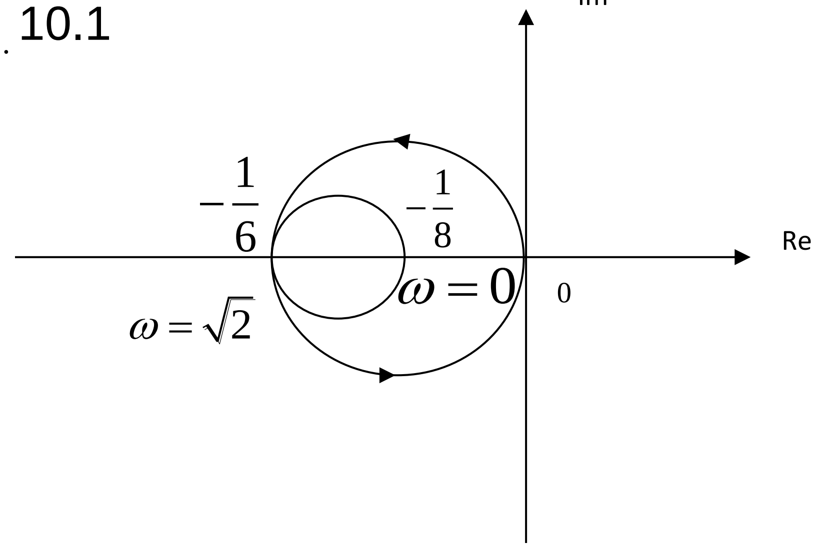
Tut 10 Solutions

10.1



Crossing points with real-axis:

$$G(j\omega) = \frac{j\omega - 1}{(j\omega - 2)(j\omega - 4)} = \frac{j\omega - 1}{8 - \omega^2 - j6\omega}$$

$$= \frac{(j\omega - 1)(8 - \omega^2 + j6\omega)}{(8 - \omega^2)^2 + 36\omega^2}$$

$$= \frac{-8 - 5\omega^2 + j\omega(2 - \omega^2)}{(8 - \omega^2)^2 + 36\omega^2}$$

$$\text{Im}\,\{G(j\omega)\} = 0 \quad \Rightarrow \quad \omega = 0 \quad or \quad \omega = \pm\sqrt{2}$$

$$G(j0) = \frac{-8}{8^2} = -\frac{1}{8}$$

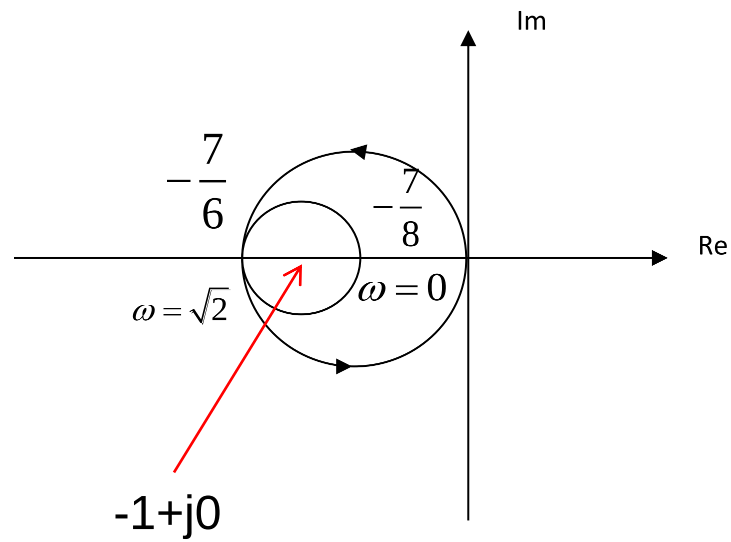
$$G(j\sqrt{2}) = \frac{-8 - 5 \times 2}{6^2 + 36 \times 2} = -\frac{1}{6}$$

The open-loop system has two poles on RHP, *P*=2.

