Tut 11 Solutions

11.1 • From the Bode plots,

$$\omega_g = 10 \text{ rad/s}, \quad \phi_0 = 0^0$$

- Since the gain-crossover frequency is to be kept at 10 rad/s, to achieve the phase margin of 50^0 , the phase lead to be added from the compensator is chosen to be

$$\phi = 50^0$$ (Explain why there is no need to add additional 5^0 to 10^0)

$$\alpha = \frac{1-\sin 50^0}{1+\sin 50^0} = 0.1325$$

- Set

$$\omega_m = \frac{1}{\sqrt{\alpha}T} = 10 \quad \Rightarrow \quad T = 0.2747$$

- At $\omega = \omega_m = 10$ rad/s, the gain of the lead compensator is

$$-10\lg\alpha \approx 8.8 \text{ dB}$$

- To keep $\omega_g^{'} = \omega_g = 10$ rad/s, the compensator gain should be chosen as

$$20\lg K = -8.8 \quad \Rightarrow \quad K = 0.3631$$

- The designed compensator is then

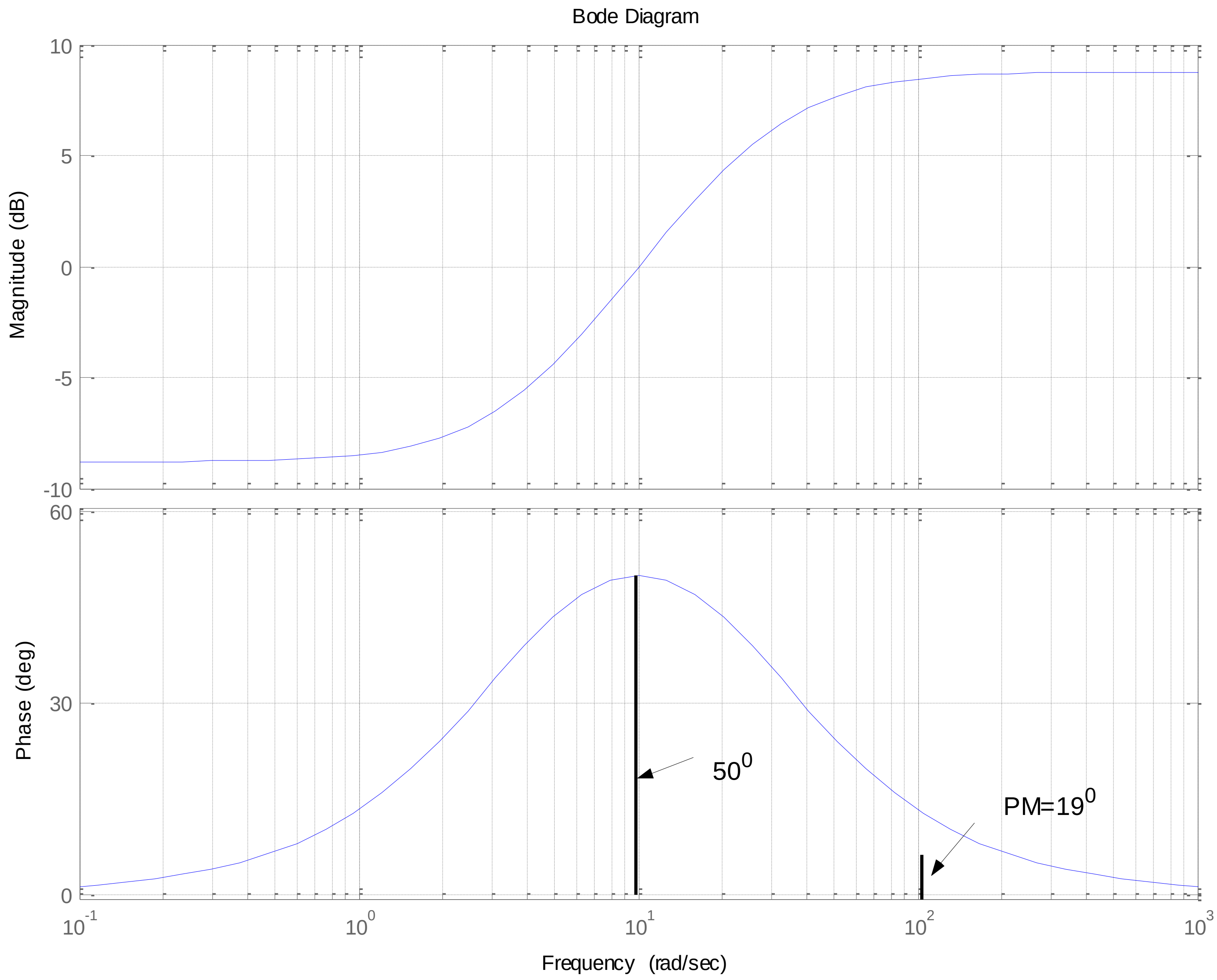
$$C(s) = 0.3631\frac{0.2747s+1}{0.0364s+1}$$

