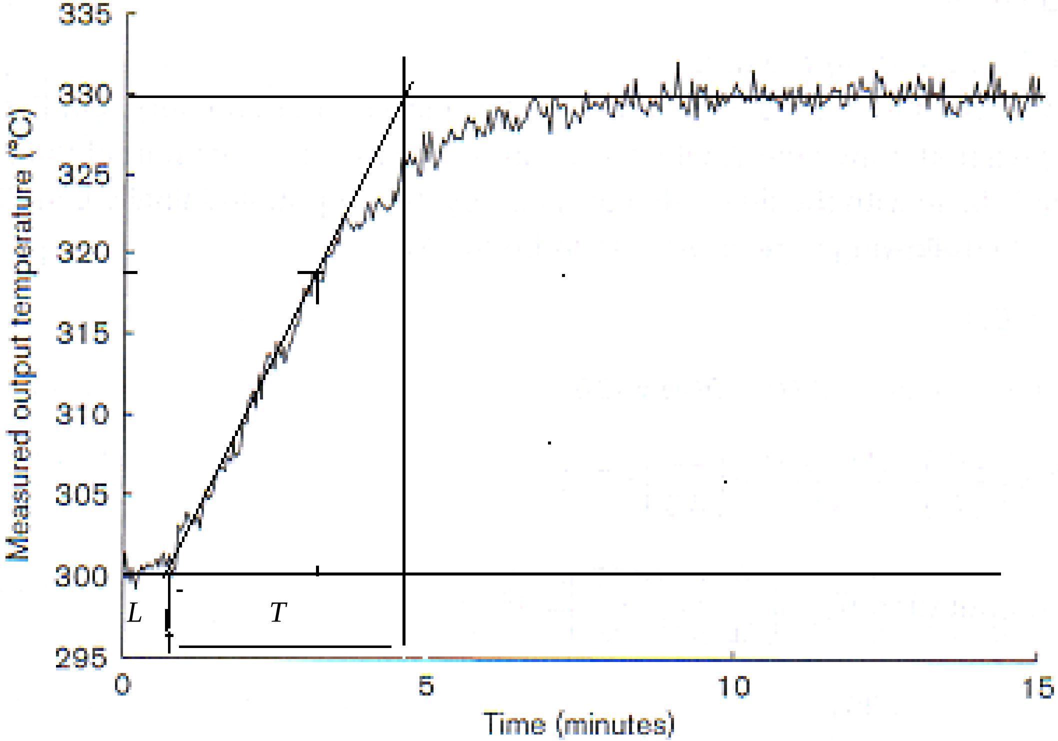
Tut 12 Solutions

12.1



From the above, L=0.4min,

$$K = \frac{y_\infty - y_0}{u_\infty - u_0} = \frac{330-300}{110-100} = 3, \quad T = 4.5\,\text{min}$$

By applying the tuning rule, PI controller parameters are:

$$K_P = 0.9 \times \frac{T}{KL} = 3.37, \quad T_i = 3L = 1.2$$

The PI controller is obtained as

$$G_c(s) = 3.37(1 + \frac{1}{1.2s}) = 3.37 + \frac{2.81}{s}$$

(b) The first order model is obtained as

$$G(s) = \frac{Ke^{-Ls}}{Ts+1} = \frac{3e^{-0.4s}}{4.5s+1}$$

12.2

- With the presence of an integrator in the plant, the step response method is not applicable. We adopt the frequency response method.

- Form a feedback control with P-controller, the closed-loop transfer function is

$$M(s) = \frac{K_P}{s(s+1)(s+20) + K_P}$$

- The C.E.: $$s^3 + 21s^2 + 20s + K_P = 0$$

- To obtain the critical gain and frequency of oscillation, we apply the Routh Array approach:

$$\begin{array}{lcc} s^3 & 1 & 20 \\ s^2 & 21 & K_P \\ s & \dfrac{420 - K_P}{21} & \\ s^0 & K_P & \end{array}$$

The critical gain is $K_{cr} = 420$ and the oscillation frequency can be found from the auxiliary equation:

$$21s^2 + 420 = 0 \quad \Rightarrow \quad s = \pm j\sqrt{20}$$

The period of oscillation is $P_{cr} = \dfrac{2\pi}{\sqrt{20}} = 1.4050$

- Referring to the Frequency Response Method, we have

$$K_P = 0.6K_{cr} = 252$$
$$T_i = 0.5P_{cr} = 0.7025$$
$$T_d = 0.125P_{cr} = 0.1756$$

