Tut # 8 Solutions

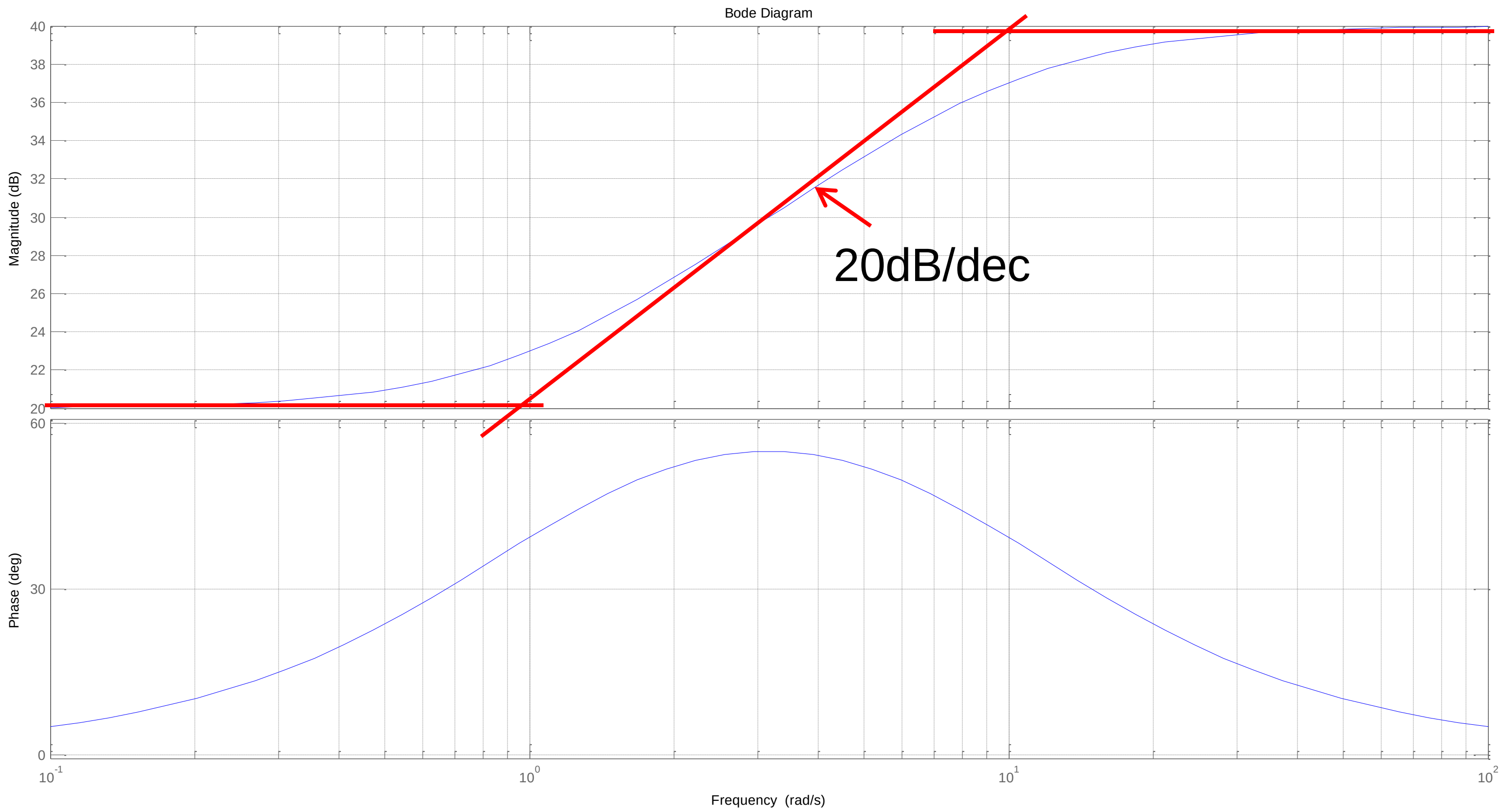
8.1 (a) $G_1(s) = \dfrac{100(s+1)}{s+10} = \dfrac{10(s+1)}{(0.1s+1)}$

Basic factors:

- 10 (20dB)
- $s+1$ (corner frequency at 1 rad/s)
- $0.1s+1$ (corner frequency at 10 rad/s)

