EE3011 MODELLING & CONTROL
Tutorial 1

Open-Loop and Closed-Loop Systems

1. Many open loop and closed-loop control systems may be found in homes. List several examples and describe them.

2. Give an example of a feedback control system in which a human acts as a controller. Describe briefly the actions of the controller in the system.

3. Figure 1 shows a flexible read/write head of a disk drive. M_c is the motor torque and M_D is the disturbance torque. Discuss the problems associated with the control of such disk drive. What are the desired performance specifications for the disk drive?

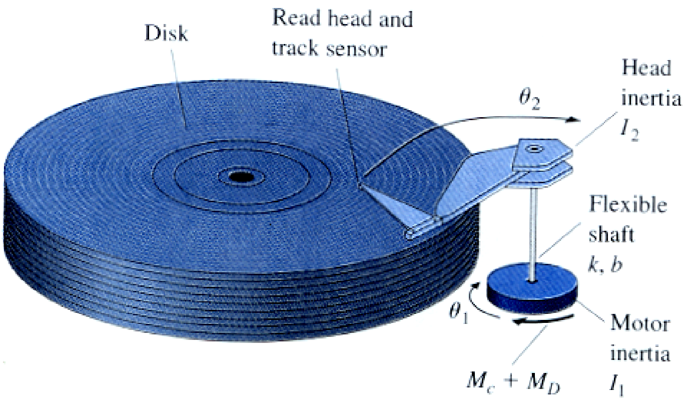

Figure 1 Flexible read/write disk drive head

4. The block diagram of a linear system is shown in Figure 2.

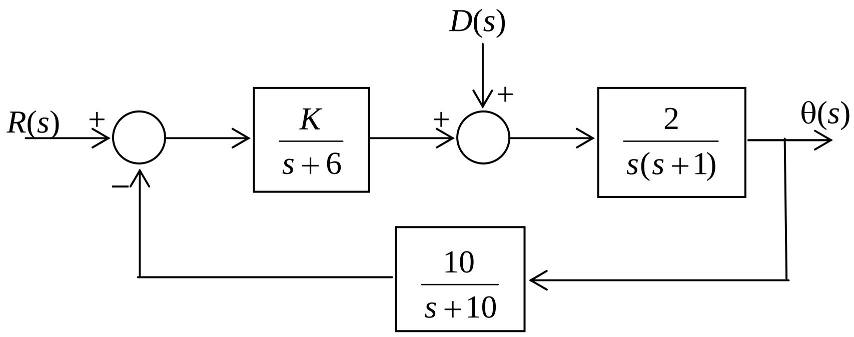

Figure 2

Find the two transfer functions, $\left.\frac{\theta(s)}{R(s)}\right|_{D=0}$ and $\left.\frac{\theta(s)}{D(s)}\right|_{R=0}$ in terms of the system gain K.

Obtain the output as a function of the 2 inputs.

[Ans: $\frac{2K(s+10)}{s(s+1)(s+6)(s+10)+20K}$, $\frac{2(s+6)(s+10)}{s(s+1)(s+6)(s+10)+20K}$]

5. Investigate the effect of external unit-step disturbance $T_L(s) = 1/s$ on open loop and closed-loop systems given in figures 3 and 4:

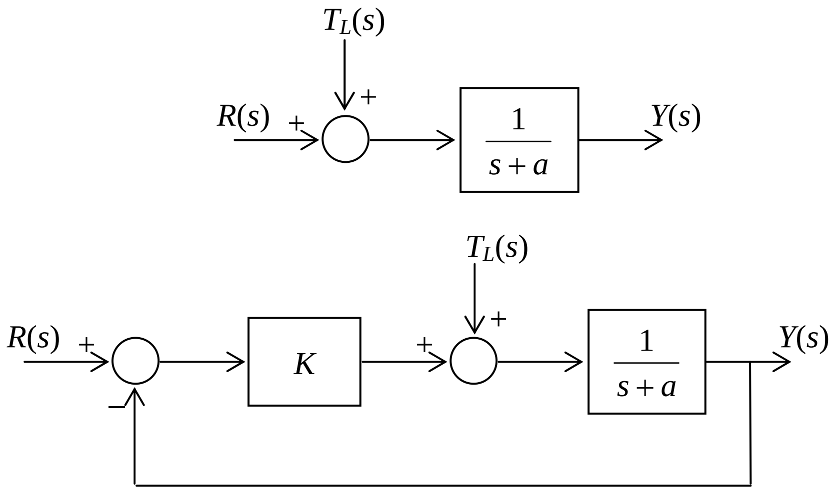

Figure 3 Disturbance entering at the input

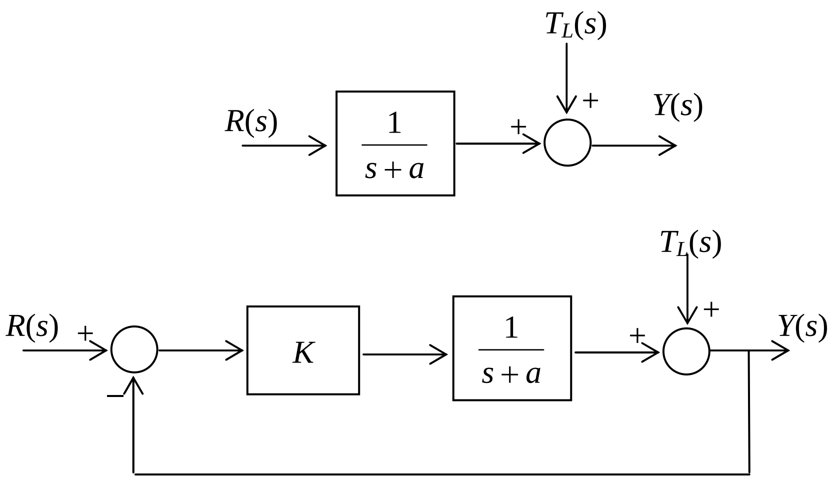

Figure 4 Disturbance entering at the output

You may use the following Laplace transform pairs:

$$\frac{1}{s+\alpha}\cdot\frac{1}{s} \;\Leftrightarrow\; \frac{1}{\alpha}(1-e^{-\alpha t}) \quad ; \quad \frac{s+\beta}{s+\alpha}\cdot\frac{1}{s} \;\Leftrightarrow\; \frac{1}{\alpha}(\beta-(\beta-\alpha)e^{-\alpha t})$$

EE3011 MODELLING & CONTROL
Tutorial 2

System Modelling

1. A control system is as shown in Figure 1. Derive the closed-loop transfer function $Y(s)/R(s)$.

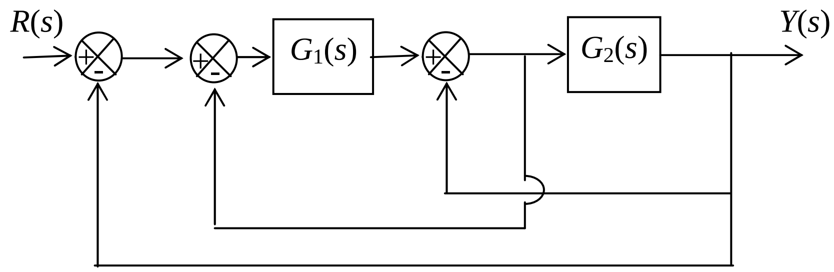

Figure 1

[Ans: $\dfrac{Y}{R} = \dfrac{G_1G_2}{1+G_1+G_2+G_1G_2}$]

2. Find the transfer function, $T(s) = C(s)/R(s)$, for the system shown in Figure 2. If the feedback path of H_2 fails, what's the resultant transfer function?

