

Tutorial 9: Sequence Components and Sequence Networks

9.1 Given $V_a = 1\angle 0^\circ$, $V_b = 1\angle -90^\circ$, $V_c = 2\angle 135^\circ$, find the sequence voltages of three unbalanced voltages and Prove that:

$$V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)}.$$

Solution:

$$\begin{aligned} \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} &= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 1\angle 0^\circ \\ 1\angle -90^\circ \\ 2\angle 135^\circ \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 1\angle 0^\circ + 1\angle -90^\circ + 2\angle 135^\circ \\ 1\angle 0^\circ + 1\angle 30^\circ + 2\angle 375^\circ \\ 1\angle 0^\circ + 1\angle 150^\circ + 2\angle 255^\circ \end{bmatrix} = \begin{bmatrix} 0.195\angle 135^\circ \\ 1.311\angle 15^\circ \\ 0.494\angle -105^\circ \end{bmatrix} \end{aligned}$$

$$\begin{aligned} V_a &= V_a^{(0)} + V_a^{(1)} + V_a^{(2)} = 0.195\angle 135^\circ + 1.311\angle 15^\circ + 0.494\angle -105^\circ \\ &= -0.137886 + j0.137886 + 1.266329 + j0.339312 \\ &\quad -0.127857 - j0.477167 \approx 1 \end{aligned}$$

Homework:

$$\begin{bmatrix} V_b^{(0)} \\ V_b^{(1)} \\ V_b^{(2)} \end{bmatrix} = ?; \quad \begin{bmatrix} V_c^{(0)} \\ V_c^{(1)} \\ V_c^{(2)} \end{bmatrix} = ?$$

Prove: $V_b = 1\angle -90^\circ$

$$V_c = 2\angle 135^\circ$$

9.2 The component parameters of the following system are given in Table 2.1. The power rating for each apparatus is 200MVA. Sketch the sequence networks and find the Thevenin equivalent reactances of each sequence network seen from bus 4. Assume that pre-fault voltage at bus 4 is $1\angle 0^\circ$ pu. (Exercise: the same question for Bus 5)

