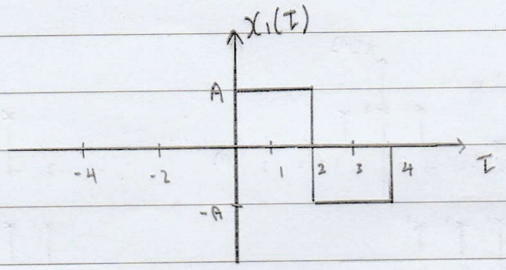
Duration of rect pulse

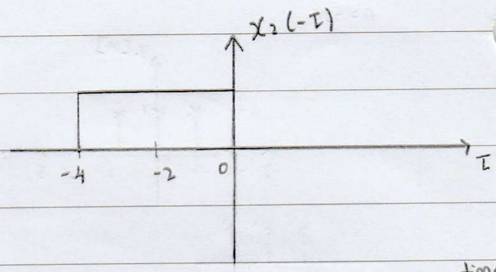
$y(t) = x_1(t) * x_2(t)$

$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$

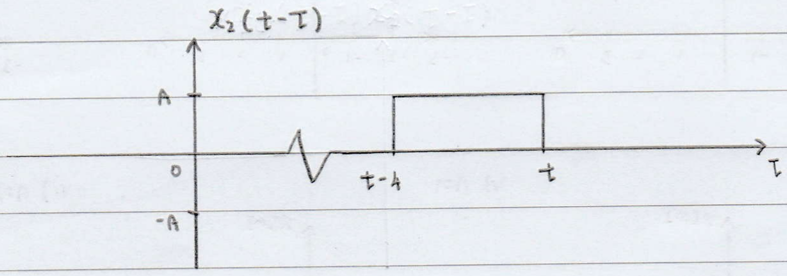






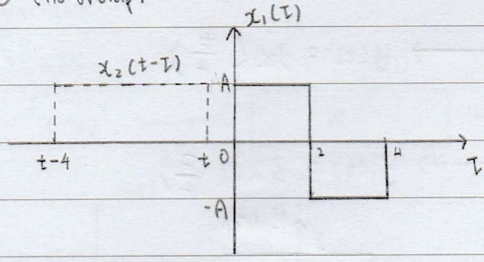


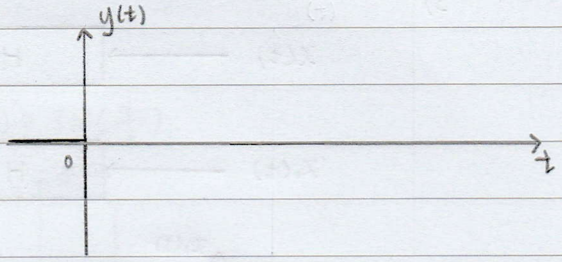
$x_2(t-\tau) \rightarrow x_2(-(\tau - t))$ time shift by t



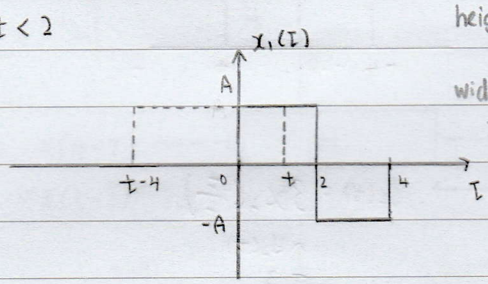
No.: Date:

i) $t < 0$ (no ovelap)

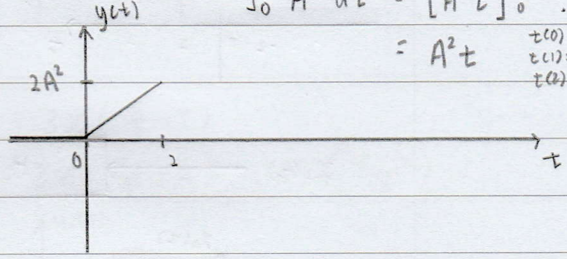




ii) $0 \leq t < 2$



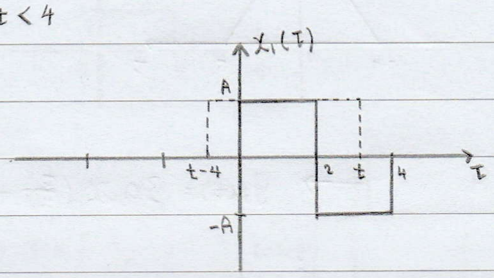
height $= A \times A$
$= A^2$
width(τ) = 2
$2 \times A^2 = 2A^2$

