TUTORIAL 8: THREE-PHASE POWER SYSTEMS

Summary

a) **Balanced voltage sources:**

Phase voltages: $V_{ag} = |V_p|\angle 0°$; $V_{bg} = |V_p|\angle 0°$; $V_{cg} = |V_p|\angle 0°$

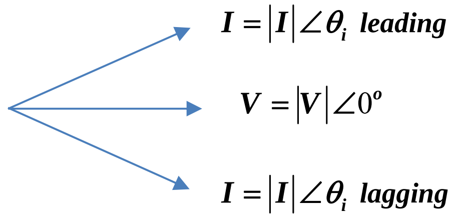
Line-line voltages:

$$V_{ab} = V_{ag} - V_{bg} = \sqrt{3}|V_p|\angle 30°; \quad V_{bc} = \sqrt{3}|V_p|\angle(30° - 120°); \quad V_{ca} = \sqrt{3}|V_p|\angle(30° - 240°)$$

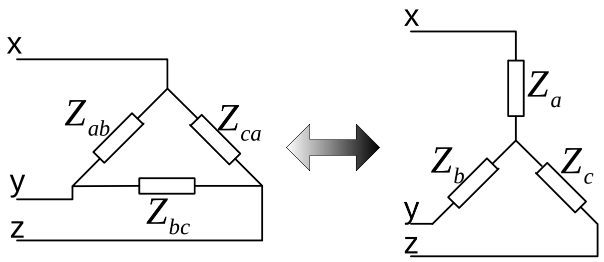
b) **Three phase power and power factor**

Power triangle:
$$S = VI^* = |V||I|\angle(-\theta_i)$$
$$S = P + jQ = |S|\angle\theta_s (pu)$$

Power factor: $pf = \cos\theta_s = \cos\theta_i$



c) **Δ-Y transform:**



$$Z_a = \frac{Z_{ab}Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}, \quad Z_b = \frac{Z_{ab}Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}}, \quad Z_c = \frac{Z_{bc}Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

d) **Thevenin Equivalent Viewed from Bus k**

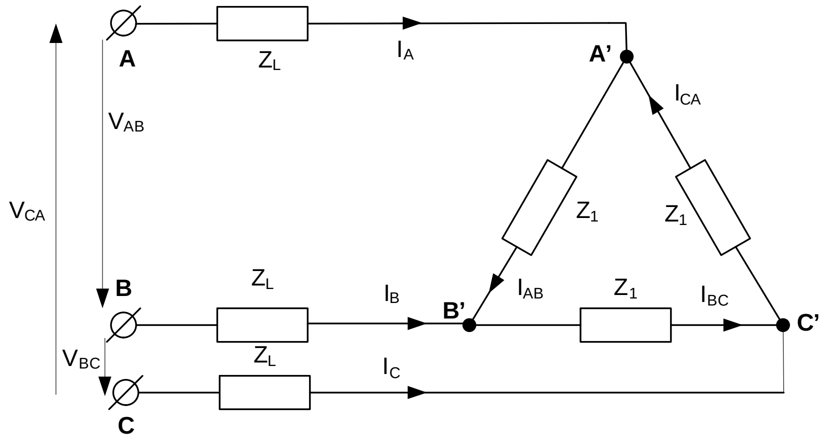
e) **KCL and KVL**

f) **Current divider**

8.1: Three (30 + j30) Ω identical impedances are connected in Delta and supplied by a 173V 3-phase system through three conductors each having impedance of (0.8 + j0.6) Ω. Find:

1) the current magnitude in each of the delta-connected impedances, and
2) the voltage cross each of Delta impedance.

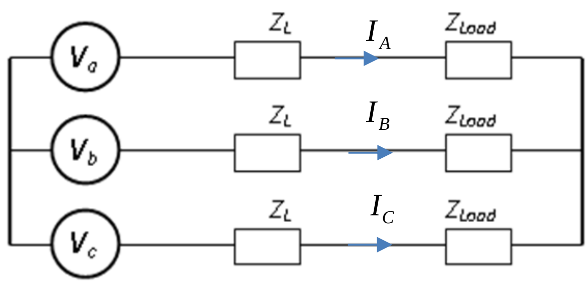
Solution: (Single phase calculation for a three phase balanced system)



For single phase calculation both Δ loads and sources have to be connected into Y.