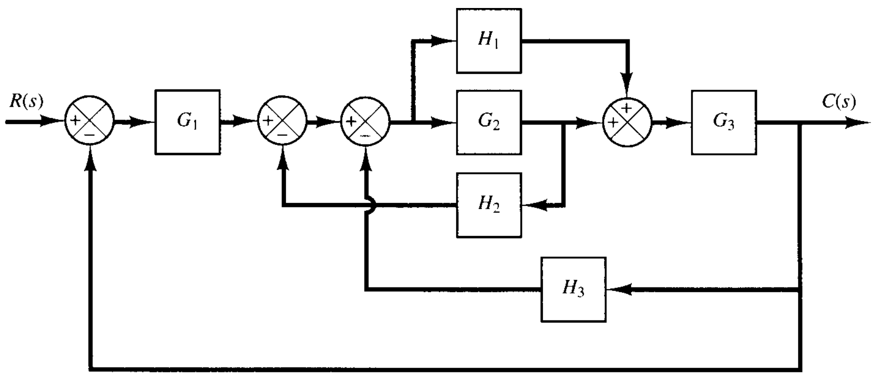

Figure 2 Block diagram of a control system

[Ans: $\dfrac{G_1G_2G_3 + G_1G_3H_1}{1+G_2H_2+G_2G_3H_3+G_3H_1H_3+G_1G_2G_3+G_1G_3H_1}$]

3. In a satellite orientation control problem, the reference satellite orientation and the actual orientation are $\theta_r(t)$ and $\theta_y(t)$, respectively. These orientations are measured via sensors that convert them into voltage signals $v_i(t)$ and $v_o(t)$, respectively. A controller, with impulse response $g_1(t)$, acts on the error signal to produce a voltage $v(t)$ that excites the thruster to produce a torque $T(t)$. The thruster is known to have an impulsive response of $g_2(t)$. The torque will ultimately cause the satellite to orientate to the desired orientation. The satellite has a moment of inertia J and it has negligible frictional torque. The relevant equations of motion are given by

$$v_i(t) = K_1\theta_r(t)\,;\quad v_o(t) = K_1\theta_y(t)$$
$$e(t) = v_i(t) - v_o(t)$$
$$V(s) = G_1(s)E(s)$$
$$T(s) = G_2(s)V(s)$$
$$J\ddot{\theta}_y(t) = T(t)$$

In the above equations, K_1 denotes the gain of the sensor used, $G_1(s)$ and $G_2(s)$ are respectively the transfer functions of the controller and the thruster, while $E(s)$, $V(s)$ and $T(s)$ are the Laplace transforms of $e(t)$, $v(t)$ and $T(t)$, respectively. Derive a block diagram that gives the functional description of the above orientation control problem.

4. In large disk drive systems containing linear actuators, the motion is control by a voice-coil motor (VCM), as shown in Figure 3. The force F produced is proportional to the current i in the coil. The link between the head (M_2) and the servo body (M_1) is flexible with spring constant K. Draw a block diagram of the system and obtain the transfer function from input e_c to output y. The relevant equations are given below.

Mechanical system: $F = M_1\dfrac{d^2q}{dt^2} + K(q-y)$ and $0 = M_2\dfrac{d^2y}{dt^2} + K(y-q)$

Current flowing in magnetic field produces the force: $F = a_1 i$

Motion in magnetic field produces e_{coil} across the coil: $e_{coil} = a_2\dfrac{dq}{dt}$

Electrical circuit: $L\dfrac{di}{dt} + Ri = e_c - e_{coil}$

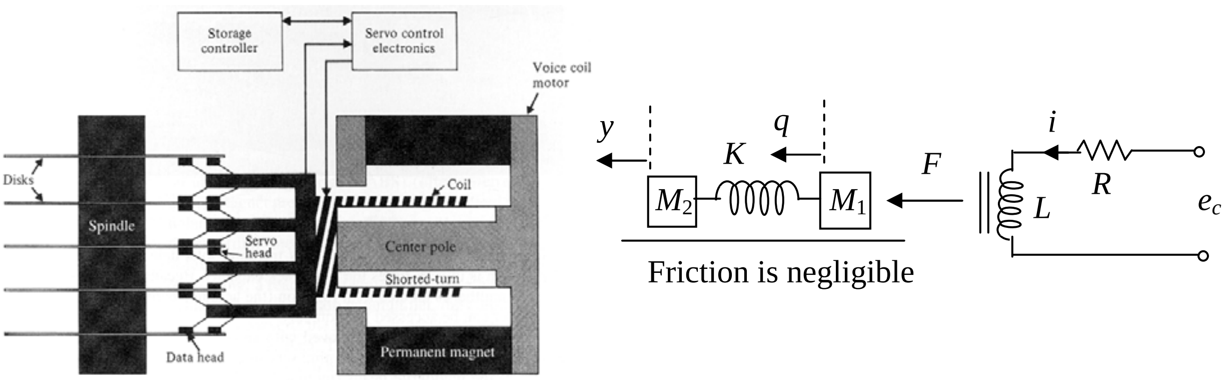

Figure 3

5. Find the transfer function of each subsystem for the antenna azimuth control system shown in Figure 4. Obtain the overall transfer function. Note that the transfer function of the motor with no load attached is given in the lecture note. Here, a load is connected to the motor through a gear train, so it is given by

$$\frac{\theta_M(s)}{E_a(s)} = \frac{K_T / R_a}{s(Js + B + \frac{K_T K_b}{R_a})}$$

where $J = J_M + \left(\frac{N_1}{N_2}\right)^2 J_L$ and $B = B_M + \left(\frac{N_1}{N_2}\right)^2 B_L$

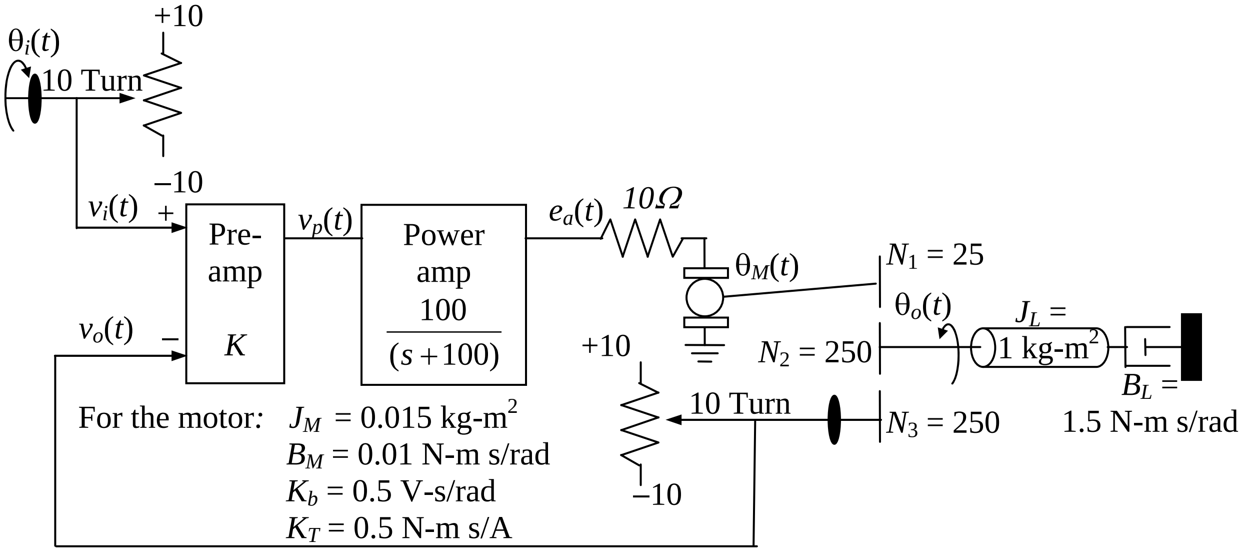

Figure 4 Azimuth control system.

The input pot is model as $\frac{V_i(s)}{\theta_i(s)} = \frac{20}{10(2\pi)} = \frac{1}{\pi}$. The output pot is similar.

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Tutorial 3

Time Domain Analysis

1. A thermometer requires 1 min to indicate 98% of the response to a step input. Assuming the thermometer to be a first-order system, find the time constant.

   If the thermometer is placed in a bath, the temperature of which is changing linearly at a rate of 10 °/min, how much error does the thermometer show?

2. A second-order under-damped system has the overall transfer function as follows:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

   It is possible to achieve a response that does not exhibit any overshoot and undershoot by an appropriate choice of the input function $r(t)$ to give the desired output as shown in Figure 1. Determine the value of t_1, A and B when $\zeta = 0.4$, $\omega_n = 1.0$.

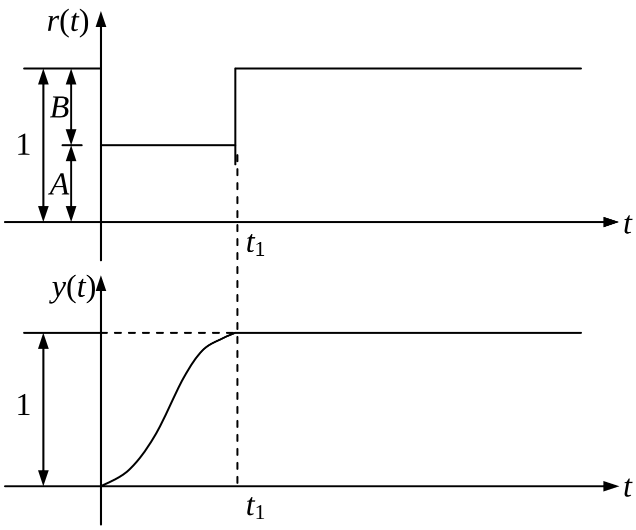

Figure 1

[Ans: 3.43 secs., 0.8, 0.2]

3. Not all second order systems are designed to give a standard 2nd order response. Consider the power steering for an automobile. The feedback system can be modelled as the block diagram shown in Figure 2. For a unit step input $A(s)$, find values of K_1 and K_2 for which the response $w(t)$ is critically damped and has a steady-state gain of 0.3 unit. Repeat for a damping ratio of 0.7 and a steady-state gain of 0.2 unit.

