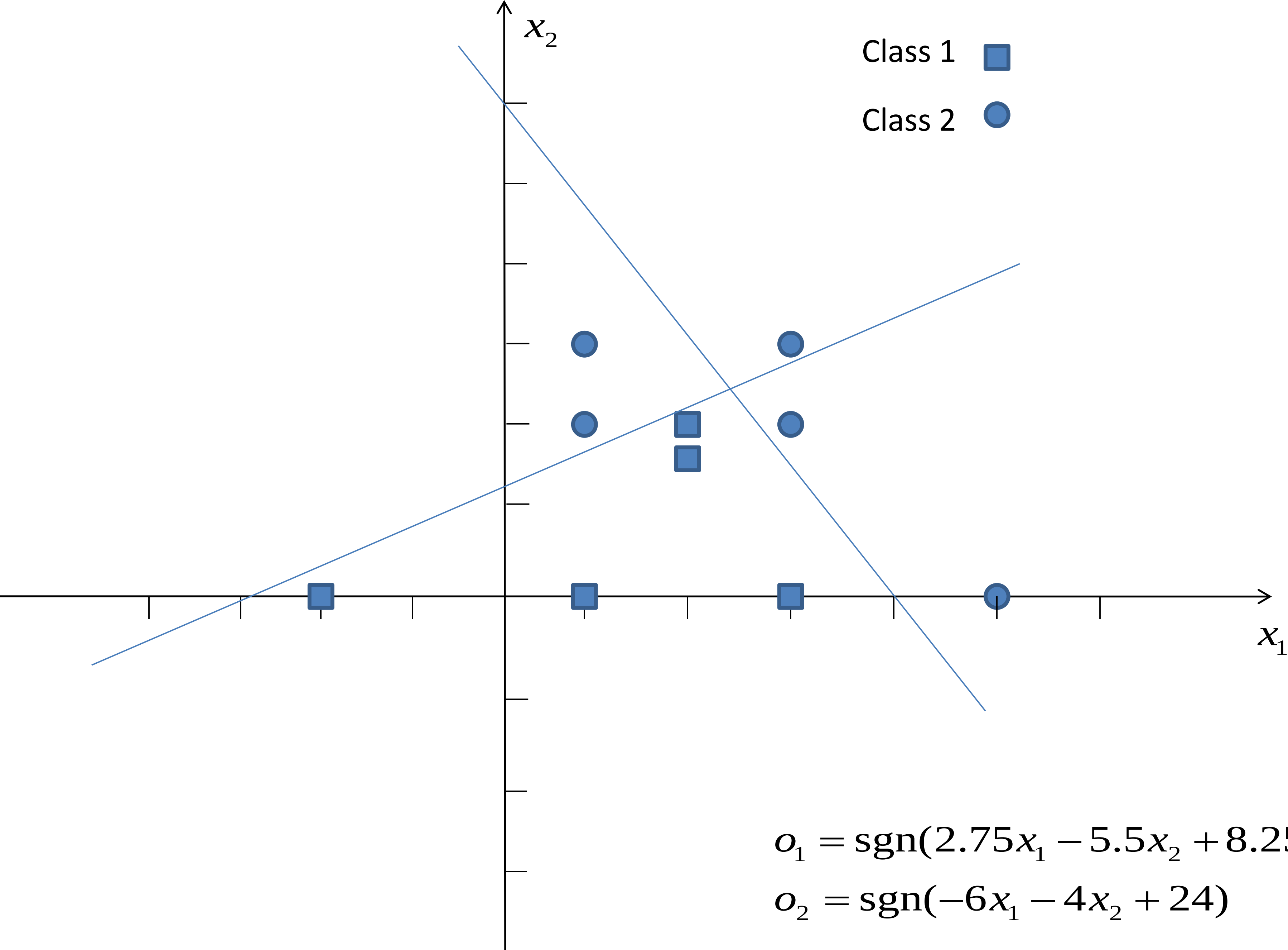
# Tutorial 5.1

The following linearly non-separable patterns below have to be classified into two classes using a layered neural network and an appropriate pattern-image space transformation. Design a two-layer neural classifier with the bipolar discrete time TLU neuron based on the appropriate space mapping.
NB: Using the minimum number of TLU units.

