EEE3002 Microprocessors Tutorial 1 (2014 Semester 1)

1) Find the two's complement representation for the following numbers, assuming that they are represented as a 16-bit number. Write the value in both binary and hexadecimal.
   a) −93
   b) 1034
   c) 492
   d) −1094

2) Using the smallest data size possible, either a byte, a halfword (16 bits), or a word (32 bits), convert the following values into two's complement representations:
   a) −18,304
   b) −20
   c) 114
   d) −128

3) Convert the following hexadecimal values to base ten:
   a) 0xFE98
   b) 0xFEED
   c) 0xB00
   d) 0xDEAF

4) Convert the following decimal numbers into hexadecimal
   a) 256
   b) 1000
   c) 4095
   d) 42

5) Convert the following fractions into decimal numbers
   a) Binary number : 101.111
   b) Hexadecimal number : 101.111

6) Convert the following numbers into binary numbers
   a) Decimal number: 8.625
   b) Hexadecimal number : A1.E8

7) List the advantages of using the two's complement representation over the more intuitive sign-magnitude representation.

EE3002 Microprocessors Tutorial 1 Solutions (2014 Semester 2)

1) Steps
   - First convert the magnitude number into binary.
   - If number is positive, stop, else proceed to next step.
   - Compute the 1's complement by inverting the bits.
   - Add 1 to the 1's complement to obtain the 2's complement number.

   a) 93 = 1 0 1 1 1 0 1 B
   Can be converted by hand using repeated division by 2 and keeping track of the remainders.

   2∟93
   2∟46 Remainder 1 (Least Significant bit)
   2∟23 Remainder 0
   2∟11 Remainder 1
   2∟5 Remainder 1
   2∟2 Remainder 1
   2∟1 Remainder 0
   0 Remainder 1

   In 16-bit format, 93 can be represented as

   0 0 0 0 0 0 0 0 0 1 0 1 1 1 0 1 B

   Invert the bits to get the 1's complement : 1 1 1 1 1 1 1 1 1 0 1 0 0 0 1 0 B
   Add 1 to obtain the 2's complement.

   1 1 1 1 1 1 1 1 1 0 1 0 0 0 1 0 B
   \+ 1 B
   1 1 1 1 1 1 1 1 1 0 1 0 0 0 1 1 B (2's complement representation of −93 in Binary)

   F F A 3 or 0xFFA3 in 16-bit hexadecimal format

   b) 1034 = 1 0 0 0 0 0 0 1 0 1 0 B
   In 16-bit format
   0 0 0 0 0 1 0 0 0 0 0 0 1 0 1 0 B

   0 4 0 A or 0x040A in 16-bit hexadecimal format

c) 492 = 1 1 1 1 0 1 1 0 0 B
In 16-bit format
0 0 0 0 0 0 0 1 1 1 1 0 1 1 0 0 B

0 1 E C or 0x01EC in 16-bit hexadecimal format

d) 1094 = 1 0 0 0 1 0 0 0 1 1 0 B
In 16-bit format
0 0 0 0 0 1 0 0 0 1 0 0 0 1 <u>1 0</u> B

When doing by hand, another way of doing 2's complement is to start from the right, and do bit by bit, all the bits will be unchanged until the first 1 is encountered. Then the bits after the first 1 will be inverted. In the example above, the bits that are unchanged are underlined, the rest will be negated.

1 1 1 1 1 0 1 1 1 0 1 1 1 0 1 0 B is the 2's complement

F B B A or 0xFBBA in 16-bit hexadecimal format

2) Two's complement integer ranges
Byte : −128 to 127
Halfword : −32,768 to 32,767
Word: −2,147,483,648 to 2,147,483,647

a) −18,304 can be represented using Halfword or 16 bits
In 16-bit format, 18304 can be represented by

0 1 0 0 0 1 1 1 <u>1 0 0 0 0 0 0 0</u> B

1 0 1 1 1 0 0 0 1 0 0 0 0 0 0 0 B (2's complement form)

b) −20 can be represented by a byte or 8 bits
In 16-bit format, 20 can be represented by

