Tut #7 Solutions

7.1

- The closed-loop transfer function is

$$T(s)=\frac{C(s)}{R(s)}=\frac{10}{s+11},\quad T(j\omega)=\frac{10}{j\omega+11}$$

$$|T(j\omega)|=\frac{10}{\sqrt{\omega^2+121}},\quad \angle T(j\omega)=-\tan^{-1}\frac{\omega}{11}$$

Then,

$$|T(j1)|=0.905,\ \angle T(j1)=-5.2^0;\quad |T(j2)|=0.894,\ \angle T(j2)=-10.3^0$$

$$(a)\ y_{ss}(t)=1\times 0.905\sin(t+30^0-5.2^0)$$

$$(b)\ y_{ss}(t)=2\times 0.894\cos(2t-45^0-10.3^0)$$

$$(c)\ y_{ss}(t)=0.905\sin(t-24.8^0)-1.79\cos(2t-55.3^0)$$

(superposition)

7.2

(a) $$G_1(j\omega)=\frac{10(j\omega-1)}{j\omega+1},$$

$$|G_1(j\omega)|=\frac{10|-1+j\omega|}{|1+j\omega|}=10\frac{\sqrt{(-1)^2+\omega^2}}{\sqrt{1^2+\omega^2}}=10$$

$$\angle G_1(j\omega)=\angle 10(-1+j\omega)-\angle(1+j\omega)=\angle(-1+j\omega)-\angle(1+j\omega)$$

$$=180^0-\tan^{-1}\omega-\mathrm{an}^{-1}\omega=180-2\tan^{-1}\omega$$

$$|G_1(j3)|=10,\ \angle G_1(j3)=36.9^0$$

(b)

$$G_2(j\omega)=\frac{10(-\omega^2-1)}{-\omega^2(j\omega+1)}=\frac{10(\omega^2+1)}{\omega^2}\cdot\frac{1}{1+j\omega}$$

$$|G_2(j\omega)|=\frac{10(\omega^2+1)}{\omega^2}\frac{1}{\sqrt{1+\omega^2}}=\frac{10\sqrt{1+\omega^2}}{\omega^2}$$

$$\angle G_2(j\omega)=\angle\frac{10(\omega^2+1)}{\omega^2}+\angle\frac{1}{1+j\omega}=0-\tan^{-1}\omega=-\tan^{-1}\omega$$

$$|G_2(j3)|=3.51,\ \angle G_2(j3)=-71.6^0$$

7.3 Recall that $y_{ss}(t) = A_i\,|G(j\omega)|\sin(\omega t + \phi_i + \angle G(j\omega))$

$$|G(j\omega)| = \left|\frac{1}{(j\omega+1)(-\omega^2 + j\omega+1)}\right| = \frac{1}{\sqrt{\omega^2+1}\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$\angle G(j\omega) = -\angle(j\omega+1) - \angle(1-\omega^2 + j\omega) = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{1-\omega^2}$$ (For w<1)

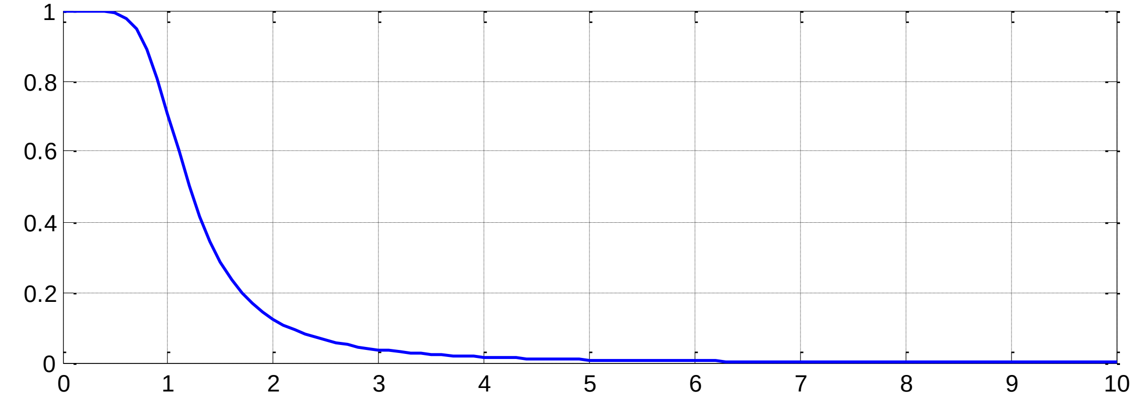
ω (rad/s)	0.1	1	10
$\lvert G(j\omega)\rvert$	1	0.707 $\sin(t-24.8^0)$	0.001
$\angle G(j\omega)\,(^0)$	-11.5	-135	-258.5

