

**EE3015 (Power Systems & Protection)****Tutorial 1/Week 2 (PU System I)**

1.1 A 3-phase generator rated 100 MVA, 13.2 kV,  $X = 20\%$  is connected through a  $\Delta/Y$  transformer to a transmission line whose series reactance is 40 ohms per phase. Assume that the base values in the line circuit are 200 MVA and 115 kV.

(a) Find the pu reactance of the transmission line.

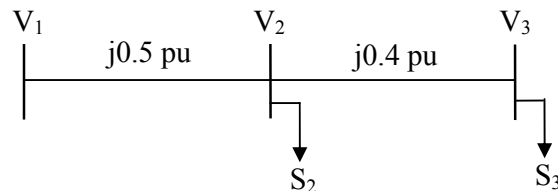
(b) Find the generator and transformer reactances in pu for the following cases:

(i) The transformer is a 3-phase unit rated 100MVA, 13.8 $\Delta$ /120Y kV,  $X = 8\%$ .

(ii) The transformer is composed of three single-phase units, each rated 35 MVA, 13.8/69 kV,  $X = 8\%$ .

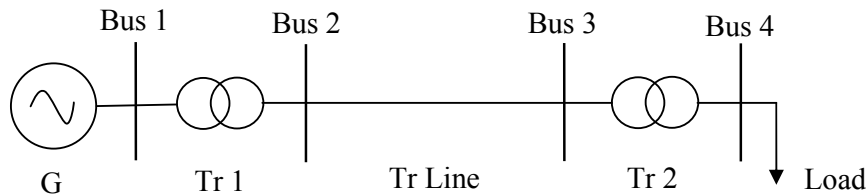
Ans.: (a) 0.6049 pu; (b) (i) 0.3985 pu, 0.1742 pu; (ii) 0.3952 pu, 0.1646 pu

1.2 The one-line diagram of a three-phase power system is shown below; impedances marked are in per unit on a 100-MVA, 400-kV base. Given that loads  $S_2 = (15.93 - j33.4)$  MVA, and  $S_3 = (77 + j14)$  MVA, and that the voltage at bus 3 is maintained at 400 kV, determine the magnitude of the voltage at bus 1.



Ans.: 480 kV

1.3 Obtain the impedance diagram in per unit for the system shown below. Assume a base of 200 MVA, 11 kV in the generator circuit. Without modeling the generator, and given that the transmission line  $X = 50$  ohms and  $R = 2.5$  ohms; transformer ratings for Tr 1 are 50 MVA, 11/132 kV,  $X = 12\%$ ,  $R = 2\%$ , and the corresponding Tr 2 values are 40 MVA, 132/33 kV,  $X = 10\%$ ,  $R = 1\%$ . The load connected at bus 4 is 40 MW at 0.8 pf lag, at a voltage of 30 kV.



Ans.:  $Z_{Tr 1} = 0.08 + j0.48$  pu,  $Z_{Line} = 0.029 + j0.574$  pu,  $Z_{Tr 2} = 0.05 + j0.50$  pu,  $Z_{load} = 3.305 \angle 36.8^\circ$  pu

**EE3015 (Power Systems & Protection)****Tutorial 2/Week 3 (PU System II)**

2.1 The single line diagram of a three-phase power system shown below has the following component ratings:

Generator  $G_1$ : 12 kV, 50 MVA,  $X = 8\%$

Generator  $G_2$ : 12 kV, 40 MVA,  $X = 8\%$

Transformer  $T_1$ : three single-phase units, each rated 13/80 kV, 15 MVA,  $X = 9\%$

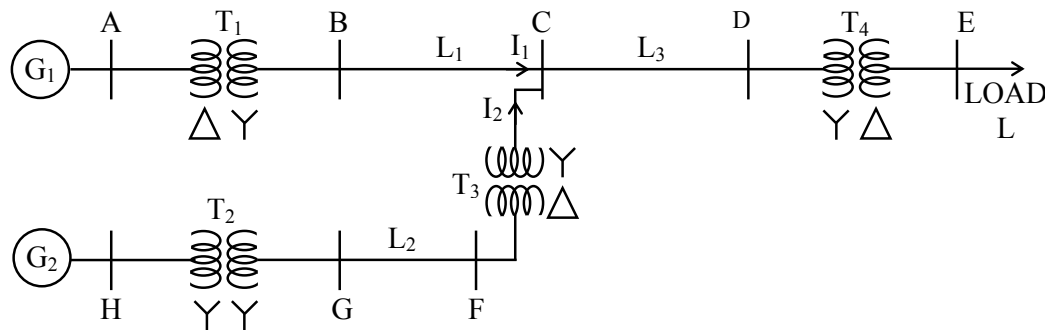
Transformer  $T_2$ : three single-phase units, each rated 6.9/39.8 kV, 25 MVA,  $X = 8\%$

Transformer  $T_3$ : three single-phase units, each rated 70/80 kV, 25 MVA,  $X = 13\%$

Transformer  $T_4$ : 12/138 kV, 80 MVA,  $X = 12\%$

Line  $L_1$ :  $X = 30 \Omega/\text{phase}$ ; Line  $L_2$ :  $X = 30 \Omega/\text{phase}$ ; Line  $L_3$ :  $X = 50 \Omega/\text{phase}$

Load L: 60 MW, 0.9 pf lag at 11 kV (resistance and reactance in series)



- (a) Selecting bases of 100 MVA and 140 kV in line  $L_3$ , draw the per unit impedance diagram. Clearly mark the values of all impedances on this diagram.
- (b) Assume that current phasors  $I_1$  and  $I_2$  are equal when load L is drawing 60 MW at 0.9 pf lag and 11 kV. Calculate the magnitude of the voltage at the terminals of each generator of the system.

Ans: (a)  $G_1$ :  $j0.1335$ ,  $G_2$ :  $j0.1916$ ,  $L_1$ :  $j0.1531$ ,  $L_2$ :  $j0.5998$ ,  $L_3$ :  $j0.2551$ ,  $T_1$ :  $j0.1959$ ,  $T_2$ :  $j0.1013$ ,  $T_3$ :  $j0.1698$ ,  $T_4$ :  $j0.1457$ ,  $Z_L = 1.1022 + j0.5338$ ; (b)  $V_1$ : 15.1527 kV,  $V_2$ : 15.9066 kV

2.2 The single line diagram of a 3-phase power system shown below has the following component ratings:

T. Line  $X = 7.2 \Omega/\text{phase}$ ;  $R = 1.5 \Omega/\text{phase}$

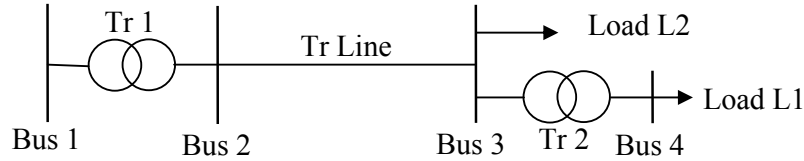
Load L1: Y-connected  $R = 32 \Omega/\text{phase}$

Load L2: 4 MW, 0.8 pf lag, 22 kV

Tr 1: 10 MVA, 230/22 kV,  $Z = (2 + j8)\%$

Tr 2: 5 MVA, 22/12 kV,  $Z = (1.2 + j6)\%$

- (a) Sketch the impedance diagram of the power system showing all quantities in pu. Assume 10 MVA and 22 kV as base values in the transmission line circuit.
- (b) If the voltage at Bus 3 is maintained at 22 kV, calculate the voltages at Buses 2 and 1.