- From the Bode plots of $C(s)$, it can be seen that

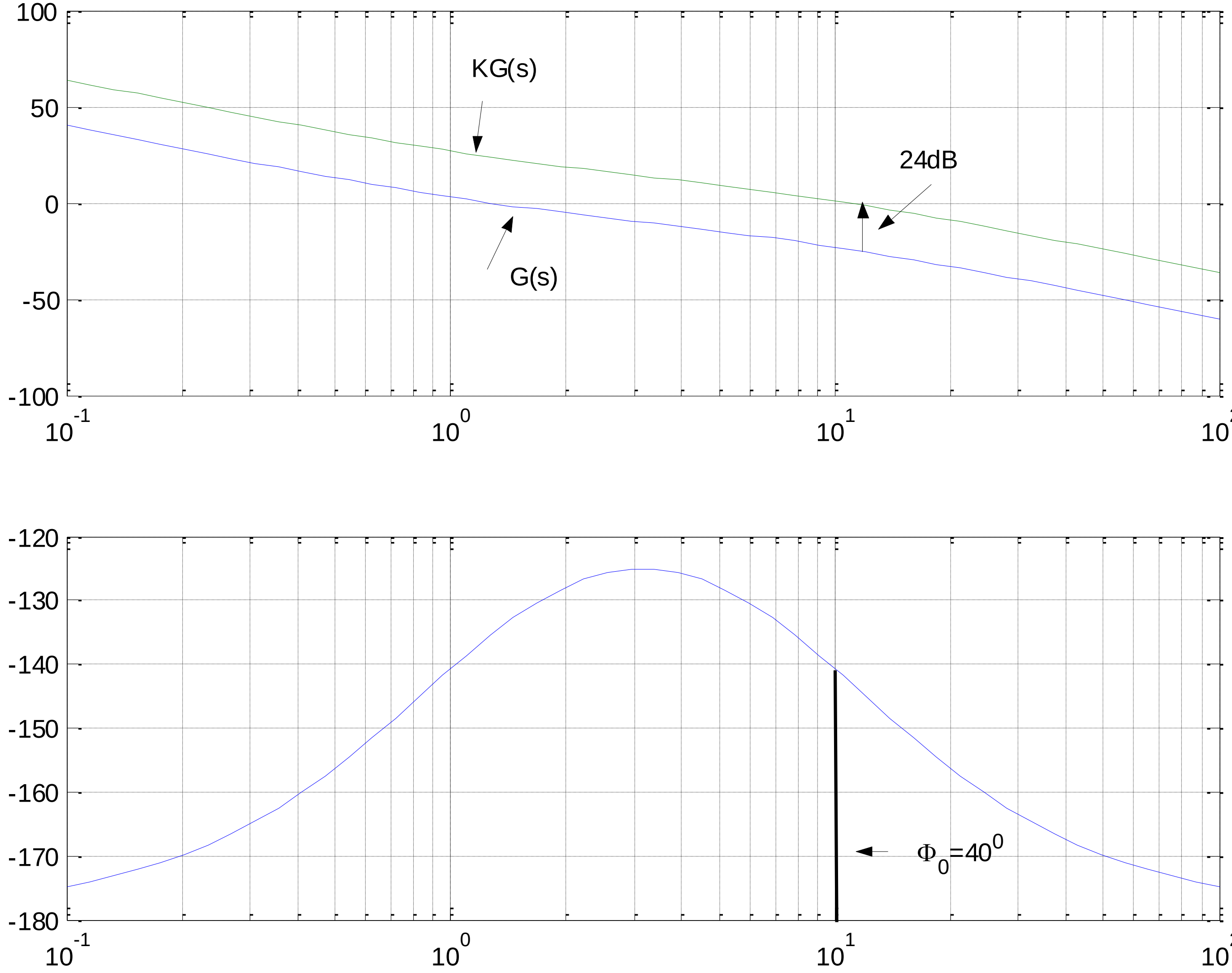
$$20\lg|C(j10)| = 0, \quad \angle C(j10) = 50^0$$

