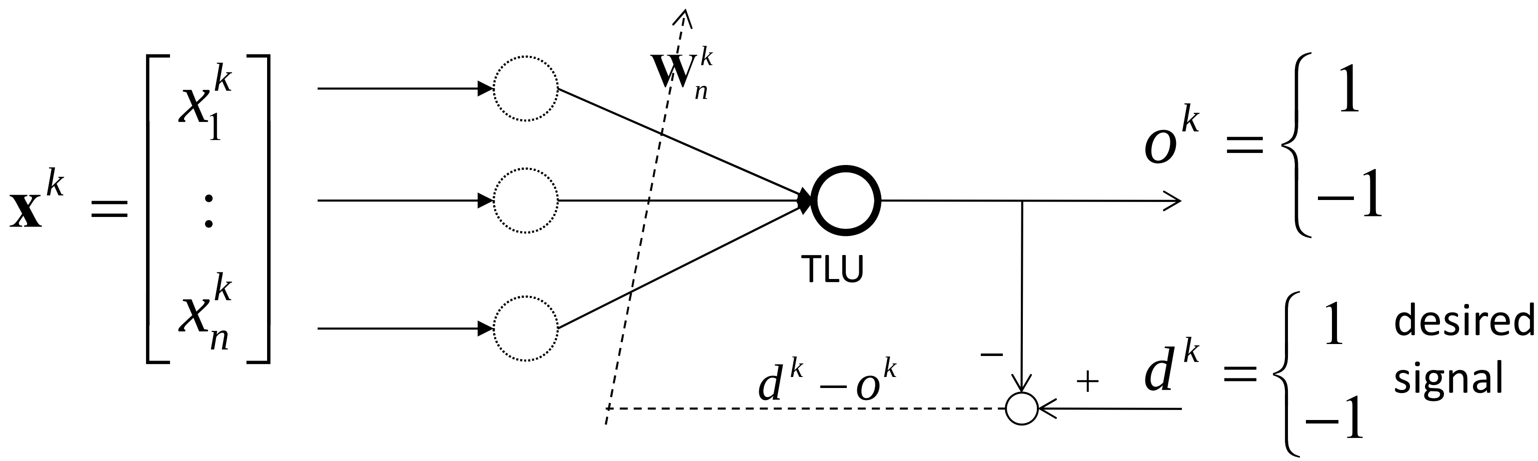
3.1 Implement the perceptron training of the following network (*c*=1)



and the following data specifying the initial weight vector and the two training pairs with the learning rate *c*=1:

$$\mathbf{w}^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \qquad \mathbf{y}_1 = \mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, d_1 = -1; \qquad \mathbf{y}_2 = \mathbf{x}_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, d_2 = 1.$$

Find the final convergent weight.

Step *k=1*; Training pattern pair $\mathbf{y}^1 = \mathbf{y}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, d^1 = d_1 = -1$ is used based on the initial weight

$$\mathbf{w}^1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^t$$

$$o^1 = \operatorname{sgn}([\mathbf{w}^1]^t \mathbf{y}^1) = \operatorname{sgn}(\begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}) = 1$$

$$d^1 - o^1 = -2$$

$$\mathbf{w}^2 = \mathbf{w}^1 - \mathbf{y}^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Updated decision line

$$[\mathbf{w}^2]^t \mathbf{y} = \begin{bmatrix} -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = -2y_1 + y_3 = 0$$

Step *k=2*; Training pattern pair $\mathbf{y}^2 = \mathbf{y}_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, d^2 = d_2 = 1$ is used based on the last updated weight

$$\mathbf{w}^2 = \begin{bmatrix} -2 & 0 & 1 \end{bmatrix}^t$$

$$o^2 = \mathrm{sgn}([\mathbf{w}^2]^t \mathbf{y}^2) = \mathrm{sgn}(\begin{bmatrix} -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}) = -1$$

$$d^2 - o^2 = 2$$

$$\mathbf{w}^3 = \mathbf{w}^2 + \mathbf{y}^2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