Convert Δ loads into Y: $Z_{Load} = Z_1/3 = (10 + j10)\Omega$

Y sources phase voltages: $V_a = \frac{173V}{\sqrt{3}}\angle 0°$; $V_b = \frac{173V}{\sqrt{3}}\angle -120°$; $V_c = \frac{173V}{\sqrt{3}}\angle -240°$



The total (per phase) impedance: $Z_T = Z_2 + Z_{Load} = 10.8 + j10.6 = 15.133\angle 44.46^o\Omega$

The phase currents: $I_A = \frac{V_a}{Z_T} = 6.60\angle -44.46°(A)$; $\boldsymbol{I_B} = \boldsymbol{I_A}\angle -120°$; $\boldsymbol{I_C} = \boldsymbol{I_A}\angle -240°$

The current in each of the delta load:

$$\boldsymbol{I_{AB}} = \frac{\boldsymbol{I_A}}{\sqrt{3}}\angle +30^o;\quad \boldsymbol{I_{BC}} = \boldsymbol{I_{AB}}\angle -120^o;\quad \boldsymbol{I_{CA}} = \boldsymbol{I_{AB}}\angle -240^o$$

The voltages of the delta connected load:

$$\boldsymbol{V_{A'B'}} = \boldsymbol{I_{AB}}\boldsymbol{Z_1} = 161.64\angle 30.54^o\,(V);\ \boldsymbol{V_{B'C'}} = \boldsymbol{V_{A'B'}}\angle -120^o\,(V);\ \boldsymbol{V_{C'A'}} = \boldsymbol{V_{A'B'}}\angle -240^o\,(V)$$

8.2: A 415V, 50 HZ, 4-wire three-phase balanced power supply with sequence RYB is connected to the following loads:

A single resistance of 12 Ω between R phase and neutral;

An inductive impedance of (2 + j8) Ω between B phase and neutral;

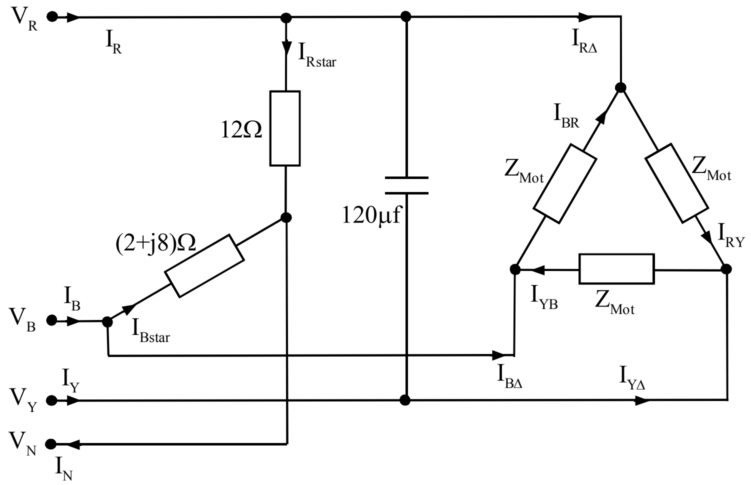
A capacitor of 120μF between R and Y phase;

A three-phase, delta connected induction motor operating at 10kW and 0.75 p.f. lagging.

Calculate:

1) the magnitude and phase angle of the current in the four lines of the supply, and
2) the total power from the supply.

Solutions: (unbalanced three phase system)



The phase supply voltages (**using phase R voltage as the reference**):

$$V_R = \frac{415V}{\sqrt{3}}\angle 0° \qquad V_Y = \frac{415V}{\sqrt{3}}\angle -120° \qquad V_B = \frac{415V}{\sqrt{3}}\angle -240°$$

The line to line supply voltages:

$$V_{RY} = 415V\angle 30° \qquad V_{YB} = 415V\angle -90° \qquad V_{BR} = 415V\angle -210°$$

The current in <u>12Ω load</u> :

$$I_{R\_star} = \frac{V_{RN}}{R} = \frac{415\angle 0^0}{\sqrt{3}12} = 19.9667\,A\angle 0^0$$

The current in the (2 + j8) Ω <u>inductor</u>:

$$I_{B\_star} = \frac{V_{BN}}{Z} = \frac{415\angle -240^0}{\sqrt{3}(2+j8)} = 29.0558\,A\angle 44.0362^0$$

The current in the 120μF <u>capacitor</u>:

$$I_{RY\_cap} = \frac{V_{RY}}{Z_c} = \frac{415\angle 30^0}{1/j(2\pi 50)(120\mu f)} = 15.6451A\angle 120^0$$

The <u>inductive motor load</u> 10kW at 0.75pf lagging:

The power factor angle: $\phi = \cos^{-1}(0.75) = 41.41^0$

The reactive power: $Q = P\tan\varphi = 10{,}000\tan\left(41.41^0\right) = 8.8192\,kVar$