Therefore, the desired specifications are achieved.

Bode plots of $C(s)$



11.2



100
50
0
-50
-100
KG(s)
G(s)
24dB
10^{-1}
10^{0}
10^{1}
10^{2}
-120
-130
-140
-150
-160
-170
-180
$\Phi_0=40^0$
10^{-1}
10^{0}
10^{1}
10^{2}

(a) To achieve $\omega_g = 10$ rad/s, the magnitude plot is to be shifted up by 24 dB,

$$20\lg K = 24 \quad \Rightarrow \quad K = 15.8$$

With $K = 15.8$, the system has the phase margin of 40^0 and infinite gain margin. Hence, the system is stable.

(b) To achieve the phase margin $\phi_M = 60^0$ and gain cross over frequency of no less than 10 rad/s, a lead compensator is adopted (why not a lag compensator?)

$$\phi = \phi_M - \phi_0 + 8^0 = 28^0$$

$$\alpha = \frac{1 - \sin\phi}{1 + \sin\phi} = 0.3618$$

Since $10\lg\alpha = -4.4$ dB, from the Bode plots we know the new gain crossover frequency is

$$\omega_g^{'} \approx 13 \text{ rad/s}$$

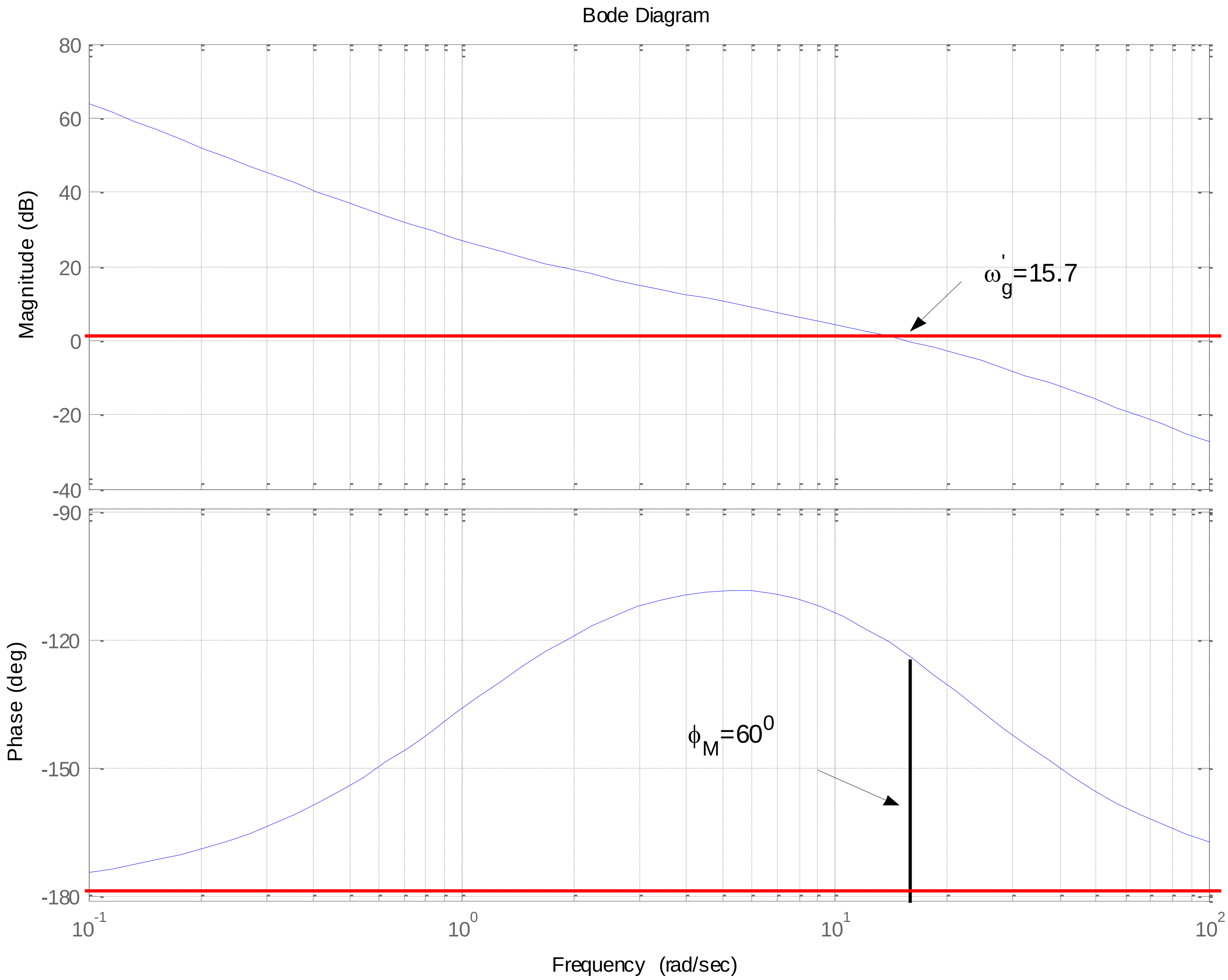
Set $\frac{1}{\sqrt{\alpha T}} = 13 \quad \Rightarrow \quad T = 0.1279$

Hence, the compensator is $C(s) = 15.8\frac{0.1279s+1}{0.0463s+1}$

From the Bode plots of $C(s)G(s)$, it can be seen that the gain crossover frequency is more than 10 rad/s and the phase margin is 60 degrees.

(c) No. The PI controller will bring the phase plot down. Then, the phase margin and the gain crossover frequency cannot be met.

Bode plots of $C(s)G(s)$



11.3

- Find compensator gain to meet $e_{ss} \leq 0.1$

$$K_V = \lim_{s\to 0} sKG(s) = K, \quad e_{ss} = \frac{1}{K} \leq 0.1 \quad \Rightarrow \quad K \geq 10$$

Choose K=10.

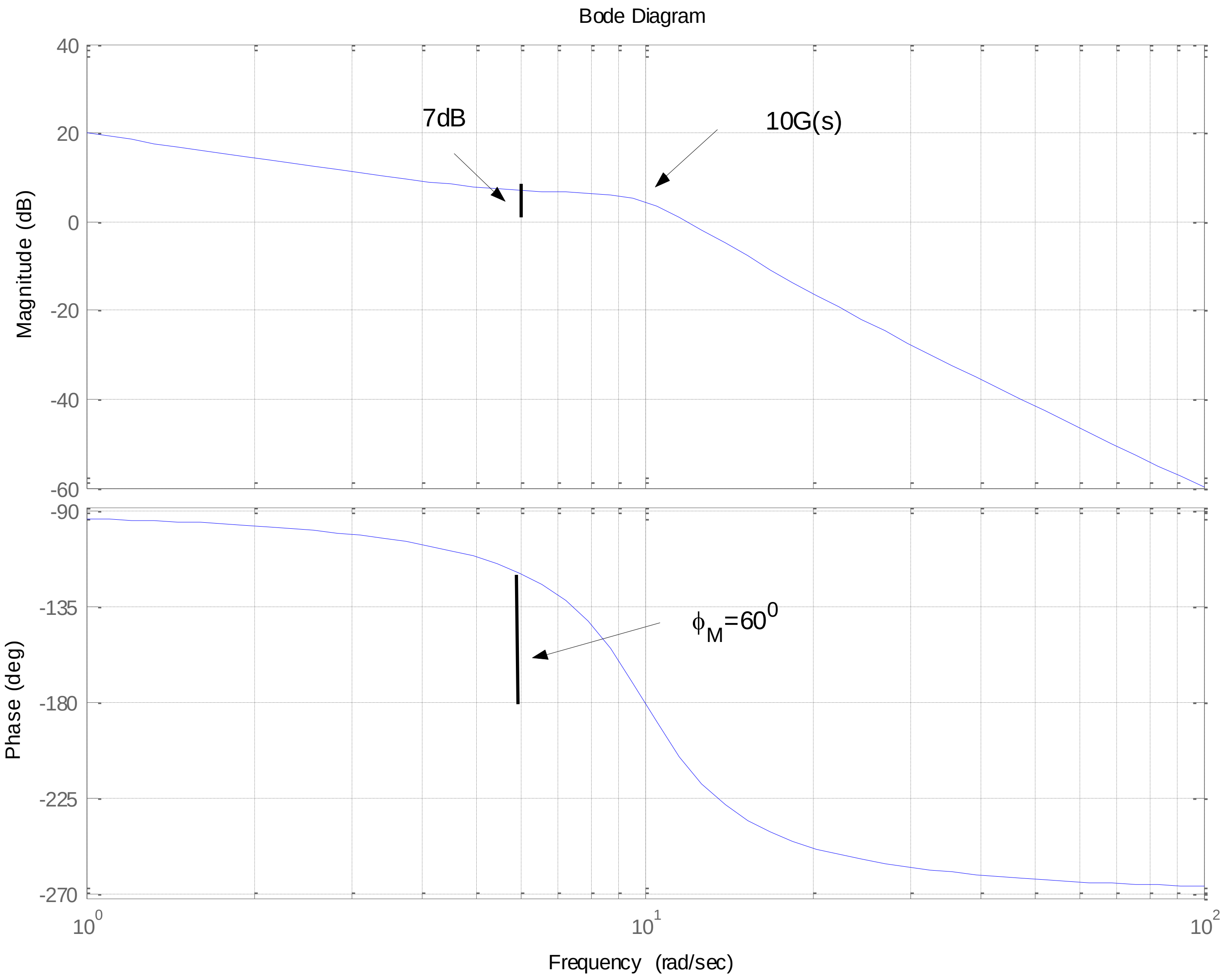
- Bode plots of KG(s) (shift the magnitude plot of G(s) by 20dB). See next slide.

- From the Bode plots of KG(s), to meet $\omega_g^{'} \leq 6$ rad/s, a lag compensator is used. Since the required PM= 50^0 , take

$$\phi_M = 50^0 + 10^0 = 60^0$$

From the Bode plots of KG(s), we know that $\omega_g^{'} = 6$ rad/s.

