Tutorial 1.1: Two-dimensional (2-D) input patterns for Apple and Orange Classes

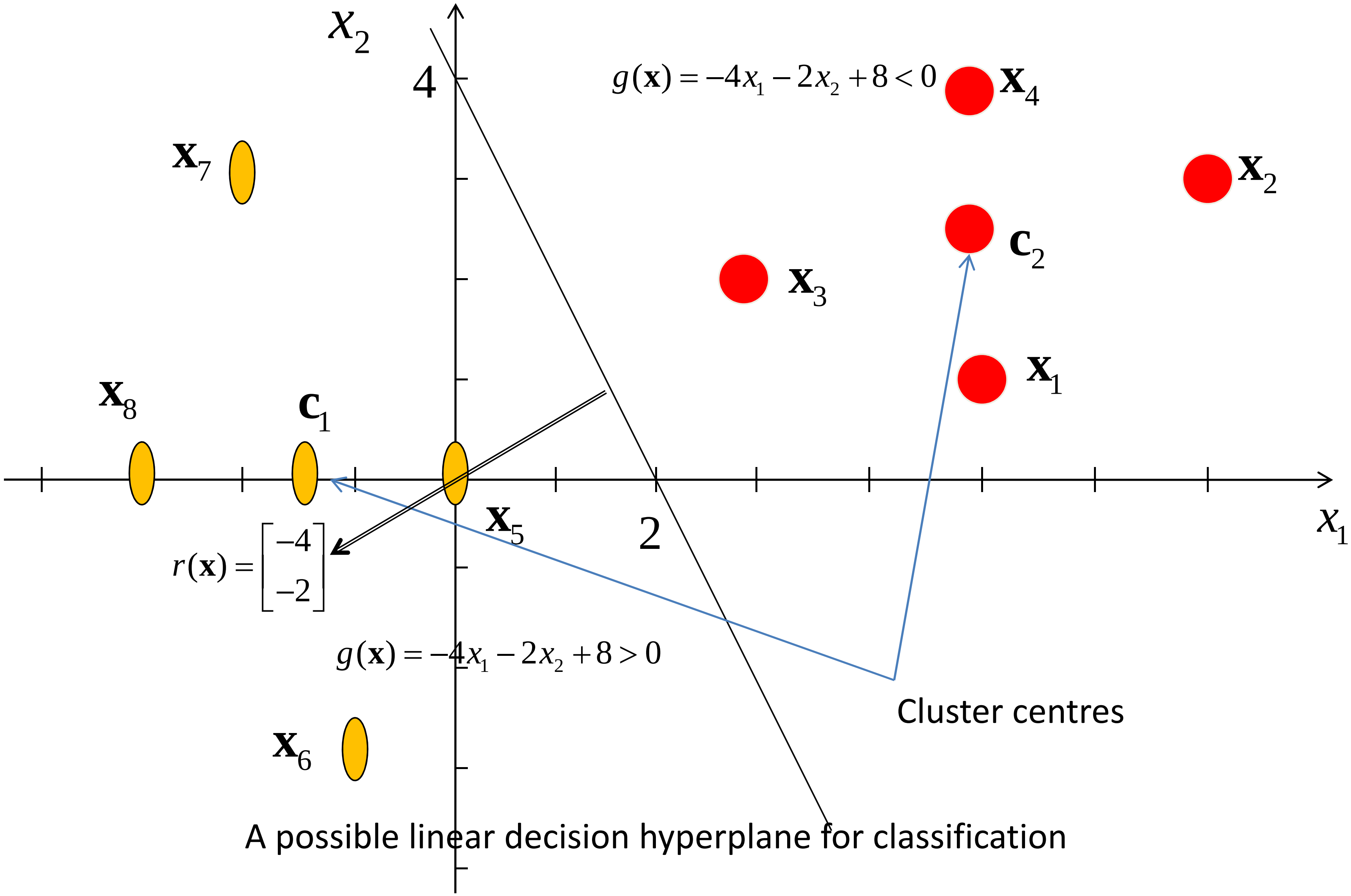
2-D input patterns of apple (cluster 2) and orange (cluster 1) are given as

Cluster 2 $\mathbf{x}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ $\mathbf{x}_3 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\mathbf{x}_4 = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Cluster 1 $\mathbf{x}_5 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\mathbf{x}_6 = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$ $\mathbf{x}_7 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ $\mathbf{x}_8 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$

a) Draw the input patterns in the pattern space to see if they are linearly separable and derive the linear decision hyperplane which is across the points $(2,0)$ and $(0,4)$;
b) Determine the cluster centers of each cluster using the centre gravity concepts.