Class 2 $\quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \mathbf{x}_2 = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \mathbf{x}_5 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \mathbf{x}_{10} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

Class 1 $\quad \mathbf{x}_4 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \mathbf{x}_6 = \begin{bmatrix} 2 \\ 1.5 \end{bmatrix} \quad \mathbf{x}_7 = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \quad \mathbf{x}_8 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \mathbf{x}_9 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

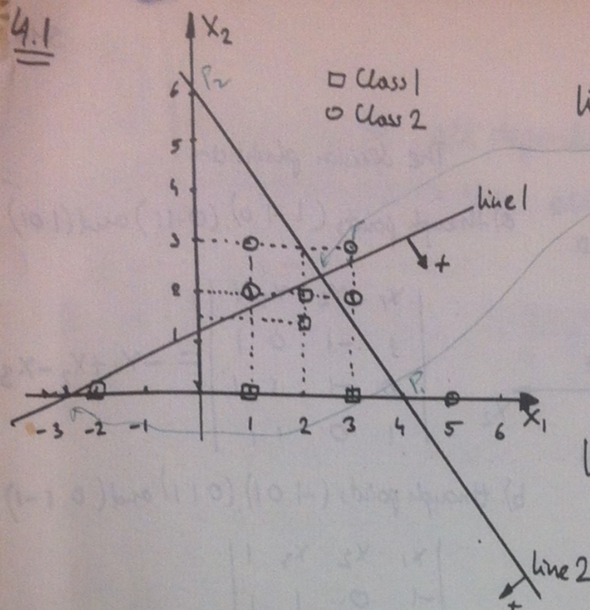



$$o_1 = \mathrm{sgn}(2.75x_1 - 5.5x_2 + 8.25)$$

$$o_2 = \mathrm{sgn}(-6x_1 - 4x_2 + 24)$$

$$o_3 = \mathrm{sgn}(o_1 + o_2 - 1.5)$$

## 4.1




line 1: $p_1 = (3, 2.75)$

$p_2 = (-2.5, 0)$

$$\begin{vmatrix} x_1 & x_2 & 1 \\ 3 & 2.75 & 1 \\ -2.5 & 0 & 1 \end{vmatrix} = 2.75x_1 - 5.5x_2 + 6.875$$

$$\bar{n} = \begin{bmatrix} 2.75 \\ -5.5 \end{bmatrix}$$

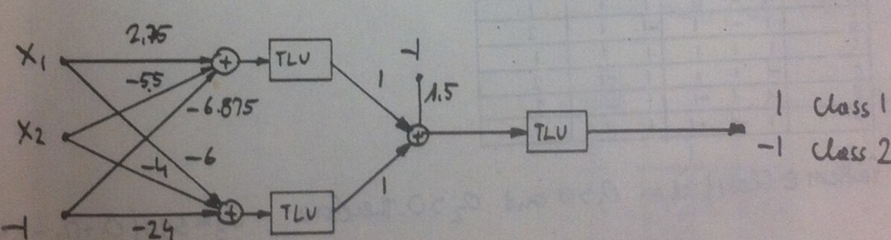
line 2: $p_1 = (4, 0)$

$p_2 = (0, 6)$

$$\begin{vmatrix} x_1 & x_2 & 1 \\ 4 & 0 & 1 \\ 0 & 6 & 1 \end{vmatrix} = (-6x_1 - 4x_2 + 24)$$