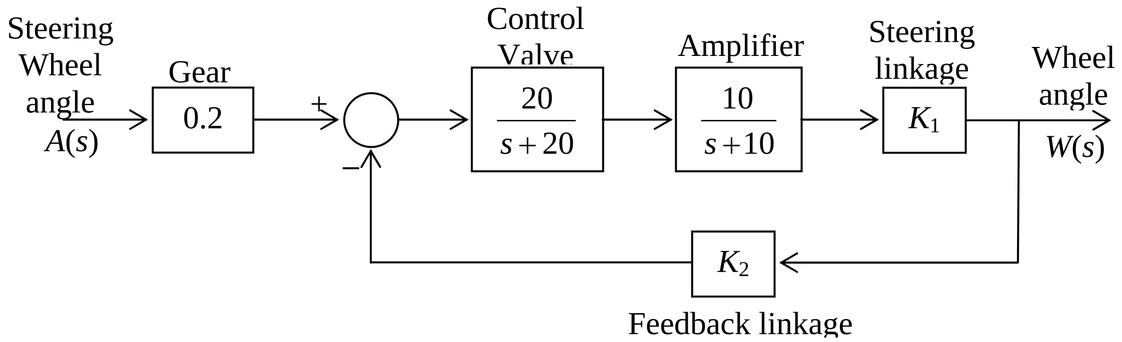

Figure 2

[Ans: 1.69, 0.074; 2.30, 0.56]

4. We have learnt that the step response of a non-standard 2nd order system with a zero is more complicated. Nevertheless, it is still easy to determine the peak time, the maximum overshoot, and the rise time of such a system. Consider the following critically damped system

$$\frac{Y(s)}{R(s)} = \frac{9(\alpha s+1)}{s^2+6s+9}$$

Determine the value of α for which there is an overshoot. Hence find the peak time the maximum overshoot and the rise time as a function of α when a unit-step is applied to the system.

If the non-standard 2nd order system is under-damped, then the calculation is more involved. If you have some time to kill, consider the system

$$\frac{Y(s)}{R(s)} = \frac{2(\alpha s+1)}{s^2+2s+2}$$

and determine the peak time, maximum overshoot and rise time for α = 0, 0.5 and 1.

[Ans: $\alpha > 1/3$]

EE3011 MODELLING & CONTROL
Tutorial 4

Stability Analysis

1. The characteristic equation of a 3rd order system is $q(s) = s^3 + a_2 s^2 + a_1 s + a_0$. It's known that the parameters a_0, a_1 and a_2 are positive. Use the Routh array to determine the relationship between a_0, a_1 and a_2 such that the system is stable.

   [Ans: $a_2 a_1 > a_0$]

2. For the following polynomials, use the Routh-Hurwitz Criterion to test the presence of the roots in the left-half plane, right-half plane and on the imaginary-axis of *s*-plane. Where applicable, show that the auxiliary equation is a factor of the original characteristic equation.
   (a) $s^4 + 2s^3 + 10s^2 + 20s + 5$
   (b) $s^6 + s^5 + 4s^4 + 4s^3 + 5s^2 + 4s + 2$

3. For each of the characteristic equations of feedback control system given, find the range of K for which the closed-loop system is stable. At what value of K will the system have undamped oscillation, if any, and what is the corresponding frequency of oscillation? Determine the range of K, if any, for which all the roots of the characteristic equations lie to the left of $s = -1.0$
   (a) $s^4 + 10s^3 + 25s^2 + 50s + K = 0$
   (b) $s^3 + (K + 3)s^2 + 3Ks + 2K = 0$

   [Ans: $0 < K < 100$, 100, $\sqrt{5}$, nil, $K > 0$, nil, $K > \frac{3+\sqrt{17}}{2}$]

4. To determine the stability of systems with 2 unknown parameters, we need to represent the 2 parameters in a plane and apply Routh array to identify appropriate regions that will yield certain responses. This is not too difficult for low-order systems. Consider the block diagram of a disk storage data-head positioning system as shown in Figure 1. Derive the relationship between K_P and K_D for which the closed-loop system is stable. Construct a parameter plane of K_P versus K_D and show the following regions in the plane.

   (a) stable and unstable regions.
   (b) trajectory on which the system will have undamped oscillation.
   (c) the point in which the undamped oscillation frequency is 2 rad/s.
   (d) region where all the poles are to the left of $s = -1.0$ in the *s*-plane. (This is more difficult.)

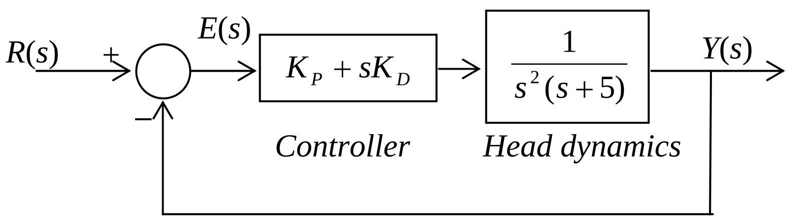

Figure 1

EE3011 MODELLING & CONTROL
Tutorial 5

System Performance

1. For the system shown in Figure 1, the nominal value of *a* is *a* = 5.

   (a) Find $\dfrac{Y(s)}{R(s)}$.
   (b) What type of system does it represent?
   (c) Find the position, velocity and acceleration error constants.
   (d) Find the steady state value of $y(t)$ if $r(t)$ is a unit step.
   (e) Repeat the above computations when $a = 2$.

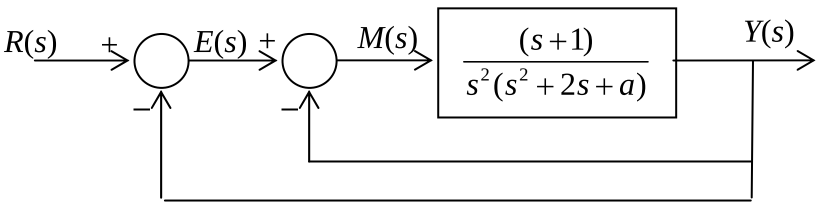

Figure 1

2. Consider the simplified satellite attitude control problems shown in Figures 2a and 2b, respectively. Determine the effect of K_P and K_I on a step disturbance input.

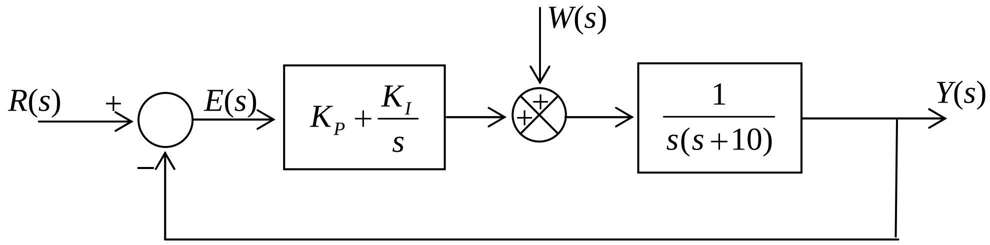

Figure 2a

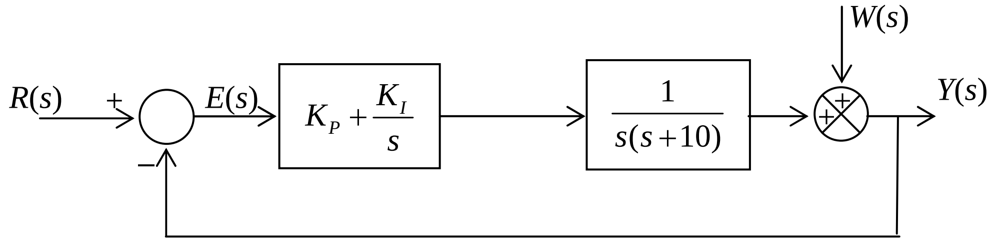

Figure 2b

3. Consider Figure 3 of a unity-feedback control system. Derive the sensitivities of the closed-loop transfer function with respect to parameters K, a and b.

