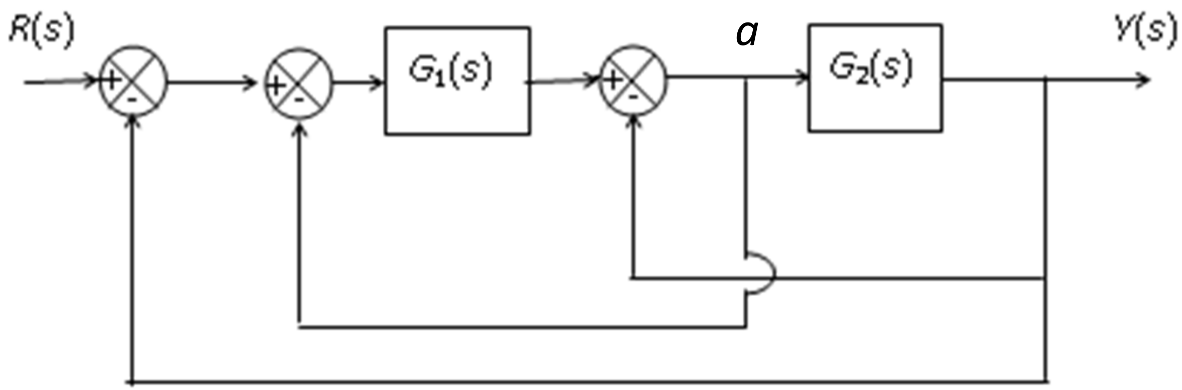
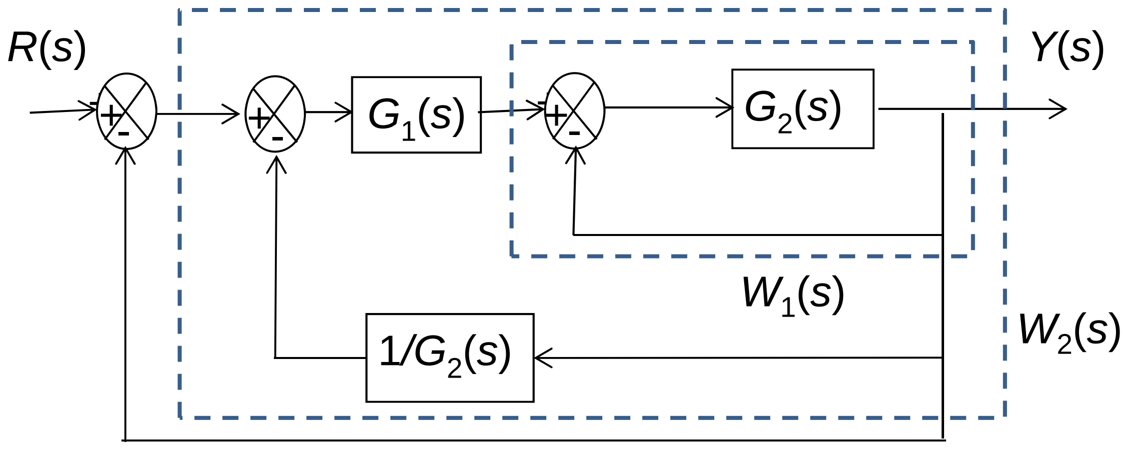
EE3011 MODELLING & CONTROL
Tutorial 2 (Solutions) System Modelling

1. Move the take-off point *a* forward:



We get



We have:

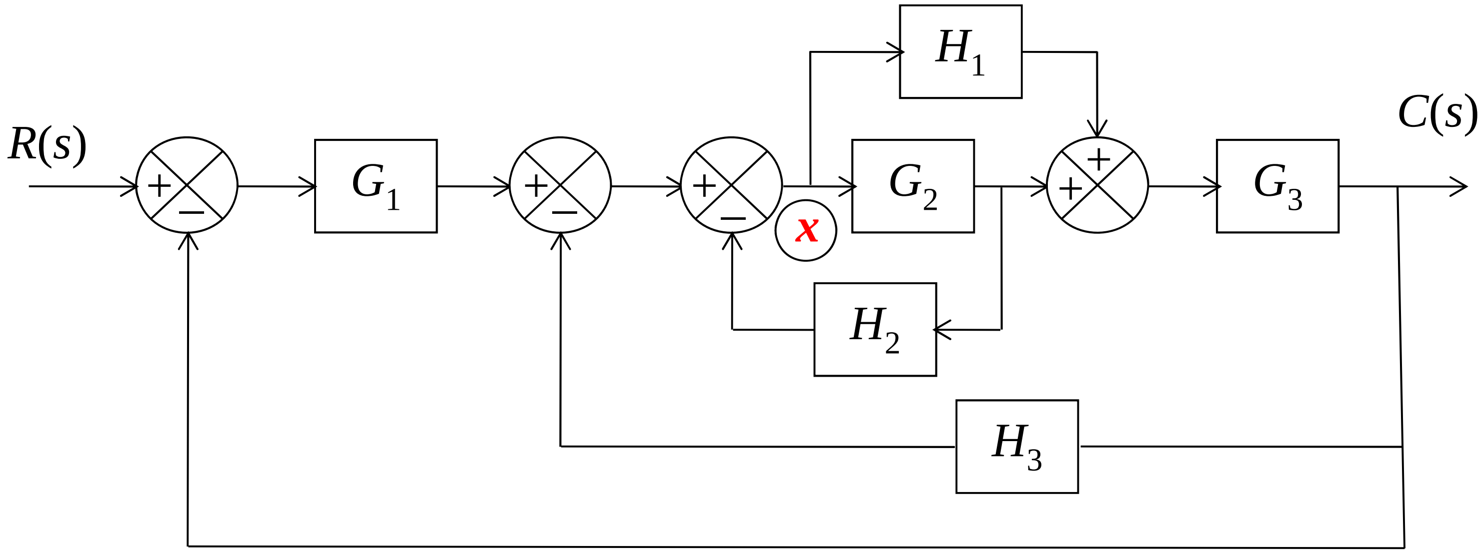
$$W_1 = \frac{G_2}{1+G_2}$$

$$W_2 = \frac{G_1 W_1}{1+\dfrac{G_1 W_1}{G_2}} = \frac{G_1 G_2 \dfrac{G_2}{1+G_2}}{G_2 + G_1 \dfrac{G_2}{1+G_2}} = \frac{G_1 G_2}{1+G_1+G_2}$$