Phase: $\angle G_1(j\omega) = 0^0 + \tan^{-1}\omega - \tan^{-1}(0.1\omega)$

(rad/s)	0.2	0.6	2	4	10	20	50
$\angle G_1(^0)$	10.6	27.5	52.1	54.1	39.3	23.7	10.2


Bode Diagram
Magnitude (dB)
40
38
36
34
32
30
28
26
24
22
20
20dB/dec
Phase (deg)
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

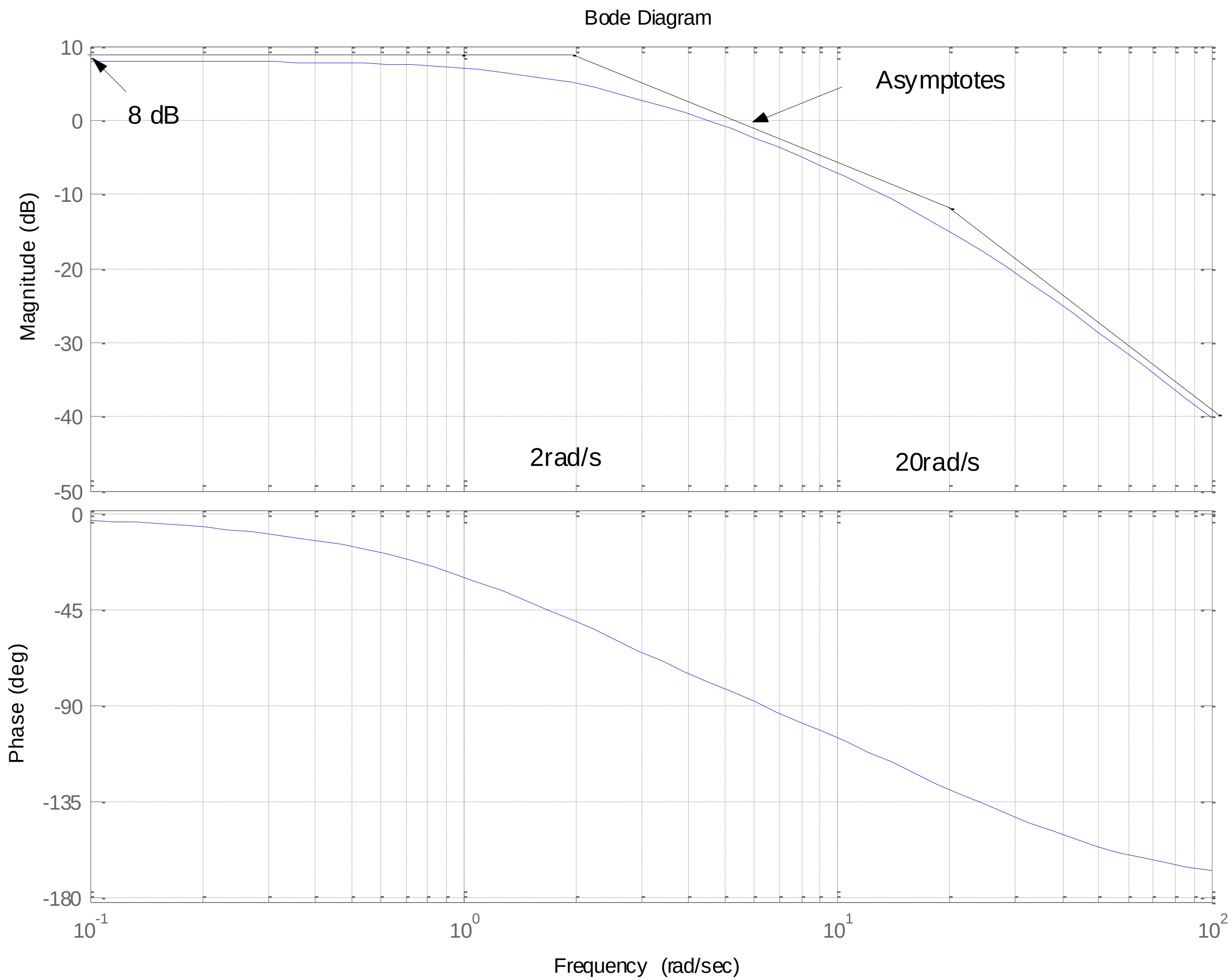
8.2 (b) Normalization: $$G_1(s) = \frac{2.5}{(0.5s+1)(0.05s+1)}$$