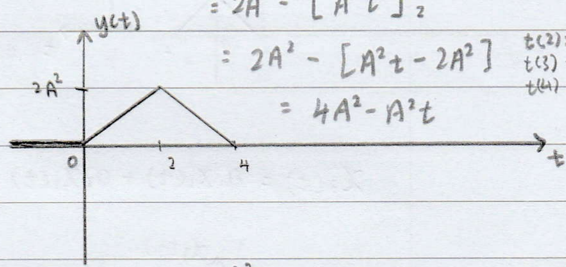


$\int_0^t A^2 \, d\tau = [A^2 \tau]_0^t$
$= A^2 t$

$t(0) = 0$
$t(1) = A^2$
$t(2) = 2A^2$

iii) $2 \leq t < 4$



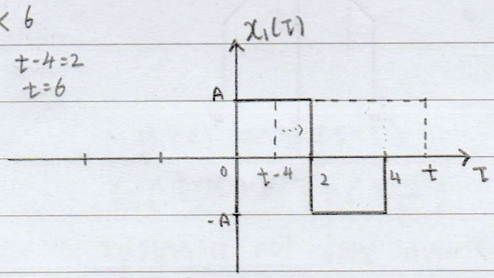


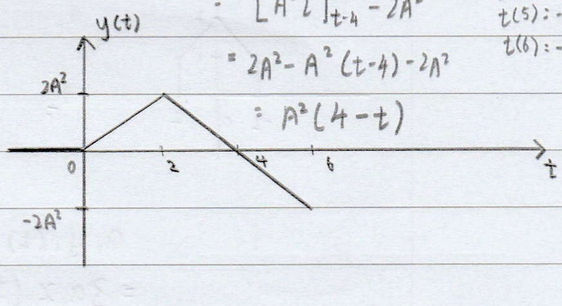
$2A^2 + \int_2^t -A^2 \, d\tau$
$= 2A^2 - [A^2 \tau]_2^t$
$= 2A^2 - [A^2 t - 2A^2]$
$= 4A^2 - A^2 t$

$t(2) = 2A^2$
$t(3) = A^2$
$t(4) = 0$

iv) $4 \leq t < 6$

$t-4 = 2$
$t = 6$



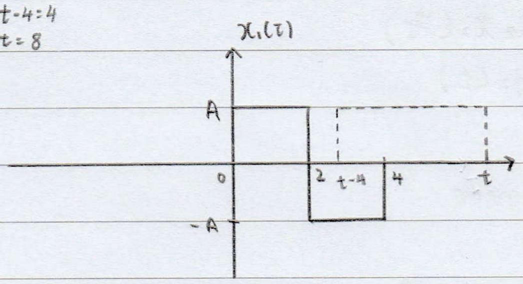


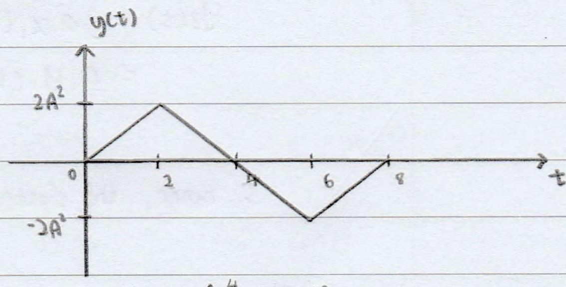
$\int_{t-4}^2 A^2 \, d\tau + (-2A^2)$
$= [A^2 \tau]_{t-4}^2 - 2A^2$
$= 2A^2 - A^2(t-4) - 2A^2$
$= A^2(4-t)$

$t(4) = 0$
$t(5) = -A^2$
$t(6) = -2A^2$

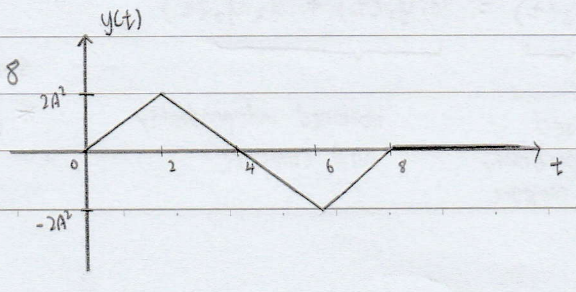
v) $6 \leq t < 8$

$t-4 = 4$
$t = 8$





vi) $t \geq 8$



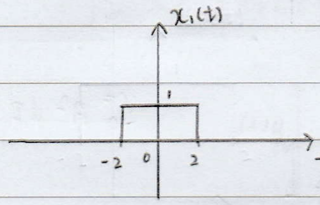
$\int_{t-4}^4 -A^2 \, d\tau$
$= -[A^2 \tau]_{t-4}^4$
$= -[A^2(4) - A^2(t-4)]$
$= -[4A^2 - A^2 t + 4A^2]$
$= A^2 t - 8A^2$

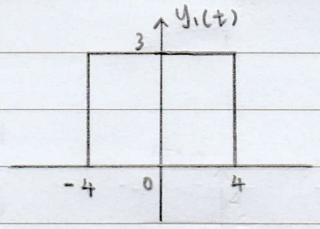
$t(6) = -2A^2$
$t(7) = -A^2$
$t(8) = 0$

3)

