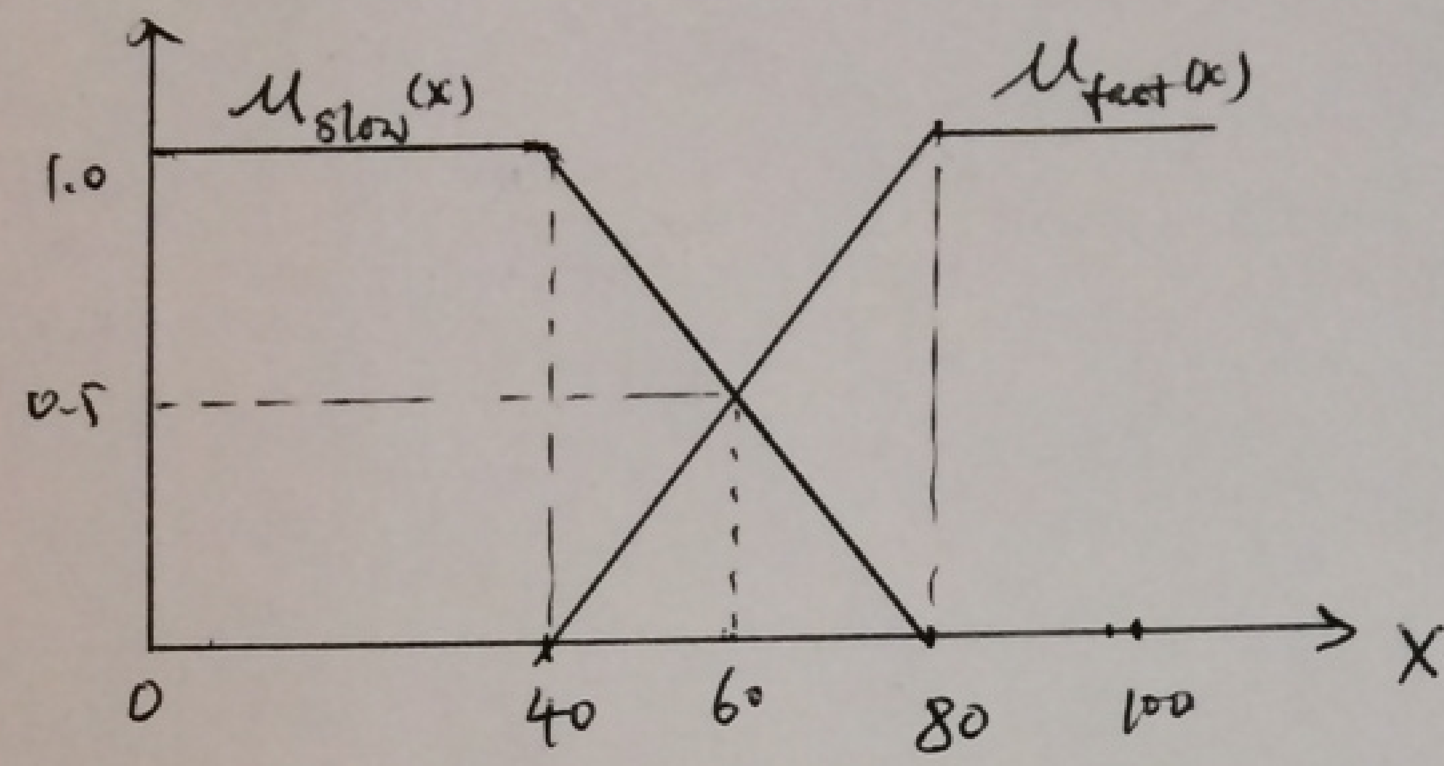
Tut 3

Q1:



(a).

$$\mu_A(x) = \mu^2_{fast}(x) = \begin{cases} 0 & \text{if } 0 \le x < 40 \\ \frac{1}{1600}(x-40)^2 & \text{if } 40 \le x < 80 \\ 1.0 & \text{if } 80 \le x \le 100 \end{cases}$$

(b).

$$\mu_B(x) = (1-\mu_{fast}(x)) \wedge (1-\mu_{slow}(x)) = \begin{cases} 0 & \text{if } 0 \le x < 40 \text{ or } 80 \le x \le 100 \\ 0.025(x-40) & \text{if } 40 \le x < 60 \\ -0.025(x-80) & \text{if } 60 \le x < 80 \end{cases}$$

(c).