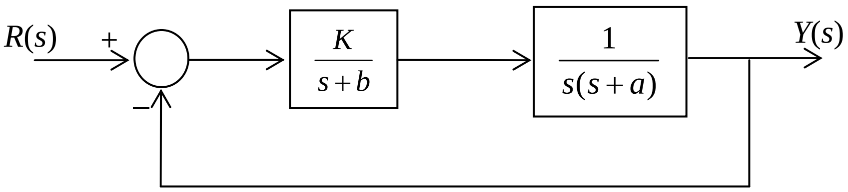

Figure 3

4. Consider the control system given in Figure 4. The parameter α has a nominal value of 10. The system is open-loop when $H(s)$ = 0, and closed-loop when $H(s)$ = 1. Determine the sensitivity of both the open-loop and closed-loop transfer functions to α about the nominal value of α. Compare and comment on the sensitivity of the system for K = 1 and K = 100.

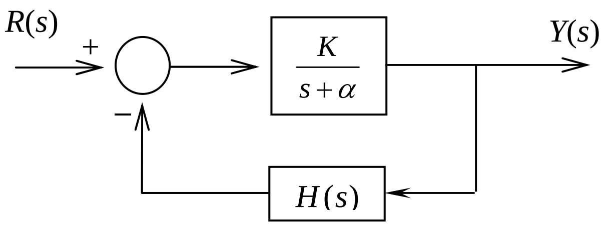

Figure 4

5. Consider the open-loop and closed-loop systems given in Figure 5. Show that the 2 systems have the same transfer function when K = 2. Compare the sensitivities of the systems with respect to deviation of K from K = 2.

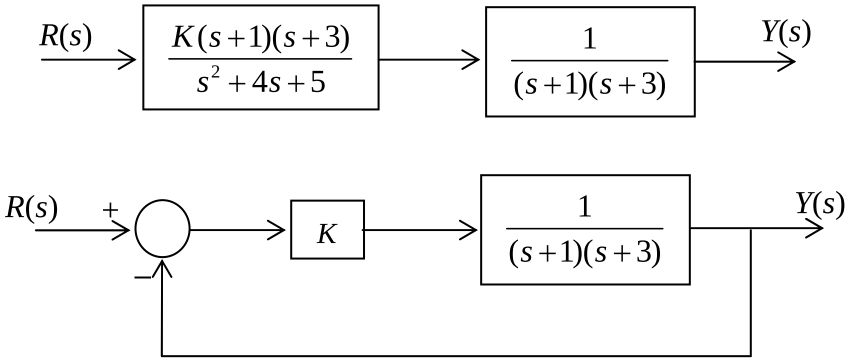

Figure 5

EE3011 MODELLING & CONTROL
Tutorial 6

PID Controller Design and Simple Root Locus

1. Consider Figure 1 of an automatic numerically controlled machine-tool position control system, using a punched tape reader to supply the reference signal.

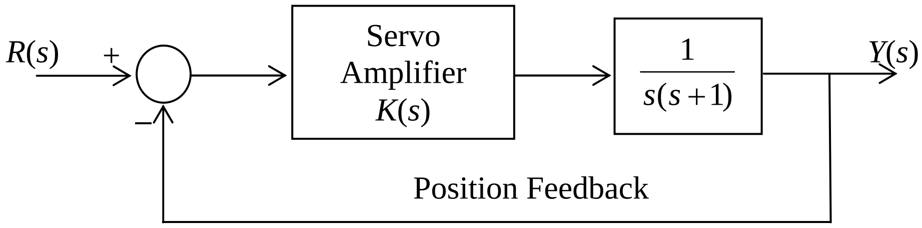

Figure 1

Design a proportional controller K_P such that the resultant damping ratio $\zeta = 0.25$, and determine the resultant undamped natural frequency ω_n. Compute the percentage overshoot M_P and 0-100% rise time t_r resulting from a unit step input. Find also the steady-state error resulting from the application of unit ramp input.

With the K_P obtained above, design $K(s) = K_P + sK_D$ such that $\zeta = 1.0$ (i.e. critically-damped response). Would there be any overshoot with respect to a unit-step input?

[Ans: 4; 2.0; 44.4%; 0.94 sec; 1/4; 3.0]

2. A control system with a type-0 process and PID controller is shown in Figure 2. Design the parameters of the PID controller so that the following specifications are satisfied:
   - e_{ss} (unit step input) = 0
   - Rise time, t_r = 0.5 sec
   - A damping ratio of $\zeta = 0.8$.

[Additional exercise: Examine the resultant closed-loop systems and comment on the pole-zero locations. Can you suggest a simpler approach to the controller design?]

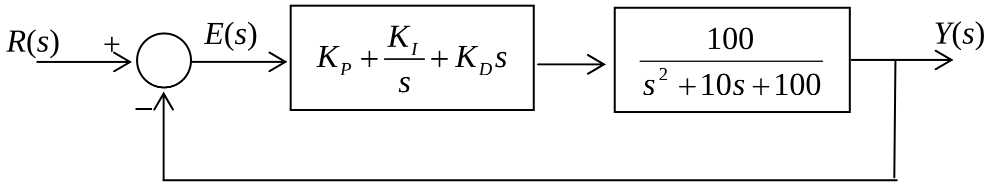

Figure 2

[Possible Ans: K_P = 8.57, K_D = 0.7, K_I = 46.22]

3. Consider the control system shown in Figure 3.

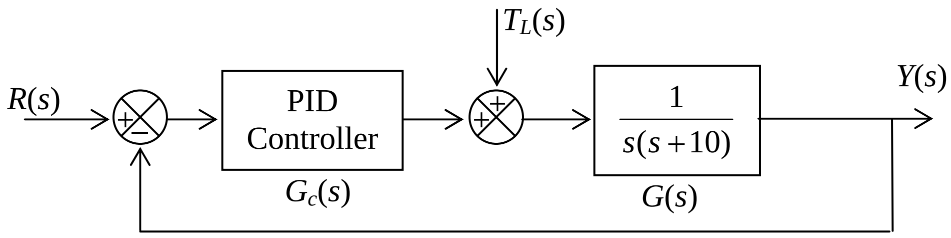

Figure 3

a) Design a PID controller that is able to provide an overshoot of no more than 5% (or equivalently damping ratio of 0.7) for the closed-loop system and at the same time able to reject step disturbance at steady-state. What if you also want to ensure that the rise-time is approximately 1.0 sec with respect to a step input?

b) Design a PI controller to yield 2 closed loop poles at $s = -1 \pm j1$, and at the same time able to reject a step disturbance at steady state.

[Ans: K_P = 116.55, K_D = 17.59, K_I = 248.14; K_P = 18, K_I = 16]

4. Consider the following open-loop transfer functions:

(a) $KG(s)H(s) = \dfrac{K}{s+2}$

(b) $KG(s)H(s) = \dfrac{K(s+2)}{s(s+4)}$

(b) $KG(s)H(s) = \dfrac{K(s+2)}{s^2}$

Sketch the locus of the closed-loop poles as K is vary from 0 to infinity.

EE3011 MODELING AND CONTROL
Tutorial 7

Frequency Responses

7.1 Consider the unity-feedback system with the open-loop transfer function

$$G(s)=\frac{10}{(s+1)}$$

Obtain the steady-state output of the system when it is subjected to each of the following inputs:

(a) $r(t)=\sin(t+30^0)$

(b) $r(t)=2\cos(2t-45^0)$

(c) $r(t)=\sin(t+30^0)-2\cos(2t-45^0)$

[Ans: $(a)\ 0.905\sin(t-24.8^0);\ (b)\ 1.79\cos(2t-55.3^0)$]

7.2 Calculate the magnitude and phase of the following transfer function at the frequency of 3 rad/s:

(a) $G_1(s)=\dfrac{10(s-1)}{(s+1)}$

(b) $G_2(s)=\dfrac{10(s^2-1)}{s^2(s+1)}$

[Ans: $(a)\ 10,\ 36.9^0;\ (b)\ 3.51,\ -71.6^0$]

7.3 Find the magnitude and phase responses of the following transfer function

$$G(s)=\frac{1}{(s+1)(s^2+s+1)}$$

Compute the steady-state responses to sinusoid $r(t)=2\sin(\omega t+30^0)$ with frequency $\omega=0.1,\ 1,\ 10$ rad/s, respectively. Explain the results.

[Ans: $2\sin(0.1t+18.5^0);1.414\sin(t-105^0);0.002\sin(10t-228.5^0)$]

7.4 A control system for a flexible structure is shown in Figure 7.1 where $d(t)$ is an output disturbance due to an external vibration source. Evaluate the effect of the disturbance on the output if the disturbance is of frequency 1 rad/s and 100 rad/s, respectively.

