Tut 2:

Q1: (a). First, determine wether there exists any dominence relationship between $\mu_A(x)$ and $\mu_B(x)$.

Check wether $\mu_A(x) = \mu_B(x)$ has solutions other than $x=0, 6$.

Assume: $\frac{1}{1+(x-3)^2} - \frac{1}{10} = 1 - \frac{x-3}{3}$ (consider $x>3$)

Then we have.

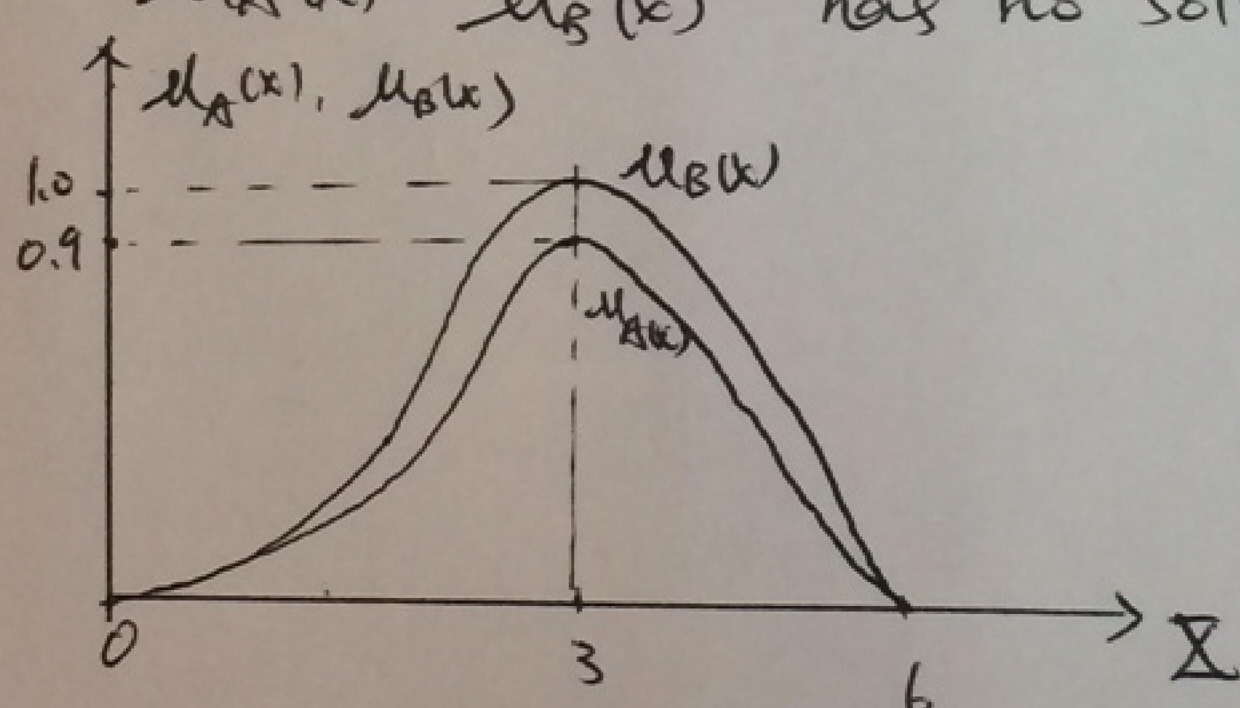
$$-10x^3 + 123x^2 - 478x + 600 = 0.$$

we know that $x=6$ is one solution. So we can use factorizn.

$$-10x^3 + 123x^2 - 478x + 600 = (x-6)\cdot(-10x^2+63x-100) = 0.$$

Since. $63^2 - 4(-10)(-100) = 3969 - 4000 = -31 < 0$.

we know that $\mu_A(x) = \mu_B(x)$ has no solution other then $x=0, 6$.



$\therefore \mu_{A\cup B}(x) = \mu_B(x) \Rightarrow A \cup B = B$.