Basic factors:
- 2.5 (8 dB),
- 1st order pole with corner frequency of 2 rad/s
- another 1st order pole with corner frequency of 20 rad/s

Phase: $$\angle G_1(j\omega) = -\tan^{-1}(0.5\omega) - \tan^{-1}(0.05\omega)$$

ω (rad/s)	0.2	0.6	1	3	5	20	100
$\angle G_1$	-6.3^0	-18.4^0	-29.4^0	-64.8^0	-105.3^0	-129.3^0	-167.5^0

Magnitude and phase plots:



8.2 (c) Normalized transfer function: $$G_2(s)=\frac{1}{s\left[(s/10)^2+2\times0.1\times(s/10)+1\right]}$$

Basic factors:
- 1/s
- 2nd order complex pole with $\omega_n=10$ and magnitude correction

$$20\lg\frac{1}{2\zeta}=20\lg\frac{1}{0.2}=14 \text{ dB}$$

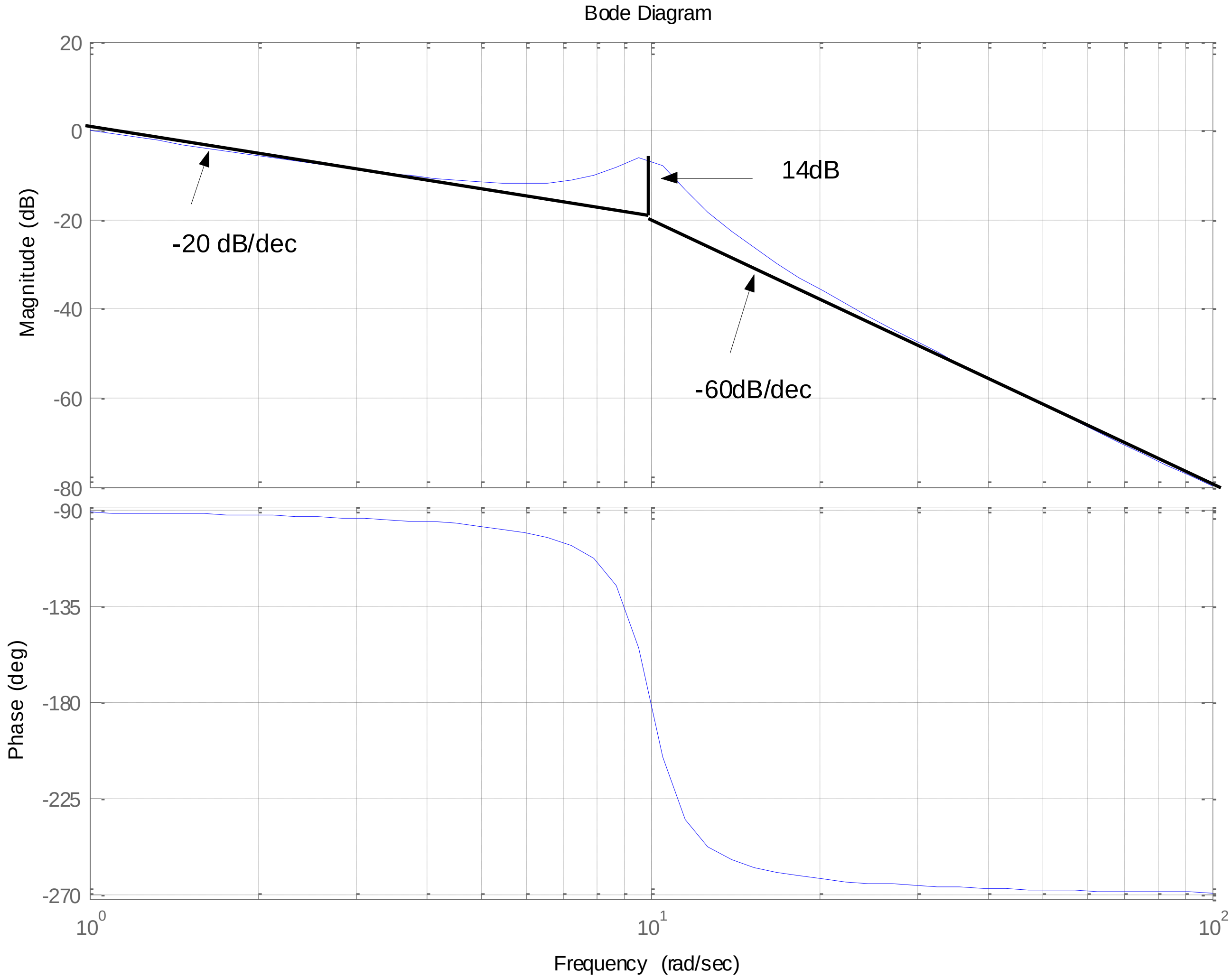
Phase: $$\angle G_2(j\omega)=-90^0-\tan^{-1}\frac{2\times0.1\times\left(\frac{\omega}{10}\right)}{1-\left(\frac{\omega}{10}\right)^2}$$