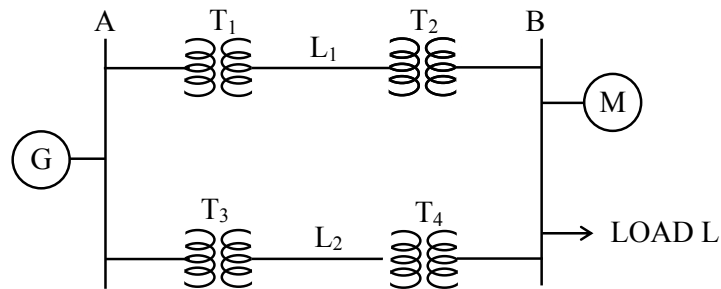


Ans.: (a) Tr 1:  $0.02 + j0.08$ , Tr Line:  $0.0309 + j0.1488$ , Tr 2:  $0.024 + j0.12$ ,  $Z_{L1} = 2.222$ ,  $Z_{L2} = 2 \angle 36.87^\circ$ ; (b)  $V(\text{Bus2}) = 23.77 \text{ kV}$ ,  $V(\text{Bus1}) = 260.12 \text{ kV}$

2.3 The single-line diagram of a 3-phase power system is shown below. The component ratings are as follows:

Generator G: 90 MVA, 22 kV,  $X = 18\%$   
 Transformer T1: 50 MVA, 220/22 kV,  $X = 10\%$   
 Transformer T2: 40 MVA, 11/220 kV,  $X = 6\%$   
 Transformer T3: 40 MVA, 110/22 kV,  $X = 6.4\%$   
 Transformer T4: 40 MVA, 11/110 kV,  $X = 8\%$   
 Synchronous motor M: 66.5 MVA, 10.45 kV,  $X = 18.5\%$   
 Load L: 57 MVA, 0.6 pf lag, 10.45 kV (R & X in series)  
 Line L1:  $X = 48.4 \text{ ohms per phase}$   
 Line L2:  $X = 65.43 \text{ ohms per phase}$

- (a) Selecting a base of 100 MVA and 22 kV in the generator circuit, sketch the per unit impedance diagram of the system (with all values in per unit).
- (b) Assuming that the motor M operates at full-load 0.8 pf leading at a terminal voltage of 10.45 kV, find
- the voltage at the generator bus
  - the generator and motor internal emfs.



Ans.: (a) G:  $j0.2$ , T1:  $j0.2$ , L1:  $j0.1$ , T2:  $j0.15$ , T3:  $j0.16$ , L2:  $j0.5407$ , T4:  $j0.2$ , M:  $j0.2511$ ,  $Z_L = 0.95 + j1.267$ ; (b) (i) 22.145 kV; (ii) 23.817 kV, 11.713 kV

**EE3015 (Power Systems & Protection)****Tutorial 3/Week 4 (Synchronous Generators I)**

3.1 A three-phase, 125-MVA, 0.8 pf (lag), 13.2-kV, 1800-rpm synchronous generator has a synchronous reactance of  $1.7 \Omega$  per phase and negligible stator resistance.

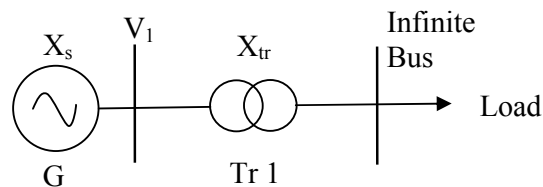
- If the machine is connected to an infinite bus operating at 13.2 kV, and it delivers 100 MW at unity power factor, find the magnitude of the generated voltage, the power angle, and the reactive power.
- Assume that the prime mover input to the generator of part (a) is reduced by 50% without changing the excitation. Find the new operating conditions.
- Starting at the condition of part (a), assume that the excitation is adjusted to provide 100 MW at 0.8 pf leading. Evaluate the new operating conditions.
- Sketch a single phasor diagram to depict the three operating conditions in parts (a) – (c).

Ans.: (a) 18.442 kV,  $44.295^\circ$ , 0; (b)  $20.437^\circ$ , 31.687 Mvar produced by the generator; (c) 13.357 kV,  $74.63^\circ$ , 75 Mvar absorbed by the generator

3.2 A three-phase Y-connected synchronous generator has a synchronous reactance of 10 ohms and negligible armature resistance. At unity power factor the generator supplies 188 A to a 13.2-kV infinite bus. When the generator excitation is increased by 25%, without changing the mechanical power input, determine the generator line current and power factor. Use the per unit system for your calculations. Sketch a phasor diagram to show the two operating conditions.

Ans.: 275.15 A, 0.683 lag

3.3 A 120-MVA, 22-kV synchronous generator has  $X_s = 1.16$  pu. The generator supplies power to a large power system (Infinite bus) through a 150 MVA, 22/345 kV,  $X_{tr} = 0.06$  pu transformer, as shown below.



- Determine the induced emf and the terminal voltage of the generator when the load supplied to the infinite bus is 80 MW at 0.8 pf leading, at the rated voltage of 345 kV.
- If the generator excitation is increased by 20% while supplying the constant load of 80 MW, calculate:
  - the generator terminal voltage

- (ii) the reactive power output of the generator at its terminals, and
  - (iii) the reactive power delivered to the infinite bus.
- (c) Determine the maximum real powers the generator can supply to the infinite bus at the excitation levels in (a) and (b) respectively. In addition, find the power factors of the load at the infinite bus while delivering these maximum powers.

Ans.: (a) 19.743 kV, 21.4835 kV; (b) 21.762 kV, 25.43 Mvar absorbed by the generator, 28.31 Mvar flowing towards the transformer; (c) 89.146 MW, 0.6678 lead, 106.975 MW, 0.7328 lead

**EE3015 (Power Systems & Protection)**

**Tutorial 4/Week 5 (Synchronous Generators II)**

4.1 A 24-MVA, 17.32-kV, 60-Hz, Y-connected, 3-phase synchronous generator has a synchronous reactance of 5 ohms per phase, and negligible armature resistance.