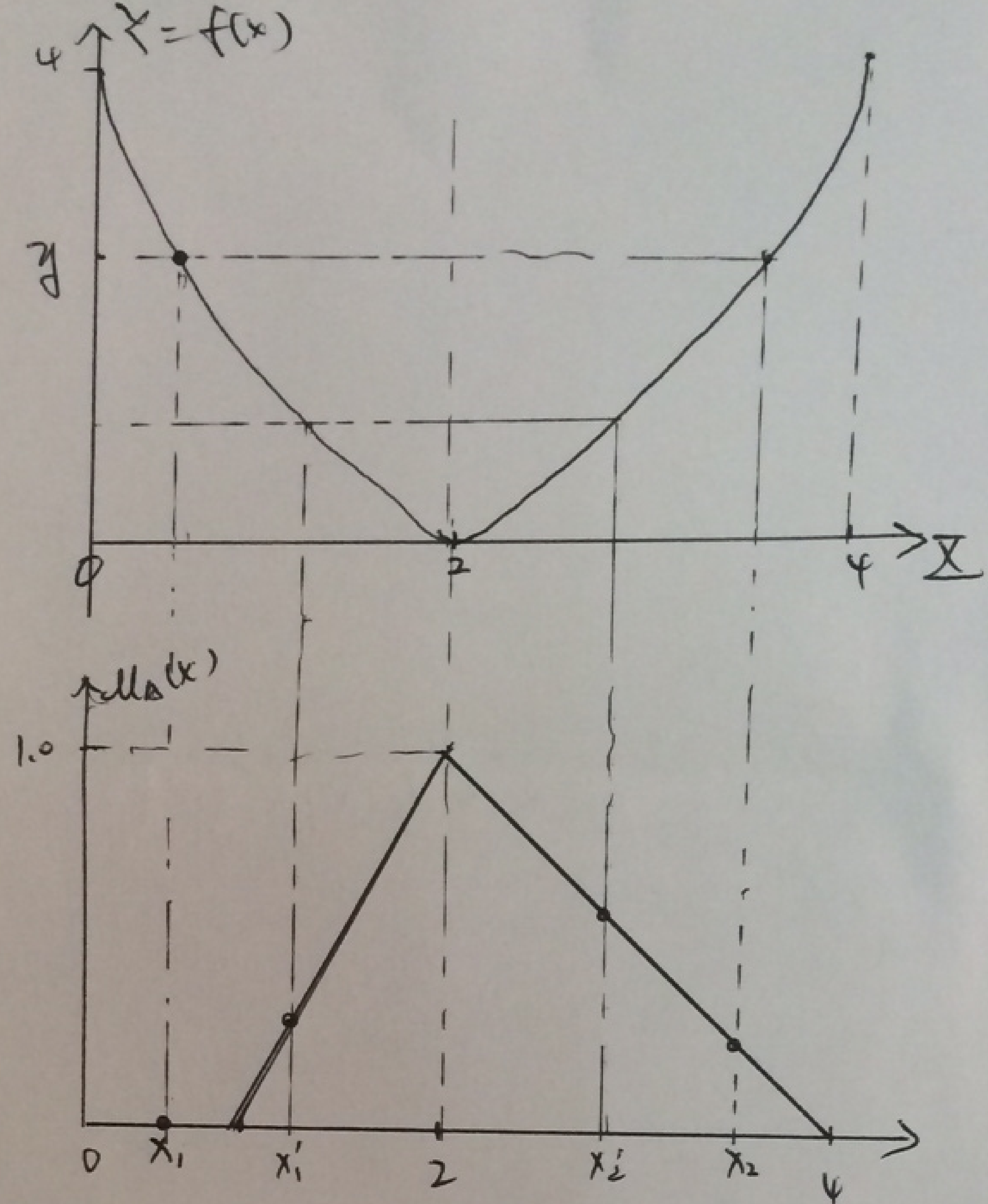
$\mu_{A\cap B}(x) = \mu_A(x) \Rightarrow A \cap B = A$.

(b). $D = A \times C \Rightarrow \mu_D(x,y) = \mu_A(x) \wedge \mu_C(y)$.

$$\mu_{D_X}(x) = \bigvee_y \mu_D(x,y) = \bigvee_y [\mu_A(x) \wedge \mu_C(y)] = \mu_A(x)$$

$$\mu_{D_Y}(y) = \bigvee_x \mu_D(x,y) = \bigvee_x [\mu_A(x) \wedge \mu_C(y)] = 0.9\, \mu_C(y)$$

Q2:



$$\therefore \quad \mu_B(y) = \max_{x\in f^{-1}(y)} \mu_A(x) = \begin{cases} -0.5(2+\sqrt{y}-4) = \mu_A \circ \tilde{f}^{-1}(y), & 0 \le y \le 4 \\ 0, & y > 4 \end{cases}$$

where: $y = \tilde{f}(x) = (x-2)^2$, when $x \geq 2$

$$\therefore \quad \tilde{f}^{-1}(y) = 2 + \sqrt{y}$$

Q3: To show $R\circ(S\cap T) \subseteq (R\circ S)\cap(R\circ T)$ with $R \subseteq X\times Y$, $S, T \subseteq Y\times Z$

$$\begin{aligned}
\mu_{R\circ(S\cap T)}(x,z) &= \bigvee_y \left[\mu_R(x,y) \wedge \mu_{S\cap T}(y,z)\right] \\
&= \bigvee_y \left[\mu_R(x,y) \wedge \left[\mu_S(y,z) \wedge \mu_T(y,z)\right]\right] \\
&= \bigvee_y \left[(\mu_R(x,y) \wedge \mu_S(y,z)) \wedge (\mu_R(x,y) \wedge \mu_T(y,z))\right] \\
&\le \left(\bigvee_y \left[\mu_R(x,y) \wedge \mu_S(y,z)\right]\right) \wedge \left(\bigvee_y \left[\mu_R(x,y) \wedge \mu_T(y,z)\right]\right) \\
&= \mu_{R\circ S}(x,z) \wedge \mu_{R\circ T}(x,z) \\
&= \mu_{R\circ S \cap R\circ T}(x,z)
\end{aligned}$$