$$\mu_C(x) = (1-\mu_{fast}(x)) \vee \mu^2_{slow}(x) = \mu_{slow}(x).$$

(d).

$$\mu_D(x) = \mu^{0.5}_{slow}(x) = \begin{cases} 1.0 & \text{if } 0 \le x < 40 \\ \sqrt{0.025(80-x)} & \text{if } 40 \le x < 80 \\ 0 & \text{if } 80 \le x \le 100 \end{cases}$$

Q2:

$$A = \begin{bmatrix} 0 \\ 0.2 \\ 0.4 \\ 0.8 \\ 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.9 \\ 0.5 \\ 0.1 \end{bmatrix}$$

(a).

$$R_1 = A \rightarrow C = \begin{bmatrix} 0 \\ 0.2 \\ 0.4 \\ 0.8 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.5 & 0.6 & 0.9 & 0.5 & 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.1 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.5 & 0.6 & 0.8 & 0.5 & 0.1 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.1 \end{bmatrix}$$

$$R_2 = B \rightarrow C = \begin{bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{bmatrix} \begin{bmatrix} 0.5 & 0.6 & 0.9 & 0.5 & 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0.6 & 0.9 & 0.5 & 0.1 \\ 0.5 & 0.6 & 0.8 & 0.5 & 0.1 \\ 0.5 & 0.6 & 0.6 & 0.5 & 0.1 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.1 \end{bmatrix}$$

(b).

$$R3 = A \times B \rightarrow C = \left(\mu_A(x) \otimes I_{5\times5}\right) \mu_{R_2}(x) = \begin{bmatrix} 0 \wedge \mu_{R_2}(x) \\ 0.2 \wedge \mu_{R_2}(x) \\ 0.4 \wedge \mu_{R_2}(x) \\ 0.8 \wedge \mu_{R_2}(x) \\ 0.5 \wedge \mu_{R_2}(x) \end{bmatrix}$$

(c).

$$A' = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.2 \\ 0.8 \\ 0. \end{bmatrix}, \quad B' = \begin{bmatrix} 0.7 \\ 0.2 \\ 0.4 \\ 0.9 \\ 10 \end{bmatrix}$$

$$\bigvee_x \mu_A(x) \wedge \mu_{A'}(x) = \begin{bmatrix} 0.2 & 0.4 & 0.2 & 0.8 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2 \\ 0.4 \\ 0.8 \\ 0.5 \end{bmatrix} = 0.8$$

$$\bigvee_y \mu_B(x) \wedge \mu_{B'}(x) = \begin{bmatrix} 0.7 & 0.2 & 0.4 & 0.9 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 \\ 0.6 \\ 0.4 \\ 0.2 \end{bmatrix} = 0.7.$$

$\therefore$ the firing strength is $0.8 \wedge 0.7 = 0.7$.

$$\mu_{C'}(z) = 0.7 \wedge \mu_C(z) = \begin{bmatrix} 0.5 \\ 0.6 \\ 0.7 \\ 0.5 \\ 0.1 \end{bmatrix}$$

Q3: $R \subseteq X \times Y$, $S, T \subseteq Y \times Z$.

$$\mu_{R\circ(S\cap T)}(x,z) = \bigvee_y [\mu_R(x,y) \cdot \mu_{S\cap T}(y,z)]$$

$$= \bigvee_y [\mu_R(x,y) \cdot \mu_S(y,z) \cdot \underline{\mu_T(y,z)}]$$

$$\mu_{(R\circ S)\cap(R\circ T)}(x,z) = \mu_{R\circ S}(x,z) \cdot \mu_{R\circ T}(x,z)$$

$$= \left[\bigvee_y \mu_R(x,y) \cdot \mu_S(y,z)\right] \left[\bigvee_{y'} \mu_R(x,y') \cdot \mu_T(y',z)\right]$$

$$= \bigvee_y \left[\mu_R(x,y)\mu_S(y,z) \cdot \underline{\left[\bigvee_{y'} \mu_R(x,y')\mu_T(y',z)\right]}\right].$$

Depending on the choice of $\mu_R(x,y)$ and $\mu_T(y,z)$, it is either.

$\mu_T(y,z) < \bigvee_{y'} \mu_R(x,y')\mu_T(y',z)$, or. $\mu_T(y,z) = \bigvee_{y'} \mu_R(x,y')\mu_T(y',z)$

or $\mu_T(y,z) > \bigvee_{y'} \mu_R(x,y')\mu_T(y',z)$.

Thus, the weak distribution over intersection does not hold with max-product composition and algebraic product for intersection.