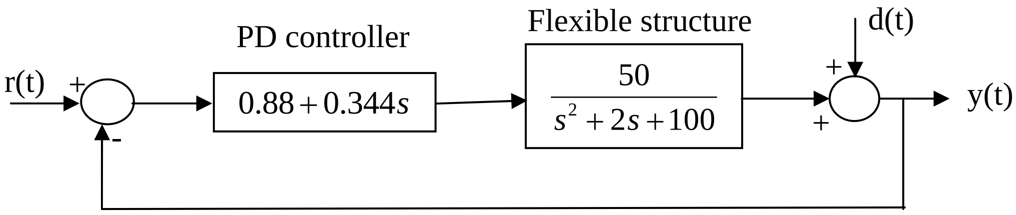

Figure 7.1

[Ans: 3.2dB attenuation; 0dB attenuation]

EE3011 MODELING AND CONTROL
Tutorial 8

Bode Plots/Frequency Domain Modeling

8.1 Sketch the Bode plots of the following simple transfer functions:

(a) $G_1(s) = \dfrac{100(s+1)}{s+10}$

(b) $G_2(s) = \dfrac{100}{(s+2)(s+20)}$

(c) $G_3(s) = \dfrac{100}{s(s^2+2s+100)}$

(d) $G_4(s) = \dfrac{s^2+2\zeta_1\omega_n s+\omega_n^2}{s^2+2\zeta_2\omega_n s+\omega_n^2}$ $(\zeta_1 = 0.1, \zeta_2 = 0.6, \omega_n = 10)$ (notch filter)

Verify the Bode plots using Matlab.

8.2 The magnitude response plot and its asymptotes of a minimum phase system are shown in Figure 8.1. Which of the following transfer functions best approximates the frequency response of the system? Explain your answer.

(i) $\dfrac{500(s+10)}{(s+1)(s+50)}$ (ii) $\dfrac{100(s+10)}{s(s+50)}$ (iii) $\dfrac{500(s+10)}{s(s+50)}$

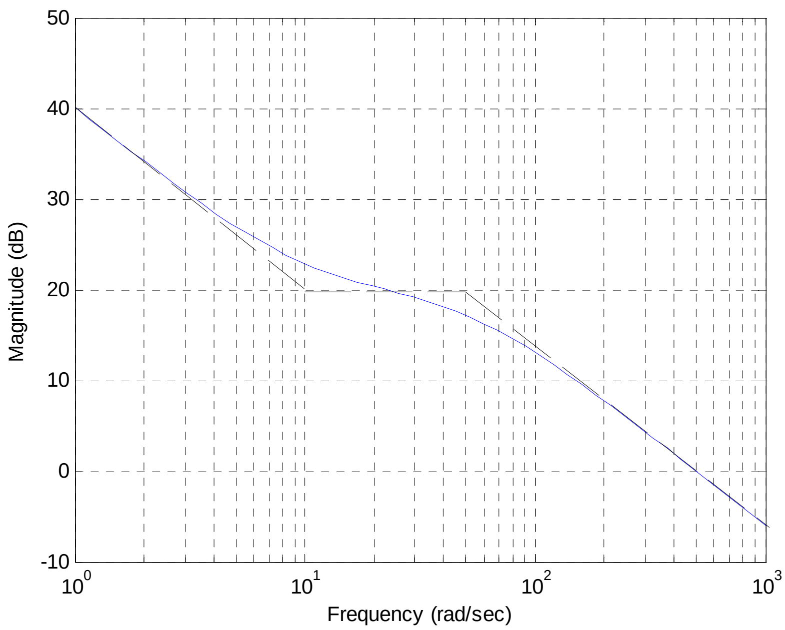

Figure 8.1

8.3 The magnitude response of a first order system is given in Figure 8.2. Find the gain and the time constant of the system.

[Ans: 10; 10 units]

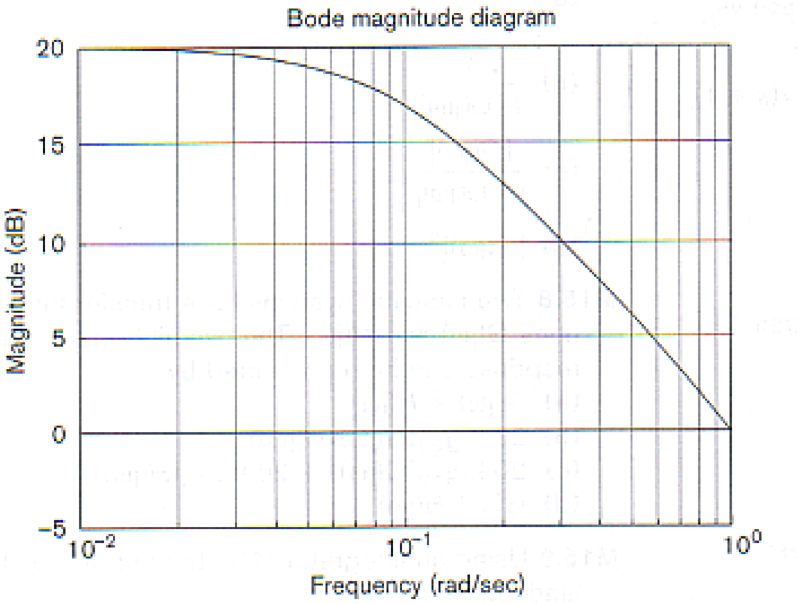

Figure 8.2

EE3011 MODELING AND CONTROL
Tutorial 9

Frequency Domain Modeling/Nyquist Stability Criterion

9.1 The Bode magnitude plot of a minimum phase system is shown in Figure 9.1. Determine an approximate mathematical model in the form of transfer function for the system.

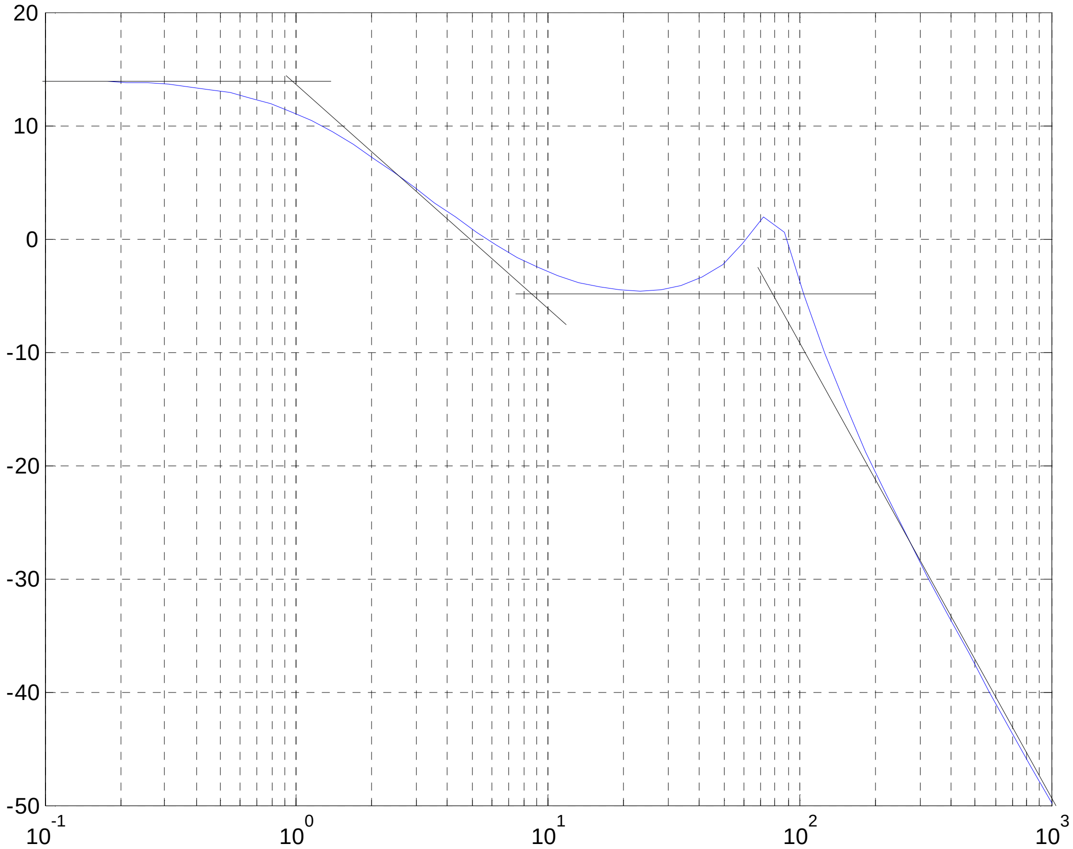

Figure 9.1

[Ans: $\dfrac{5(1/9s+1)}{(s+1)\left[\left(\dfrac{s}{80}\right)^2+0.5\dfrac{s}{80}+1\right]}$]

9.2 The Bode magnitude plot of a minimum phase system whose transfer function is of the form

$$G(s)=\frac{K(s+a)}{s^N(s+0.5)(s^2+2\zeta\omega_n s+\omega_n{}^2)}$$

is given below. Estimate the values of K, N, a, ζ and ω_n.