- (i) At a certain excitation, the generator delivers rated MVA at 0.8 power factor lagging, to an infinite bus operating at 17.32 kV. Determine the magnitude of the internal emf per phase and the power angle of the generator.
- (ii) The internal emf and terminal voltage are maintained constant at 13.4 kV/phase and 10 kV/phase respectively. What is the maximum three-phase real power this generator can develop before it pulls out of synchronism? What are the armature current and reactive power under this condition?

Ans.: (i) 12.806 kV, 14.47°; (ii) 80.4 MW, 3344.106 A, 60 MVar absorbed by the generator

4.2 Two generators, rated 3 MW each, are operating in parallel to supply a total load of 4 MW at 0.8 pf (lag). The generators have no-load frequencies of 52 Hz and 51 Hz respectively. The frequencies of both generators fall to 49 Hz at full load.

- (a) What is the system frequency and how much power is supplied by each generator?
- (b) Find the pf of the second generator if the first generator is operating at 0.85 (lag).
- (c) Determine the no-load frequency setting of the second generator to bring the system back to 50 Hz at this load.

Ans.: (a) 49.8 Hz, 2.2 MW, 1.8 MW; (b) 0.74 lag; (c) 51.33 Hz

4.3 A 480-V, 200-kW, 2-pole, 3-phase, 50-Hz synchronous generator's prime mover has a no-load speed of 3040 rpm, and a full-load speed of 2975 rpm. It is operating in parallel with a 480-V, 150-kW, 4-pole, 3-phase, 50-Hz synchronous generator whose prime mover has a no-load speed of 1500 rpm, and a full-load speed of 1485 rpm. The total load supplied by the two generators is 200 kW at 0.85 pf lagging.

- (a) Find the operating frequency of the power system.
- (b) Find the power being supplied by each of the two generators.

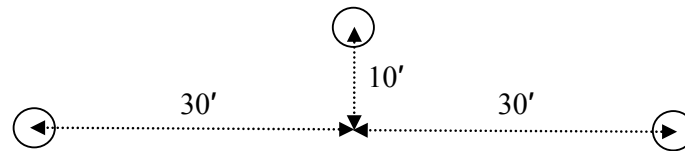
Ans.: (a) 49.843 Hz; (b) 153 kW, 47 kW

4.4 Two three-phase, 6.6-kV, Y-connected synchronous generators operating in parallel supply a load of 3 MW at 6.6 kV and 0.8 pf lagging. The synchronous impedances of the machines A and B are  $(0.5 + j10) \Omega$  and  $(0.4 + j12) \Omega$  respectively. The excitation of machine A is adjusted so that it delivers 150 A at a lagging pf, and the governors of the two machines are so set that the load (real power) is shared equally between the machines. Determine the armature current of the second machine, and also the power factor, internal voltage and power angle of each machine. Sketch a phasor diagram showing all currents and voltages.

Ans.: 180.63 A, 0.8747 lag, 8.274 kV,  $15.49^\circ$ , 0.7264 lag, 9.640 kV,  $15.90^\circ$

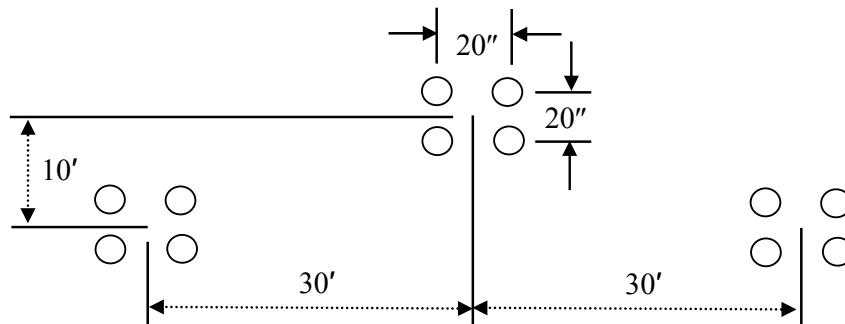
**EE3015 (Power Systems & Protection)****Tutorial 5/Week 6 (Transmission Lines)**

- 5.1(a) A three-phase transmission line consisting of stranded ACSR conductors is shown below. Given that the system frequency is 50 Hz,  $R = 0.1204$  ohms/mile,  $GMR = 0.0403$  ft, conductor diameter = 1.196 inches, calculate the resistance, inductance, series reactance, capacitance, and shunt admittance per km of the line.



Ans.:  $0.0748 \Omega$ ;  $1.3758$  mH;  $0.432 \Omega$ ;  $0.008341 \mu\text{F}$  to neutral;  $j2.62 \mu\text{S}$

- 5.1(b) Each phase of the line in part (a) above consists of a bundle of 4 stranded conductors as shown below. Find the new  $R$ ,  $L$ ,  $X$ ,  $C$  and  $Y$  for the line.



Ans.:  $0.0187 \Omega$ ,  $0.8002$  mH,  $0.2514 \Omega$ ,  $0.01409 \mu\text{F}$ ,  $j4.43 \mu\text{S}$

- 5.2 A three-phase transposed transmission line is composed of one ACSR conductor per phase with flat horizontal spacing of 11 m between adjacent conductors. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm each. The line is to be replaced by a 3-conductor bundle of ACSR conductors having a diameter of 2.1793 cm and a GMR of 0.8839 cm each. The replaced line will also have a flat horizontal configuration, but it is to be operated at a higher voltage and therefore the phase spacing is increased to 14 m between adjacent bundles. The spacing between the conductors in the bundle is 45cm ( $\epsilon = 8.85 \times 10^{-12}$  F/m).

Determine:

- the percentage change in the line inductance, and
- the percentage change in the line capacitance.

Ans.: (i) 27.53% (decrease); (ii) 35.25% (increase)



5.3 A 3-phase, 765-kV, 60-Hz transmission line is composed of four ACSR conductors per phase with a flat horizontal spacing of 14 m between adjacent conductors. The conductors have a diameter of 3.625 cm, and a GMR of 1.439 cm. The bundle spacing is 45 cm, and the line is 400 km long. Assume the line to be lossless and model it as a nominal- $\pi$  equivalent with base values of 765 kV and 2000 MVA ( $\epsilon = 8.85 * 10^{-12}$  F/m).