Since: $S = P + jQ = \sqrt{3}V_{l-l}I_p^* = 3V_{phs}I_p^*$

The terminal currents of the motor load:

$$\mathrm{I}_{R\_\Delta} = \left(\frac{\mathrm{S}}{\sqrt{3}\mathrm{V}_{l\text{-}l}}\right) = \left(\frac{10{,}000 + \mathrm{j}8819.2}{415\times\sqrt{3}\angle 0°}\right)^* = 18.55\mathrm{A}\angle -41.41°$$

$$\mathrm{I}_{Y\_\Delta} = 18.55\mathrm{A}\angle -161.41°$$

$$\mathrm{I}_{B\_\Delta} = 18.55\mathrm{A}\angle 78.59°$$

The four supply line currents (**unbalance**):

$$I_R = I_{R\_star} + I_{RY\_cap} + I_{R\_\Delta} = 26.09\,A\angle 2.82^0$$

$$I_Y = -I_{RY\_cap} + I_{Y\_\Delta} = 21.77\,A\angle -116.63^0$$

$$I_B = I_{B\_star} + I_{B\_\Delta} = 45.56\,A\angle 57.39^0$$

$$I_N = I_{R\_star} + I_{B\_star} = 45.57\,A\angle 26.31^0$$

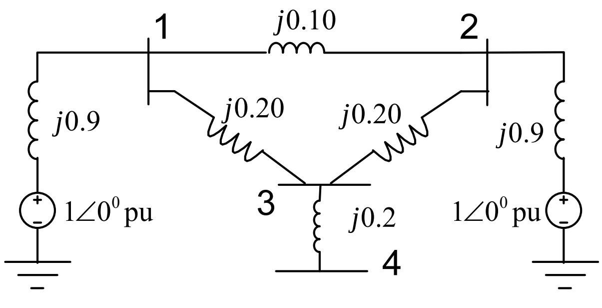
The total power taken from the supply (**unbalance**):

$$S = V_R I_R^* + V_Y I_Y^* + V_B I_B^* = 16.473kW + j\ 9.080kVar$$

8.3: The figure below shows the per-phase per-unit network of a 4-bus power system.

Develop Thevenin equivalent circuits as viewed

1) from bus 4 and ground, and
2) from bus 2 and ground.



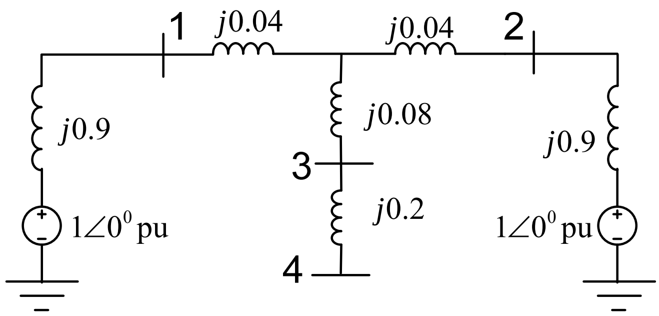
Solutions:

1) From bus 4 and ground:

Equivalent network of the original circuit using delta-Y transform:

$$Z_a = \frac{Z_{ab}Z_{ca}}{Z_{ab}+Z_{bc}+Z_{ca}} = \frac{j0.20\times j0.20}{j0.20+j0.20+j0.10} = j0.08$$

$$Z_b = Z_c = \frac{Z_{ab}Z_{bc}}{Z_{ab}+Z_{bc}+Z_{ca}} = \frac{j0.20\times j0.10}{j0.20+j0.20+j0.10} = j0.04$$



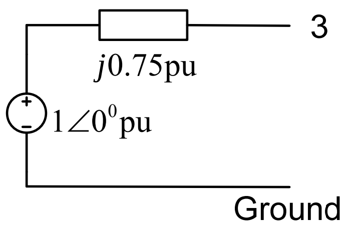
Thevenin impedance (kill all the sources in the original circuit):

$$Z_{3,th} = j0.28 + (j0.04 + j0.9) \,\|\, (j0.04 + j0.9) = j0.75$$

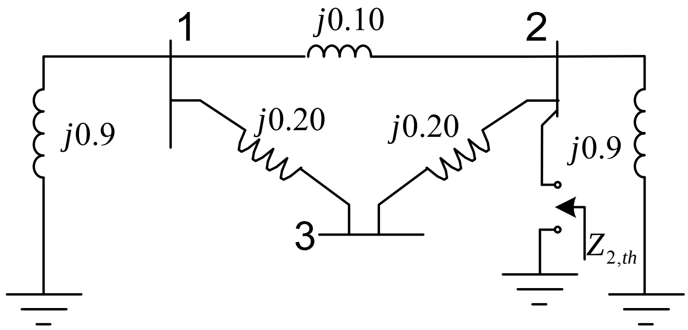
Thevenin Voltage (Open circuit voltage):

$$\mathrm{V_{3,th}} = 1\angle 0^0 \,\mathrm{pu.}$$

Thevenin equivalent circuit:



2) from bus 2 and ground:



Thevenin impedance:

$$Z_{2,th} = j0.9 \,\|\, \left[(j0.2 + j0.2) \,\|\, j0.1 + j0.9 \right] = j0.9 \,\|\, j0.98 = j0.4692\text{pu}=\text{Z}_{22}$$

Open-circuit voltage as viewed from bus 2:

$$\text{V}_{2,\text{th}} = 1\angle 0^0\,\text{pu}$$

Thevenin equivalent circuit: