Exercise Questions for Chapter 3: Empirical Modeling

1. An operator introduces a step change in the flow rate q_i from 500 to 540 gal/min to a particular process at 3.05am. The first change in the process temperature T (initially at 120^0F) comes at 3.09am. After that, the response in T is quite rapid, slowing down gradually until it appears to reach a steady state value of 124.7^0F. The operator notes in the logbook that there is no change after 3.34 am. What approximate transfer function might be used to relate the temperature to flow rate for this process in the absence of more accurate information? What should operator do next time to obtain a better estimate?

2. The non-isothermal CSTR shown in Figure Q2a is considered in this problem.

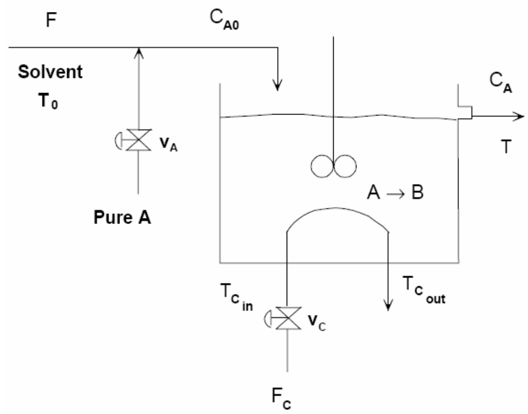

Figure Q2a. The non-isothermal CSTR

Two experiments have been performed to identify the model relating the reactor concentration, C_A, and the coolant valve opening, v_c. Two step changes of +20% and –20% were introduced in v_c, and reactor concentration was measured using an analyzer. The process reaction curve is shown in Figure Q2b. Discuss the good and poor aspects of these experiments for use with the process reaction curve modeling method.

a. Critique the testing results carefully.

b. Determine the parameters for the first order with dead time model using two different sets of experimental data.

c. Compare the parameter values in part b obtained from two different experiments, and explain any differences.

d. Discuss experimental designs that could help identify the problem encountered in question

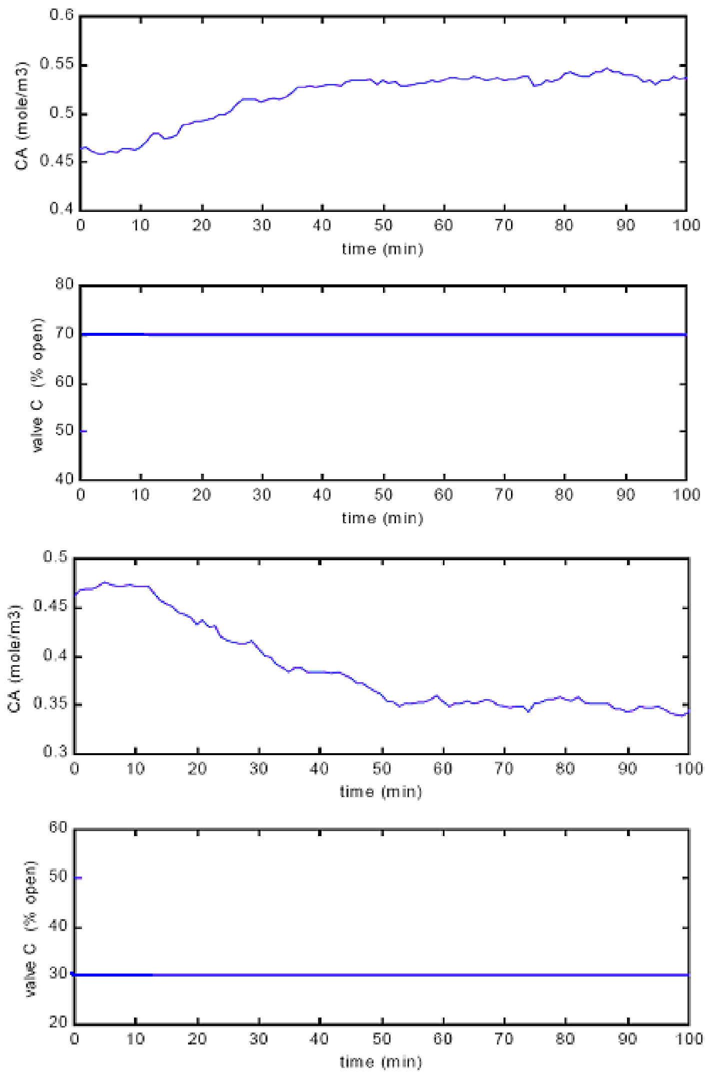

Figure Q2b. Process reaction curves for the CSTR without any unmeasured disturbances