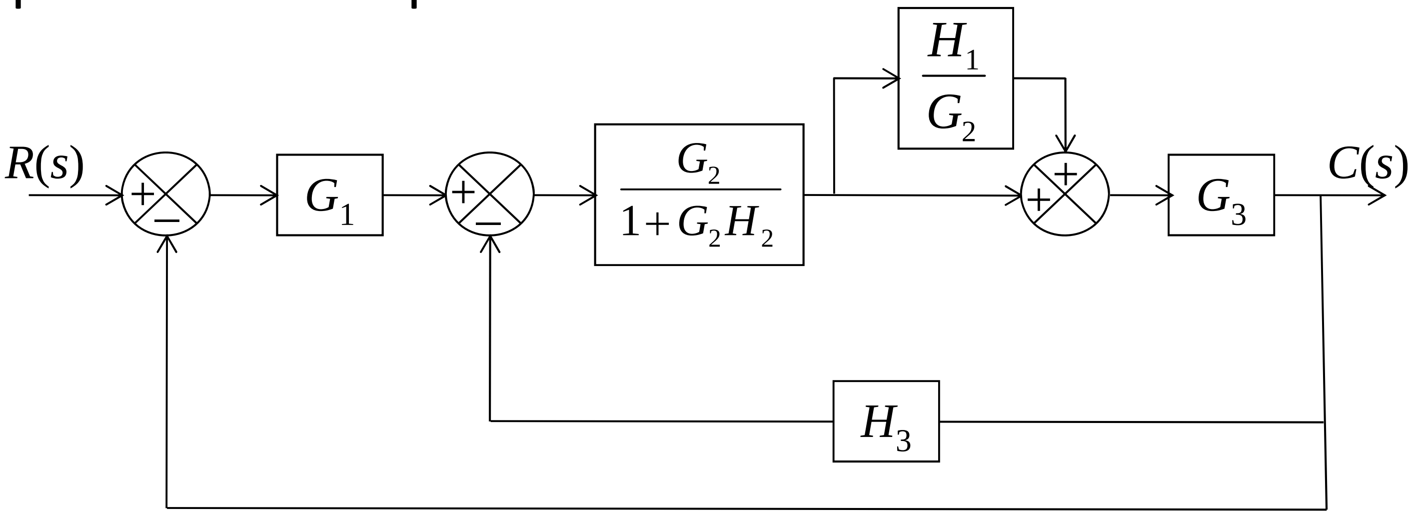
Hence

$$\frac{Y}{R} = \frac{W_2}{1+W_2} = \frac{G_1 G_2}{1+G_1+G_2+G_1 G_2}$$

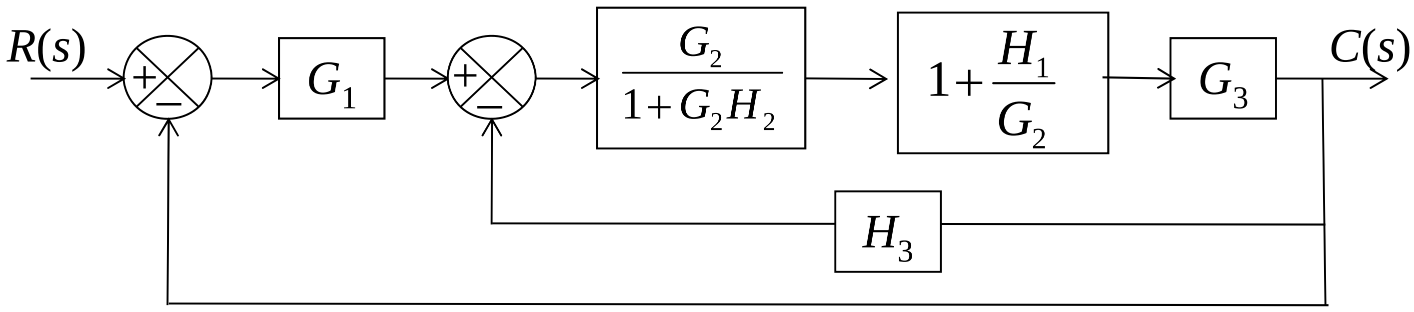
2. Interchange 2 summers (comparators)



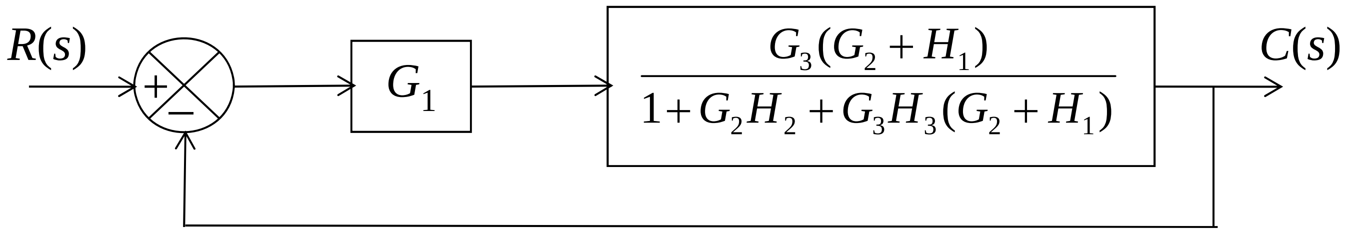
Shift point *x* and simplified



Simplify the inner feedback loop



Thus, we have



$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)+G_1G_3(G_2+H_1)}$$

If feedback path H_2 fails $\Rightarrow H_2 \equiv 0$ (assume complete failure), then

$$\frac{C(s)}{R(s)} = \frac{G_1G_3(G_2+H_1)}{1+G_3H_3(G_2+H_1)+G_1G_3(G_2+H_1)}$$

