Tutorial 2.1 For a minimum distance two-class classifier, the weight and augmented pattern vectors are

$$\mathbf{w} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

a) Find the equation of the decision surface in the pattern space;

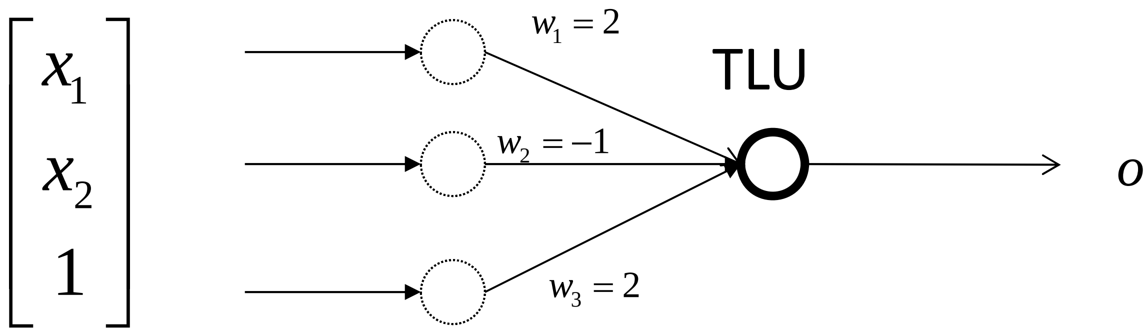
b) Find the equation if the decision surface in the augmented pattern space;

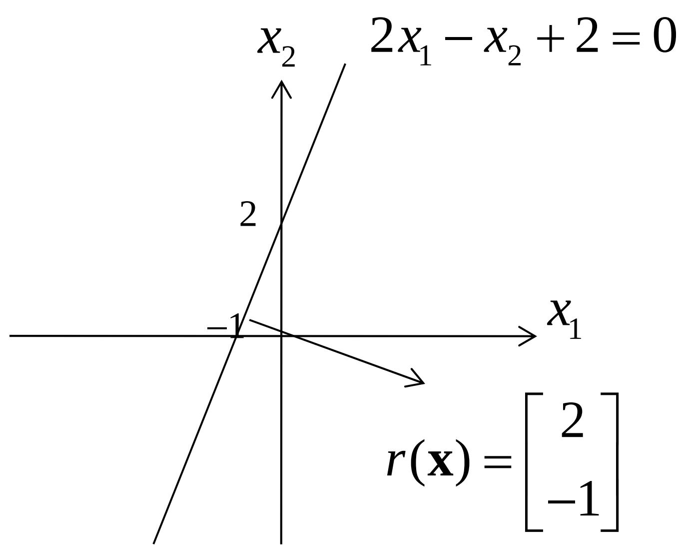
c) Compute the new solution weight vector if the two class prototype points are

$$\mathbf{x}_1 = \begin{bmatrix} 2 & 5 \end{bmatrix}^t \text{ and } \mathbf{x}_2 = \begin{bmatrix} -1 & -3 \end{bmatrix}^t$$

d) Sketch the decision surfaces for each case in parts a), b) and c).

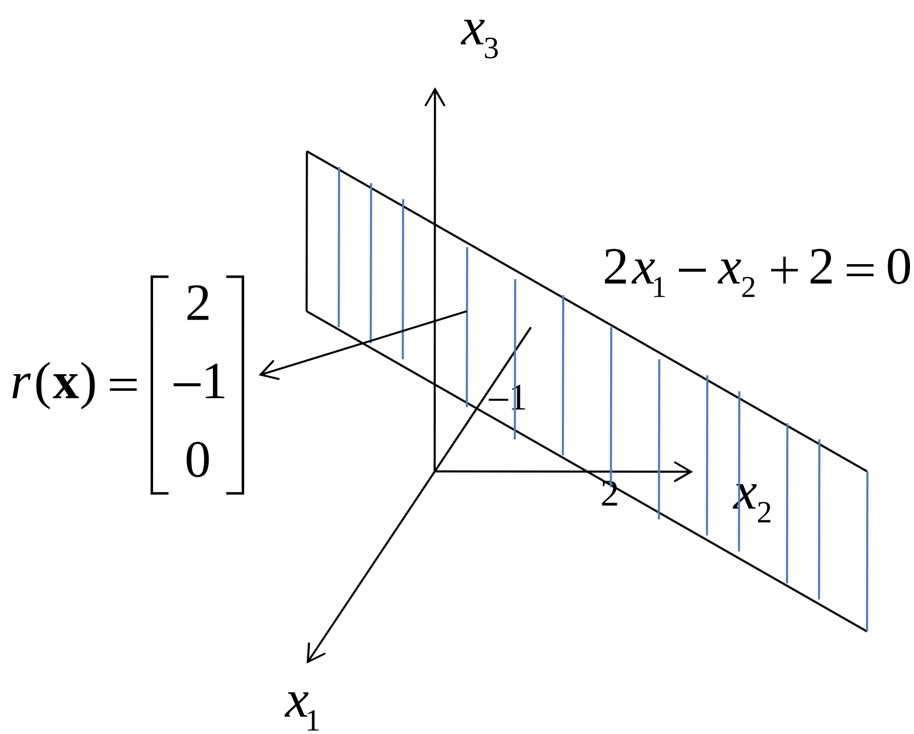
Solution: a): The decision line equation in 2-D pattern space is $2x_1 - x_2 + 2 = 0$.
The decision surface in the 3-D augmented plane is the same equation.