3. The overall transfer function of a process is given by

$$G = G_1 G_2$$

where

$$G_1(s) = \frac{K_1}{\tau_1 s + 1}; \quad G_2(s) = \frac{K_2}{\tau_2 s + 1}; \quad \tau_1 = 5; \quad \tau_2 = 3$$

(a) What is the overall gain of *G*?

(b) For a first-order system, the time constant is the time for a step change *M* when *y* reaches *63.2* of the steady state. Is the equivalent time constant for the step response of the second-order system *G* (time when the response reaches *63.2* of the steady state) equal to $\tau_1 + \tau_2$? Show supporting calculations for the step response formula.

$$y(t) = KM\left(1 - \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{\tau_1 - \tau_2}\right)$$

(c) Can *y(t)* show oscillations for a step input? Explain.

4. A single-tank process has been operating for a long period of time with the inlet flow rate q, equal to $30.4\, ft^3 / \min$. After the operator increase the flow rate suddenly by 10%, the liquid level in the tank changes as shown in Table Q4.

Table Q4

Time (min)	f (ft)	Time (min)	f (ft)
0	5.50	1.4	6.37
0.2	5.75	1.6	6.40
0.4	6.93	1.8	6.43
0.6	6.07	2.0	6.45
0.8	6.18	3.0	6.50
1.0	6.26	4.0	6.51
1.2	6.32	5.0	6.53

Assuming that the process dynamics can be described by a first-order model, calculate the steady-state gain and the time constant using the following three methods:

(a) From the time required for the output to reach 63.2 of the total change.

(b) From the initial slope of the response curve.

(c) From the slope of the Log curve.

5. A heat exchanger used to heat a glycol solution with hot oil is known to exhibit first-order-plus-time-delay behavior, $G_1(s) = T'(s)/Q'(s)$ where T' is the outlet temperature deviation and Q' is the hot oil flow rate deviation. A thermocouple is placed *3m* downstream from the outlet of the heat exchanger. The average velocity of the glycol in the outlet pipe is *0.5 m/s*. The thermocouple also is known to exhibit first-order behavior; however, its time constant is expected to be considerably smaller than the heat exchanger time constant,

(a) Data from a unit step test in Q' on the complete system are shown in Figure Q5. Using a method of your choice, calculate the time constants of this process from the step response.

(b) From your empirical model, find transfer functions for the heat exchanger, for the pipe, and for the thermocouple. Think of the model as the product of three transfer functions: process, pipe flow, and sensor. What assumptions do you have to make to obtain these individual transfer functions from the overall transfer function?

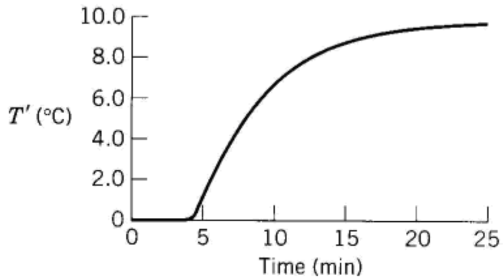

Figure Q5

6. The process input and output curves in a relay test is shown as in Figure Q6, where $p_u = 10s$, *a=0.3*, and the first relay switch time of *0.2s* are obtained from measuring the curves of Figure 1

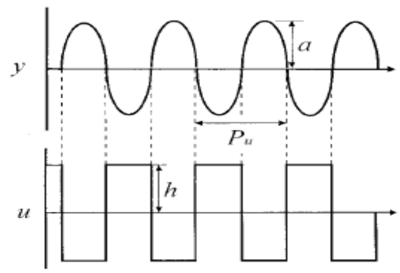

Figure Q6

Assume that the amplitude of the relay is *h=0.1*, and the process can be sufficiently model by

$$G_p(s) = \frac{K_p}{Ts+1} e^{-Ls}$$

determine ultimate gain and ultimate frequency of the process and, consequently, the transfer function parameters *K, T* and *L*.