[Ans: 64, 1, 2, 0.2, 8]

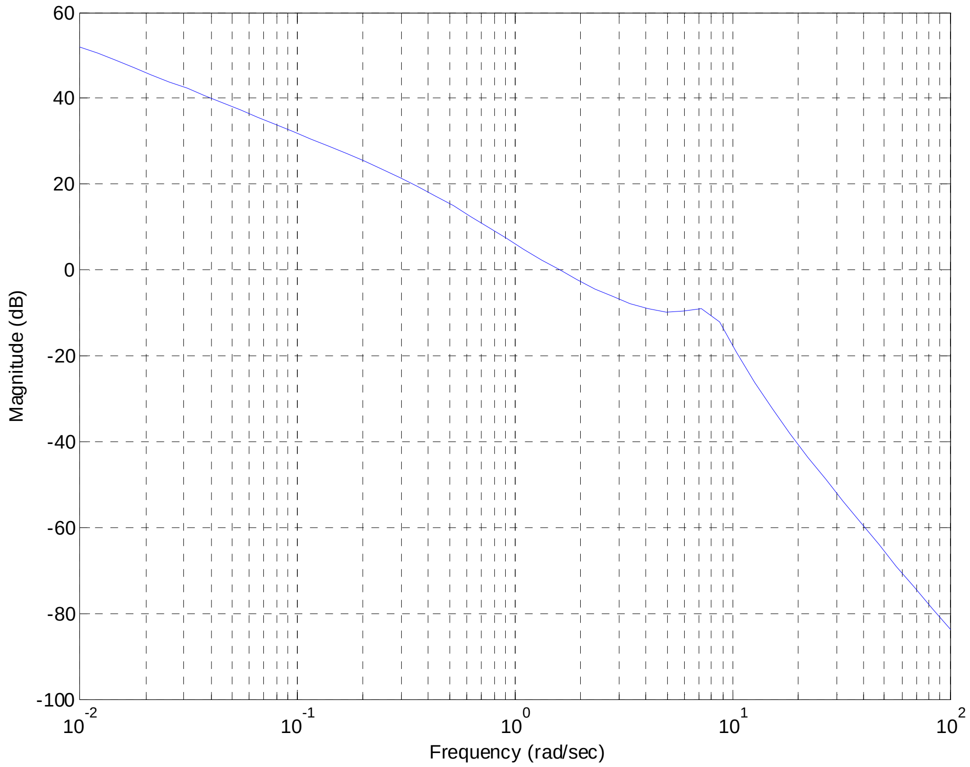

Figure 9.2

9.3 The Nyquist plot of $G(s)=\dfrac{K(s-1)}{(s-2)(s-4)}$ for $K=1$ is shown in Figure 9.3. Verify the crossing points of the Nyquist plot with the real-axis and discuss the stability of the system when $K=1$ and $K=7$, respectively (use unit feedback). Further, determine the range of K such that the closed-loop system is stable.

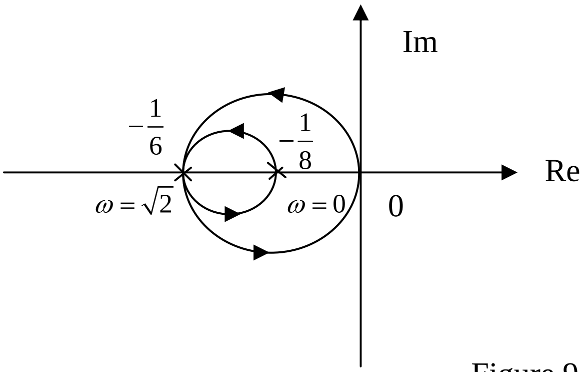

Figure 9.3

9.4 A speed control for a gasoline engine is shown in Figure 9.4. The lag τ_t is equal to 1 second. The engine time constant τ_e is equal to 3 seconds. The speed measurement time constant τ_m is 0.4 second.

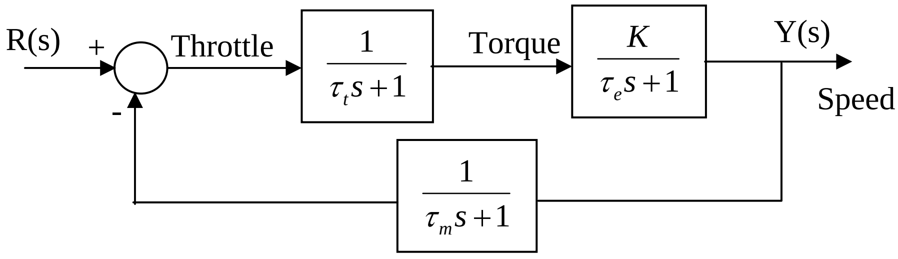

Figure 9.4

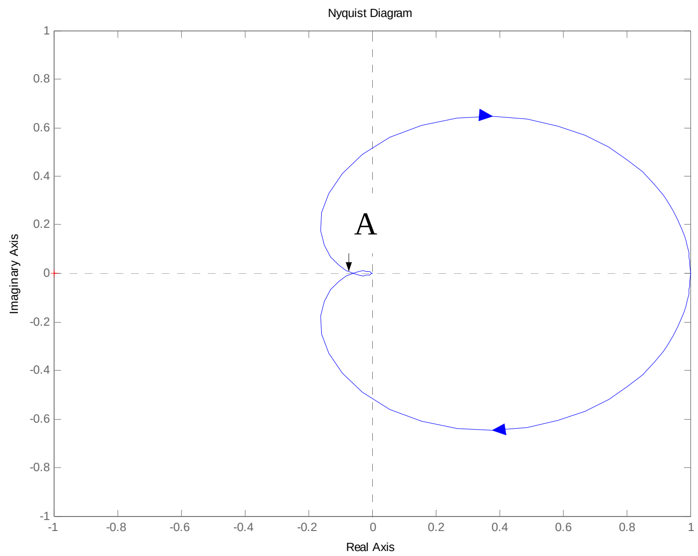

Figure 9.5

(a) With $K=1$, the Nyquist plot of the open-loop transfer function is shown in Figure 9.5. Determine the coordinates of the crossing point A and investigate the stability of the system.

(b) Determine the range of K such that the system is stable.

[Ans: -0.063; stable; 0<K<15.873]

EE3011 MODELING AND CONTROL
Tutorial 10

Relative Stability

10.1 A feedback control system is as shown in Figure 10.1.

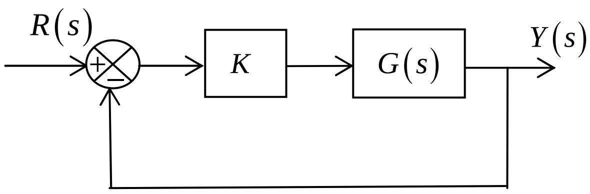

Figure 10.1

Assume that $G(s)=\dfrac{1}{s(Ts+1)}$, where $T>0$ is the time constant. Given that the phase margin of the feedback system is 45^0 and the gain crossover frequency is 3 rad/sec, find the controller gain K and the time constant T. Can one find a suitable value of gain K so as to achieve a velocity error constant of no less than 5 and a phase margin of 45^0 for the system?

[Ans: 4.24; 1/3]

10.2 A synchronous generator excitation control system is shown in Figure 10.2. The parameters available are:

$$K_E = 0.05,\quad K_G = K_R = 1,\quad T_A = 0.1,$$
$$T_E = 0.5,\quad T_G = 1,\quad T_R = 0.05$$

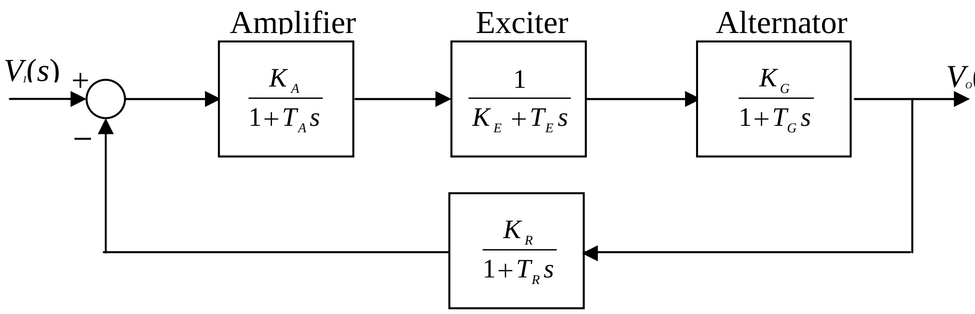

Figure 10.2 Excitation Control System

(i) The Bode plots of the open-loop transfer function with $K_A = 40$ are given in Figure 10.3. Determine the gain and phase margins of the system. Is the system stable?