b)

The original decision plane in augmented pattern space



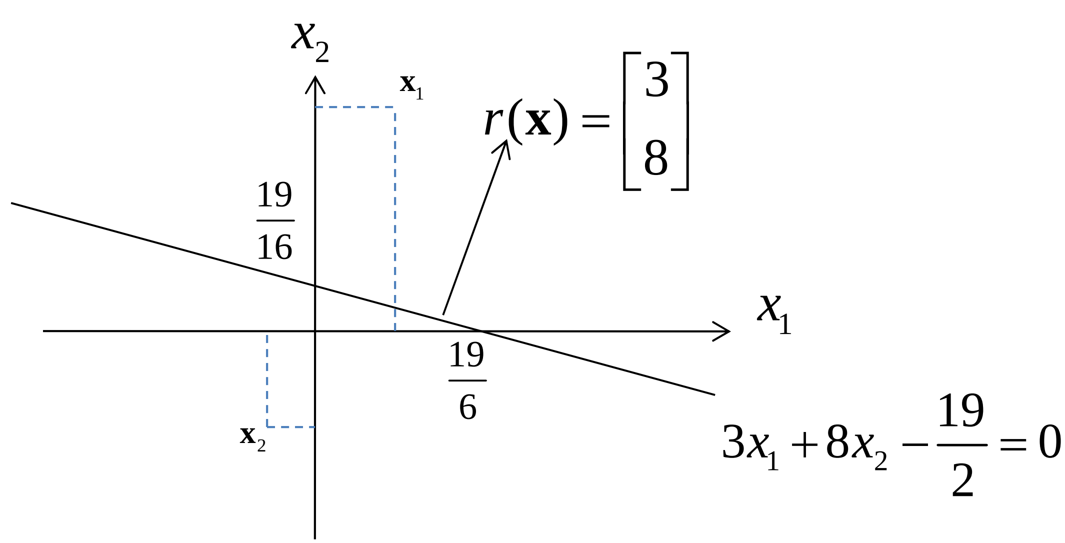
c)

$$\mathbf{x}_1 = \begin{bmatrix} 2 & 5 \end{bmatrix}^t \text{ and } \mathbf{x}_2 = \begin{bmatrix} -1 & -3 \end{bmatrix}^t$$

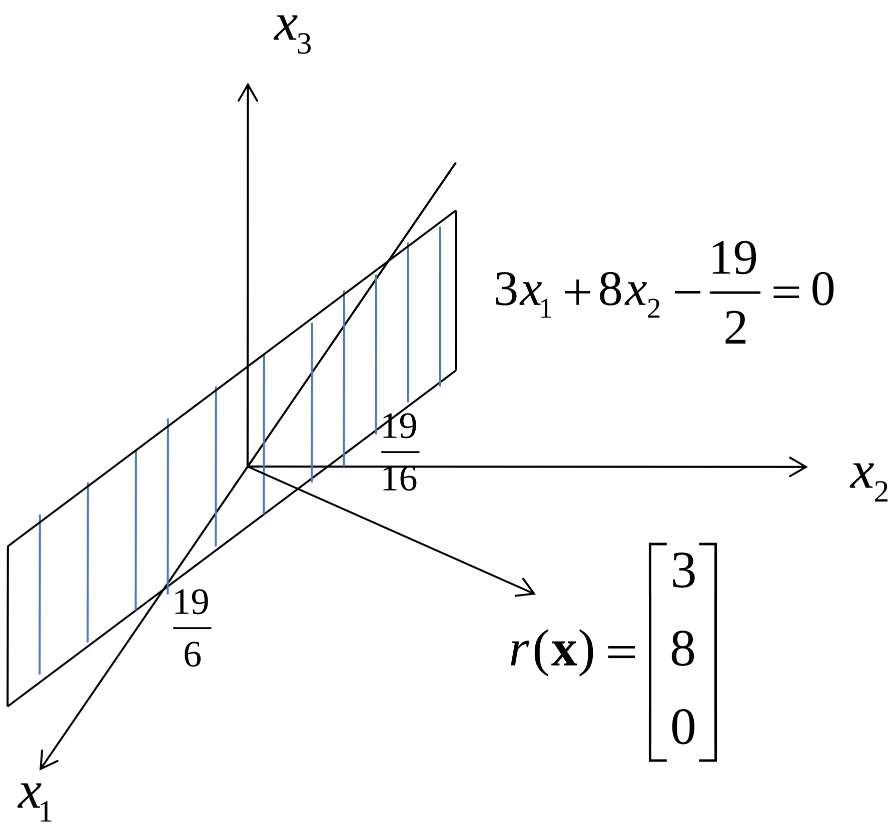
$$(\mathbf{x}_1 - \mathbf{x}_2)^t \mathbf{x} + \frac{1}{2}(\|\mathbf{x}_2\|^2 - \|\mathbf{x}_1\|^2) = 0$$