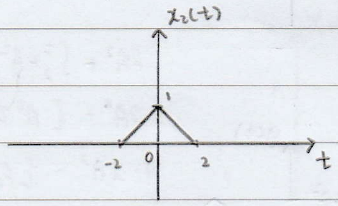
$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = 3x\left(\tfrac{t}{2}\right)$$

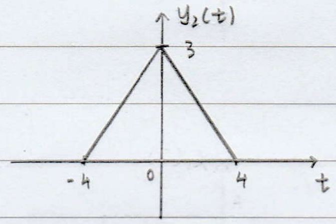
$$x_1(t) \longrightarrow \boxed{H} \longrightarrow y_1(t) = 3x_1\left(\tfrac{t}{2}\right)$$



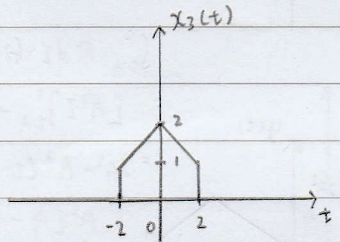


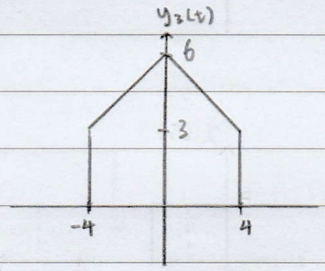
$$x_2(t) \longrightarrow \boxed{H} \longrightarrow y_2(t) = 3x_2\left(\tfrac{t}{2}\right)$$





$$x_3(t) = a_1 x_1(t) + a_2 x_2(t) \longrightarrow \boxed{H} \longrightarrow y_3(t) = 3a_1 x_1\left(\tfrac{t}{2}\right) + 3a_2 x_2\left(\tfrac{t}{2}\right)$$





$$a_1 y_1(t) + a_2 y_2(t)$$
$$= 3a_1 x_1\left(\tfrac{t}{2}\right) + 3a_2 x_2\left(\tfrac{t}{2}\right)$$

$$y_3(t) = 3a_1 x_1\left(\tfrac{t}{2}\right) + 3a_2 x_2\left(\tfrac{t}{2}\right)$$
$$= a_1 y_1(t) + a_2 y_2(t)$$

$\therefore$ hence, the system is linear

$$* \quad \underbrace{y_3(t)}_{\text{obtained independently and compare}} = \underbrace{a_1 y_1(t) + a_2 y_2(t)}_{\text{obtained independently and compare}}$$

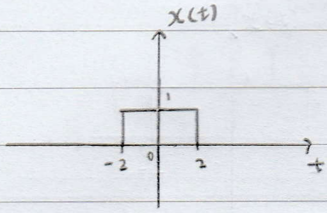
same = linear

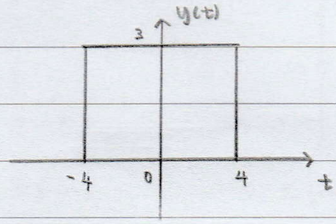
$*$ $a_{1,2}$ = Any real number

No.: Date:

Time invariant

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = 3x\left(\frac{t}{2}\right)$$



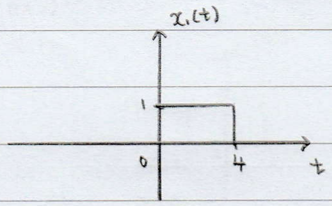


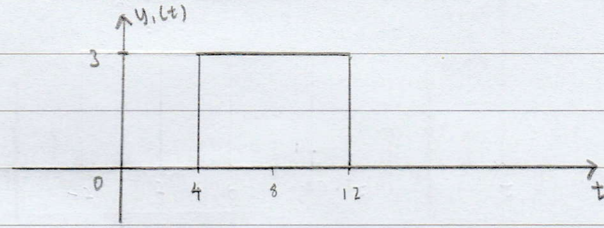
* replace t by t-T

$$x_1(t) = x(t-T) \longrightarrow \boxed{H} \longrightarrow y_1(t) = 3x\left(\frac{t}{2} - T\right)$$

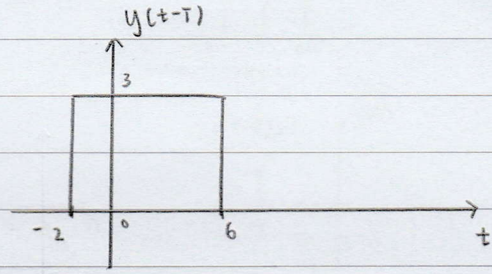
$\approx x(t-2)$ shift right by 2

$$\approx 3x\left(\frac{t}{2} - 2\right) = 3x\left(\frac{t-4}{2}\right)$$





$$y(t-T) = 3x\left(\frac{t-T}{2}\right) = 3x\left(\frac{t-2}{2}\right)$$



$$y_1(t) = y(t-T)$$

$$3x\left(\frac{t-4}{2}\right) \neq 3x\left(\frac{t-2}{2}\right)$$

$\therefore$ system is not time invariant