EE3011 MODELLING & CONTROL
Tutorial 1 (Solutions) Open-Loop and Closed-Loop Systems

1. Open-loop control systems found in homes:

(a) **Washer/dryer** for clothes (some new and modern ones have closed-loop control!)

(b) **Dish washer**

(c) **Electric fan**

Closed-loop control systems found in homes:

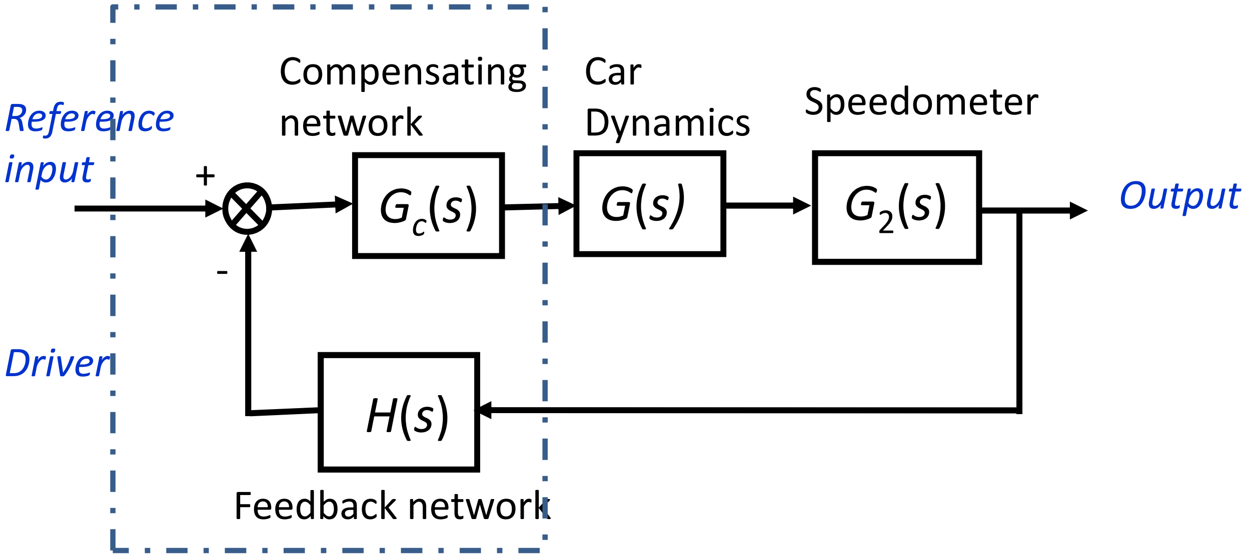
(a) **Room temperature control system.** Room temperature is kept constant regardless of the number of people in the room or the outside temperature.

(b) **Refrigerator.** Keeps the temperature in the refrigerator constant regardless of the amount of food in it or the outside temperature.

(c) **Oven temperature control.** The oven temperature is controlled at the set-point regardless of the amount of food in the oven or the outside temperature.

2. **Driving a car**

Suppose that the desired output is a certain speed, 80 km/hr say. The input is the driver’s foot on the accelerator (which translates into a desired speed), and the human driver is the controller. To maintain the speed, he/she monitors the speedometer and either accelerates or lift foot off the accelerator to slow down.



Disturbance?

Well, the sighting of a police officer will cause the controller to react by touching the brake.

Humps, child rushing across the road etc are other forms of disturbance..

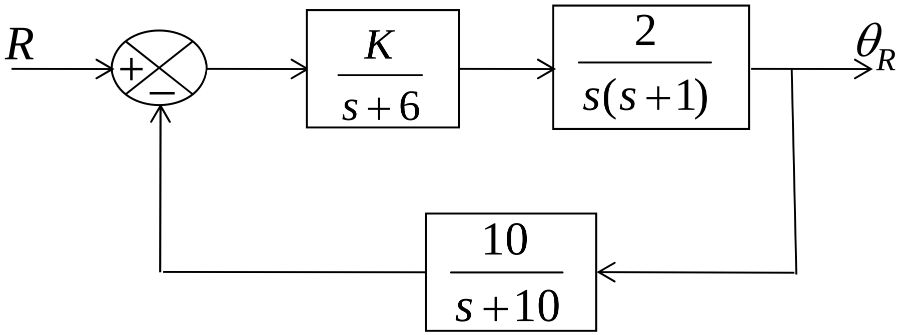
How about riding a bicycle?

