# Exercise Questions for Chapter 4: Fundamentals of PID

**1.** Analog proportional-derivative controllers sometimes are formulated with a transfer function of the form:

$$G_\alpha(s) = K_c\left(\frac{\tau_D s + 1}{\alpha\tau_D s + 1}\right)$$

where $\alpha \approx 0.05$ to $0.2$. The ideal PD transfer functions obtained when $\alpha = 0$ .

$$G_i(s) = K_c\left(\tau_D s + 1\right)$$

(a) Analyze the accuracy of this approximation for step and ramp responses. Treat $\alpha$ as a parameter and let $\alpha \to 0$ .

(b) Why might it be difficult to construct an analog device with exactly this ideal transfer function?

(c) Is there any advantage in not being able to obtain the ideal transfer function?

**2**. The physically realizable form of the PD controller transfer function is given in the tutorial question 1.

(a) Show how to obtain this transfer function with a parallel arrangement of two much simpler functions in Fig. Q2:

(b) Find expressions for $K_1$, $K_2$. and $\tau_1$ that can be used to obtain desired values of $K_c$; $\tau_D$ and $\alpha$.

(c) Verify the relations for $K_c = 3$, $\tau_D = 2$, $\alpha = 0.1$.

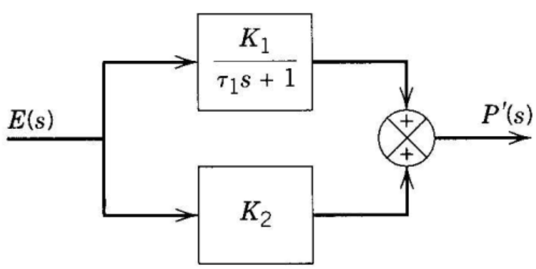


**Figure Q2**

**3.** The parallel form of the PID controller has the transfer function given by

$$G_i(s) = K_c'\left(1 + \frac{1}{\tau_I' s} + \tau_D' s\right)$$

Many commercial analog controllers can be described by the series form given by

$$G_a(s) = K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{\tau_D s+1}{\alpha\tau_D s+1}\right)$$

(a) For the simplest case, $\alpha \to 0$, find the relations between the settings for the parallel form $\left(K_c^{'}, \tau_I^{'}, \tau_D^{'}\right)$ and the settings for the series form $\left(K_c, \tau_I, \tau_D\right)$.

(b) Does the series form make each controller setting $\left(K_c, \tau_I, \tau_D\right)$ larger or smaller than would be expected for the parallel form?

(c) What are the magnitudes of these interaction effects for $K_c$ = 4, $\tau_I$ = 10 min, $\tau_D$ = 2 min?

(d) What can you say about the effect of nonzero $\alpha$ on these relations? (Discuss only first-order effects.)

**4.** If the input $y_m$ to a PI controller changes stepwise ($y_m$(s) =2/s) and the controller output changes initially as in Fig Q4, what are the values of the controller gain and integral time?

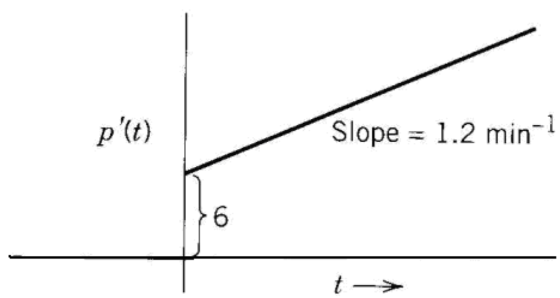


Fig Q4. Initial controller output change

**5.** You are required to implement a level controller on a troublesome process tank that contains a boiling liquid. Someone told you that a level transmitter used with such a system has a very noise output and that a *P* or *PI* controller will require a noise filter on the measurement.

(a) Show how a measurement noise filter can be implemented with a *PI* controller by drawing a block diagram of the controller, modified with a first-order transfer function (time constant = $\tau_f$ ( where $\tau_f << \tau_I$) in the appropriate location.

(b) How would Fig. Q4 in Tutorial Question 4 be modified to illustrate the response of a PI controller to a step change in the filtered measurement? If necessary, calculate the time response.

(c) A company purchased a digital control system that provides a first-order digital filter as one of its many features. Can you suggest why this required noise filter ought to be implemented using an analog instrument, instead of using the available digital implementation? Assume that the digital controller utilizes a sampling interval of 1s.

**6.** a) Derive the differential equation model of the series PID controller.

b) Qualitatively describe its response to a step change in *e(t)*.

**7.** (a) Derive a differential equation model represents the parallel PID controller with a derivative filter?

(b) Repeat for the series PID controller with a derivative filter.

(c) Simulate the time response of each controller for a step change in *e(t)*.

**8.** For the process

$$g_0(s) = \frac{(-0.3s+1)(0.08s+1)}{(2s+1)(s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

Find the first-order-plus-time delay model.

**9.** For the process

$$G_0(s) = \frac{(0.17s+1)}{s(s+1)^2(0.028s+1)}$$

Using half rule method to reduce the process to simplest forms

**10.** A process consists of two stirred tanks with input $q$ and outputs $T_1$and $T_2$. (see Fig. Q6). To test the hypothesis that the dynamics in each tank are basically first order, a step change in $q$ is made from 82 to 85 with output responses given in Table Q6.

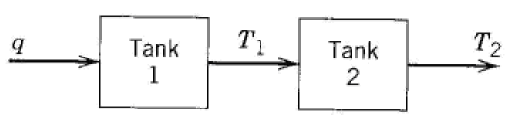


Figure Q6

(a) Find the transfer functions $T_1^{'}(s)/Q^{'}(s)$ and $T_2^{'}(s)/T_1^{'}(s)$. Assume that they are of the form $K_i/(\tau_i s+1)$.

(b) Calculate the model responses to the same step change in $q$ and plot with the experimental data.

Table Q6

| Time | $T_1$ | $T_2$ | Time | $T_1$ | $T_2$ |
|---|---|---|---|---|---|
| 0 | 10.00 | 20.00 | 11 | 17.80 | 25.77 |
| 1 | 12.27 | 20.65 | 12 | 17.85 | 25.84 |
| 2 | 13.89 | 21.79 | 13 | 17.89 | 25.88 |

| | | | | | |
|---|---|---|---|---|---|
| 3 | 15.06 | 22.83 | 14 | 17.92 | 25.92 |
| 4 | 15.89 | 23.68 | 15 | 17.95 | 25.94 |
| 5 | 16.49 | 24.32 | 16 | 17.96 | 25.96 |
| 6 | 16.91 | 24.79 | 17 | 17.98 | 25.98 |
| 7 | 17.22 | 25.13 | 18 | 17.99 | 25.98 |
| 8 | 17.44 | 25.38 | 19 | 17.99 | 25.99 |
| 9 | 17.60 | 25.55 | 20 | 17.99 | 25.99 |
| 10 | 17.71 | 25.68 | 50 | 18.00 | 26.00 |