(ii) By reducing the amplifier gain K_A, it is possible to improve the phase margin of the system. If the system is required to have a phase margin of 10^0, what should be the K_A?

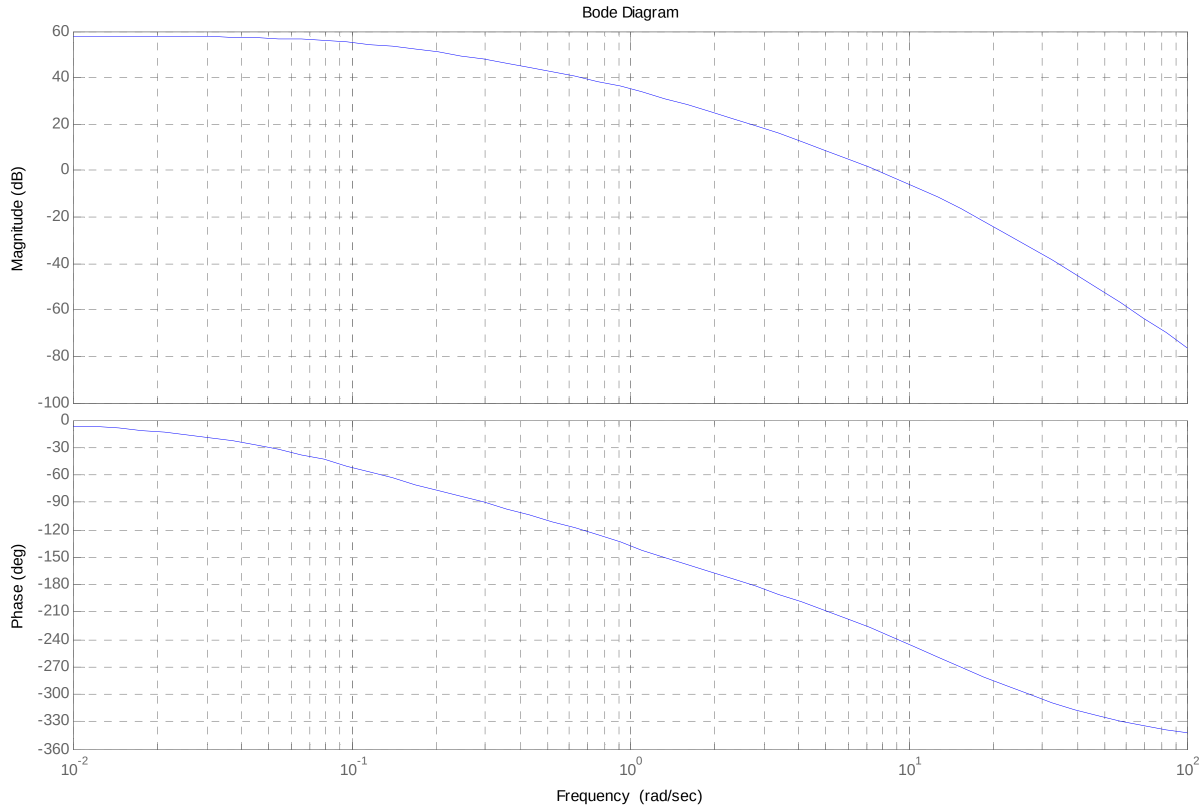

Figure 10.3

[Ans: -20 dB, -50^0; 2.52

EE3011 MODELING AND CONTROL
Tutorial 11

Compensator Design

11.1 The frequency response of an industrial process is as shown in Figure 11.1. Design a lead compensator that will yield a cross frequency of 10 rad/s and a phase margin of 50^0.

