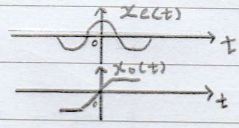
No.: Date:

Esmund Lim Signals Tutorial 1

Even / odd signal

$x_e(t) = x_e(-t)$

$x_o(t) = -x_o(-t)$



Any deterministic signal $x(t)$ can be decomposed

$$x(t) = x_e(t) + x_o(t)$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

even . even = even

odd . odd = even

even . odd = odd

$$\int_{-T_0}^{T_0} x_e(t)\,dt = 2\int_0^{T_0} x_e(t)\,dt$$

$$\int_{-T_0}^{T_0} x_o(t)\,dt = 0$$

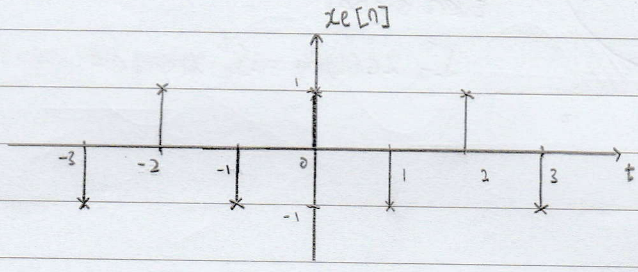
1) $x[n] = n + (-1)^n$

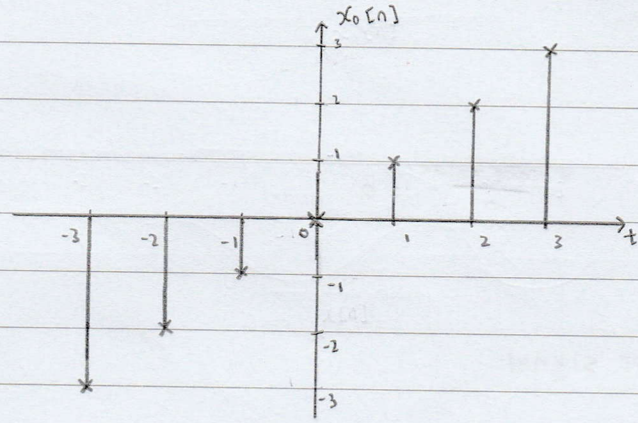
The even component of $x[n]$

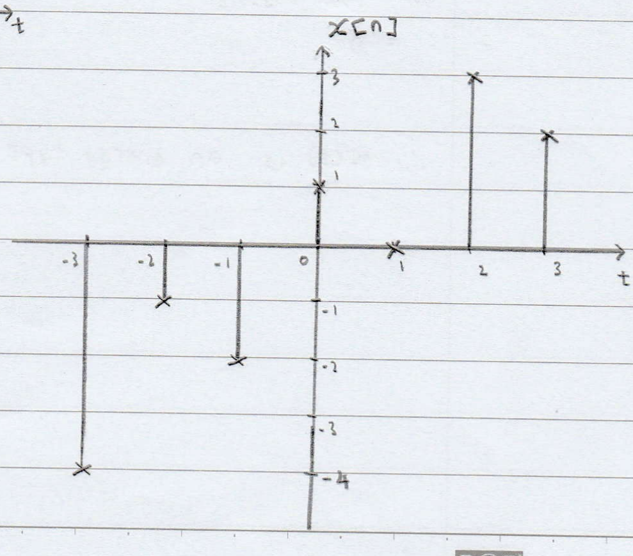
$$\begin{aligned} x_e[n] &= \frac{1}{2}[x[n] + x[-n]] \\ &= \frac{1}{2}[(n+(-1)^n) + (-n+(-1)^{-n})] \\ &= \frac{1}{2}[(-1)^n + (\tfrac{1}{-1})^n] \\ &= \frac{1}{2}[(-1)^n + (-1)^n] \\ &= \frac{1}{\cancel{2}}[\cancel{2}(-1)^n] \\ &= (-1)^n \end{aligned}$$

The odd component of $x[n]$

$$\begin{aligned} x_o[n] &= \frac{1}{2}[x[n] - x[-n]] \\ &= \frac{1}{2}[(n+(-1)^n) - (-n+(-1)^{-n})] \\ &= \frac{1}{2}[(n+(-1)^n) - (-n+(\tfrac{1}{-1})^n)] \\ &= \frac{1}{2}[2n] \\ &= n \end{aligned}$$







No.: .... Date: ....