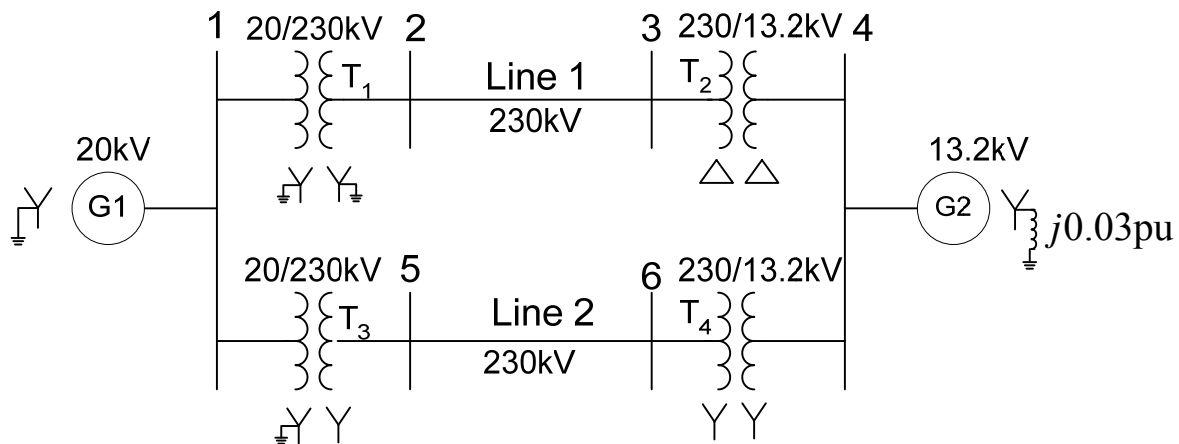


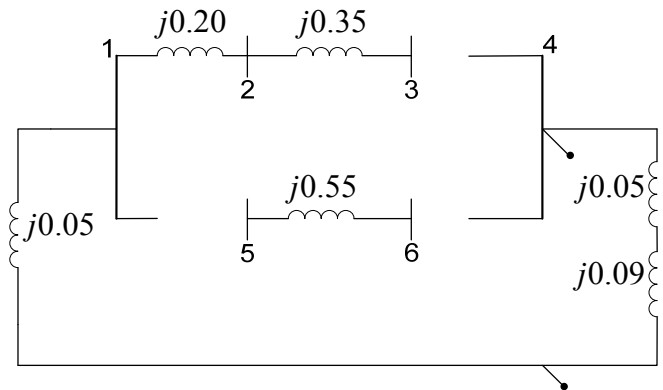
Table 2.1

Component	kV Rating	$X_1(\text{pu})$	$X_2(\text{pu})$	$X_0(\text{pu})$
G_1	20	0.20	0.14	0.05
G_2	13.2	0.20	0.14	0.05
T_1	20/230	0.20	0.20	0.20
T_2	230/13.2	0.30	0.30	0.30
T_3	20/230	0.25	0.25	0.25
T_4	230/13.2	0.35	0.35	0.35
Line 1	230	0.15	0.15	0.35
Line 2	230	0.22	0.22	0.55

Solutions:

Zero-sequence network:

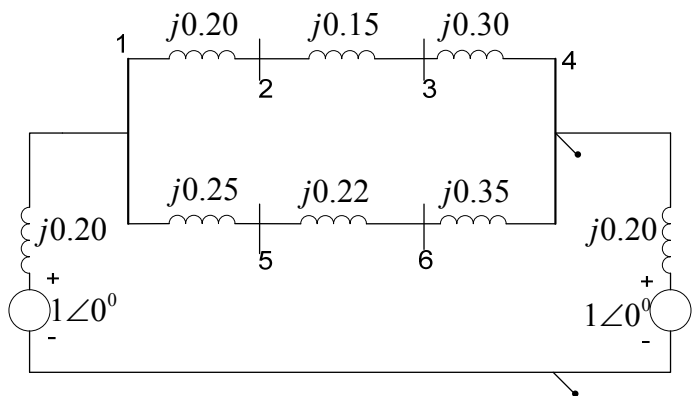
$$Z_{0,th} = j0.05 + j0.09 = j0.14\text{pu}$$



Positive-sequence network:

$$Z_{1,th} = j0.20 \parallel [j0.20 + j0.65 \parallel j0.82]$$

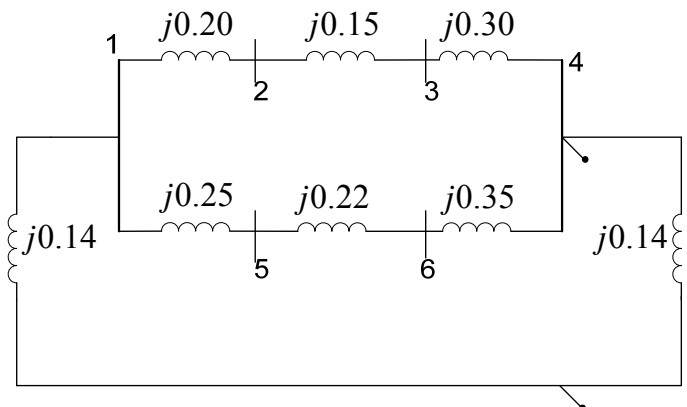
$$= j0.1475$$



Negative-sequence network:

$$Z_{2,th} = j0.14 \parallel [j0.14 + j0.65 \parallel j0.82]$$

$$= j0.1095$$



9.3 Part of a power system is shown in the figure. $V_f = 1\angle 0^\circ$ pu. $X_N = 5\%$. For a bolted double line-to-ground fault occurs on phase b and c at bus 1, the three-phase currents out of G_1 :

$$I_a = 0.4008\angle 90^\circ \text{ pu}$$

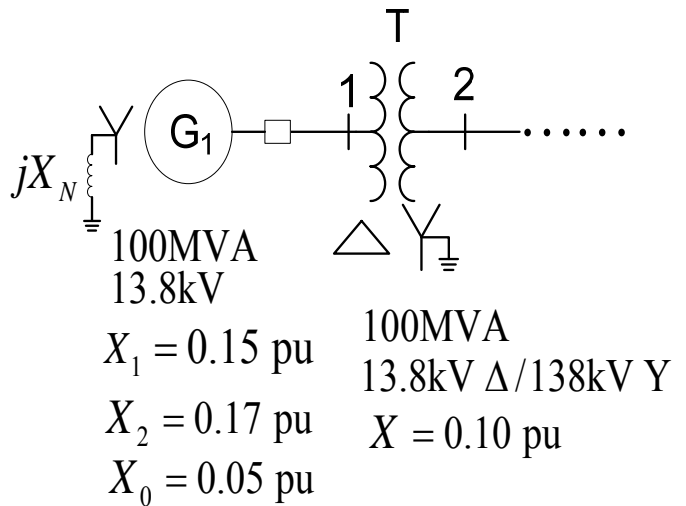
$$I_b = 6.1817\angle 152.86^\circ \text{ pu}$$