ω (rad/s)	2	5	9	10	20	40	100
$\angle G_2$	-92.4^0	-97.6^0	-133.5^0	-180^0	-262.4^0	-266.9^0	-268.9^0

Note: For $\omega>10$,

$$\tan^{-1}\frac{2\times0.1\times\left(\frac{\omega}{10}\right)}{1-\left(\frac{\omega}{10}\right)^2}=180^0-\tan^{-1}\frac{2\times0.1\times\left(\frac{\omega}{10}\right)}{\left(\frac{\omega}{10}\right)^2-1}$$

Magnitude (sketch and actual) and Phase Plots:



8.2 (d)

Normalized transfer function:

factor 2:

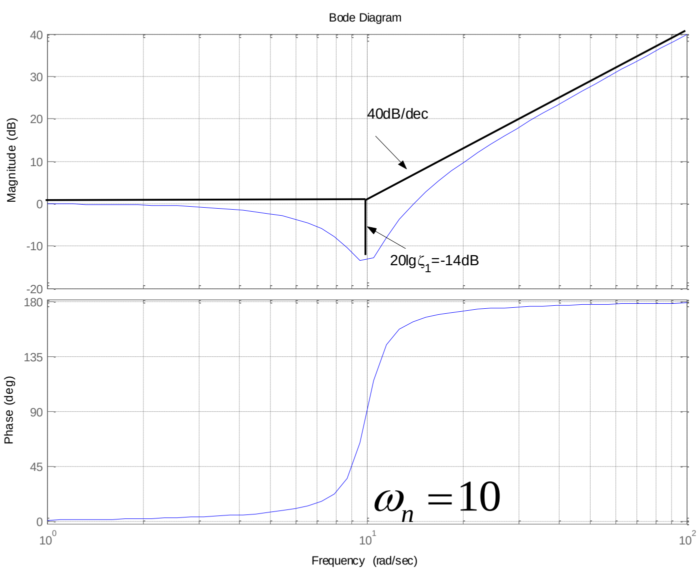
$$G_3(s)=\frac{\left(\frac{s}{\omega_n}\right)^2+2\zeta_1\frac{s}{\omega_n}+1}{\left(\frac{s}{\omega_n}\right)^2+2\zeta_2\frac{s}{\omega_n}+1}$$

Factor 1:

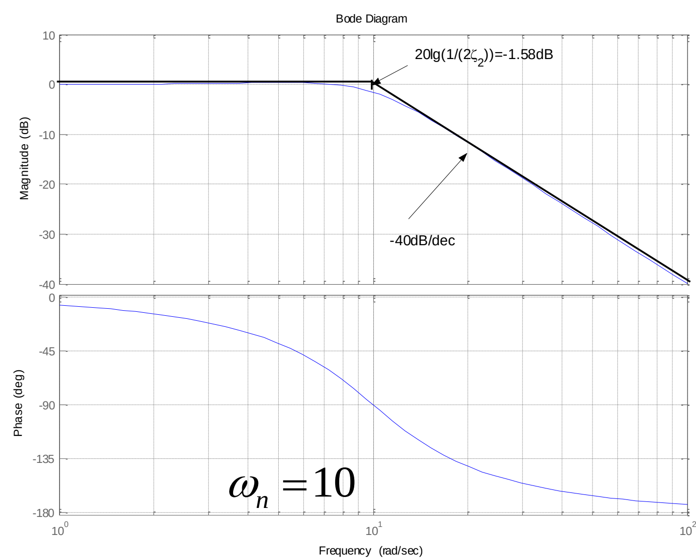
$$\left(\frac{s}{\omega_n}\right)^2+2\zeta_1\frac{s}{\omega_n}+1$$