3. Apply Laplace Transform to all the differential equations:

$$v_i(t) = K_1\theta_r(t) \quad \Rightarrow \quad V_i(s) = K_1\theta_r(s)$$

$$v_o(t) = K_1\theta_y(t) \quad \Rightarrow \quad V_o(s) = K_1\theta_y(s)$$

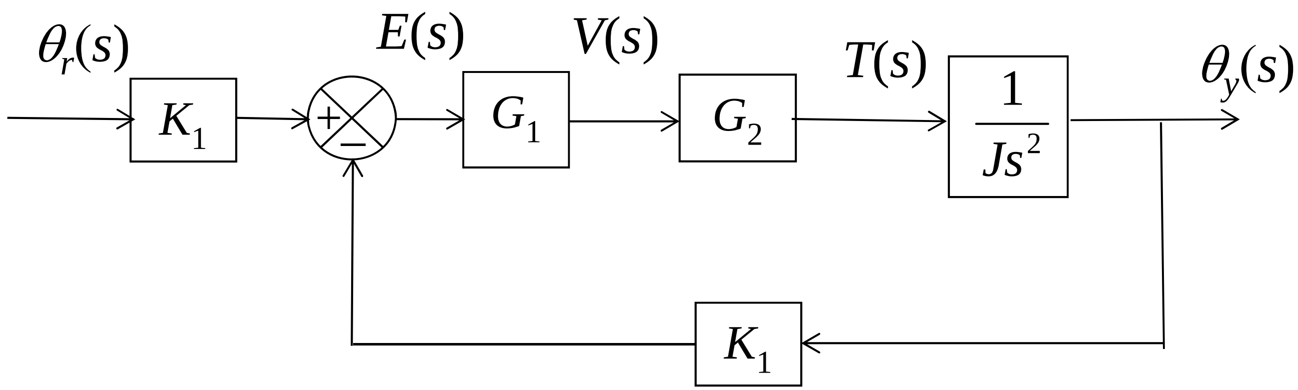
$$e(t) = v_i(t) - v_o(t) \quad \Rightarrow \quad E(s) = V_i(s) - V_o(s)$$

$$V(s) = G_1(s)E(s)$$

$$T(s) = G_2(s)V(s)$$

$$J\ddot{\theta}_y(t) = T(t) \quad \Rightarrow \quad Js^2\theta_y(s) = T(s) \quad \Rightarrow \quad \theta_y(s) = \frac{1}{Js^2}T(s)$$

Thus, we have



4. The derivation for this translational motion is similar to the one derived for rotational motion.

Mechanical system:

$$F = M_1\ddot{q} + K(q - y)$$

$$0 = M_2\ddot{y} + K(y - q)$$

Coupling between mechanical & electrical systems:

Motion with velocity in magnetic field produces a voltage across the coil, i.e.

$$e_{\text{coil}} = a_2\dot{q} \qquad \text{(law of generator)}$$

Current flowing in magnetic field produces a force

$$F = a_1 i \qquad \text{(law of motor)}$$

Electric circuit:

$$L\frac{di}{dt} + Ri = e_c - e_{\text{coil}}$$

In *s*-domain (zero initial conditions):