Updated decision line

$$[\mathbf{w}^3]^t \mathbf{y} = \begin{bmatrix} -2 & -1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = -2y_1 - y_2 = 0$$

Step *k=3*; Training pattern pair $\mathbf{y}^3 = \mathbf{y}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, d^3 = d_1 = -1$ is used based on the last updated weight

$$\mathbf{w}^3 = \begin{bmatrix} -2 & -1 & 0 \end{bmatrix}^t$$

$$o^3 = \operatorname{sgn}([\mathbf{w}^3]^t \mathbf{y}^3) = \operatorname{sgn}(\begin{bmatrix} -2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}) = -1$$

$$d^3 - o^3 = 0$$

$$\mathbf{w}^4 = \mathbf{w}^3 = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

Step *k=4*; Training pattern pair $\mathbf{y}^4 = \mathbf{y}_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, d^4 = d_2 = 1$ is used based on the last updated weight

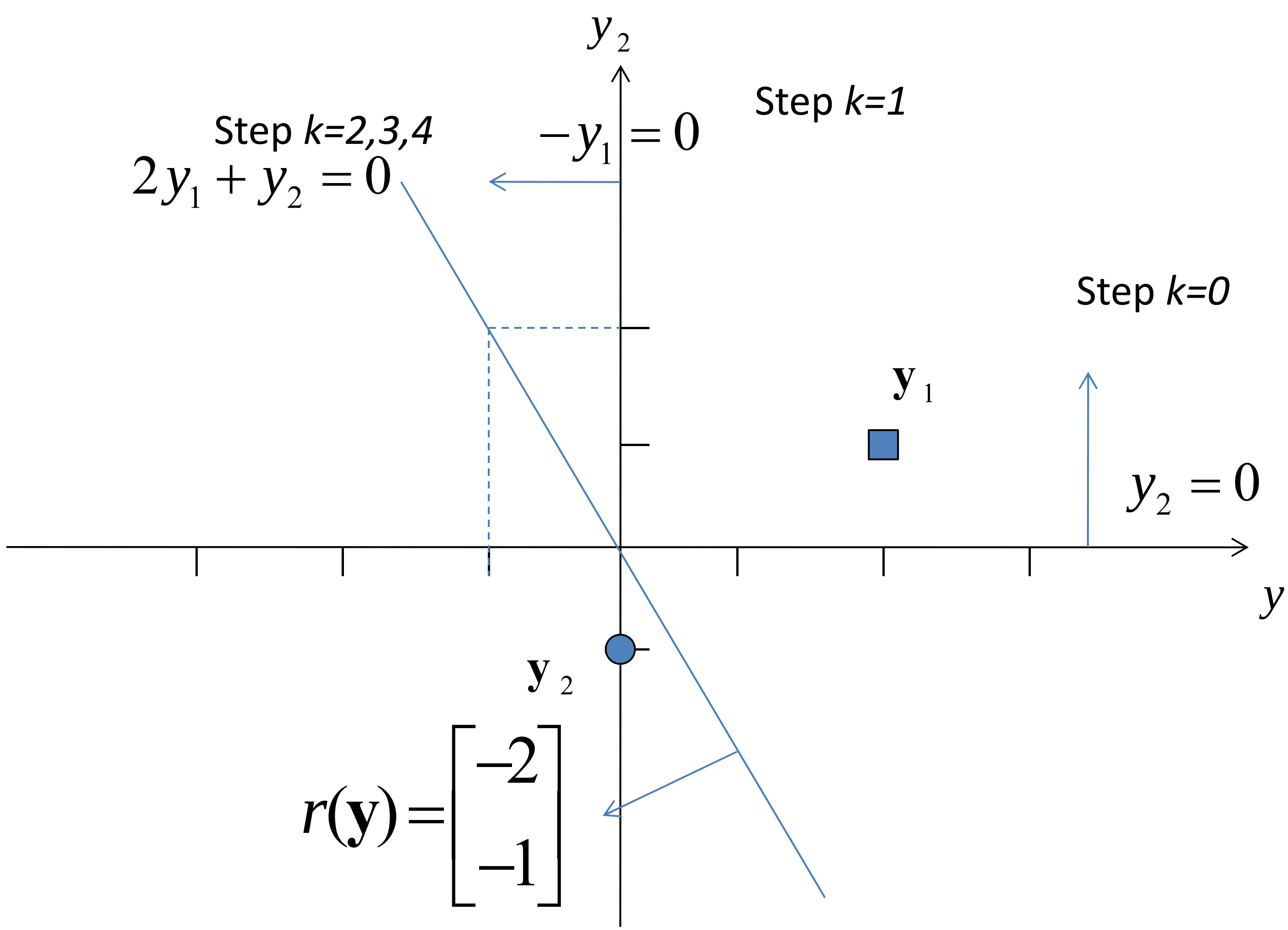
$$\mathbf{w}^3 = \begin{bmatrix} -2 & -1 & 0 \end{bmatrix}^t$$