Stability analysis:

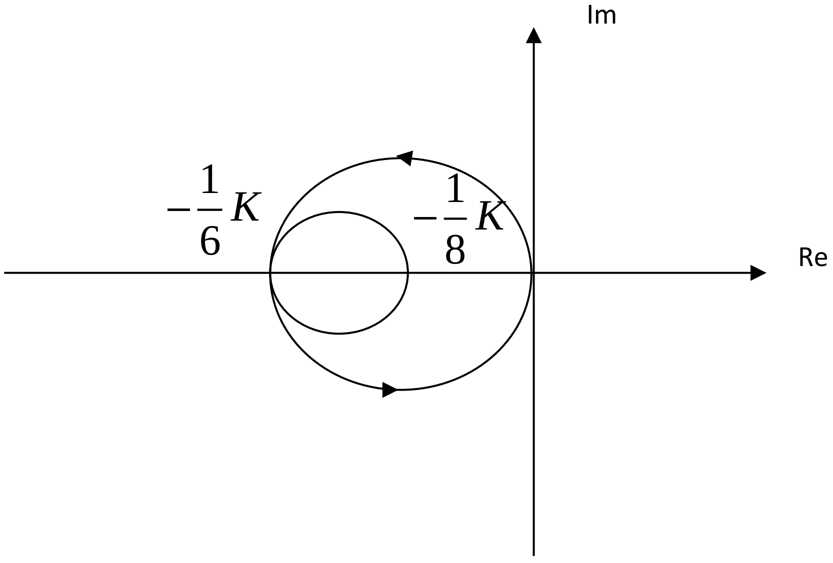
(i) For $K = 1, N = 0, N + P = 2 \Rightarrow$ unstable

(ii) For $K = 7, N = -2, N + P = 0 \Rightarrow$ stable



(iii) The system is stable if and only if

$$-\frac{1}{6}K < -1 < -\frac{1}{8}K \quad \Rightarrow 6 < K < 8.$$



Question: What if $K > 8$?

10.2 (a) For $K=1$, the open-loop transfer function is $G(s)=\dfrac{1}{(s+1)(3s+1)(0.4s+1)}$

$$G(j\omega)=\frac{1}{(j\omega+1)(j3\omega+1)(j0.4\omega+1)} \qquad P=0$$

$$=\frac{1}{1-4.6\omega^2+j\omega(4.4-1.2\omega^2)}$$

The intersection with the real axis means that $\mathrm{Im}\{G(j\omega)\}=0$, implying

$$\omega(4.4-1.2\omega^2)=0 \quad\Rightarrow\quad \omega=0,\pm1.9149$$

At A, $\omega=\pm1.9149$ and $G(j1.9149)=G(-j1.9149)=-0.063$

Since there is no encirclement of (0,-1) point, $N=0,\ N+P=0$, the system is stable.

(b) The Nyquist plot will not encircle -1+j0 if

$$-0.063K>-1 \quad\Rightarrow\quad K<15.873$$

Range of K for stability: $0<K<15.873$

Note: In stability analysis using Nyquist stability criterion, $K>0$ is assumed.

10.3

$$G_{op}(s) = \frac{K}{s(Ts+1)}, \quad |G_{op}(j\omega)| = \frac{K}{\omega\sqrt{T^2\omega^2+1}}$$

At the gain cross-over frequency,

$$|G_{op}(\omega_g)| = 1, i.e., K = \omega_g\sqrt{T^2\omega_g^2+1} = 3\sqrt{9T^2+1} \qquad \text{------ (1)}$$

