Exercise Solutions for Chapter 1: Fundamental Concepts

1. Consider a home heating system consisting of a natural gas-fired furnace and a thermostat. The process consists of the interior space to be heated. The thermostat contains both the measuring element and the controllers. The furnace is either on (heating) or off. Draw a schematic diagram for this control system. On your diagram, identify the controlled variables, manipulated variables, and disturbance variables. Include as many as possible sources of disturbances that can affect room temperature.

Answer:

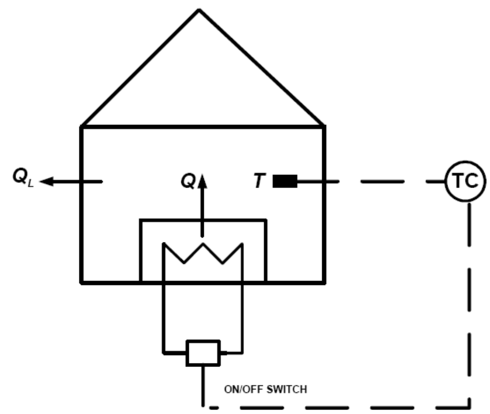


Figure S1. Schematic diagram of home heating control system

- Controlled variable- T (house interior temperature)
- Manipulated variable- Q (heat from the furnace)
- Disturbance variable- Q_L (heat lost to surroundings);

Other possible sources of disturbances are:

- *loss of gas pressure;*
- *the outside door opening;*
- *change in outside temperature;*
- *change in outside wind velocity (external heat transfer coefficient);*
- *the opening of doors or windows into the house;*
- *the number of people inside (each one generating and transmitting energy into the surrounding air);*
- *other electric lights and appliances of any nature are being used.*

2. In addition to a thermostatically-operated home heating system, identify two other feedback control systems that can be found in most residences. Describe briefly how each of them works: include sensor, actuator, and controller information.

Answer:

The ordinary kitchen oven (either electric or gas), the water heater, and the furnace, all work similarly, generally using a feedback control mechanism and an electronic on-off controller. For example, the oven uses a thermal element similar to a thermocouple to sense temperature; the sensor's output is compared to the desired cooking temperature (input via dial or electronic set-point/display unit); and the gas or electric current is then turned on or off depending on whether the temperature is below or above the desired value. Disturbances include the introduction or removal of food from the oven, etc. A non-electronic household appliance that utilizes built-in feedback control is the water tank in a toilet. Here, a float (ball) on a lever arm closes or opens a valve as the water level rises and falls above the desired maximum level. The float height represents the sensor; the lever arm acting on the valve stem provides actuation; and the on-off controller and its set point are built into the mechanical assembly.

3. Consider the following transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{5}{10s+1}$$

(a) What is the steady-state gain?

(b) What is the time constant?

(c) If $U(s) = 2/s$, what is the value of the output y(t) when $t \to \infty$?

(d) For the same *U(s)*, what is the value of the output when *t = 10*? What is the output when expressed as a fraction of the new steady-state value?

(e) If $U(s) = (1-e^{-s})/s$. that is, the unit rectangular pulse, what is the output when $t \to \infty$?

(f) If $u(t) = \delta(t)$, that is, the unit impulse at t = 0, what is the output when $t \to \infty$?

(g) If $u(t) = 2\sin 3(t)$ what is the value of the output when $t \to \infty$?

Answer:

a) 5

b) 10

c)

$$Y(s) = \frac{10}{s(10s+1)}$$

From the Final Value Theorem, $y(t) = 10$ when $t \to \infty$

d) $y(t) = 10(1-e^{-t/10})$, then $y(10) = 6.32 = 63.2\%$ of the final value.

e)

$$Y(s) = \frac{5}{(10s+1)}\frac{1-e^{-s}}{s}$$

From the Final Value Theorem, $y(t) = 0$ when $t \to \infty$

f)

$$Y(s) = \frac{5}{(10s+1)}$$

From the Final Value Theorem, $y(t) = 0$ when $t \to \infty$

g)

$$Y(s) = \frac{5}{(10s+1)}\frac{6}{(s^2+9)}$$

Then $y(t) = 0.33e^{-0.1t} - 0.33\cos(3t) + 0.011\sin(3t)$

The sinusoidal input produces a sinusoidal output and $y(t)$ does not have a limit when $t \to \infty$.