3. With a flexible shaft, the assembly of R/W head will be smaller and lighter and can give a faster response. However, it will give rise to higher order dynamics. Further, the actual displacement θ_2 will be different from the displacement of the motor, θ_1. Thus, some form of estimation is required.

Performance specifications:

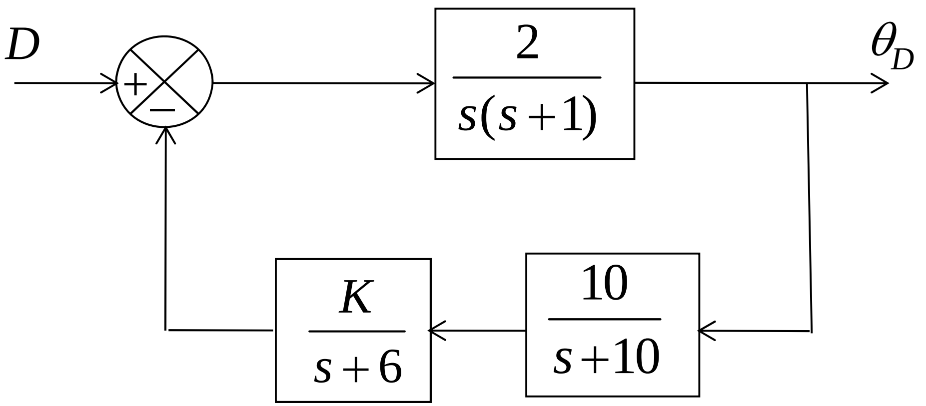
- Fast response to give fast access time.
- Steady-state error of less than 1/2 the track width to give correct tracking.
- Very small overshoot to prevent R/W head hitting the casing.
- Ability to reject disturbance torque.

4. To get $\dfrac{\theta_R(s)}{R(s)}$, set $D(s) \equiv 0$, then



$$\frac{\theta_R(s)}{R(s)} = \frac{\frac{K}{s+6}\cdot\frac{2}{s(s+1)}}{1+\frac{K}{s+6}\cdot\frac{2}{s(s+1)}\cdot\frac{10}{s+10}} = \frac{2K(s+10)}{s(s+1)(s+6)(s+10)+20K} \quad (:= T_1(s))$$

To get $\dfrac{\theta_D(s)}{D(s)}$, set $R(s) \equiv 0$, then



$$\frac{\theta_D(s)}{D(s)} = \frac{\dfrac{2}{s(s+1)}}{1+\dfrac{2}{s(s+1)}\cdot\dfrac{K}{s+6}\cdot\dfrac{10}{s+10}} = \frac{2(s+6)(s+10)}{s(s+1)(s+6)(s+10)+20K} \quad (:= T_2(s))$$

With both *R*(*s*) and *D*(*s*) acting, $\theta(s) = T_1(s)R(s) + T_2(s)D(s)$, i.e.

$$\theta(s) = \frac{2K(s+10)}{s(s+1)(s+6)(s+10)+20K}R(s) + \frac{2(s+6)(s+10)}{s(s+1)(s+6)(s+10)+20K}D(s)$$

5. **Case (a)**

Let $Y_d(s)$ denote the output due to disturbance (i.e. $R(s) \equiv 0$)

For the open loop system, $Y_d(s) = \dfrac{1}{s+a} \cdot T_L(s)$ ------ (1)