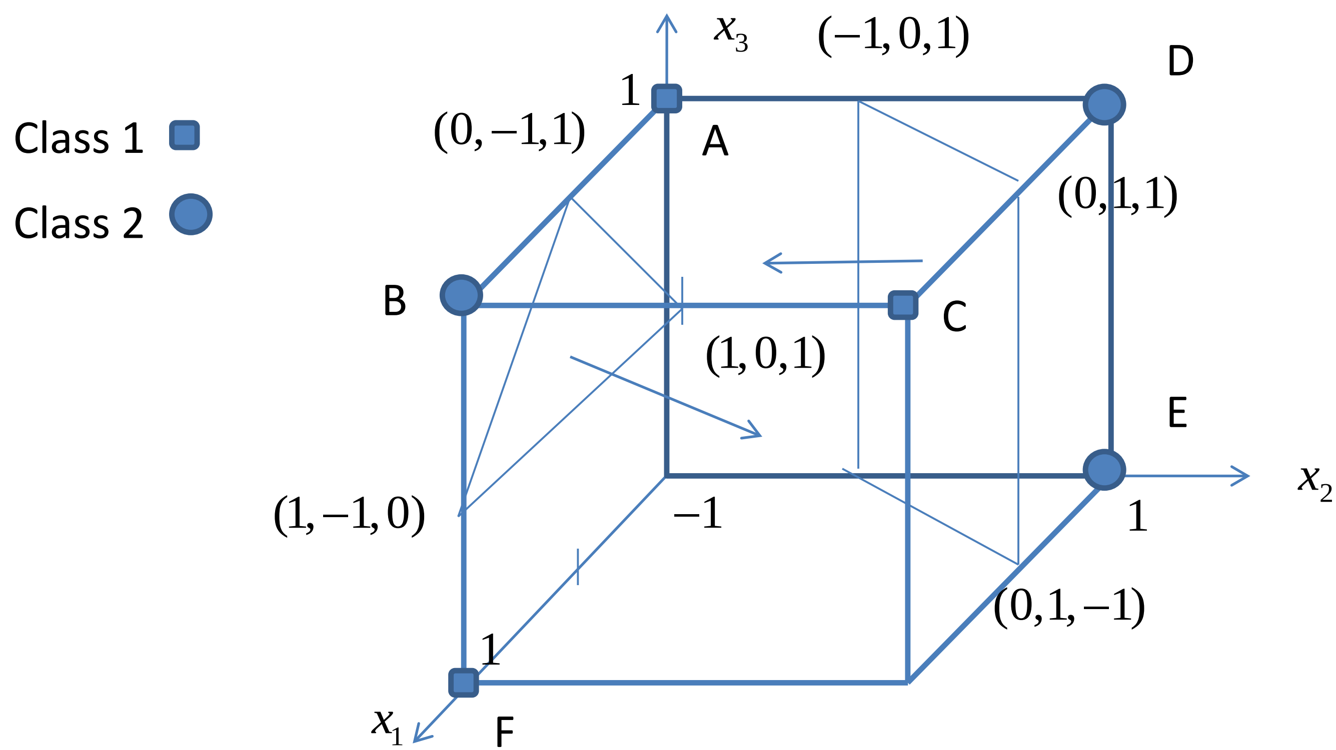
$$\bar{n} = \begin{bmatrix} -6 \\ -4 \end{bmatrix}$$

Pattern ∈ Class 1 when both orientations are positive and thus using bipolar TLUs we obtain



# Tutorial 5.2

Linearly non-separable patterns as shown in the figure below have to be classified into two classes using a layered neural network. Construct the separating planes in the pattern space and draw patterns in the image space. Calculate all weight and threshold values of related TLU neuron units.
NB: Using the minimum number of TUL units.

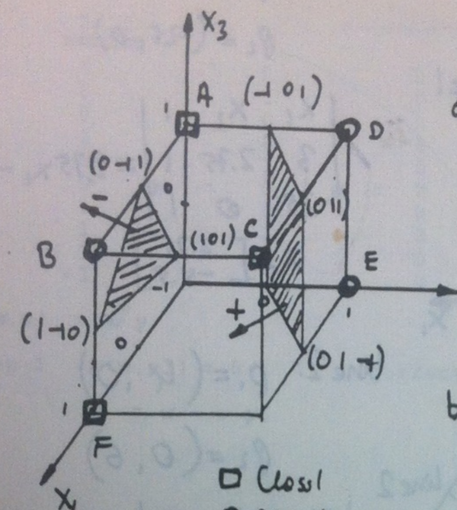


$$o_1 = \mathrm{sgn}(-x_1 + x_2 - x_3 + 2)$$

$$o_2 = \mathrm{sgn}(x_1 - x_2 + 1)$$

$$o_3 = \mathrm{sgn}(o_1 + o_2 - 1.5)$$

# 4.2




The decision planes are:

a) through points $(1\ -1\ 0)$, $(0\ -1\ 1)$ and $(1\ 0\ 1)$

$$\begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{vmatrix} = -x_1 + x_2 - x_3$$

b) through points $(-1\ 0\ 1)$, $(0\ 1\ 1)$ and $(0\ 1\ -1)$

$$\begin{vmatrix} x_1 & x_2 & x_3 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 \end{vmatrix} = x_1 - x_2 + 1 \quad = 0$$

$$O_1 = \mathrm{sgn}(-x_1 + x_2 - x_3 + 2)$$

$$O_2 = \mathrm{sgn}(x_1 - x_2 + 1)$$

| PATTERN | $x_1$ | $x_2$ | $x_3$ | $O_1$ | $O_2$ | CLASS |
|---|---|---|---|---|---|---|
| A | -1 | -1 | 1 | 1 | 1 | 1 |
| B | 1 | -1 | 1 | -1 | 1 | 2 |
| C | 1 | 1 | 1 | 1 | 1 | 1 |
| D | -1 | 1 | 1 | 1 | -1 | 2 |
| E | -1 | 1 | -1 | 1 | -1 | 2 |
| F | 1 | -1 | -1 | 1 | 1 | 1 |

Pattern ∈ Class 1 when $O_1 > 0$ and $O_2 > 0$. Therefore $O_3 = \mathrm{sgn}(O_1 + O_2 - 1.5)$.