$$F(s) = (s^2 M_1 + K)Q(s) - KY(s) \qquad \text{------ (1)}$$

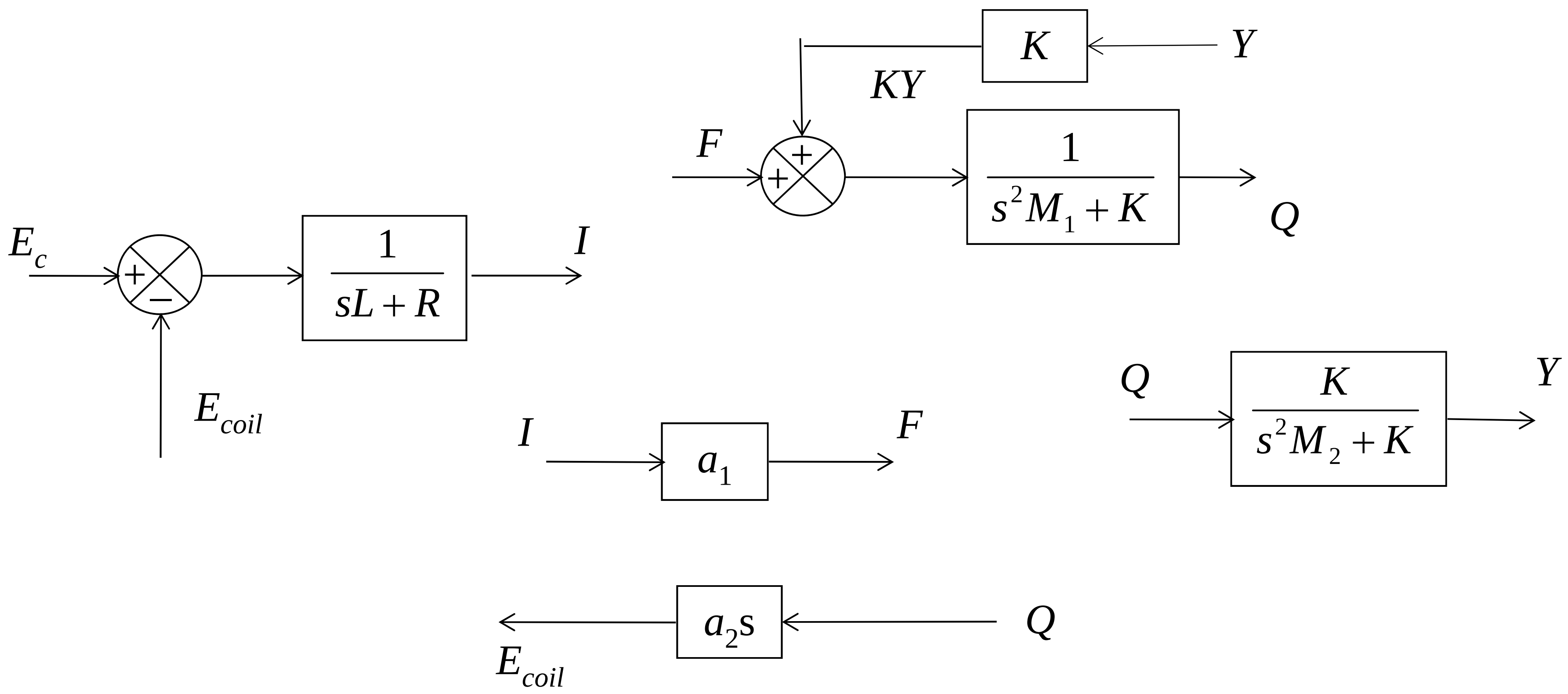
$$0 = (s^2 M_2 + K)Y(s) - KQ(s) \qquad \text{------ (2)}$$

$$F(s) = a_1 I(s) \qquad \text{------ (3)}$$

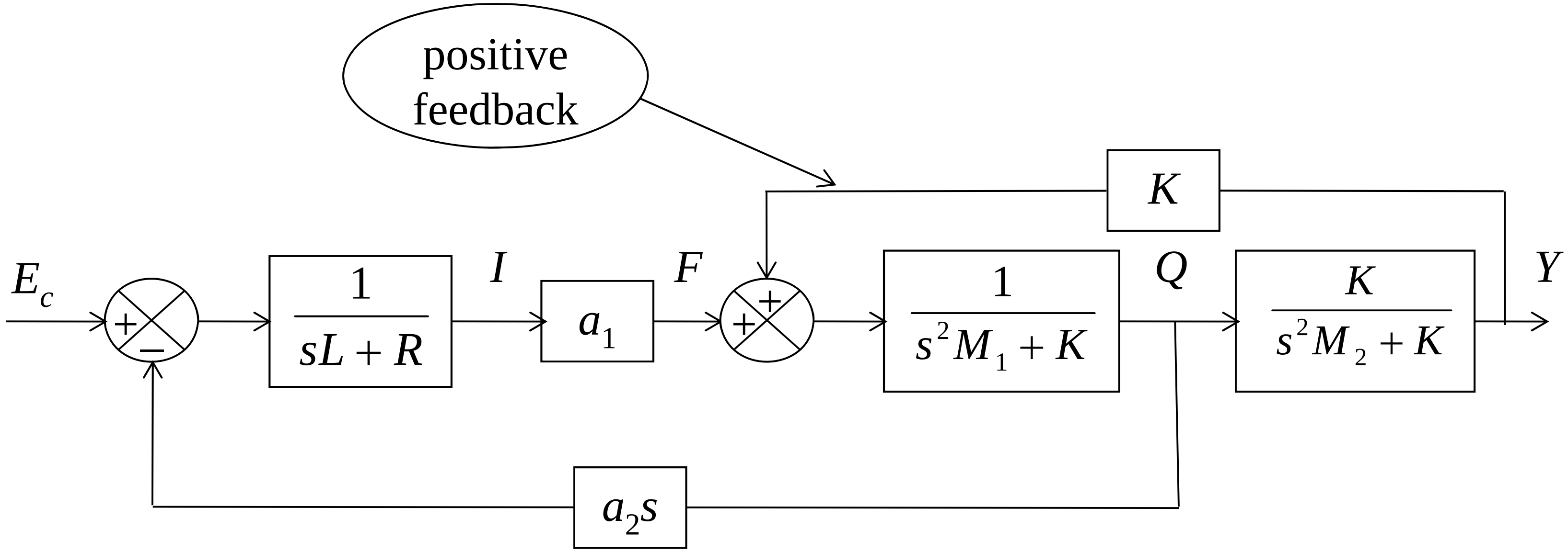
$$E_{coil}(s) = a_2 s Q(s) \qquad \text{------ (4)}$$