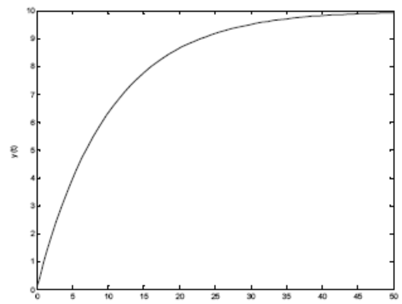


Fig S3a. *Output for part c) and d)*

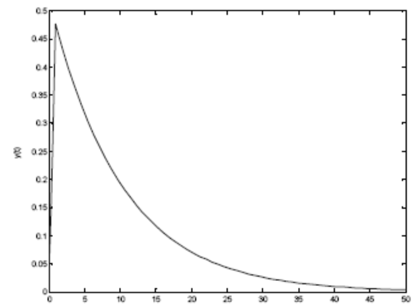


Fig S3b. *Output for part e)*

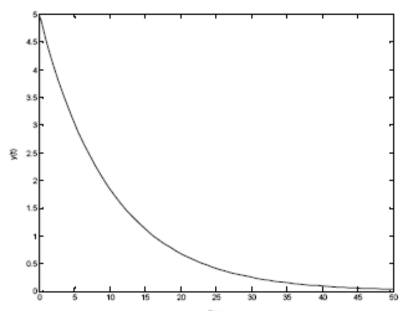


Fig S3c. *Output for part f)*

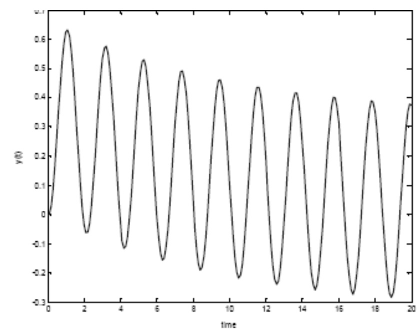


Fig S3d. *Output for part g)*

Figure S3. Output response to different inputs

4. Discuss the economic benefits achieved by reducing the variability (and, in some cases changing the average value) of the key controlled variable for the situations in the following.

a. Crude oil is distilled, and one segment of the oil is converted in a chemical reactor to make gasoline. The reactor can be operated over a range of temperatures; as the temperature is increased, the octane of the gasoline increases, but the yield of gasoline decreases because of increased by-products of lower value. (It's not really *this* simple, but the description captures the essence of the challenge.) The customer cannot determine small changes in octane. You are responsible for the reactor operation. Is there a benefit for tight temperature control of the packed bed reactor? How would you determine the correct temperature value?

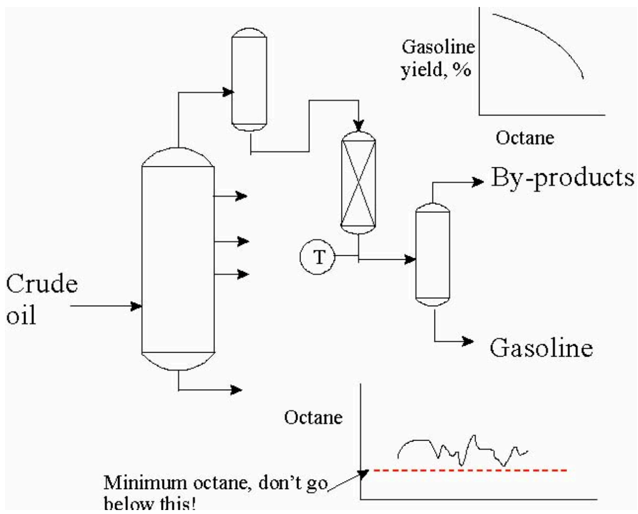


Figure Q5 Crude oil distillation column

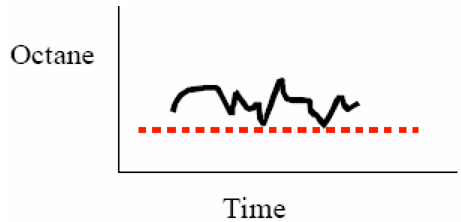
b. You are working at a company that produces large roles of paper sold to newspaper printers. Your client has many potential suppliers for this paper. Your customer can calibrate the printing machines, but after they have been calibrated, changes to paper thickness can course costly paper breaks in the printing machines. Discuss the importance of variance to your customer, what your product quality goal would be. Is this concept different from the situation in part (a) of this question?