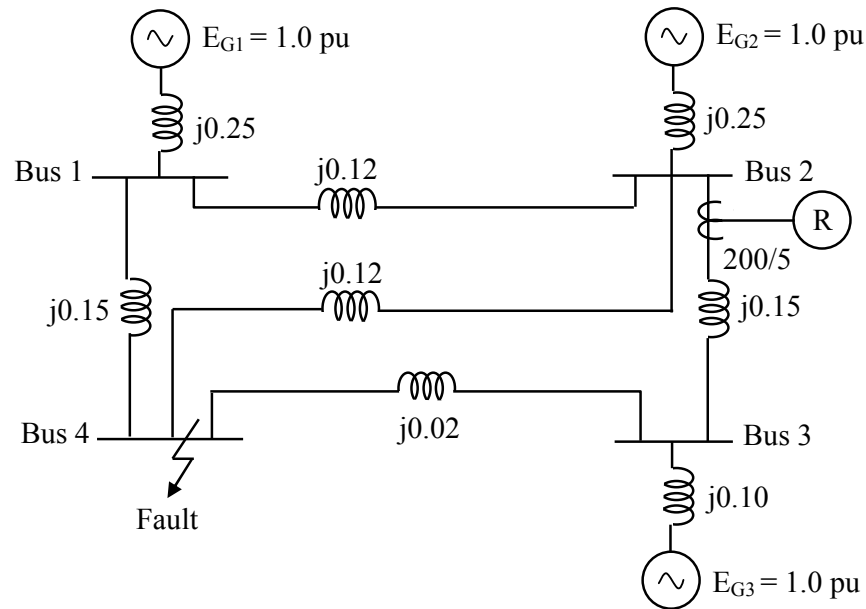
- (a) Determine the receiving-end voltage, current and complex power when 1920 MW and 600 MVar (lag) are being transmitted at 765 kV at the sending end.
- (b) If the line is energized with 765 kV at the sending end when the load at the receiving end is removed, what will be the receiving-end voltage?

Ans.: (a) 654.86 kV, 1732.31 A, 1920 MW, 417.64 Mvar (lag), 1964.86 MVA; (b) 877.39 kV

**EE3015 (Power Systems & Protection)****Tutorial 6/Week 7 (Symmetrical Faults)**

6.1 The figure below shows a 4-bus power system where all the pu values are based on 33 kV, 100 MVA bases.

- (a) Form the bus admittance matrix for the system.
- (b) Find the total fault current, the voltages at all the buses and the fault currents in all the lines, for a 3-phase fault at Bus 4 (as shown). Assume that the voltage throughout the network is 1.0 pu before the fault. The impedance matrix for the system is given below.



$$Z_{\text{Bus}} = j \begin{bmatrix} 0.0929 & 0.0532 & 0.0416 & 0.0483 \\ 0.0532 & 0.08 & 0.0467 & 0.0517 \\ 0.0416 & 0.0467 & 0.0647 & 0.06 \\ 0.0483 & 0.0517 & 0.06 & 0.0731 \end{bmatrix}$$

$$\text{Ans.: (a) } Y = -j \begin{bmatrix} 19 & -8.33 & 0 & -6.67 \\ -8.33 & 27.33 & -6.67 & -8.33 \\ 0 & -6.67 & 66.67 & -50 \\ -6.67 & -8.33 & -50 & 65 \end{bmatrix};$$

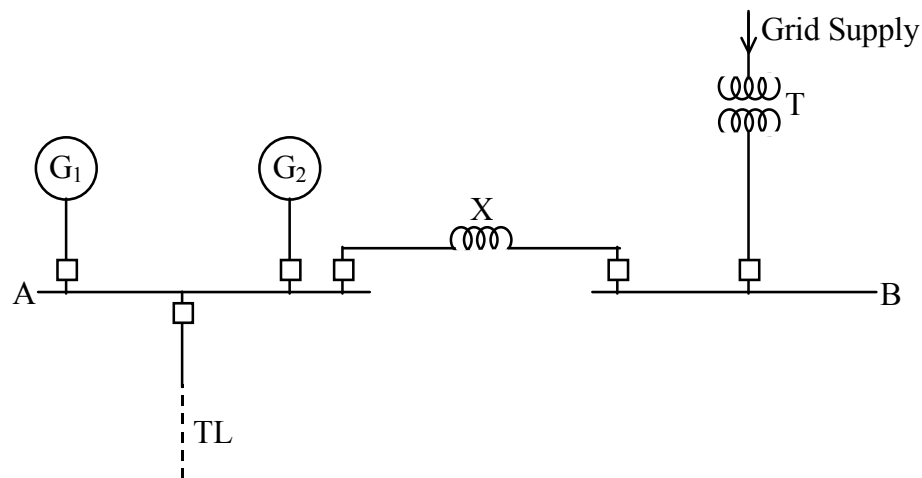
- (b)  $I_f = 23.934 \text{ kA}$ ,  $V_1 = 11.2 \text{ kV}$ ,  $V_2 = 9.66 \text{ kV}$ ,  $V_3 = 5.91 \text{ kV}$ ,  $I_{12} = 0.679 \text{ kA}$ ,  $I_{14} = 3.957 \text{ kA}$ ,  $I_{23} = 1.324 \text{ kA}$ ,  $I_{24} = 4.267 \text{ kA}$ ,  $I_{34} = 15.676 \text{ kA}$

6.2 A 3-phase synchronous generator is rated 50 MVA, 13.2 kV, 0.8 pf lagging, and has a synchronous reactance of 20%. The generator is connected to a 60 MVA, 13.4/138 kV, 3-phase transformer having a reactance of 15%. The generator is initially operating on no load and at rated terminal voltage. A 3-phase symmetrical fault occurs on the high-voltage terminals of the transformer. Determine the short-circuit current supplied by the generator and its terminal voltage.

Ans.: 6650.9 A, 5.17 kV

6.3 Consider the three-phase power system shown below. Generators  $G_1$  and  $G_2$  are connected in parallel to a 30-kV busbar A. The ratings of  $G_1$  and  $G_2$  are 100 MVA, 30 kV,  $X = 20\%$  and 50 MVA, 30 kV,  $X = 15\%$  respectively. Busbar A feeds a transmission line TL. A grid supply is connected to the station busbar B through a transformer T rated for 500 MVA, 400/30 kV,  $X = 20\%$ .

Determine the reactance (in ohms) of a current limiting reactor X to be connected between the grid system (busbar B) and the 2-generator busbar (busbar A) such that the short-circuit MVA at bus A does not exceed 1250 MVA.



Ans.: 1.8 ohms

**EE3015 (Power Systems & Protection)**

**Tutorial 7/Week 8 (Shunt Compensation)**

**Note: This Tutorial will be preceded by a 30-minute Closed Book Quiz.**

7.1 A feeder from a distribution transformer is 500 feet long. It consists of three wires having an impedance of  $0.052 + j0.069$  ohms per 1,000 feet and current capacity of 225 amps. The three-phase load is balanced and draws 120 kW at 440 volts with a 60% lagging power factor.

- (a) Determine the kVA rating of capacitors capable of correcting the power factor to allow rated current to flow in the feeder.
- (b) Draw the phasor diagram of the feeder current before and after power factor correction and compute the new power factor.
- (c) Determine the voltage at the sending end of the feeder.
- (d) Determine the voltage regulation of the feeder.