Tutorial 1: Noisy input patterns for Apple and Orange Classes (no meaningful features)



Linear decision hyperplane – two points method

A line is across the two pints: $(2,0)$ and $(0,4)$,
the straight liner equation can be determined by

$$\begin{vmatrix} x_1 & x_2 & 1 \\ 2 & 0 & 1 \\ 0 & 4 & 1 \end{vmatrix} = x_1 \begin{vmatrix} 0 & 1 \\ 4 & 1 \end{vmatrix} - x_2 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix}$$

$$= -4x_1 - 2x_2 + 8 = 0$$

$$g(\mathbf{x}) = -4x_1 - 2x_2 + 8 = 0 \Leftrightarrow \text{linear equation with:}$$

$$\begin{cases} x_1 = 0 \\ x_2 = 4 \end{cases} \quad and \quad \begin{cases} x_2 = 0 \\ x_1 = 2 \end{cases}$$

Centres of gravity

Class 2 $$\mathbf{c}_2 = \begin{bmatrix} \dfrac{5+7+3+5}{4} & \dfrac{1+3+2+4}{4} \end{bmatrix}^t = \begin{bmatrix} 5 & 2.5 \end{bmatrix}^t$$

Class 1 $$\mathbf{c}_1 = \begin{bmatrix} \dfrac{0-1-2-3}{4} & \dfrac{0-3+3+0}{4} \end{bmatrix}^t = \begin{bmatrix} -1.5 & 0 \end{bmatrix}^t$$

Tutorial 1.2 Decision regions in a 3-dimensional cube

Write down line equations along with the constraint inequalities for the decision planes subdividing the cube into four decision regions as shown in the Figure below. The pattern vectors' memberships in classes are:

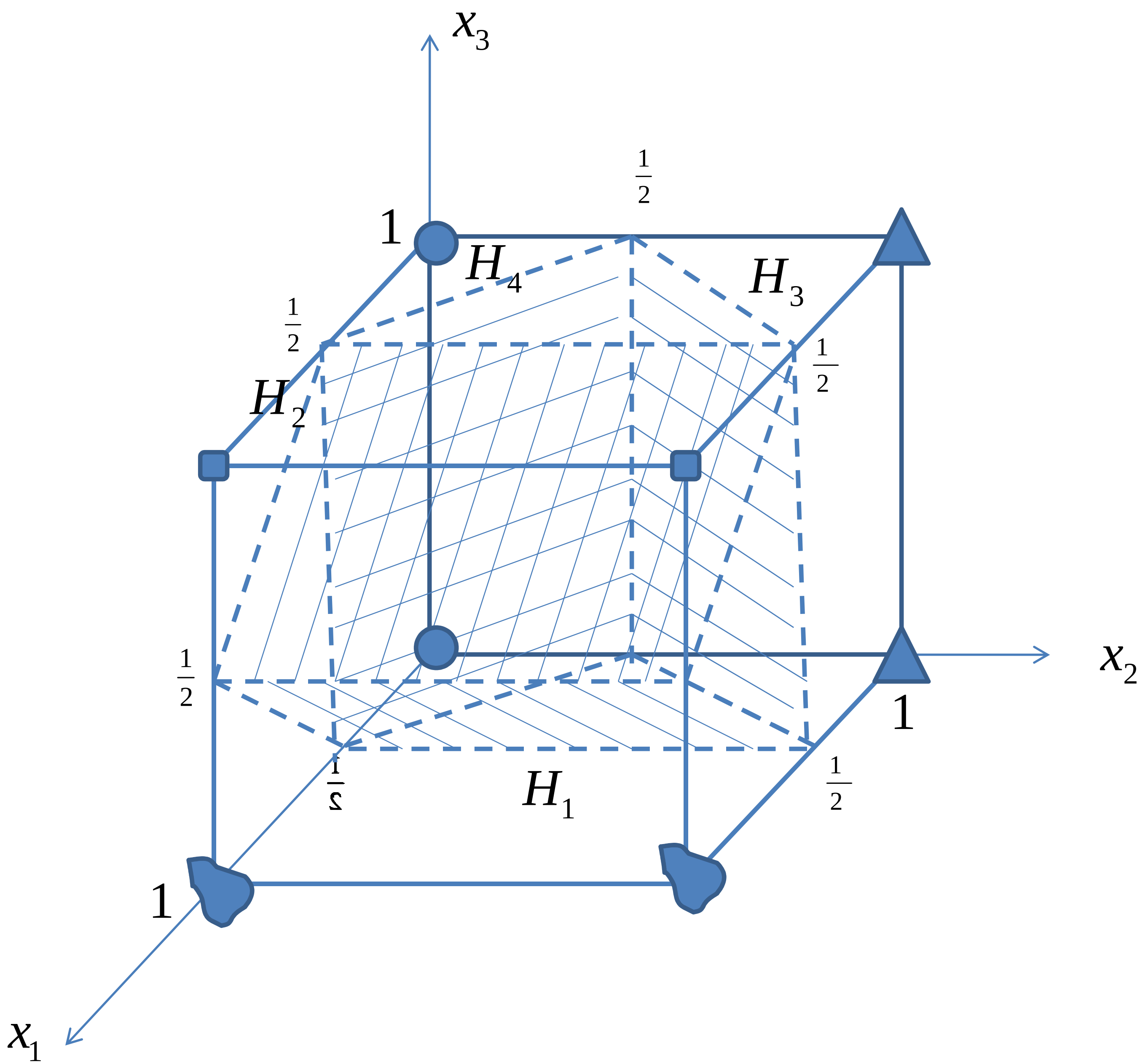
$$i_o = 1 \;\; \mathit{for}\; \mathbf{x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^t, \mathbf{x} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^t$$