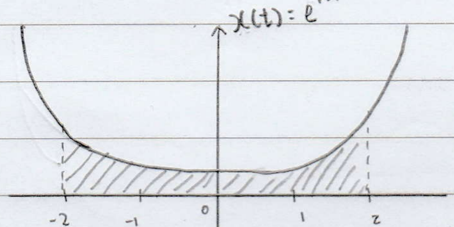
2a) $x(t) = \begin{cases} e^{|t|}, & \text{for } -2 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$
$$= \int_{-\infty}^{\infty} |e^{|t|}|^2 \, dt$$
$$= \int_{-\infty}^{\infty} e^{2|t|} \, dt$$
$$= \int_{-\infty}^{-2} e^{2|t|} dt + \int_{-2}^{2} e^{2|t|} dt + \int_{2}^{\infty} e^{2|t|} dt$$
$$= 0 + \int_{-2}^{2} e^{2|t|} dt + 0$$
$$= \int_{-2}^{2} e^{2|t|} dt$$
$$= \int_{-2}^{0} e^{-2t} dt + \int_{0}^{2} e^{2t} dt$$
$$= \left[-\tfrac{1}{2} e^{-2t}\right]_{-2}^{0} + \left[\tfrac{1}{2} e^{2t}\right]_{0}^{2}$$
$$= \left[-\tfrac{1}{2} e^{-2(0)} - \left(-\tfrac{1}{2} e^{-2(-2)}\right)\right] + \left[\tfrac{1}{2} e^{2(2)} - \tfrac{1}{2} e^{2(0)}\right]$$
$$= -\tfrac{1}{2} + \tfrac{1}{2} e^{4} + \tfrac{1}{2} e^{4} - \tfrac{1}{2}$$
$$= e^{4} - 1$$

absolute value t for positive, $-t$ for negative.

$|t| = \begin{cases} -t, & \text{for } t < 0 \\ t, & \text{for } t \ge 0 \end{cases}$

or



Even signal

$$\int_{-T_0}^{T_0} x_e(t) dt = 2\int_{0}^{T_0} x_e(t) dt$$

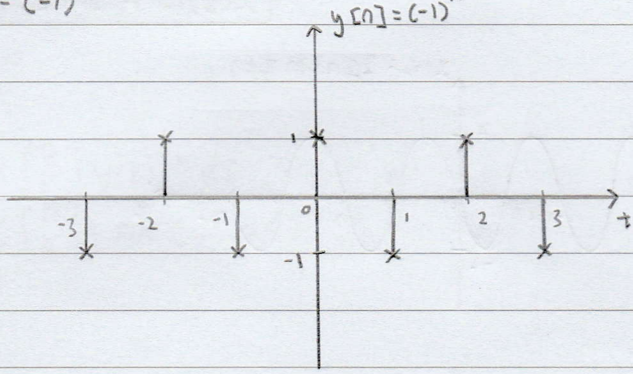
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$
$$= 2\int_{0}^{2} e^{2t} dt$$
$$= 2\left[\tfrac{1}{2} e^{4} - \tfrac{1}{2} e^{0}\right]$$
$$= e^{4} - 1$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \qquad * \frac{\text{finite value}}{\infty} = 0$$
$$= 0$$

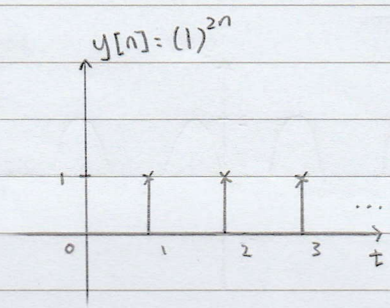
$\therefore$ $x(t)$ is an energy type signal

No.: Date:

b) $y[n] = (-1)^n$



$$E_y = \sum_{n=-\infty}^{\infty} |y[n]|^2$$
$$= \sum_{n=-\infty}^{\infty} (1)^{2n}$$
$$= \infty$$



infinite no. of 1

$$P_y = \lim_{K \to \infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |y[n]|^2$$
$$= 1$$

$\sum_{n=-K}^{K} 1 = 2K+1$

$(2K+1)$ no. of '1'