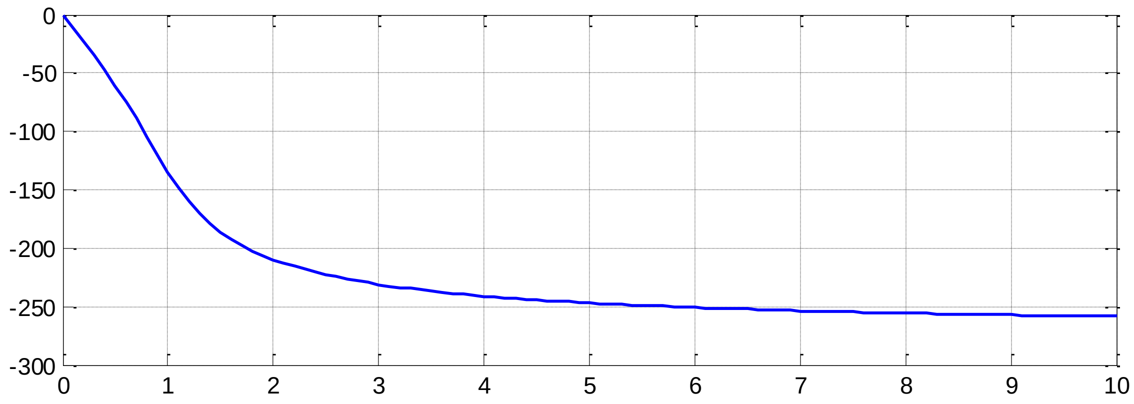
(i) For $\omega = 0.1$ rad/s, $y_{ss}(t) = 2\times1\times\sin(0.1t + 30^0 - 11.5^0) = 2\sin(0.1t + 18.5^0)$

(ii) For $\omega = 1$ rad/s, $y_{ss}(t) = 2\times0.707\times\sin(t + 30^0 - 135^0) = 1.414\sin(t - 105^0)$

(iii) For $\omega = 10$ rad/s, $y_{ss}(t) = 2\times0.001\times\sin(10t + 30^0 - 258.5^0) = 0.002\sin(10t - 228.5^0)$

The magnitude and phase plots below indicate that the system is in fact a low pass filter





7.4 The disturbance transfer function is

$$G_d(s) = \frac{Y(s)}{D(s)} = \frac{1}{1 + C(s)G(s)} = \frac{s^2 + 2s + 100}{s^2 + 19.2s + 144}$$

$$G_d(j\omega) = \frac{100 - \omega^2 + j2\omega}{144 - \omega^2 + j19.2\omega}$$

$$|G_d(j\omega)| = \frac{\sqrt{(100 - \omega^2)^2 + 4\omega^2}}{\sqrt{(144 - \omega^2)^2 + (19.2\omega)^2}}$$

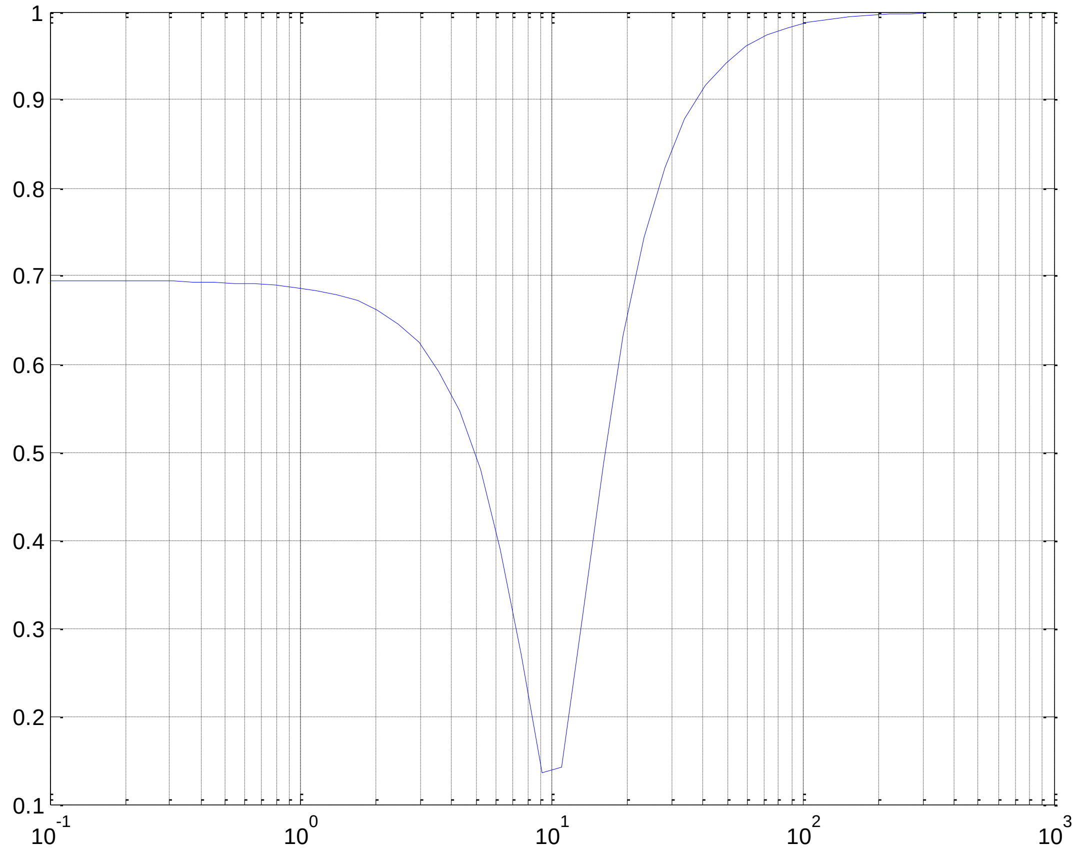
Then, $|G_d(j1)| = 0.6863$ (or -3.27 dB)

$|G_d(j100)| = 0.9891$ (or -0.09 dB)

Note that the amplitude of the output is $A_o = A_i\,|G_d(j\omega)|$

Hence, at 1 rad/s, there will be 3.27 dB of attenuation but at 100 rad/s the attenuation is almost 0 dB.

The magnitude plot of the disturbance transfer is



Clearly, for disturbance with frequency up to 2 rad/s, there will be attenuation of more than 3dB (0.7).