Ans.: (a) 37.513 kVA; (b) 0.7 lag; (c) 456.7 V; (d) 4.49%

**Extra Problems**

1. The single-line diagram of an unloaded three-phase power system is shown below. The component ratings are as follows:

Generator  $G_1$ : 20 MVA, 13.8 kV,  $X = 0.2\text{pu}$

Generator  $G_2$ : 30 MVA, 18 kV,  $X = 0.2\text{pu}$

Generator  $G_3$ : 30 MVA, 20 kV,  $X = 0.2\text{pu}$

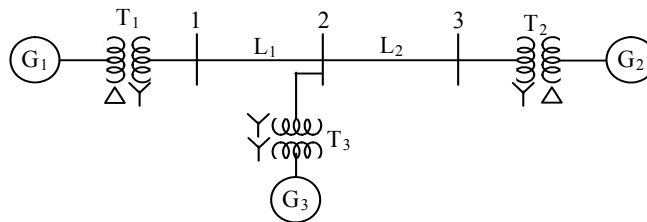
Transformer  $T_1$ : 25 MVA, 220Y/13.8 $\Delta$  kV,  $X = 10\%$

Transformer  $T_2$ : 3 single-phase units, each rated 10 MVA, 127/18 kV,  $X = 10\%$

Transformer  $T_3$ : 35 MVA, 220Y/22Y kV,  $X = 10\%$

Line  $L_1$ :  $j80\ \Omega$  per phase; Line  $L_2$ :  $j100\ \Omega$  per phase

Draw the per unit impedance diagram using 50 MVA, 13.8 kV bases in the circuit of generator  $G_1$ . Clearly mark all impedance values on the diagram.



Ans.:  $G_1$ :  $j0.5$ ,  $T_1$ :  $j0.2$ ,  $L_1$ :  $j0.0826$ ,  $L_2$ :  $j0.1033$ ,  $T_2$ :  $j0.1667$ ,  $G_2$ :  $j0.3332$ ,  $T_3$ :  $j0.1428$ ,  $G_3$ :  $j0.2755$

2. Two three-phase, Y-connected synchronous generators have per-phase generated voltages of  $120\angle 10^\circ$  V and  $120\angle 20^\circ$  V under no load, and reactances of  $j5$  ohms/phase and  $j8$  ohms/phase, respectively. They are connected in parallel to a load impedance of  $(4+j3)$  ohms/phase. Determine

- the per-phase terminal voltage
- the armature current of each generator
- the power supplied by each generator, and
- the total power output.

Ans.: (a) 82 V/ph; (b)  $I_{a1} = 9.36\angle -51.17^\circ$  A,  $I_{a2} = 7.31\angle -32.06^\circ$  A; (c)  $P_{G1} = 1624$  W,  $P_{G2} = 1617$  W; (d) 3241 W

3. A 500-kVA, 2300-V, three-phase, Y-connected synchronous generator is operated at its rated speed to obtain its rated voltage on no-load. When a short-circuit is established, the phase current is 150 A. The average resistance of each phase is 0.5 ohms.

- Determine the synchronous reactance in ohms per phase.
- Using the per unit system, determine the percentage regulation when the generator delivers 500 kVA at its rated voltage and 0.8 pf lagging.

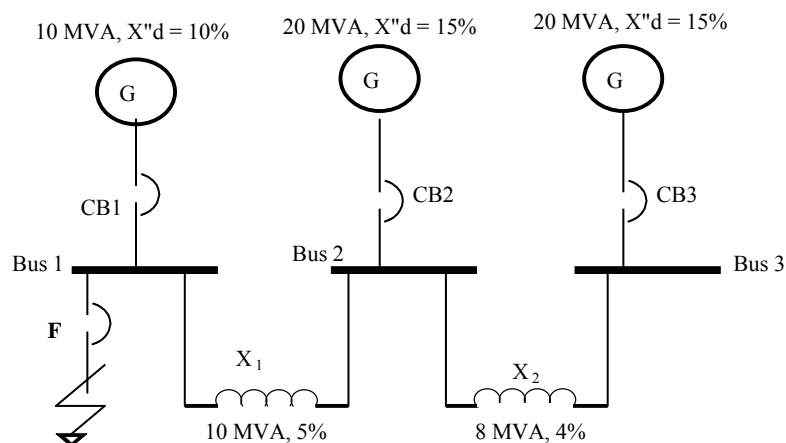
Ans.: 8.84ohms, 66.67%

4. A 3-phase, 345-kV, 50-Hz, 96 km long transmission line has a series impedance of  $(0.21 + j0.85)$  ohms/km. The load at the receiving end draws 120 MW at the rated voltage. Calculate the following for each of the three conditions (1) 0.8 pf lag, (2) unity pf, and (3) 0.8 pf lead:

- The sending end voltage.
- The sending end real and reactive powers.
- The real power loss in the line.
- Sketch the phasor diagrams for cases (1), (2) and (3).

Ans.: (1) 374 kV, 123.81 MW, 105.4 MVar, 3.81 MW; (2) 353.1kV, 122.43MW, 9.9MVar, 2.43MW; (3) 332.4kV, 123.81MW, -74.6MVar, 3.81MW

5. Three generators are interconnected through fault-current-limiting reactors as shown below. If the busbar line-to-line voltage is 11 kV, calculate the fault level and the fault current for a 3-phase symmetrical fault at F. Assume a base MVA of 20 MVA.



Ans.: 203.23 MVA, 10.67 kA

Apr/May 2014

**Q1 Ans.:** (a)  $X_s = 1.45$  pu,  $X_{tr} = 0.06$  pu,  $Z = j0.07429$  pu,  $Y = j0.4896$  pu,  $Z_L = 2\angle 36.87^\circ$  pu; (b)  $V_s = 346.57$  kV,  $I_s = 0.1096$  kA,  $|S_s| = 65.78$  MVA,  $P_s = 60$  MW,  $Q_s = 26.953$  Mvar flowing towards the transformer; (c)  $V_1 = 21.870$  kV,  $E = 20.882$  kV

**Q2 Ans.:** (a)  $E_2 = 19.511$  kV,  $\delta_2 = 26.81^\circ$ ,  $Q_2 = 0.417$  Mvar absorbed by the generator,  $\theta_2 = 27.529^\circ$  lead because  $Q_2$  is negative or absorbed by the generator; (b)  $\delta = 29.069^\circ$

Nov/Dec 2013

**Q1 Ans.:** (a)  $X_G = 1.093$  pu,  $X_{T1} = 0.4$  pu,  $X_{T2} = 0.667$  pu,  $X_{Line} = 0.386$  pu,  $X_{M1} = 3.267$  pu,  $X_{M2} = 3.267$  pu; (b)  $V_t = 13.753$  kV,  $E_G = 14.85$  kV,  $E_{M1} = E_{M2} = 11.698$  kV; (c)  $Q_C = -2.125$  Mvar