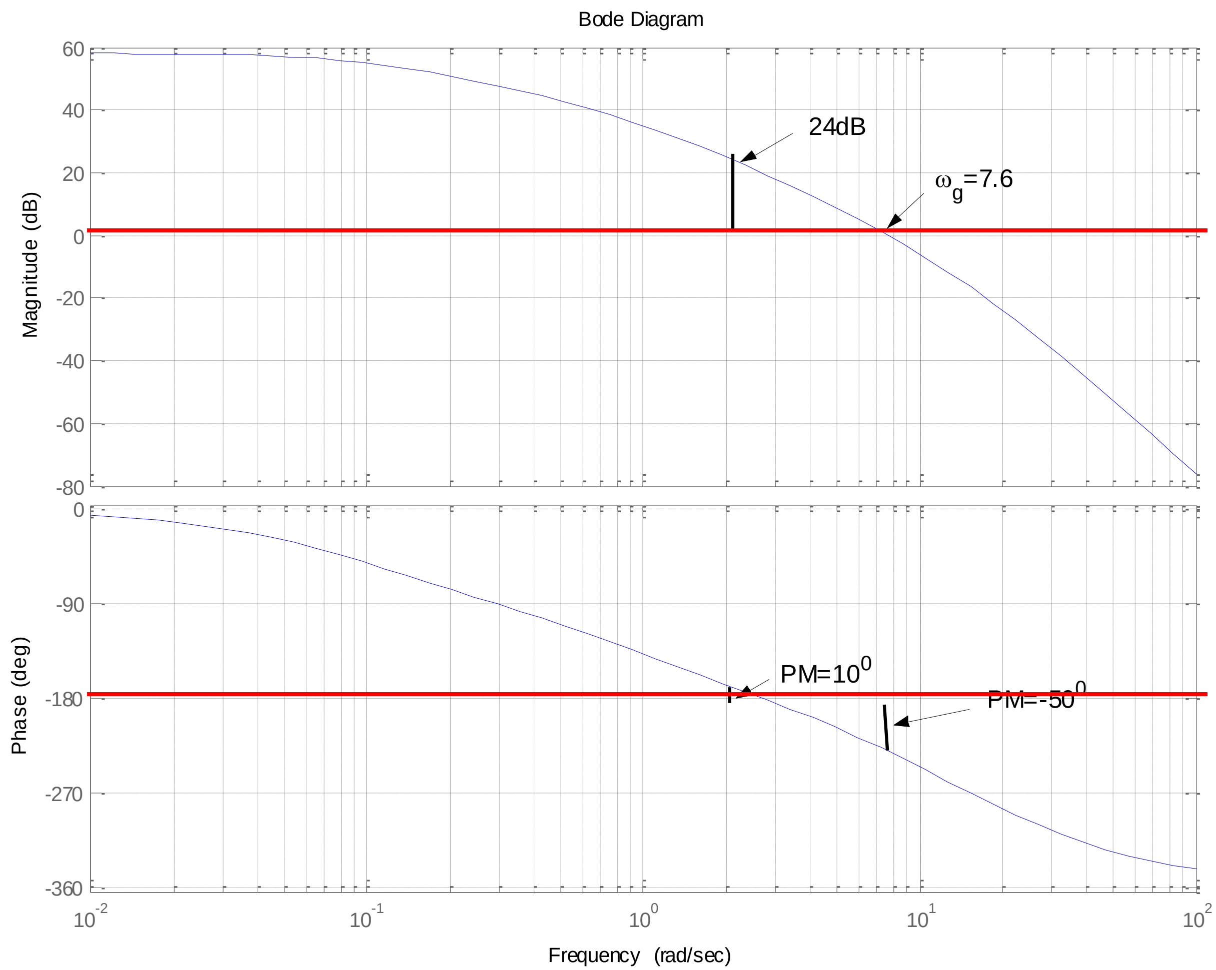
Also, note that
$$\angle G_{op}(j\omega_g) = -90^0 - \tan^{-1}(\omega_g T)$$
$$PM = 180^0 + \angle G_{op}(j\omega_g) = 90^0 - \tan^{-1}(\omega_g T) = 45^0$$

Then,
$$\tan^{-1}(\omega_g T) = 45^0 \Rightarrow \omega_g T = 1 \Rightarrow T = \frac{1}{3}$$

From (1), we get $K = 4.24$

To achieve $K_V \geq 5$, $K \geq 5$. Since the phase of $G_{op}(j\omega)$ is decreasing with the increase of ω. With the higher gain *K*, the gain cross-over frequency will be higher and phase margin will be smaller than the required.

10.4 Observe from the Bode plot



$\omega_g \approx 7.6$ rad/s, $\omega_\phi = 2.67$ rad/s; $PM \approx -50^0$, GM=-20dB

The system is unstable.

(b) From the phase plot, to achieve the phase margin of 10^0 , the new gain crossover frequency is estimated to be

$$\omega_g^{'} \approx 2.1 \text{ rad/s}$$

For $K_A = 40,$ the magnitude of the open-loop system is about 24 dB at $\omega = 2.1$ rad/s.

To achieve $\omega_g^{'} \approx 2.1$, the magnitude plot needs to be shifted down by 24 dB, i.e.

$$20\lg 40 - 20\lg K_A = 24 \quad \Rightarrow \quad K_A = 2.52$$