Factor 2:

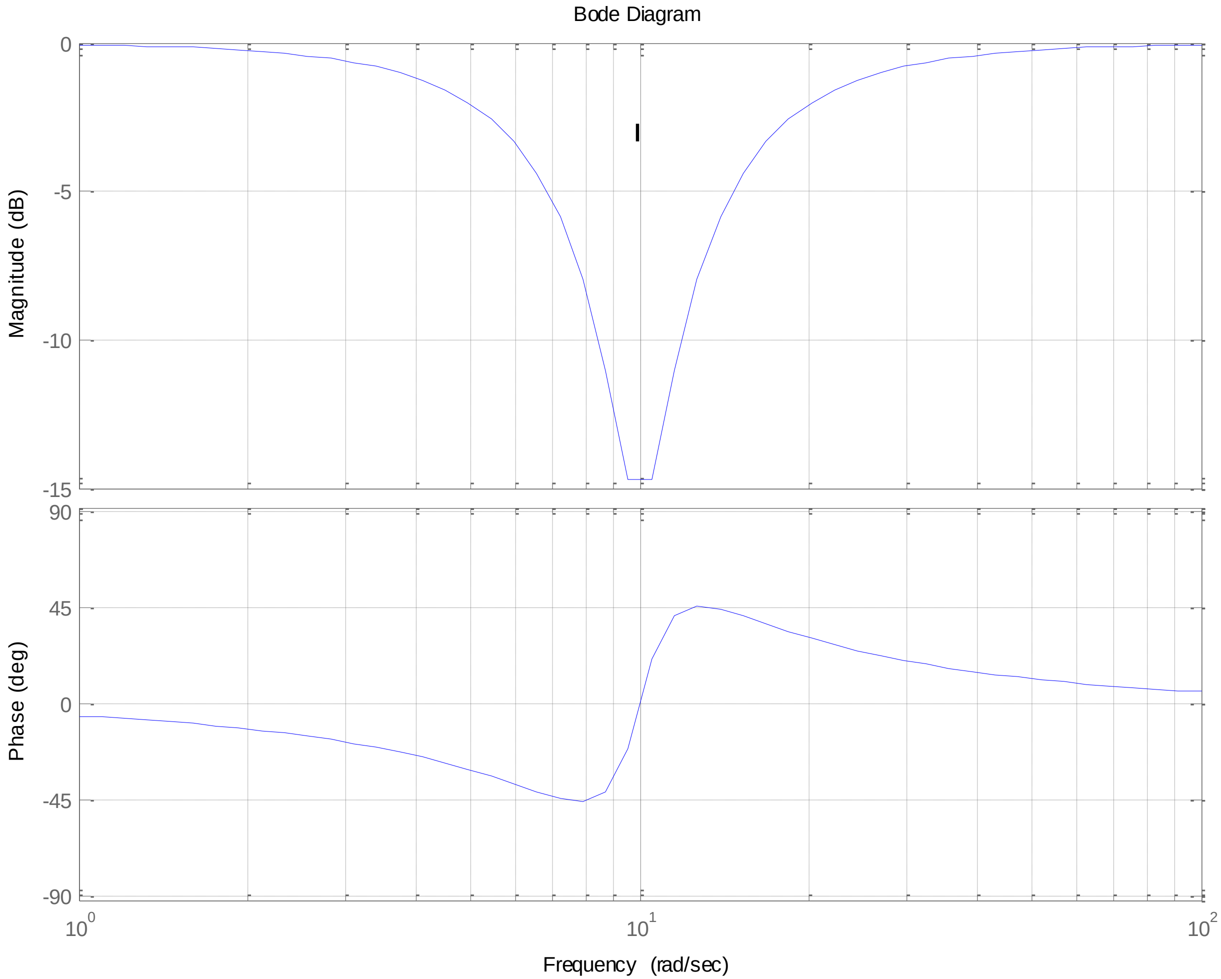
$$\frac{1}{\left(\frac{s}{\omega_n}\right)^2+2\zeta_2\frac{s}{\omega_n}+1}$$



$\zeta_1=0.1$



$\zeta_2=0.6$



Note: Notch filter has a lot of applications in control and signal processing. For example, it can be used to remove disturbance of known frequency or to remove resonance peaks in disk drives and other flexible bodies.