**Q2 Ans.:** (a)  $f_{nLA} = f_{nLB} = 62.206$  Hz,  $f_{nLC} = 61.177$  Hz; (b)  $f_{sys} = 59.804$  Hz, (c)  $f_{nLSWG} = 61.765$ ; (d)  $f_{sys} = 59.118$  Hz but at this frequency,  $P_A = P_B = 105$  MW are overloaded and their protective relays may trip

Apr/May 2013

**Q1 Ans.:** (a)  $X_G = 0.2$  pu,  $X_T = 0.05$  pu,  $Z = 0.1712\angle 83.66^\circ$  pu,  $Y/2 = j0.045$  pu,  $Z_{LD} = 2\angle 36.87^\circ$  pu; (b)  $V_2 = 242.17$  kV,  $\eta = 98.95\%$ ; (c)  $I_f = 1.498$  kA

**Q2 Ans.:** (a)  $E = 19.68$  kV,  $VR = -10.56\%$ ; (b)  $\delta_2 = 45.69^\circ$ ,  $Q_2 = 0.7504$  MVar absorbed

Nov/Dec 2012

**Q1 Ans.:** (a)  $X_{12} = X_{13} = X_{23} = 0.1$  pu,  $X_{GT1} = 0.15$  pu,  $X_{GT2} = 0.075$  pu; (b) Diagonal matrix elements = -j26.67, -j33.33, -j20 pu, off-diagonal matrix elements = j10 pu for all; (c)  $I_f = 1.2374$  kA,  $V_1 = 103.673$  kV,  $V_2 = 123.106$  kV,  $V_3 = 0$ ,  $I_{12} = I_{21} = 0.1061$  kA,  $I_{13} = I_{31} = 0.5657$  kA,  $I_{23} = I_{32} = 0.6718$  kA; (d)  $I_f = 1.2374$  kA

**Q2 Ans.:** (a)  $E = 36.204$  kV,  $\delta = 27.113^\circ$ ; (b)  $E_2 = 39.8247$  kV,  $\delta_2 = 24.476^\circ$ ,  $Q_2 = 43.169$  MVar generated,  $pf = 0.7569$  lag; (c)  $\delta_3 = 30.206^\circ$ ,  $P_3 = 60.716$  MW,  $Q_3 = 37.628$  Mvar generated; (d)  $P_{max} = 120.681$  MW

Apr/May 2012

**Q1 Ans.:** (a)  $X_{G1} = 0.4$  pu,  $X_{G2} = 0.6$  pu,  $X_T = 0.16$  pu,  $X_{TL} = 0.1$  pu,  $Z_{LD} = 2\angle 36.87^\circ$  pu; (b)  $V_1 = 32.49$  kV; (c)  $I_f = 320.75$  A,  $S_F = 222.222$  MVA; (d)  $S_{G1} = 133.333$  MVA,  $S_{G2} = 88.888$  MVA

**Q2 Ans.:** (a)  $A = D = 0.9924\angle 0.0491^\circ$  pu,  $B = 0.1705\angle 83.684^\circ$  pu,  $C = j0.08993$ ; (b)  $V_s = 248.208$  kV,  $I_s = 0.4962$  kA,  $S_s = 163.912$  MW + j136.505 Mvar; (d)  $\eta = 97.613\%$

Nov/Dec 2011

**Q1 Ans.:** (a)  $X_{G1} = 0.15$  pu,  $X_{G2} = 0.25$  pu,  $X_{T1} = 0.1$  pu,  $X_{T2} = 0.1$  pu,  $X_L = 0.1736$  pu,  $Z_{LD} = 0.6667\angle 36.87^\circ$  pu; (b)  $E_{G1} = 14.475$  kV,  $E_{G2} = 15.163$  kV; (c)  $I_f = 0.6325$  kA,  $S_{G1} = 74.227$  MVA,  $S_{G2} = 57.234$  MVA

**Q2 Ans.:** (a)  $f_{sys} = 50.615$  Hz,  $P_{GA} = 18.769$  MW,  $P_{GB} = 21.231$  MW; (b)  $PF_B = 0.7125$  lagging; (c)  $f_{nLB} = 50.1667$  Hz

May 2011

**Q1 Ans.:** (a)  $X_{GT1} = 0.15$  pu,  $X_{GT2} = 0.075$  pu,  $X_{12} = X_{13} = X_{23} = 0.1$  pu; (b)  $I_f = 13.736$  pu; (c)  $S_{G1} = 333.59$  MVA,  $S_{G2} = 353.21$  MVA

**Q2 Ans.:** (a)  $A = 1$  pu,  $B = 0.06605\angle 76.905^\circ$  pu,  $C = 0$ ,  $D = 1$  pu,  $VR = 5.143\%$ ; (b)  $PF = 0.966$  leading; (c)  $98.474\%$

December 2010

**Q1 Ans.:** (a)  $X_{T1} = 0.06667$  pu,  $X_{T2} = 0.07841$  pu,  $X_L = 0.05377$  pu,  $Z_{LD} = 1.3443\angle 36.87^\circ$  pu; (b)  $V_1 = 37.9378$  kV; (c)  $I_f = 1.7888$  kA

**Q2 Ans.:** (a)  $\delta = 22.293^\circ$ ,  $P = 69.376$  MW,  $Q = 52.033$  Mvar delivered; (b)  $\delta = 50.152^\circ$ ,  $P = 140.414$  MW; (c)  $P_{max} = 182.891$  MW

Apr/May 2010

**Q1 Ans.:** (a)  $X_G = 0.15$  pu,  $X_{T1} = 0.08571$  pu,  $X_{T2} = 0.1$  pu,  $X_L = 0.1134$  pu,  $X_M = 0.2$  pu; (b)  $E_M = 12.6724$  kV,  $E_G = 14.7349$  kV; (c)  $I_f$  in LV side of T2 = 3.0851 kA,  $I_f$  in line L = 0.3353 kA

**Q2 Ans.:** (a)  $L = 10.4241 \times 10^{-2}$  H,  $C_n = 1.0911$   $\mu$ F; (b)  $A = 0.9919$  pu,  $B = j0.1644$  pu,  $C = j0.09791$  pu,  $D = 0.9919$  pu; (c)  $V_s = 348.58$  kV,  $I_s = 0.6648$  kA,  $S_s = 401.355$  MVA

Nov/Dec 2009

**Q1 Ans.:** (a)  $X_{T1} = 0.1$  pu,  $X_{T2} = 0.1$  pu,  $X_L = 0.09452$  pu; (b) and (c)  $S_f = 378.524$  MVA,  $S_{G1} = 128.523$  MVA,  $S_{G2} = 250$  MVA

**Q2 Ans.:** (a)  $E = 17.99$  kV,  $VR = 63.55\%$ ; (b)  $P_{\max} = 197.892$  MW,  $I_a = 12.174$  kA,  $PF = 0.8532$  leading,  $Q = 121$  Mvar absorbed by the generator



## TUTORIAL 8: THREE PHASE POWER SYSTEMS

8.1: Three  $(30 + j30) \Omega$  identical impedances are connected in Delta and supplied by a 173V 3-phase system through three lines each having impedance of  $(0.8 + j0.6) \Omega$ . Find:

- 1) the current magnitude in each of the delta-connected impedances, and
- 2) the voltage across each of Delta impedance.