7. The process reaction experiment has been performed on the stirred tank process shown in Figure Q7a, the input and system reaction curves are given in Figures Q7b. Using the two points method determine parameters *K, T* and *L* for the first-order-plus-delay model

$$G_p(s) = \frac{K}{Ts+1} e^{-Ls}$$

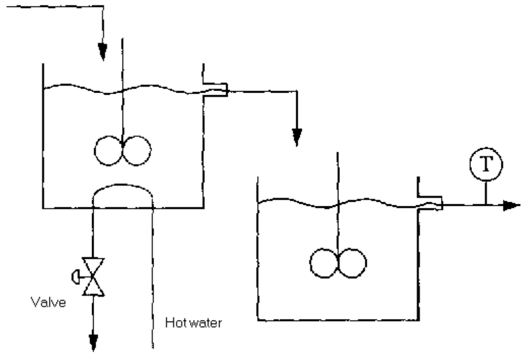

Figure Q7a Stirred Tank Process

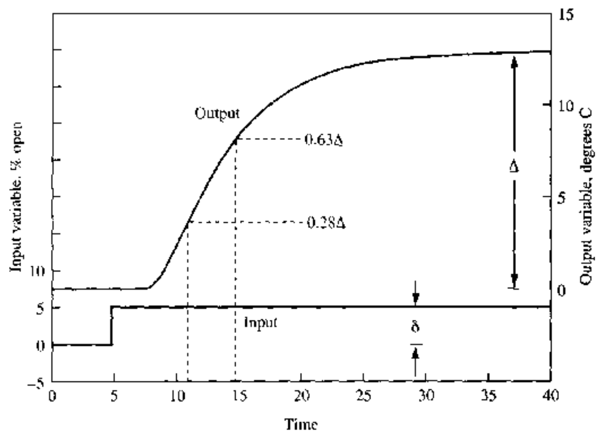

Figure Q7b Reaction Curve of Stirred Tank Process

8. The temperature of a certain industrial distillation column is regulated by the flowrate of the underflow reflux to the tray just below it. The typical steady-state operating values for these variables are 380^{0}F and 25,000 lb/hr respectively. The result of a step test carried out on this column is presented in Table Q8.

Table Q8 Step Response of the Distillation Column

Time (hr)	Reflux Flowrate (R lb/hr)	Tray #16 Temp. (T ^{0}F)	u	y	$(y_\infty - y)/y_\infty$
t <0	25,000	380.0	0	0.0	1.000
0.0	27,500	380.0	2,500	0.0	1.000
0.5	27,500	380.9	2,500	0.9	0.940
1.0	27,500	383.1	2,500	3.1	0.793
1.5	...	386.1	...	6.1	0.593
2.0	...	387.8	...	7.8	0.480
2.5	...	389.1	...	9.1	0.393
3.0	...	391.2	...	11.2	0.253
3.5	...	391.7	...	11.7	0.22
4.0	...	392.4	...	12.4	0.173
4.5	...	393.0	...	13.0	0.133
5.0	...	393.4	...	13.4	01.07
5.5	...	393.6	...	13.6	0.093
6.0	...	394.0	...	14.0	0.067
6.5	...	394.3	...	14.3	0.047
7.0	...	394.4	...	14.4	0.040

7:5	...	394.5	...	14.5	0.033
8.0	...	394.7	...	14.7	0.020
9.0	...	394.8	...	14.8	0.013
10.0	...	394.9	...	14.9	0.007
11.0	...	395.0	...	15.0	0.00
12.0	...	394.9	...	14.9	0.007
13.0	...	395.1	...	15.1	0.007
14.0	27,500	395.0	2,500	15.0	0.00

Find the parameter of FOPDT model:

$$y(s) = \frac{K}{\tau s + 1} e^{-Ls} u(s)$$