$$I_c = 6.1817\angle 27.14^\circ \text{ pu}$$

Currents I_{fb} and I_{fc} from bus 1 to ground:

$$I_{fb} = 7.8104\angle 157.25^\circ \text{ pu}$$

$$I_{fc} = 7.8104\angle 22.75^\circ \text{ pu}.$$

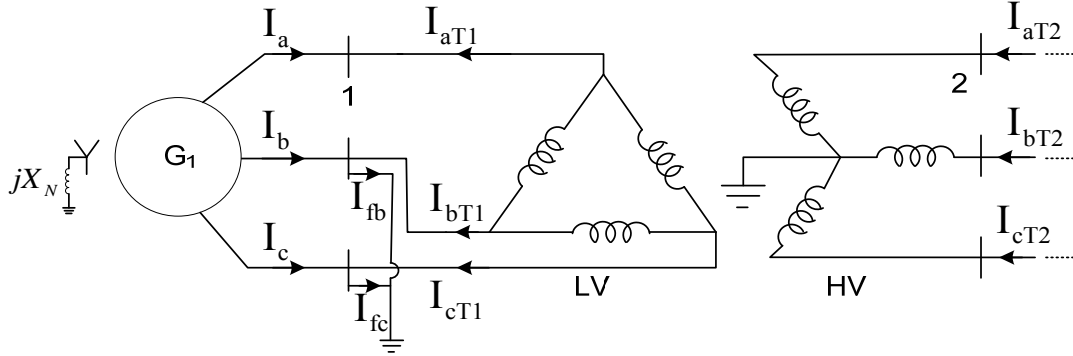


- Calculate per-unit sequence currents of phase a flowing from delta side of transformer T_1 to bus 1.
- Calculate per-unit three-phase currents flowing from bus 2 to Y side of T_1 .

Givens:

$$I_a = 0.4008 \angle 90^\circ, I_b = 6.1817 \angle 152.86^\circ, I_c = 6.1817 \angle 27.14^\circ$$

$$I_{fb} = 7.8104 \angle 157.25^\circ, I_{fc} = 7.8104 \angle 22.75^\circ.$$



a) Three-phase currents from Δ -side of T1 to bus 1 (KCL):

$$I_{aT1} = I_{fa} - I_a = 0 - 0.4008 \angle 90^\circ = 0.4008 \angle -90^\circ$$

$$I_{bT1} = I_{fb} - I_b = 7.8104 \angle 157.25^\circ - 6.1817 \angle 152.86^\circ = 1.7136 \angle 173.28^\circ$$

$$I_{cT1} = I_{fc} - I_c = 7.8104 \angle 22.75^\circ - 6.1817 \angle 27.14^\circ = 1.7136 \angle 6.72^\circ$$

The sequence currents from Δ -side of T1 to bus 1:

$$\begin{bmatrix} I_{aT1}^{(0)} \\ I_{aT1}^{(1)} \\ I_{aT1}^{(2)} \end{bmatrix} = [\mathbf{A}^{-1}] \begin{bmatrix} I_{aT1} \\ I_{bT1} \\ I_{cT1} \end{bmatrix} = \begin{bmatrix} 0 \\ -j1.1830 \\ j0.78211 \end{bmatrix} \text{ pu}$$

b) The sequence currents from bus 2 to Y-side of T1:

$$I_{aT2}^{(0)} = 0$$

$$I_{aT2}^{(1)} = I_{aT1}^{(1)} \times 1 \angle 30^\circ = 1.1830 \angle -60^\circ \text{ pu}$$

$$I_{aT2}^{(2)} = I_{aT1}^{(2)} \times 1 \angle -30^\circ = 0.7821 \angle 60^\circ \text{ pu}$$

The three phase currents from bus 2 to Y-side of T1:

$$\begin{bmatrix} I_{aT2} \\ I_{bT2} \\ I_{cT2} \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} I_{aT2}^{(0)} \\ I_{aT2}^{(1)} \\ I_{aT2}^{(2)} \end{bmatrix} = \begin{bmatrix} 1.0421 \angle -19.46^\circ \\ -1.9651 \\ 1.0421 \angle 19.46^\circ \end{bmatrix} \text{ pu}$$