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Esmund Lim   Signal Tutorial 5

1) $R_{x\delta} = \sum_{n=-\infty}^{\infty} x[n]\,\delta^*[n+m]$

$= \sum_{n=-\infty}^{\infty} x[n]\,\underbrace{\delta[n+m]}$

$= x[-m]\,\delta[-m+m]$ This term will be zero except for $n=-m$

$= x[-m]\,\delta[0]$

$= x[-m]\,(1)$

$= x[-m]$

$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n\neq 0 \end{cases}$

$= \begin{cases} 1, & n+m=0 \\ 0, & n+m\neq 0 \end{cases}$

$= \begin{cases} 1, & n=-m \\ 0, & n\neq -m \end{cases}$

Cross correlation function (two different function)

① Energy type

- $R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y^*[n+m]$
- $R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\,y^*(t+\tau)\,dt$

② Power type

- $R_{xy}[m] = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n]\,y^*[n+m]$
- $R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,y^*(t+\tau)\,dt$

* cross correlation any waveform with $x[n]$

= the mirror image of waveform $x[n]$

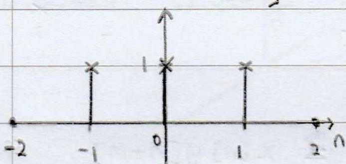
* Energy type x Power type = Energy type

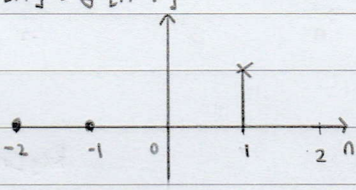
2) $x[n] = \text{rect}\left[\frac{n}{2}\right]$ $y[n] = \delta[n-1]$

$-\frac{2}{2} \leq n \leq \frac{2}{2}$

$-1 \leq n \leq 1$

$= 1$

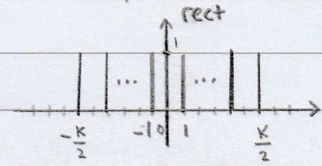




DT: $\text{rect}\left[\frac{n}{K}\right] = \begin{cases} 1, & |n| \leq \frac{K}{2} \\ 0, & \text{otherwise} \end{cases}$ $\frac{-K}{2} \leq n \leq \frac{K}{2}$

variable

even integer width of pulse

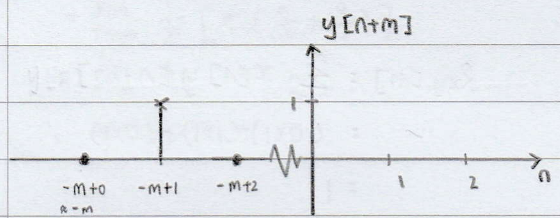


→ Both are energy type signal

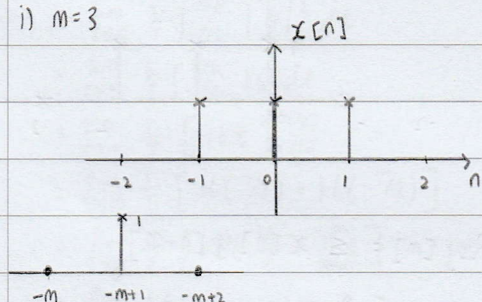
→ Both are real valued signal (ignore conjugate)

$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y^*[n+m]$

$= \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$ ← shift by $-m$



i) $m=3$



$-m+1 = -2$

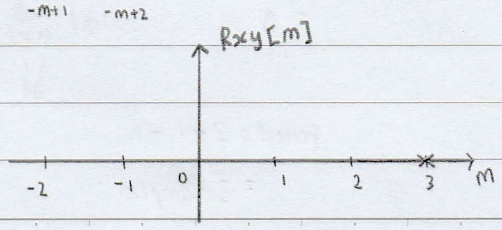
$-m = -3$

$m = 3$

$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$ multiply then sum

$= \sum_{n=-\infty}^{\infty} 0$

$= 0$



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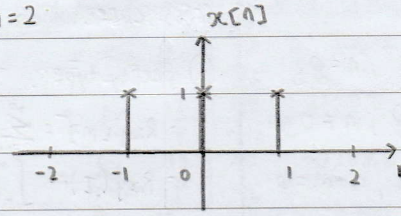
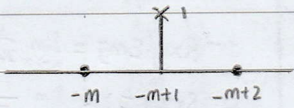
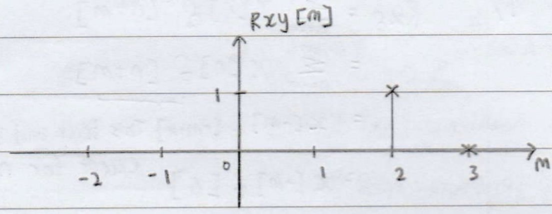
ii) $m=2$





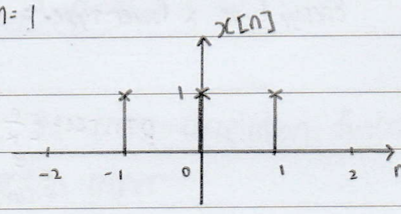


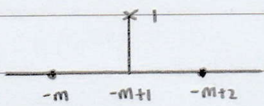
$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$$

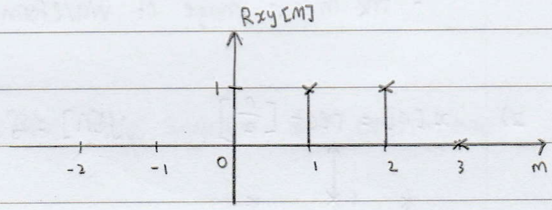
$$= (0\times0) + (1\times1) + (0\times1)$$

$$= 1$$

iii) $m=1$





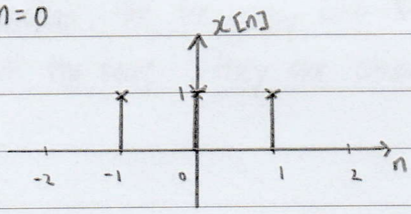


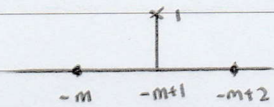
$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$$

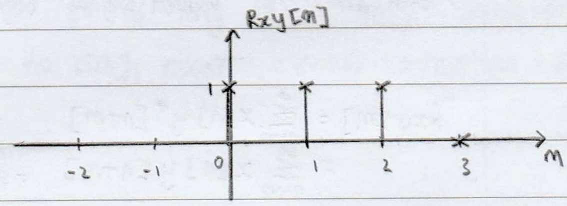
$$= (0\times1) + (1\times1) + (0\times1)$$

$$= 1$$

iv) $m=0$





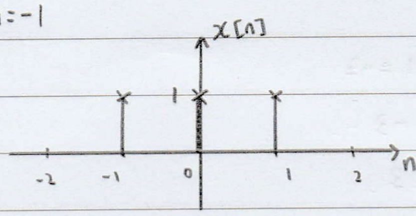


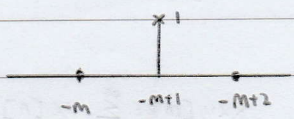
$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$$

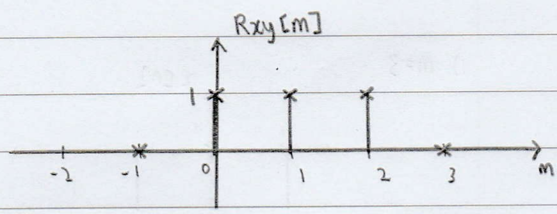
$$= (0\times1) + (1\times1) + (0\times1)$$

$$= 1$$

v) $m=-1$







$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]\,y[n+m]$$

$$= 0$$

Period $= 3+1-1$

$= 3$ sample

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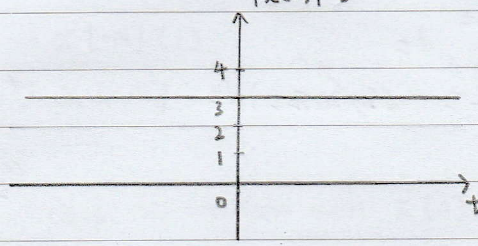
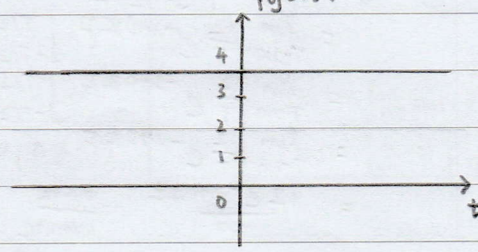
5) $x(t) = 3e^{j20\pi t}$ and $y(t) = 4e^{j40\pi t}$