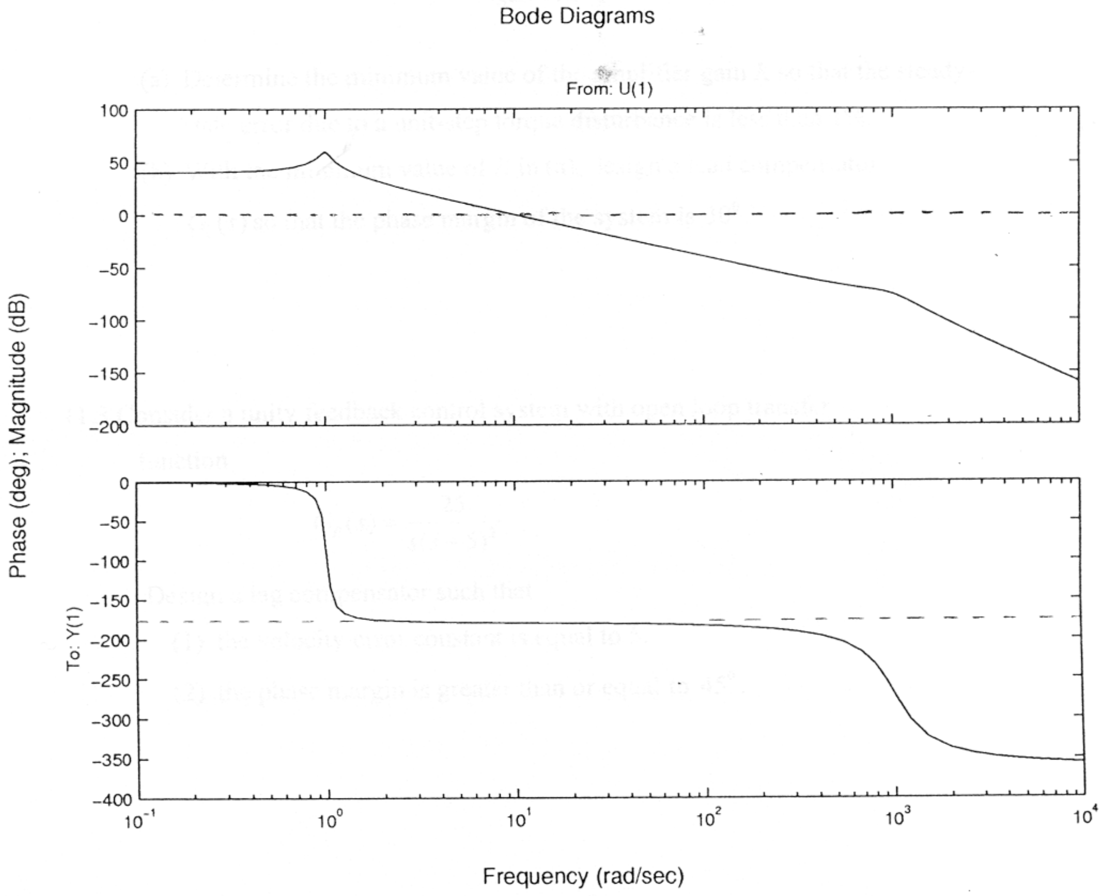

Figure 11.1

11.2 Consider the control system shown in Figure 11.2.

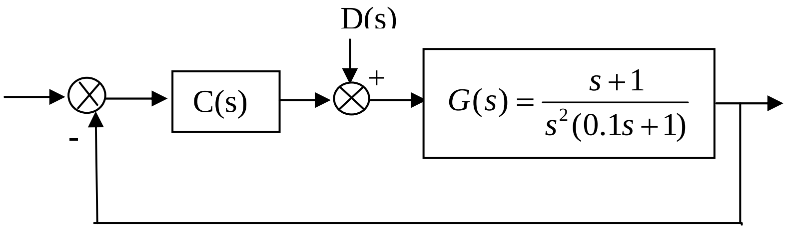

Figure 11.2

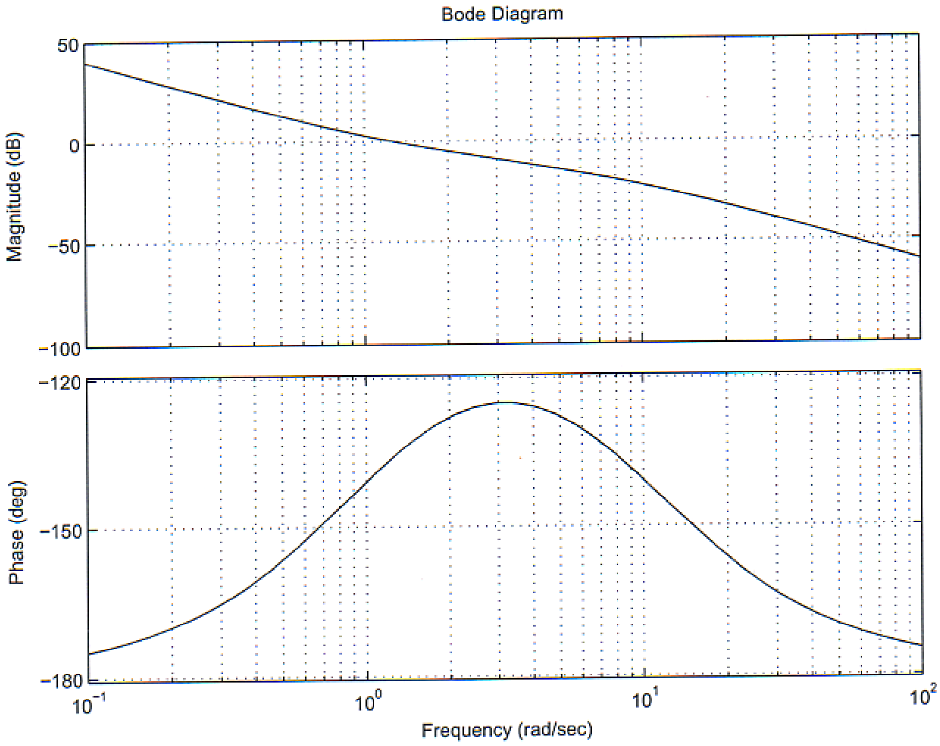

Figure 11.3

(a) Given the Bode plots of G(s) in Figure 11.3, determine a proportional controller, i.e. $C(s) = K$ such that the gain crossover frequency of the system is 10rad/s. Is the system stable?

(b) Let the compensator $C(s) = KC_0(s)$. With the value of K obtained in (a), design a suitable $C_0(s)$ such that the gain crossover frequency of the system is at least 10rad/s and the phase margin of the system is at least 60^0.

(c) Suppose that $D(s)$ is a step disturbance. To meet the disturbance rejection requirement, a student proposes to use a PI controller. Can this strategy achieve the specifications given in (b)? Explain.

11.3 Consider the control system as shown in Figure 11.4 with $a = 1$, $N = 1$ and $\omega_n = 10$ rad/s.

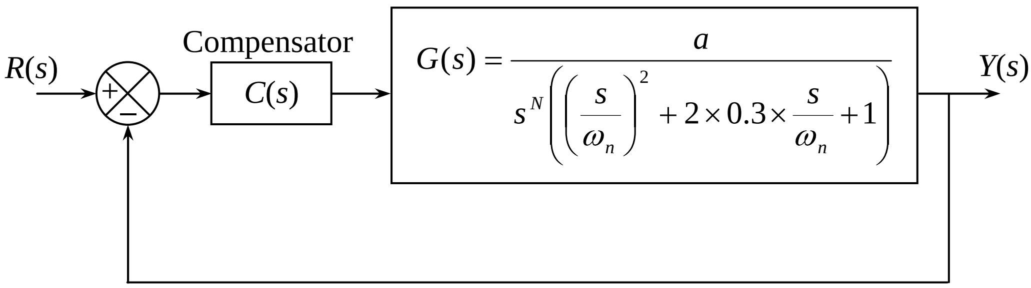

Figure 11.4

Given the Bode plots of G(s) in Figure 11.5, design a suitable compensator C(s) to meet the following specifications:

(a) The steady-state error due to a unit-ramp input is less than or equal to 0.1.
(b) The gain crossover frequency is less than or equal to 6 rad/s.
(c) The phase margin of the system is at least 50°.

If there exists a delay of 0.2 sec. in G(s), can the compensator designed maintain the stability of the system?

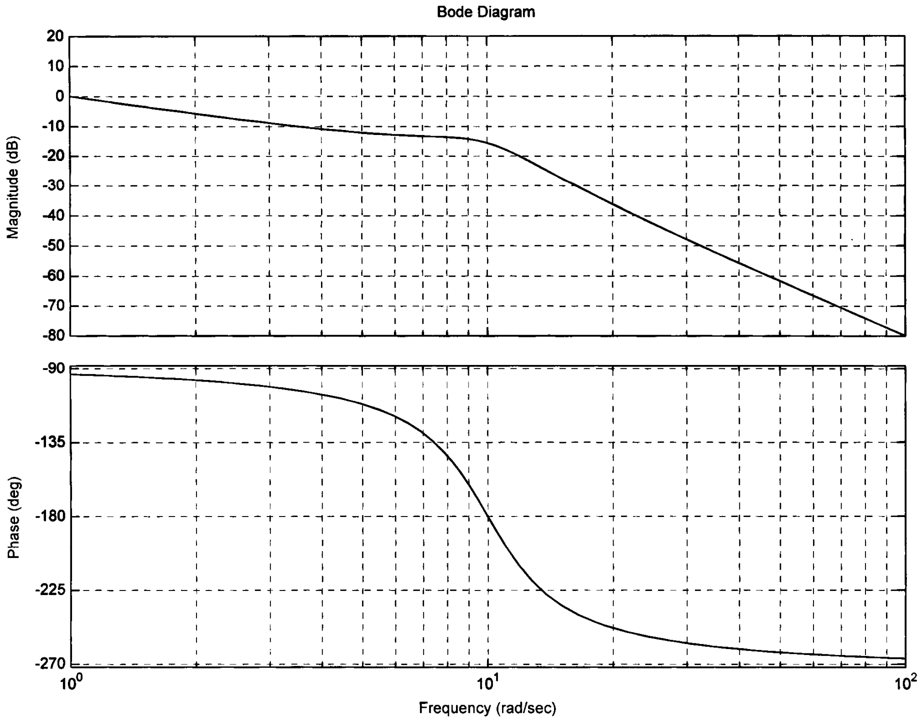

Figure 11.5

EE3011 MODELING AND CONTROL
Tutorial 12

<u>PID Tuning</u>

12.1 An engineer conducts a step response test for a furnace temperature system. The detail of the test is given in Table 12.1 and the process reaction curve is shown in Figure 12.1.

Table 12.1

Input Level-Start	100	Output Level-Start	300^0C
Input Level-End	110 (10% step)	Output Level-End	330^0C

(a) Process the data file and produces the tuning parameters for a PI controller.
(b) Fit an appropriate first-order plus dead time model transfer function given by

$$G(s) = \frac{Ke^{-Ls}}{\tau s + 1}$$

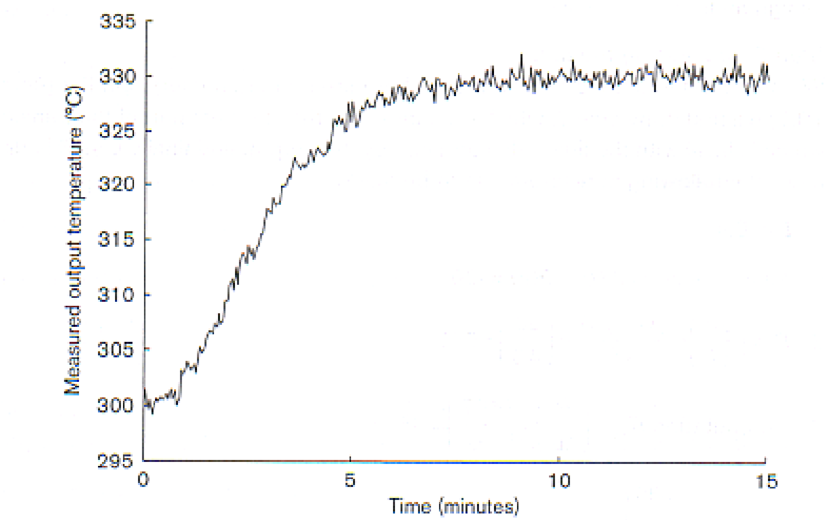

Figure 12.1

12.2 Consider the control system as shown in Figure 12.2. Using a Ziegler-Nichols tuning rule, determine the values of K_p, T_i and T_d. Obtain the unit step response of the designed system. Make fine adjustments of parameters K_p, T_i and T_d such that the maximum overshoot in the unit step response be approximately 15%.

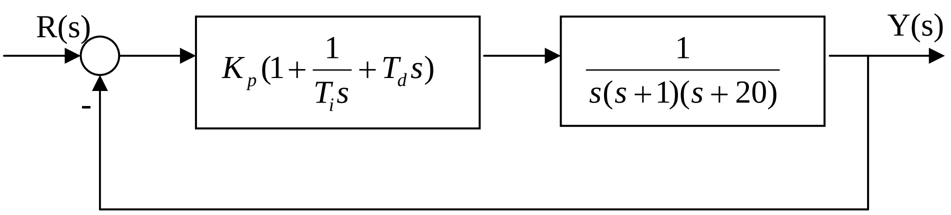

Figure 12.2