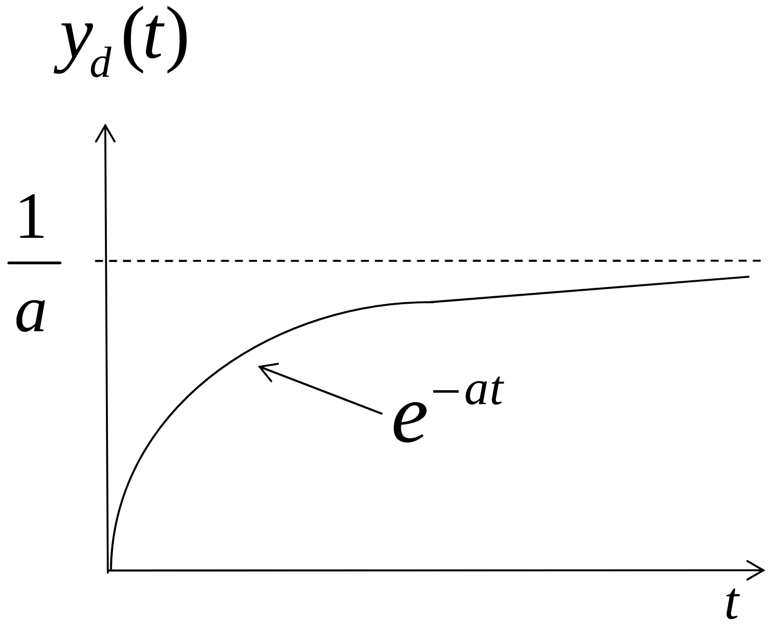
For the closed-loop system, $Y_d(s) = \dfrac{1}{s+a}\left(T_L(s) - KY_d(s)\right)$

$$\Rightarrow Y_d(s) = \frac{1}{s+a+K} T_L(s) \quad \text{------ (2)}$$

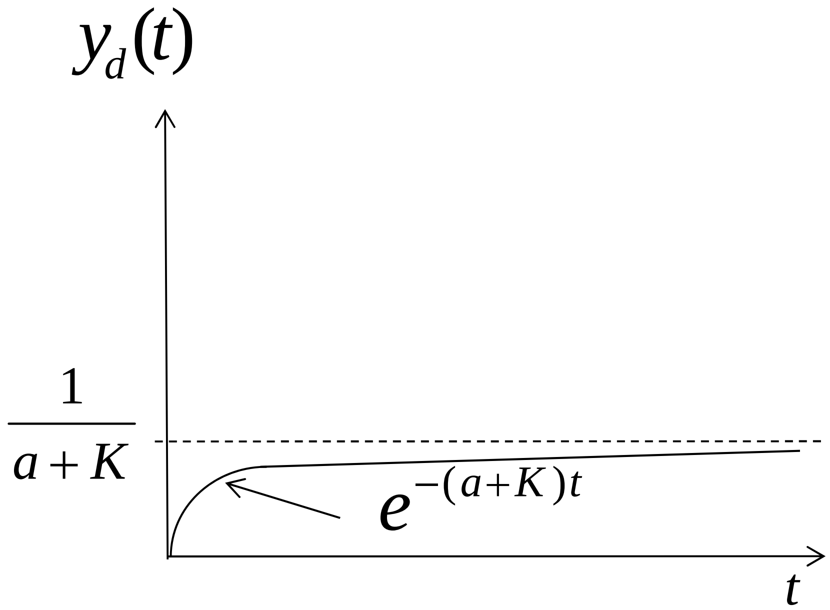
If the disturbance is a unit-step, then

From (1), $Y_d(s) = \dfrac{1}{s+a} \cdot \dfrac{1}{s}$ $\quad \Rightarrow y_d(t) = \dfrac{1}{a}\left(1 - e^{-at}\right)$

From (2), $Y_d(s) = \dfrac{1}{s+a+K} \cdot \dfrac{1}{s}$ $\quad \Rightarrow y_d(t) = \dfrac{1}{a+K}\left(1 - e^{-(a+K)t}\right)$



$y_d(t)$
$\frac{1}{a}$
e^{-at}
t



$y_d(t)$
$\frac{1}{a+K}$
$e^{-(a+K)t}$
t

Case (b) Again, let $Y_d(s)$ denote the output due to the disturbance.