a) sketch the amplitude plot

$|x(t)| = 3$

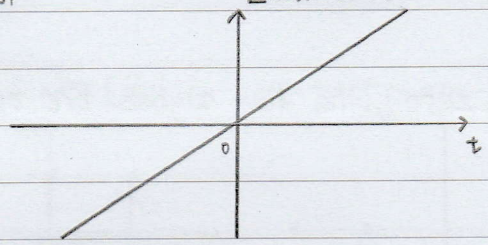


$|y(t)| = 4$

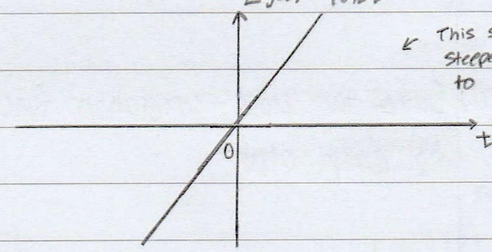


angle plot

$\angle x(t) = 20\pi t$



$\angle y(t) = 40\pi t$



← This slope is steeper as compare to $\angle x(t)$

b)
$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |3|^2\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 9\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\left[9t\right]_{-T/2}^{T/2}$$
$$= \lim_{T\to\infty} \frac{1}{T}\left[9(T/2) - 9(-T/2)\right]$$
$$= \lim_{T\to\infty} \frac{1}{T}\left[\frac{9T}{2} + \frac{9T}{2}\right]$$
$$= \lim_{T\to\infty} \frac{1}{\cancel{T}}\left[\cancel{T}\left(\frac{9}{2}+\frac{9}{2}\right)\right]$$
$$= \lim_{T\to\infty} 9$$
$$= 9$$

Power (Aperiodic Signal)

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$$
$$P_x = \lim_{K\to\infty} \frac{1}{2K+1}\sum_{n=-K}^{K} |x[n]|^2$$

$$P_y = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |y(t)|^2\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |4|^2\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 16\, dt$$
$$= \lim_{T\to\infty} \frac{1}{T}\left[16t\right]_{-T/2}^{T/2}$$
$$= \lim_{T\to\infty} \frac{1}{T}\left[16(T/2) - 16(-T/2)\right]$$
$$= \lim_{T\to\infty} \frac{1}{\cancel{T}}\left[\cancel{T}\left(\frac{16}{2}+\frac{16}{2}\right)\right]$$
$$= \lim_{T\to\infty} 16$$
$$= 16$$

* Power finite or Energy finite
Energy infinite Power zero

* $\text{Power} = \frac{\text{Energy}}{\text{time}}$

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c) $R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, y^*(t+\tau)\, dt$

$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 3e^{j20\pi t}\; 4e^{-j40\pi(t+\tau)}\, dt$

$= \lim_{T\to\infty} \frac{12}{T} \int_{-T/2}^{T/2} e^{j20\pi t}\, e^{-j40\pi t}\, e^{-j40\pi\tau}\, dt$

$= \lim_{T\to\infty} \frac{12}{T} \int_{-T/2}^{T/2} e^{-j20\pi t}\, e^{-j40\pi\tau}\, dt$

$= \lim_{T\to\infty} \frac{12}{T} e^{-j40\pi\tau} \int_{-T/2}^{T/2} e^{-j20\pi t}\, dt$

$= \lim_{T\to\infty} \frac{12}{T} e^{-j40\pi\tau} \int_{-T/2}^{T/2} (\overset{0}{\cancel{\cos 20\pi t}} - j\overset{0}{\cancel{\sin 20\pi t}})\, dt$

$= \lim_{T\to\infty} \frac{12}{T} e^{-j40\pi\tau}$

$= \frac{12e^{-j40\pi\tau}}{\infty}$ $\frac{\text{finite}}{\text{infinite}}$

$= 0$

* $e^{j40\pi t} \to e^{-j40\pi t}$ conjugate

cannot use graphical approach

→ Both waveform period = ∞

* $e^{j2\pi f_0 t}$

$\underbrace{\cos(2\pi f_0 t)}_{\text{real}} + \underbrace{j\sin(2\pi f_0 t)}_{\text{imaginary}}$

d) Since the cross-correlation function is zero, the two signals are orthogonal to each other

note

- This 2 signals have a frequency which is f_0 (10Hz) and $2f_0$ (20Hz), for any complex exponential function, the frequency are integer multiple of f_0 as long as the integer terms are not the same, they are always orthogonal to each other ∴ cross-correlation = zero