$$o^4 = \text{sgn}([\mathbf{w}^4]^t \mathbf{y}^4) = \text{sgn}(\begin{bmatrix} -2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}) = 1$$

$$d^4 - o^4 = 0$$

$$\mathbf{w}^5 = \mathbf{w}^4 = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

2-D pattern space solution



3.2 The following data specifying the three training pairs

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}, d_1 = -1; \qquad \mathbf{x}_2 = \begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix}, d_2 = 1; \qquad \mathbf{x}_3 = \begin{bmatrix} -2 \\ 0 \\ -3 \\ -1 \end{bmatrix}, d_3 = -1.$$

The final weight is obtained by using the similar perceptron training scheme as in 3.1 as

$$\mathbf{w}^4 = \begin{bmatrix} 3 & 2 & 6 & 1 \end{bmatrix}^t$$

Knowing that the corrections has been performed in each step, determine the following weights:

$\mathbf{w}^3, \mathbf{w}^2, \mathbf{w}^1$ by back-tracking the training;

3.2

Key word: each step has updating.

$$\mathbf{w}^4 = \mathbf{w}^3 - \mathbf{x}^3 \Rightarrow = \mathbf{w}^3 = \mathbf{w}^4 + \mathbf{x}^3 = \begin{bmatrix} 3 \\ 2 \\ 6 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

$$\mathbf{w}^3 = \mathbf{w}^2 + \mathbf{x}^2 \Rightarrow = \mathbf{w}^2 = \mathbf{w}^3 - \mathbf{x}^2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{w}^2 = \mathbf{w}^1 - \mathbf{x}^1 \Rightarrow = \mathbf{w}^1 = \mathbf{w}^2 + \mathbf{x}^1 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 0 \end{bmatrix}$$

3.3 Design a minimum distance classifier based on the cluster centers obtained from Tutorial 1.1 (b).

Centres of gravity at T1.1(b)

Class 2 $$\mathbf{c}_2 = \left[\frac{5+7+3+5}{4} \quad \frac{1+3+2+4}{4} \right]^t = [5 \quad 2.5]^t$$

Class 1 $$\mathbf{c}_1 = \left[\frac{0-1-2-3}{4} \quad \frac{0-3+3+0}{4} \right]^t = [-1.5 \quad 0]^t$$

$$(\mathbf{c}_1 - \mathbf{c}_2)^t \mathbf{x} + \frac{1}{2}(\|\mathbf{c}_2\|^2 - \|\mathbf{c}_1\|^2) = 0$$

$$[(-1.5)-5 \quad (0)-2.5]\mathbf{x} + \frac{1}{2}(\|5\|^2 + \|2.5\|^2 - \|1.5\|^2 - \|0\|^2)$$

$$= -6.5x_1 - 2.5x_2 + 14.5 = 0$$