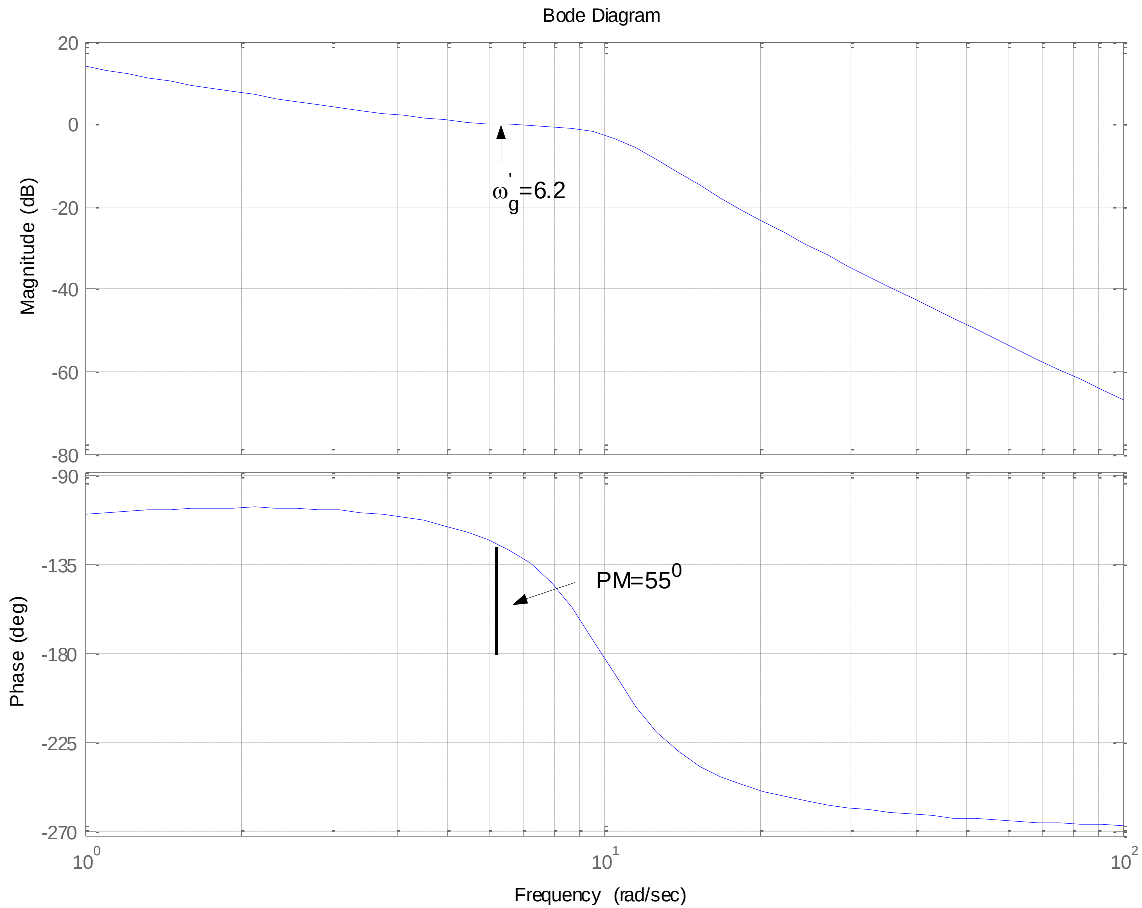
- Since $20\lg|10G(j6)| = 7$ dB, set $20\lg\beta = 7 \quad \Rightarrow \quad \beta = 2.2387$


Bode Diagram
Magnitude (dB)
40
20
0
-20
-40
-60
7dB
10G(s)
Phase (deg)
-90
-135
-180
-225
-270
$\phi_M = 60^0$
10^0
10^1
10^2
Frequency (rad/sec)

• Set $\frac{1}{T} = 0.1 \times 6 \quad \Rightarrow \quad T = 1.6667$

The compensator is obtained as $C(s) = 10\frac{1.6667s+1}{3.7312s+1}$

• Verify the performance by drawing the Bode plots of $C(s)G(s)$.



- With the delay of 0.2sec, the resulting open-loop phase is

$$\angle C(j\omega)G(j\omega) - \omega \times 0.2 \times \frac{180^0}{\pi}$$

while the magnitude plot remains the same, implying that the gain crossover frequency remains the same. Then, with delay the phase margin is reduced to

$$55^0 - 6.2 \times 0.2 \times \frac{180^0}{\pi} < 0$$

Hence, the system becomes unstable.