$$\begin{bmatrix} 2-(-1) & 5-(-3) \end{bmatrix}\mathbf{x} + \frac{1}{2}(\|1\|^2 + \|3\|^2 - \|2\|^2 - \|5\|^2)$$

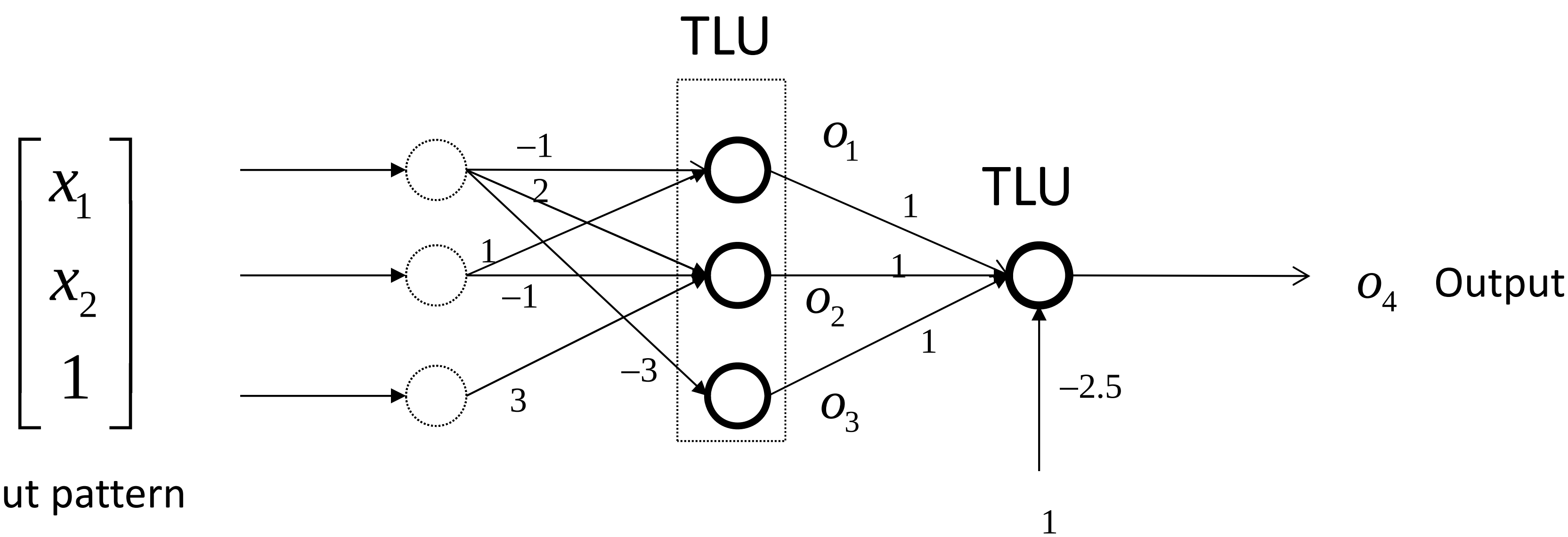
$$= 3x_1 + 8x_2 - \frac{19}{2} = 0$$



d) The minimum distance classifier in augmented pattern space



2.2 The feedforward neural network shown below is mapping the entire pattern plane x_1, x_2 into bipolar values. Find the segments (decision regions) of the x_1, x_2 plane for which $o_4 = 1$ and its complement for which $o_4 = -1$.