8.2: A 415V, 50 HZ, 4-wire three-phase balanced power supply with sequence RYB is connected to the following loads:

A single resistance of  $12 \Omega$  between R phase and neutral;

An inductive impedance of  $(2 + j8) \Omega$  between B phase and neutral;

A capacitor of  $120\mu\text{F}$  between R and Y phase;

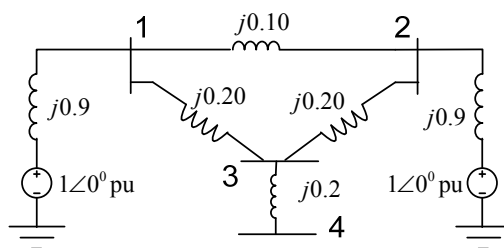
A three-phase, delta connected induction motor operating at 10kW and 0.75 p.f. lagging.

Calculate:

- 1) the magnitude and phase angle of the current in the four lines of the supply, and
- 2) the total power from the supply.

8.3: The figure below shows the per-phase per-unit network of a 4-bus power system. Develop Thevenin equivalent circuits as viewed

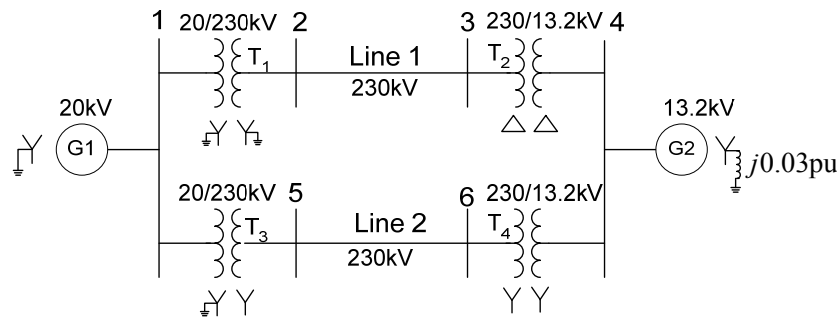
- 1) from bus 4 and ground, and
- 2) from bus 2 and ground.



## Tutorial 9: Sequence Components and Sequence Networks

9.1: The phase voltages at a bus of an unbalanced power system are  $V_a = 1\angle 0^\circ$ ,  $V_b = 1\angle -90^\circ$ ,  $V_c = 2\angle 135^\circ$ . Find the sequence voltages of three unbalanced voltages and prove that:  $V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)}$ .

9.2: The component parameters of the following system are given in Table. The power rating for each apparatus is 200MVA. Sketch the sequence networks and find the Thevenin equivalent reactance of each sequence network seen from bus 4. Assume that pre-fault voltage at bus 4 is  $1\angle 0^\circ$  pu. (Exercise: the same question for Bus 5)



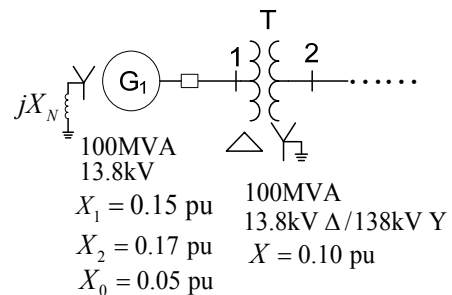
Component	kV Rating	$X_1(\text{pu})$	$X_2(\text{pu})$	$X_0(\text{pu})$
$G_1$	20	0.20	0.14	0.05
$G_2$	13.2	0.20	0.14	0.05
$T_1$	20/230	0.20	0.20	0.20
$T_2$	230/13.2	0.30	0.30	0.30
$T_3$	20/230	0.25	0.25	0.25
$T_4$	230/13.2	0.35	0.35	0.35
Line 1	230	0.15	0.15	0.35
Line 2	230	0.22	0.22	0.55

9.3: Part of a power system is shown in the figure.  $V_f = 1\angle 0^\circ$  pu.  $X_N = 5\%$ . For a bolted double line-to-ground fault occurs on phase b and c at bus 1, the three-phase currents out of  $G_1$ :

$$I_a = 0.4008\angle 90^\circ \text{ pu}, I_b = 6.1817\angle 152.86^\circ \text{ pu}, I_c = 6.1817\angle 27.14^\circ \text{ pu}$$

Currents  $I_{fb}$  and  $I_{fc}$  from bus 1 to ground:  $I_{fb} = 7.8104\angle 157.25^\circ$  pu and  $I_{fc} = 7.8104\angle 22.75^\circ$  pu.

- Calculate per-unit sequence currents of phase  $a$  flowing from delta side of transformer  $T_1$  to bus 1.
- Calculate per-unit three-phase currents flowing from bus 2 to Y side of  $T_1$ .



## Tutorial 10: Fault Analysis

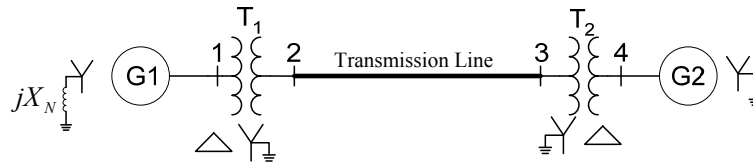
10.1: The one-line diagram for a power system is given in the following figure. The parameters of the generators and transformers are given as:

G1 and G2: 100MVA, 20kV,  $X_1 = X_2 = 20\%$ ,  $X_0 = 6\%$  and  $X_N = 5\%$

Transformers  $T_1$  and  $T_2$ : 100MVA, 20 $\Delta$  / 345Y kV and  $X = 10\%$

Line (on bases 100MVA, 345kV):  $X_1 = X_2 = 10\%$ ,  $X_0 = 25\%$

A bolted single line-to-ground fault occurs on phase  $a$  at bus 2.  $V_f = 1 \angle 0^\circ$  pu.



- Draw the sequence networks.
- Calculate the per unit sequence and phase currents at the fault point.
- Determine the per unit phase currents flowing from Y side of  $T_1$  to fault point.
- Determine the per unit current flowing out of phase  $b$  of G1.