$$(sL + R)I(s) = E_c(s) - E_{coil}(s) \qquad \text{------ (5)}$$

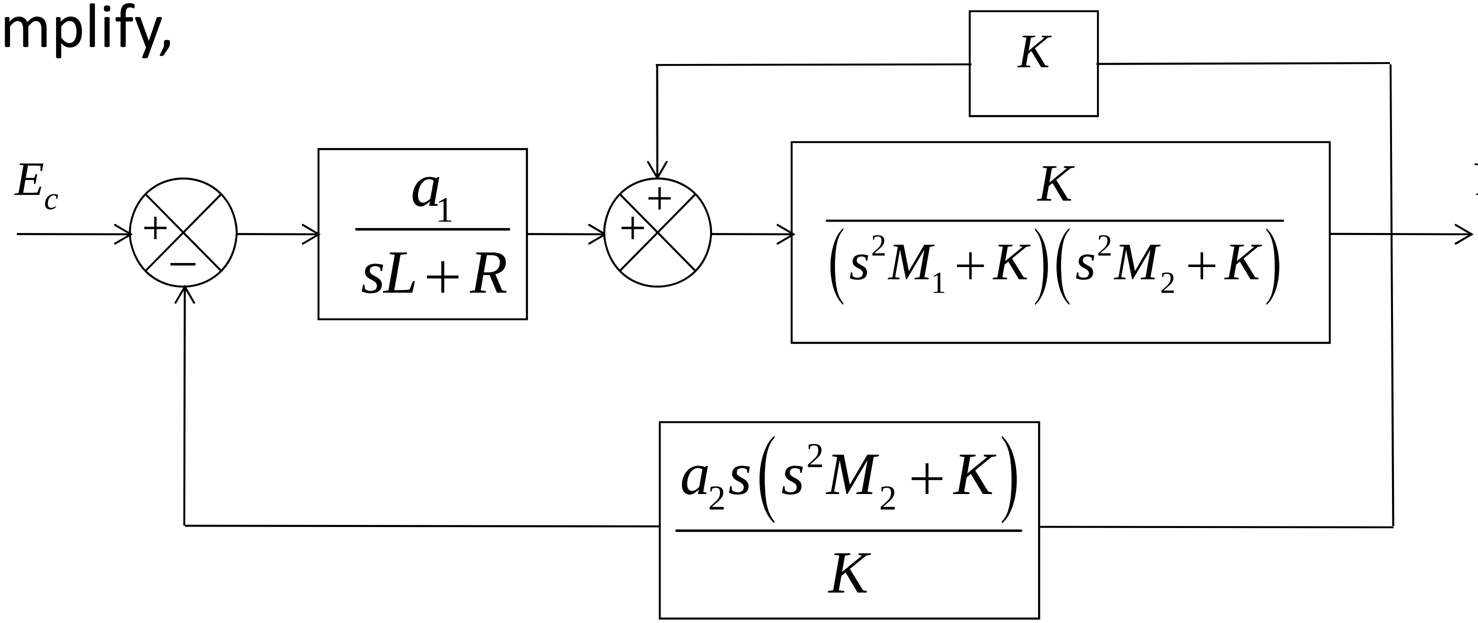
(1) – (5) can be represented by each of the following incomplete block diagrams:



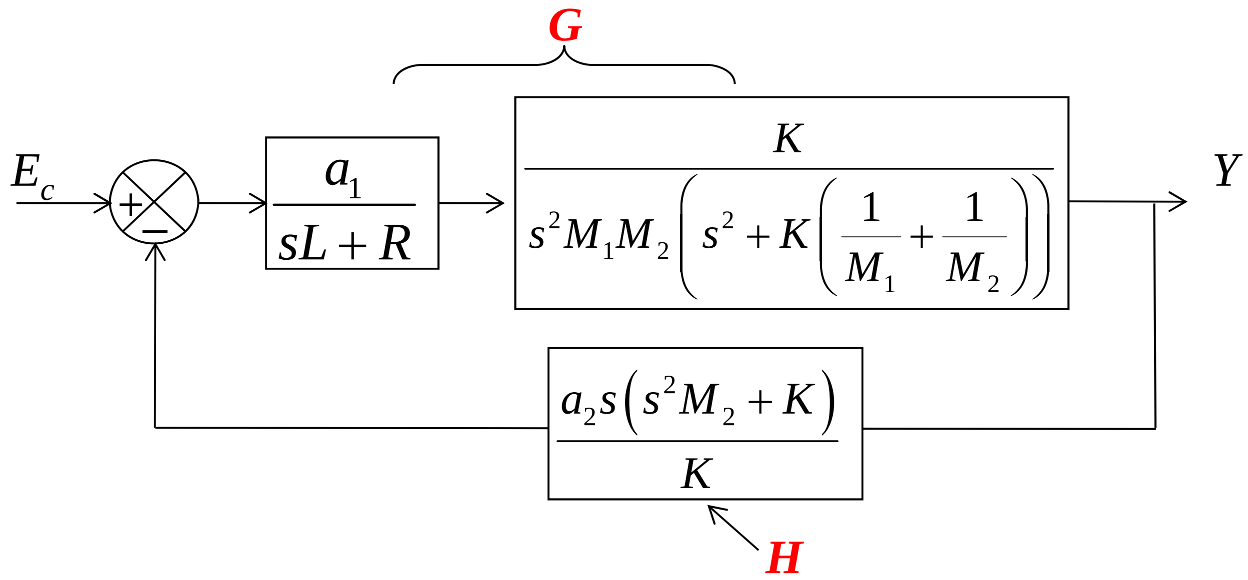
Joining the blocks, we have



Simplify,



Simplify,



$$\therefore \frac{Y}{E_c}=\frac{G}{1+GH}=\frac{a_1K}{s^2M_1M_2\left(sL+R\right)\left(s^2+K\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\right)+a_1a_2s\left(s^2M_2+K\right)}$$

5.(i) Input/output potentiometer:

max. change in displacement, $\Delta\theta$ = 10(2π) rad

max. change in voltage, ΔV = 10 – (–10) = 20 V

$$\therefore \frac{V_i(s)}{\theta_i(s)} = \frac{20}{10(2\pi)} = \frac{1}{\pi}$$

(What's the assumption?)

Similarly, $$\frac{V_o(s)}{\theta_o(s)} = \frac{1}{\pi}$$

(ii) Pre-amplifier: $$\frac{V_p(s)}{V_i(s) - V_o(s)} = K$$

(iii) Power amplifier: $$\frac{E_a(s)}{V_p(s)} = \frac{100}{s+100}$$

(iv) Motor and load:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_T / R_a}{s\left(Js + B + \dfrac{K_T K_b}{R_a}\right)}$$

$$J = J_m + \left(\frac{N_1}{N_2}\right)^2 J_L = 0.015 + \left(\frac{1}{10}\right)^2 \times 1 = 0.025$$

$$B = B_m + \left(\frac{N_1}{N_2}\right)^2 B_L = 0.01 + \left(\frac{1}{10}\right)^2 \times 1.5 = 0.025$$