For the open-loop system, $Y_d(s) = T_L(s)$ ------ (3)

For the closed-loop system, $Y_d(s) = T_L(s) - \dfrac{K}{s+a} Y_d(s)$

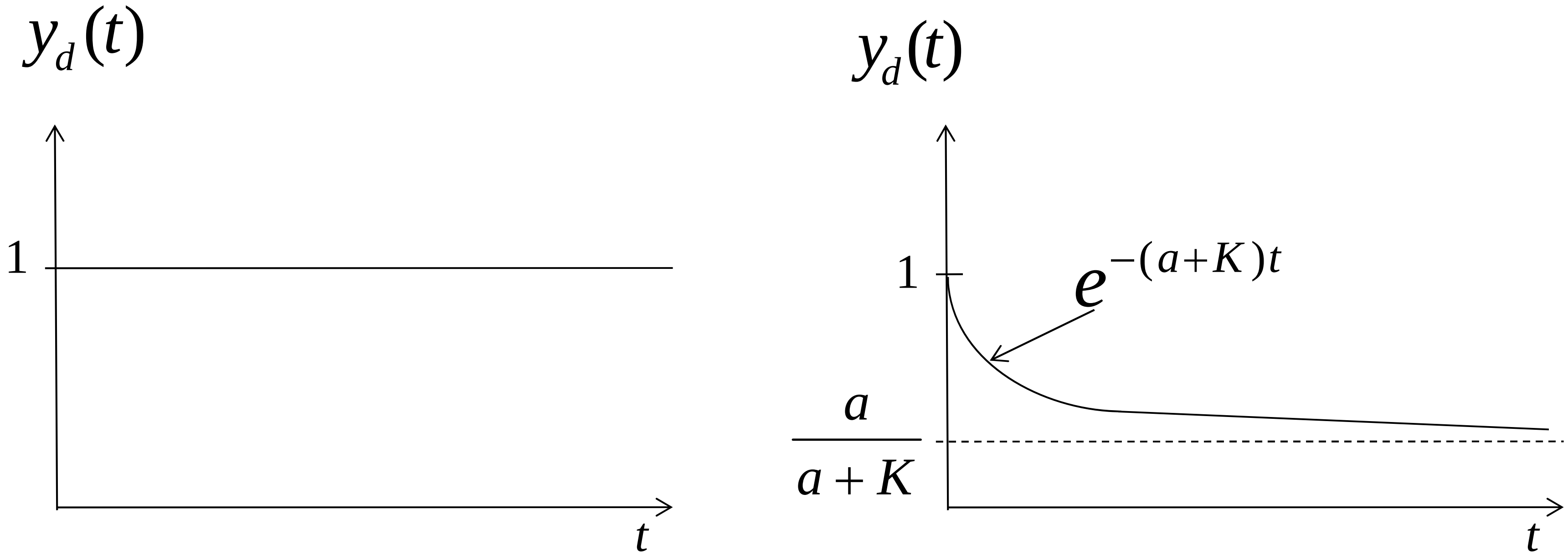
$$\Rightarrow (s+a+K)Y_d(s) = (s+a)T_L(s)$$

$$\Rightarrow Y_d(s) = \frac{s+a}{s+a+K} T_L(s) \qquad \text{------ (4)}$$

Suppose that $T_L(s)$ is a unit-step disturbance.

From (3), we have $y_d(t) = 1$.

From (4), we have $Y_d(s) = \dfrac{\frac{a}{a+K}}{s} + \dfrac{\frac{K}{a+K}}{s+a+K} \quad \Rightarrow y_d(t) = \dfrac{a}{a+K} + \dfrac{K}{a+K} e^{-(a+K)t}$


$y_d(t)$
1
t
$y_d(t)$
1
$e^{-(a+K)t}$
$\frac{a}{a+K}$
t