$$i_o = 2 \;\; \mathit{for}\; \mathbf{x} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^t, \mathbf{x} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^t$$

$$i_o = 3 \;\; \mathit{for}\; \mathbf{x} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^t, \mathbf{x} = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^t$$

$$i_o = 4 \;\; \mathit{for}\; \mathbf{x} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^t, \mathbf{x} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^t$$

Note that the faces, edges, and vertices here belong to the pattern space . Whenever the decision surface intersects the edge of the cube, the intersection point should have that edge.



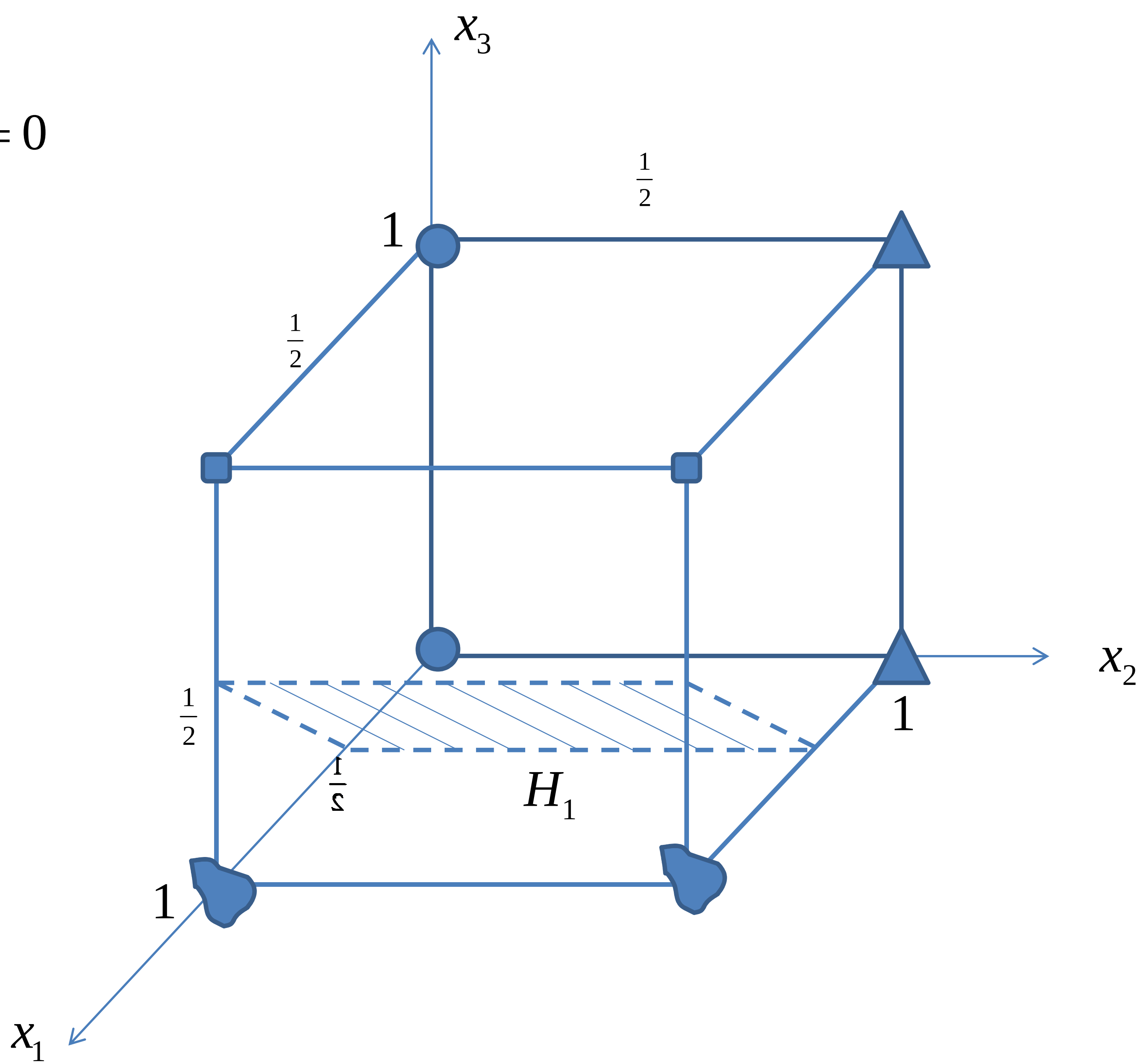
x_3
x_2
x_1
1
1
1
H_1
H_2
H_3
H_4
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$