0 0 0 1 0 <u>1 0 0</u> B

1 1 1 0 1 1 0 0 B (2's complement form)

c) 114 can be represented by a byte
0 1 1 1 0 0 1 0 B

d) −128 can be represented by a byte
1n 8-bit format, −128 can be represented by
1 0 0 0 0 0 0 0

3) a) 0xFE98
Decimal value is $8\times16^0+9\times16^1+14\times16^2+15\times16^3 = 65176$

b) 0xFEED
Decimal value is $13\times16^0+14\times16^1+14\times16^2+15\times16^3 = 65261$

c) 0xB00
Decimal value is $0\times16^0+0\times16^1+11\times16^2 = 2816$

d) 0xDEAF
Decimal value is $15\times16^0+10\times16^1+14\times16^2+13\times16^3 = 57007$

4) Can be converted using repeated division by 16 and keeping track of the remainders

a) 16∟256
16∟16 Remainder 0 (Least Significant hexadecimal)
16∟1 Remainder 0
0 Remainder 1
Therefore hexadecimal value is 0x100

b) 16∟1000
16∟62 Remainder 8 (Least Significant hexadecimal)
16∟3 Remainder E (Decimal 14)
0 Remainder 3
Therefore hexadecimal value is 0x3E8

c) 16∟4095
16∟255 Remainder F (Least Significant hexadecimal)
16∟15 Remainder F
0 Remainder F
Therefore hexadecimal value is 0xFFF

d) 16∟42
16∟2 Remainder A (Least Significant hexadecimal)
0 Remainder 2
Therefore hexadecimal value is 0x2A

5) Similar to Question 3

   a) Binary number 101.111

      Value in Decimal $= \mathbf{1} \times 2^2 + \mathbf{0} \times 2^1 + \mathbf{1} \times 2^0 + \mathbf{1} \times 2^{-1} + \mathbf{1} \times 2^{-2} + \mathbf{1} \times 2^{-3}$

      $= 4+1+0.5+0.25+0.125$

      $= 5.875$

   b) Hexadecimal 101.111

      Value in Decimal $= \mathbf{1} \times 16^2 + \mathbf{0} \times 16^1 + \mathbf{1} \times 16^0 + \mathbf{1} \times 16^{-1} + \mathbf{1} \times 16^{-2} + \mathbf{1} \times 16^{-3}$

      $= 256 + 1 + 0.0625 + 0.00390625 + 0.000244140625$

      ≈ 257.06665

6) Have to separate the number into two parts, the whole number part and the fractional part.

   a) 8.625 = 8 + 0.625

   Convert the whole number part using repeated division by 2.

   8 decimal = 1 0 0 0 Binary

   Convert the fractional part using repeated multiplication by 2.

   0.625 × 2 = 1.25 ; 1

   0.25 × 2 = 0.5 ; 0

   0.5 × 2 = 1.0 ; 1

   Hence the binary number equivalent of 8.625 will be 1 0 0 0. 1 0 1

   b) Hexadecimal to binary conversion is simple even for float numbers

A	1	.	E	8
1010	0001	.	1110	1000

      Hence answer is 10100001.11101 Binary

7) There are a number of significant advantages that two's complement has over the sign-magnitude representation

   - An adder can be used to perform subtraction when negative numbers are represented in two's complement form.
   - A sign-magnitude representation has two representations of zeros. For example a zero byte can be represented by 0 0 0 0 0 0 0 0 B or 1 0 0 0 0 0 0 0 B. Hence it is more difficult to determine the zero condition as both values must be tested.
   - Two's complement number has the advantage of sign-extension, i.e. to convert a byte to a halfword or a word, you only need to replicate the sign-bit to the left.

     For example, −20 in 8 bits is 1 1 1 0 1 1 0 0 B

     −20 in 16 bits is 1111 1111 1110 1100 B

     −20 in 32 bits is 1111 1111 1111 1111 1111 1111 1110 1100 B