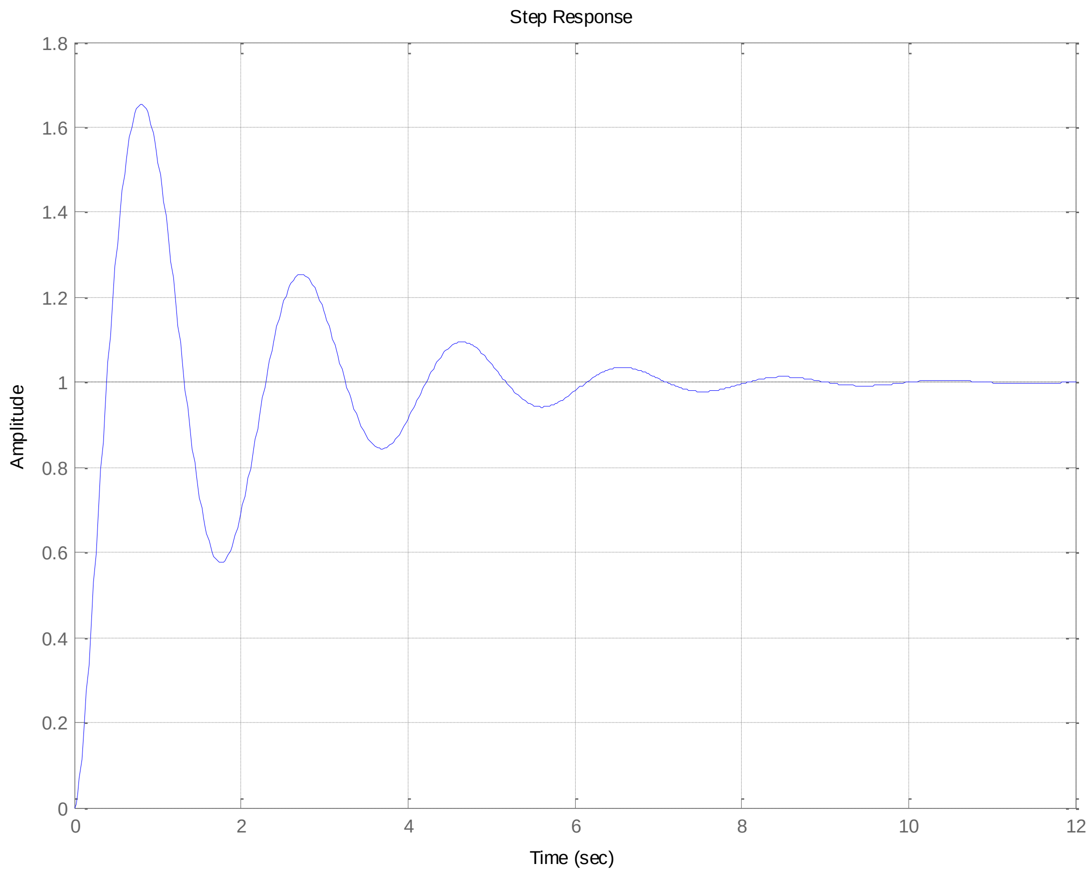
The PID controller is then

$$G_c(s) = K_P\left(1 + \frac{1}{T_i s} + T_d s\right) = \frac{44.2575(s + 2.8470)^2}{s}$$

The closed-loop transfer function is

$$M(s) = \frac{44.26s^2 + 252s + 358.72}{s^4 + 21s^3 + 64.25s^2 + 252s + 358.72}$$

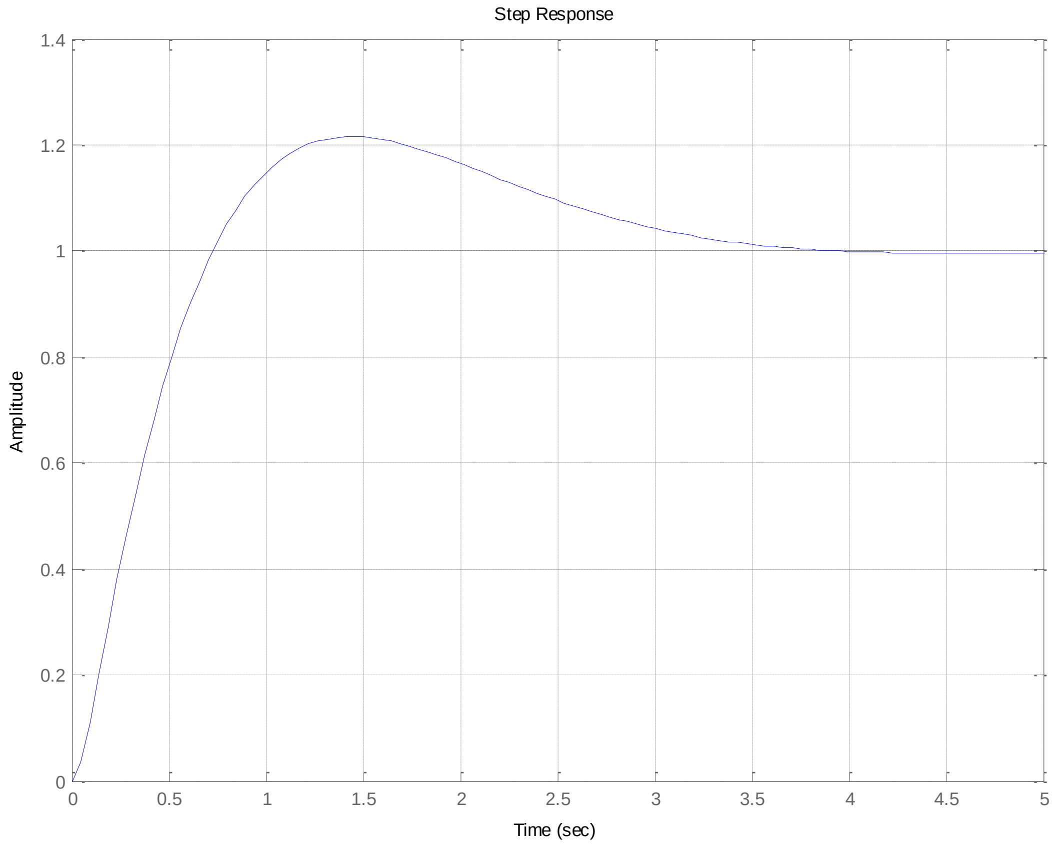
The step response is



The maximum overshoot is 68%.

To reduce the maximum overshoot, we reduce K_P and increase T_i and T_d :

$$K_P = 84, T_i = 2.1, T_d = 0.51$$



The maximum overshoot is now around 20%. Further tuning can be applied to achieve 15% of maximum overshoot.