Answer:

The economic benefits achieved by reducing the variability (and, in some cases changing the average value) of the key controlled variable for the situations in the following.

a. In this situation, the customer cannot distinguish small changes from the minimum octane when driving their automobiles. Therefore, this small deviation in product quality is acceptable. However, the variability in the octane results in a lower average yield of gasoline and a higher yield of lower valued byproducts. Tight control of reactor temperature will reduce the variability in octane and allow a higher average yield of valuable gasoline. The average temperature can be selected to achieve acceptable octane for all production within the variation.

Note that the goal here is to reduce variability and adjust the average value to increase profit.



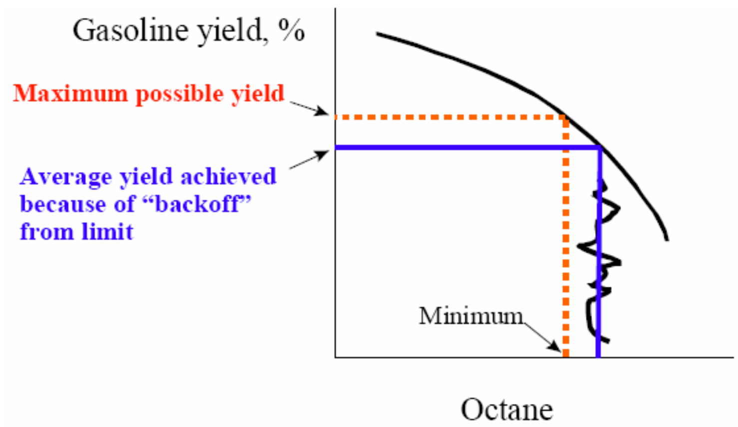


Figure S5a. Effects of controlled variable variability in crude oil distillation column

b. In this situation, the average paper thickness is not extremely important, so long as the customers can calibrate their machinery. However, after you and the customers have agreed on a thickness, essentially any variation is harmful, because it increases the likelihood of paper breaks. The customers lose production time, paper, and perhaps, the workers are subject to hazardous conditions. If you do not supply consistent thickness, the customer will find another supplier.

Therefore, the goal here is to retain the agreed average and reduce the variability to the minimum achievable.

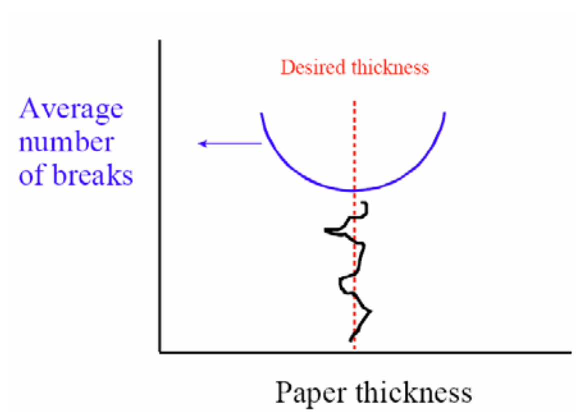


Figure S5b. Effects of average paper thickness to the economic benefits

5. A certain process whose transfer function is given as:

$$g(s) = \frac{2.0}{(2s-1)(5s+1)}$$

has proved particularly difficult to control. After careful analysis, the following was discovered about the process:

- If either P control or PI control is used on this process, then there are no values of K_c, or τ_I for which it can ever be stabilized.
- This process can only be stabilized with a PD or a PID controller.
- Furthermore, for the PID controller, the parameters must satisfy the following three conditions:

(a) $K_c > \dfrac{1}{2}$

(b) $\tau_D > \dfrac{3}{2K_c}$

(c) $\tau_I > \dfrac{20K_c}{(2K_c\tau_D - 3)(2K_c - 1)}$

Confirm or refute these statements.

Answer:

The characteristic equation is:

$$1 + \frac{2g_c}{(5s+1)(2s-1)} = 0$$

a). For $g_c = K_c$, proportional-only control, it becomes

$$10s^2 - 3s + (2K_c - 1) = 0$$

and for all values of K_c, the system is unstable because of the presence of (–3) as the coefficient of s.