8.2

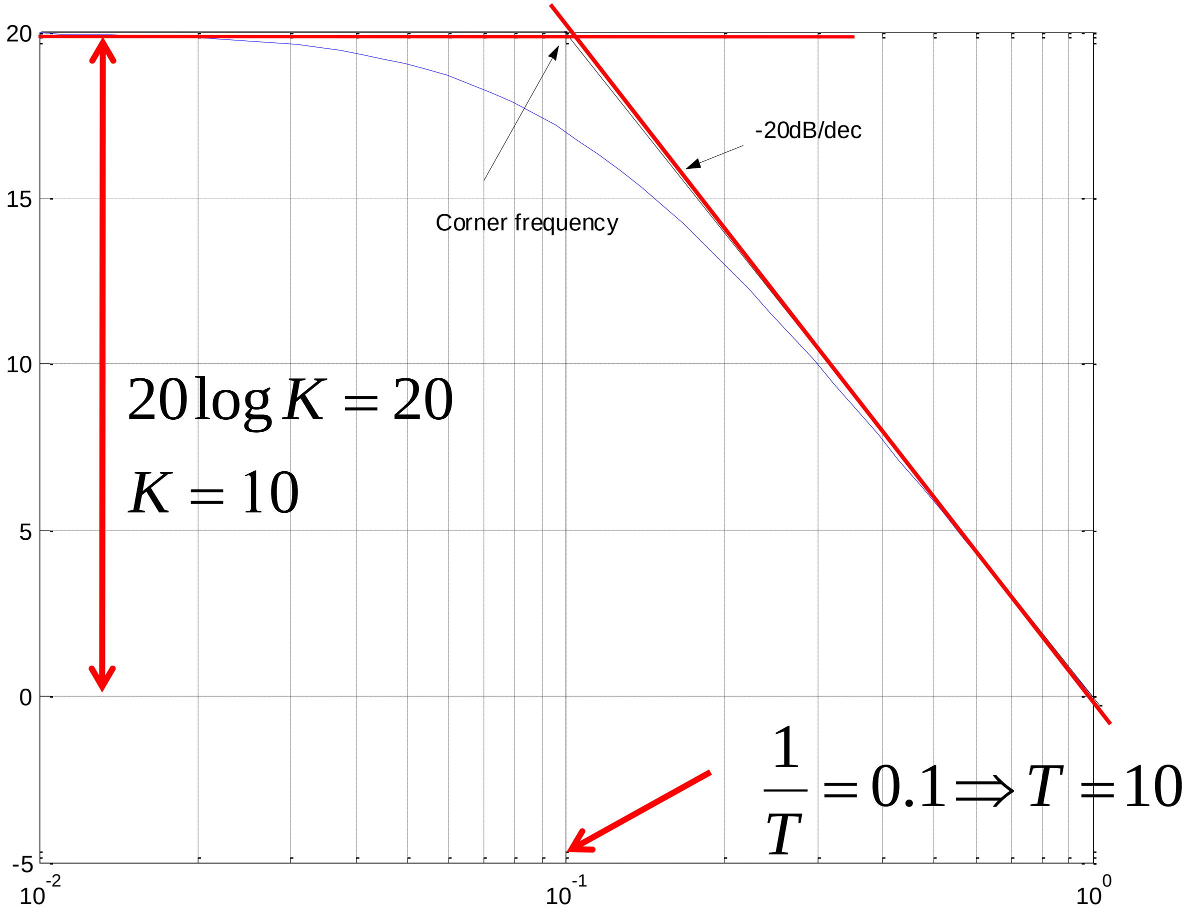
• At low frequency, the gradient of the asymptote is about -20 dB/dec, implying that there is an integrator. Hence, (i) is out.

• The extension of the low frequency asymptote inserts 0dB line at 100 rad/s, implying the normalized gain is 100.

• Between (ii) and (iii), (iii) has the normalized gain of 100. Hence, (iii) is the answer. In fact, from the asymptotes, we identify the basic factors:

$$\left.\begin{array}{ll} \checkmark & 100/s \\ \checkmark & (0.1s+1) \\ \checkmark & \dfrac{1}{1/50s+1} \end{array}\right\} \quad \text{Multiplying them leads to (iii)}$$

8.3 Approximate the magnitude by asymptotes



Transfer function for 1st order system: $\dfrac{K}{Ts+1}$