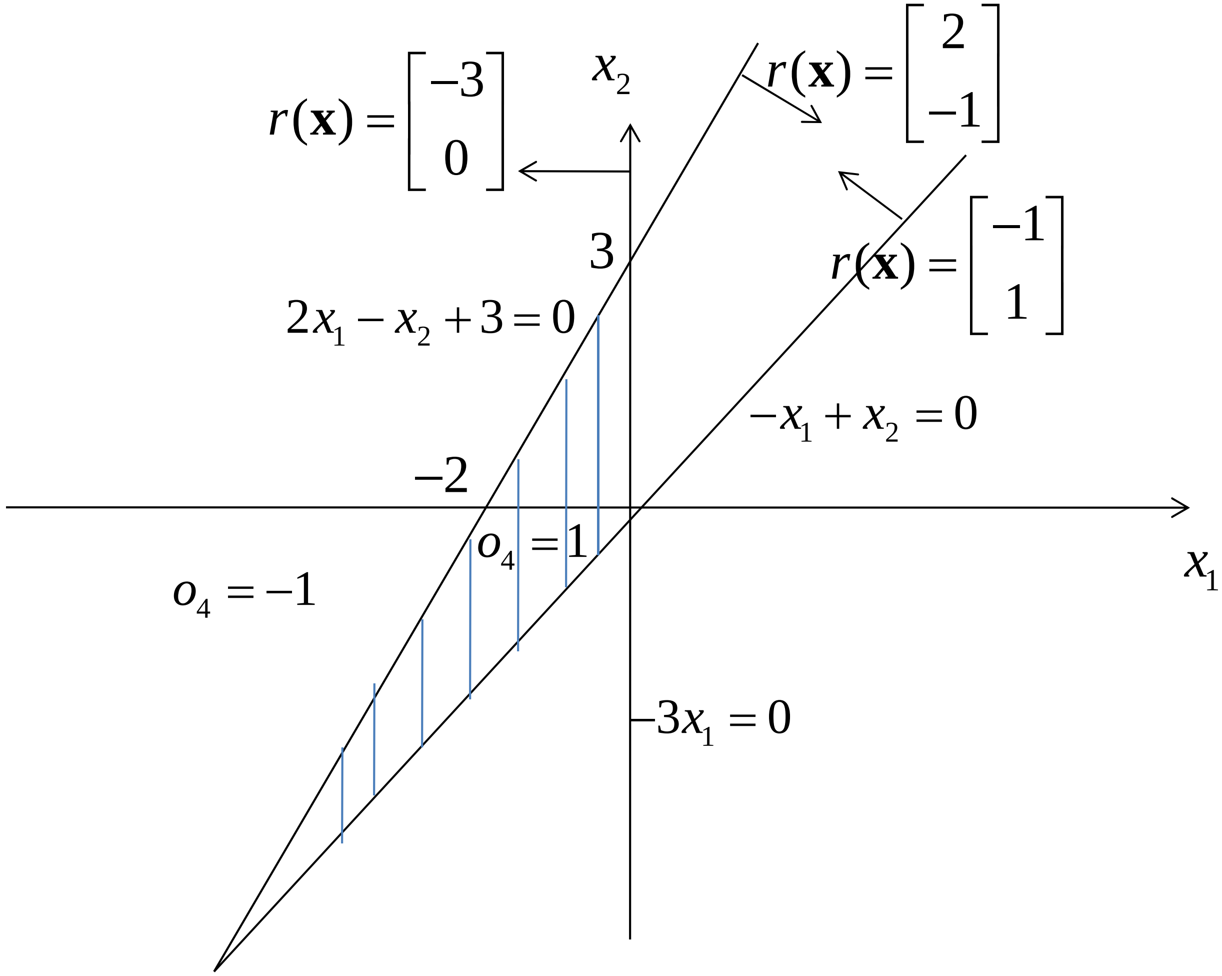
$net_1 = -x_1 + x_2$ $net_2 = 2x_1 - x_2 + 3$ $net_3 = -3x_1$ $net_4 = o_1 + o_2 + o_3 - 2.5$

For $o_4 = 1$ net_1, net_2 and net_3 must all be 1, therefore, we have

$$net_1 = -x_1 + x_2 > 0 \rightarrow x_2 > x_1$$

$$net_2 = 2x_1 - x_2 + 3 > 0 \rightarrow x_2 < 2x_1 + 3$$

$$net_3 = -3x_1 > 0 \rightarrow x_1 < 0$$



In the 2-D pattern space, the decision region for $o_4 = 1$ met the inequalities are marked with the shaded area and the rest of area is for $o_4 = -1$.