10.2: A bolted double line-to-ground fault between phase  $b$  and  $c$  occurs at bus 2 in the power system shown in question 10.1.

- Calculate per unit sequence currents at the fault point.
- Calculate the per unit phase voltages at bus 3.

10.3 (Homework) A double line fault between phase  $b$  and  $c$  with the fault impedance  $X_f = 0.1$  pu occurs at bus 1 in the power system shown in question 10.1.

- Calculate the per unit sequence currents through transmission line.
- Calculate the per unit sequence voltages at bus 3.

## Tutorial 11: Protection Zone and Overcurrent Protection

11.1: The system is shown in the following figure. Assuming that all the circuit breakers operated correctly, determine the fault locations for each of the following cases.

Case 1: B2 and B3

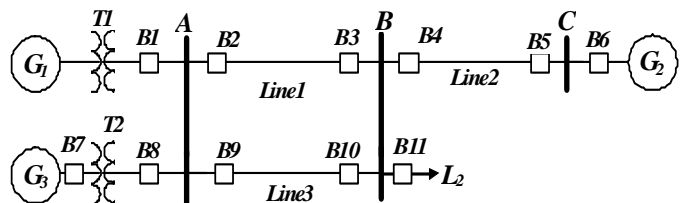
Case 2: B7, B8, B1, B2 and B9

Case 3: B6

Case 4: B11

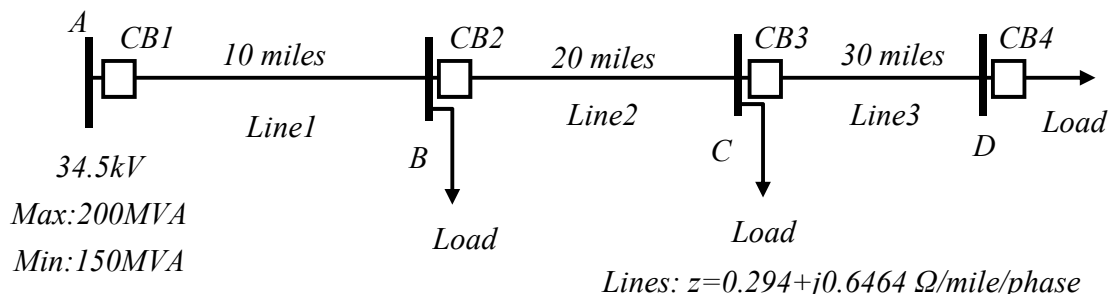
Case 5: B2, B3, B4, B10 and B11

Case 6: B3, B4, B10 and B11



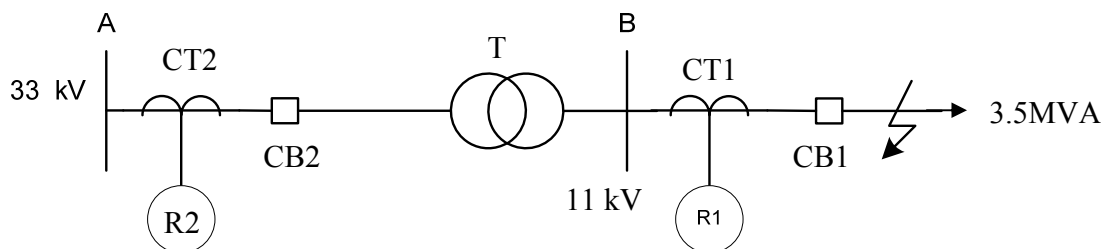
11.2: A 50-Hz radial distribution system and the data are shown in the figure. The maximum and minimum fault levels at bus A are 200 MVA and 150 MVA, respectively. The maximum and minimum loads for each of the three load points are 5 MVA and 3MVA, respectively. The power factors of the loads are the same. The CO-8 relays are selected to protect load and lines. The coordination time interval (CTI) for two adjacent relays is 0.3s. Ignore transmission losses when calculate the maximum load current. Ignore the operating time of the breakers.

- 1) Select the CT ratio for the related relay at each bus.
- 2) Select current plug settings (PSs) for all the relays.
- 3) Select time dial settings (TDSs) for the phase relays R2 at bus B and R3 at bus C.
- 4) Calculate the operating time of R2 and R3 if a three phase to ground fault occurs at 10 miles to Bus C on line3 for the minimum system capacity.



11.3: A 3.5 MVA rated load is supplied from 33 kV system bus through a 40MVA and 33 $\Delta$ /11Y kV transformer with a leakage reactance of 15%. The fault level at 33 kV is 2000MVA. The relays R1 and R2 are IDMT relays with 5A rating. The time-current curve of the relay is represented by  $t = \frac{0.14}{M^{0.02} - 1} \text{TDS}$ . The TDSs are from 0.1 to 1 in the step of 0.1. The plug settings are from 50% to 200% of the relay rating in the step of 25%. The CT ratios for CT1 and CT2 are 400:5 and 300:5 respectively.

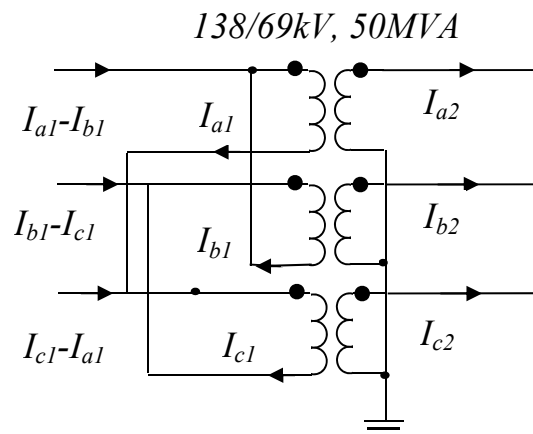
- a. Determine the PSs and the TDSs of the phase relays if the required discrimination time or CTI between R2 and R1 is 0.4s.
- b. Find the fault location and the minimum operating time of R2 based on the relay settings from part a.



## Tutorial 12: Differential Protection

12.1: A 138/69kV  $\Delta$ -Y transformer is shown in the following figure. The CT intermediate taps on high and low voltage sides are  $T_H=6$  and  $T_L=9$  respectively. The CT ratio on  $\Delta$  side is 300:5 and Y side is 500:5

- Draw the phasor diagram of the three phase terminal currents.
- Draw the connections of CTs and percentage differential relays for correct relay operation.
- Calculate current mismatch of the relay for the maximum load current with and without tap change.



12.2: A Y-connected generator is connected to a 230 kV system through a  $\Delta$ -Y 33/230 kV transformer as shown in the following figure. The system parameters are also shown in the figure. The positive and negative impedances are the same for all components. A differential protection is used to protect generator.

- Select CT ratio.
- Calculate the currents through restraint windings and operating winding for the single phase to ground faults at  $F1$  and  $F2$ .