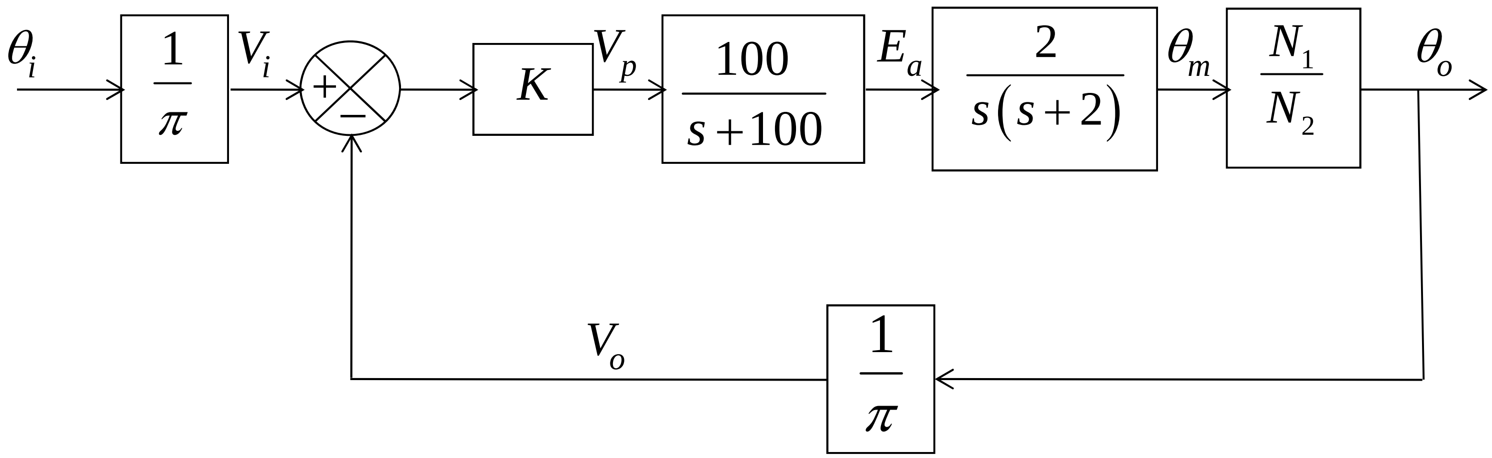
$K_T = 0.5$, $K_b = 0.5$ and $R_a = 10$

$$\therefore \frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{0.5}{10 \times 0.025}}{s\left(s + \dfrac{1}{0.025}\left(0.025 + \dfrac{0.5 \times 0.5}{10}\right)\right)} = \frac{2}{s(s+2)}$$

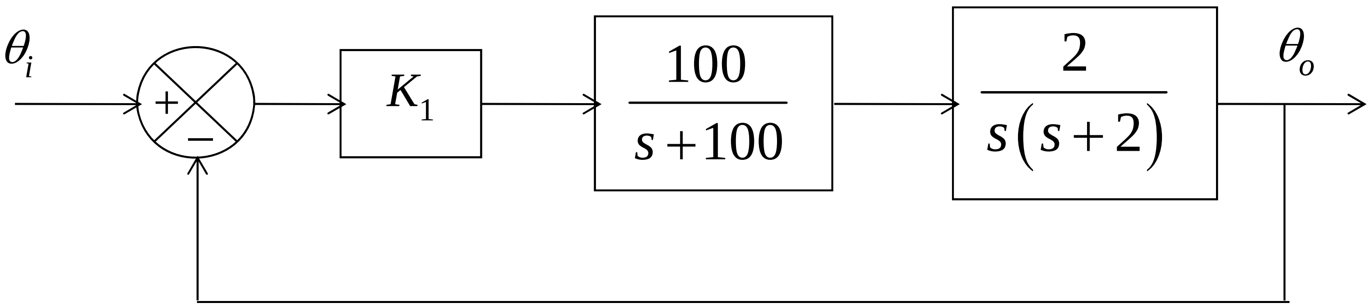
(v) Finally, the load displacement is related to the motor displacement via

$$\theta_o(s) = \frac{N_1}{N_2}\theta_m(s)$$

Hence, the block diagram representation is



Or a simplified “equivalent” one:



where $$K_1 = \frac{K}{\pi} \cdot \frac{N_1}{N_2}$$