For a PI controller,

$$g_c = K_c(1 + \frac{1}{\tau_I s}),$$

the characteristic equation becomes

$$10\tau_I s^3 - 3\tau\ s^2 + (2K_c - 1)\tau_I s + 2K_c = 0$$

and, irrespective of τ_I, no value of K_c will stabilize this system, **ESTABLISHING POINT 1**.

b). For $g_c = K_c(1+\tau_D s)$**,** a PD controller, becomes

$$10s^2 + (2K_c\tau_D - 3)s + 2K_c = 0$$

and now for $2K_c\tau_3 > 3$, the system is stabilized.
Similarly, for the PID controller,

$$g_c = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)$$

the characteristic equation becomes

$$10\tau_I s^3 + \tau_I(2K_c\tau_D - 3)s^2 + \tau_I(2K_c - 1)s + 2K_c = 0$$

given a cubic equation

$$a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$$

one may use the Roth array to establish that it will have no roots in the RHP if, and only if,

$$a_0,\ a_1,\ a_2,\ a_3 > 0$$

and

$$a_1 a_2 > a_0 a_3$$

Employing these results Routh criterion, we obtain the following as the required conditions for stability:

(i) $a_0 > 0 \Rightarrow \tau_I > 0$ (trivially satisfied)

(ii) $a_1 > 0 \Rightarrow \tau_I(2K_c\tau_D - 3) > 0$ Or $\tau_D > \dfrac{3}{2K_c}$

(iii) $a_2 > 0 \Rightarrow \tau_I(2K_c - 1) > 0$ Or $K_c > \dfrac{1}{2}$

(iv) $a_3 > 0 \Rightarrow K_c > 0$, redundant

(v) $a_1 a_2 > a_0 a_3 \Rightarrow \tau_I^2(2K_c\tau_D - 3)(2K_c - 1) > 20\tau_I K_c$

Because $\tau_I > 0$, this condition rearranges to give

$$\tau_I > \frac{20K_c}{(2K_c\tau_D - 3)(2K_c - 1)}$$

Thus confining all the statements. **ESTABLISHING POINT 2 AND 3**.

6.

(a) Find the range of K_c values for which the system with the following transfer function will remain stable under proportional feedback control:

$$y(s) = \frac{5(1-0.5s)}{(2s+1)(0.5s+1)} u(s)$$

(b) Consider now the situation in which the following unorthodox controller is utilized:

$$g_c(s) = K_c \frac{(\tau_z s+1)}{(\tau_L s+1)}$$

a proportional controller with a Lead-lag element. lf now τ_z is chosen to be 0.5, find the τ_L value required to obtain a stability range of $0 < K_c < 4$ for the proportional gain.

(c) Determine whether or not the controller will leave a steady-state offset.

Answer:

a) Characteristic equation of the closed loop system is

$$1 + \frac{5K_c(1-0.5s)}{(2s+1)(0.5s+1)} = 0$$

which is reduced to

$$s^2 + (2.5 - 2.5K_c)s + 1 + 5K_c = 0$$

Stability requires

$$2.5 - 2.5K_c > 0 \Rightarrow K_c < 1$$

$$1 + 5K_c > 0 \Rightarrow K_c > -\frac{1}{5}$$

The required K_c range is therefore

$$-\frac{1}{5} < K_c < 1$$

b) With the new controller, the characteristic equation becomes

$$1 + \frac{5K_c(0.5s+1)(1-0.5s)}{(\tau_L s+1)(2s+1)(0.5s+1)} = 0$$

or

$$2\tau_L s^2 + (2 + \tau_L - 2.5K_c)s + 1 + 5k_c = 0$$

And now the stability requires $K_c > 0$

$$2 + \tau_L - 2.5K_c > 0$$

or

$$K_c < \frac{2+\tau_L}{2.5}$$

In order to obtain a stability range $0 < K_c < 4$, choose τ_L such that

$$\frac{2+\tau_L}{2.5} = 4 \Rightarrow \tau_L = 8$$

c) The closed loop transfer function is

$$\psi(s) = \frac{5K_c(1-0.5s)}{(8s+1)(2s+1)+5K_c(1-0.5s)}$$

$$\lim_{s\to 0}\psi(s) = \frac{5K_c}{1+5K_c}$$

Regardless of specific values chosen for τ_L or τ_c, there will be a non-zero offset