$\therefore$ $y[n]$ is a power-type signal

$$\boxed{x(t) = A\sin(2\pi f_0 t + \phi)}$$

3) $x(t) = 2\sin(20\pi t)$

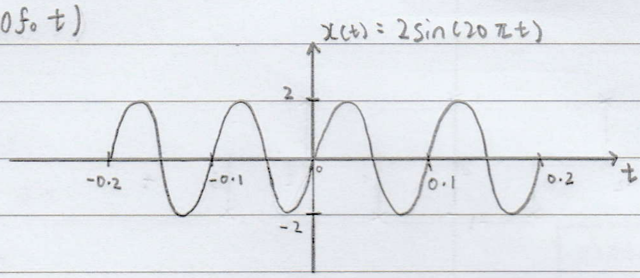
$= 2\sin(2\pi\, 10 f_0\, t)$

$f_0 = 10$

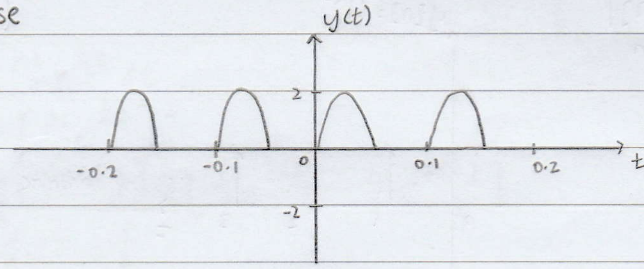
$T_0 = \frac{1}{f_0}$

$= \frac{1}{10}$

$= 0.1s$



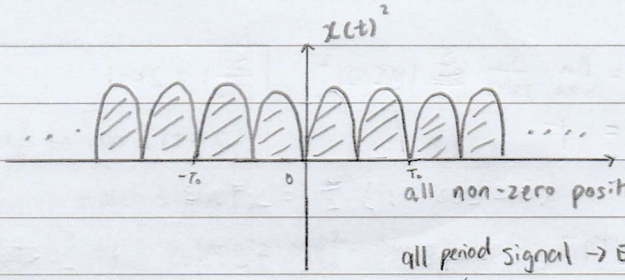
$$y(t) = \begin{cases} x(t), & \text{if } x(t) > 0, \\ 0, & \text{otherwise} \end{cases}$$



b) $\mathcal{E}_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$

$= \int_{-\infty}^{\infty} |2\sin(2\pi\, 10 f_0 t)|^2\, dt$

$= \infty$



all non-zero positive value

all period signal → Energy = ∞

always repeat itself

$P_x = \frac{1}{T_0}\int_0^{T_0} 2^2 \sin^2(20\pi t)\, dt$

$= \frac{1}{T_0}\int_0^{T_0} 4\sin^2(20\pi t)\, dt$

$= \frac{1}{T_0}\int_0^{T_0} \frac{4}{2}(1 - \cos(40\pi t))\, dt$

$= \frac{1}{T_0}\int_0^{T_0} 2(1 - \cos(40\pi t))\, dt$

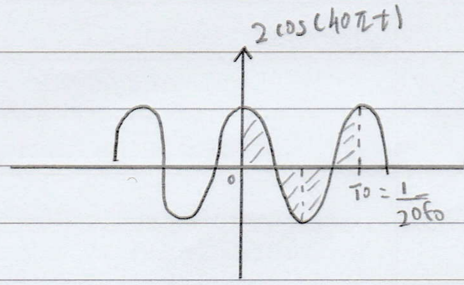
$= \frac{1}{T_0}\int_0^{T_0} 2 - 2\cos(40\pi t)\, dt$

$= \frac{1}{T_0}\int_0^{T_0} 2\, dt - \frac{1}{T_0}\int_0^{T_0} 2\cos(40\pi t)\, dt$

$= \frac{1}{T_0}[2t]_0^{T_0} - 0$

$= \frac{2(\cancel{T_0})}{\cancel{T_0}} - \cancel{\frac{0}{T_0}}$

$= 2$,,



$x(t)$ is a power type signal

No.:   Date:

b) $E_y = \int_{-\infty}^{\infty} |y(t)|^2 \, dt$

$= \infty$

$P_y = \frac{1}{T_0} \int_0^{T_0} |y(t)|^2 \, dt$

$= \frac{1}{T_0} \int_0^{T_0/2} 4 \sin^2(20\pi t) \, dt$

$= \frac{2}{2}$

$= 1$

$y(t)$ is a power signal