$$x_1 - x_3 - \frac{1}{2} = 0$$

$$\frac{1}{2} \le x_1 \le 1$$

$$0 \le x_2 \le 1$$

$$0 \le x_3 \le \frac{1}{2}$$

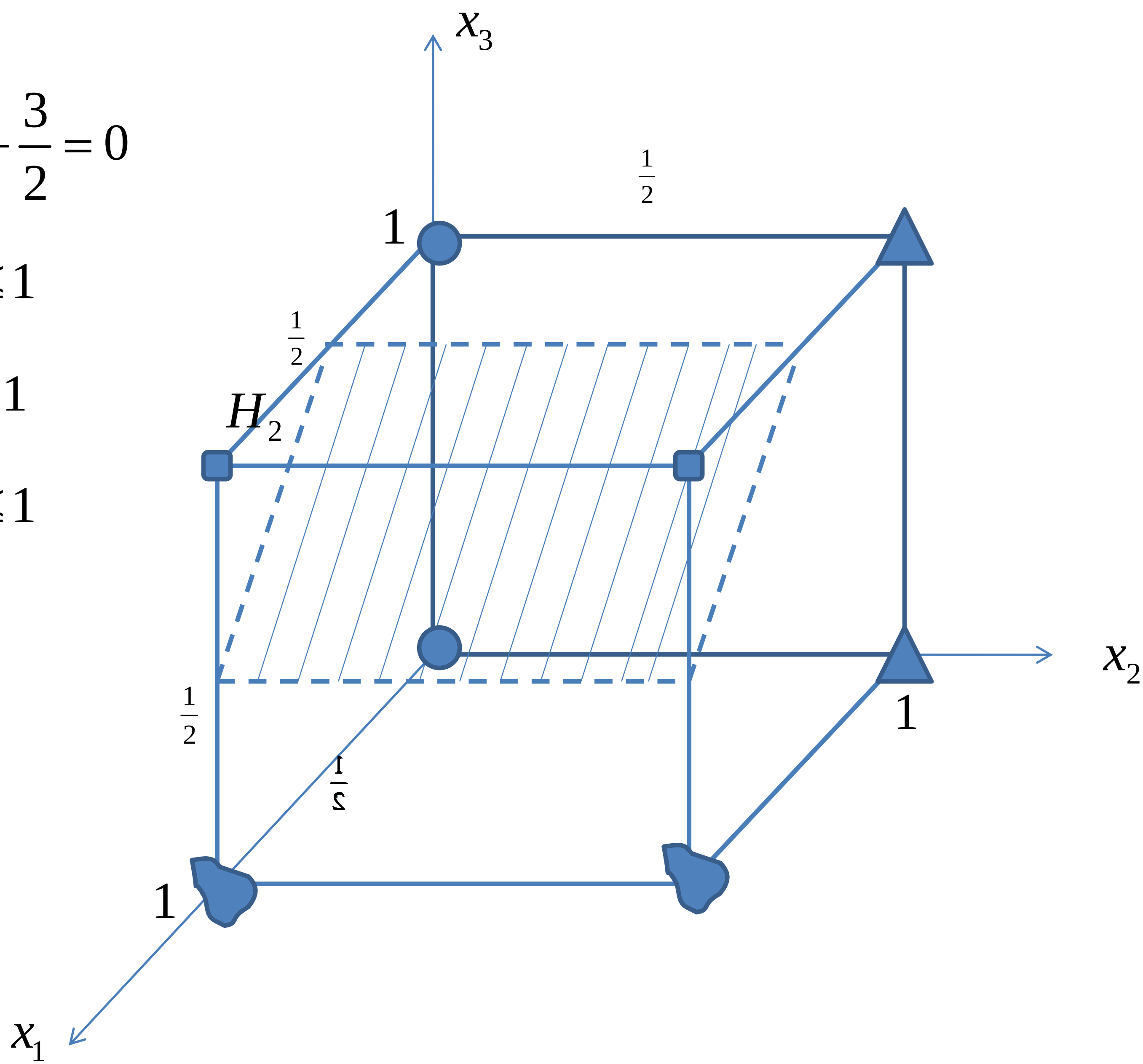


$$x_1 + x_3 - \frac{3}{2} = 0$$

$$\frac{1}{2} \leq x_1 \leq 1$$

$$0 \leq x_2 \leq 1$$

$$\frac{1}{2} \leq x_3 \leq 1$$

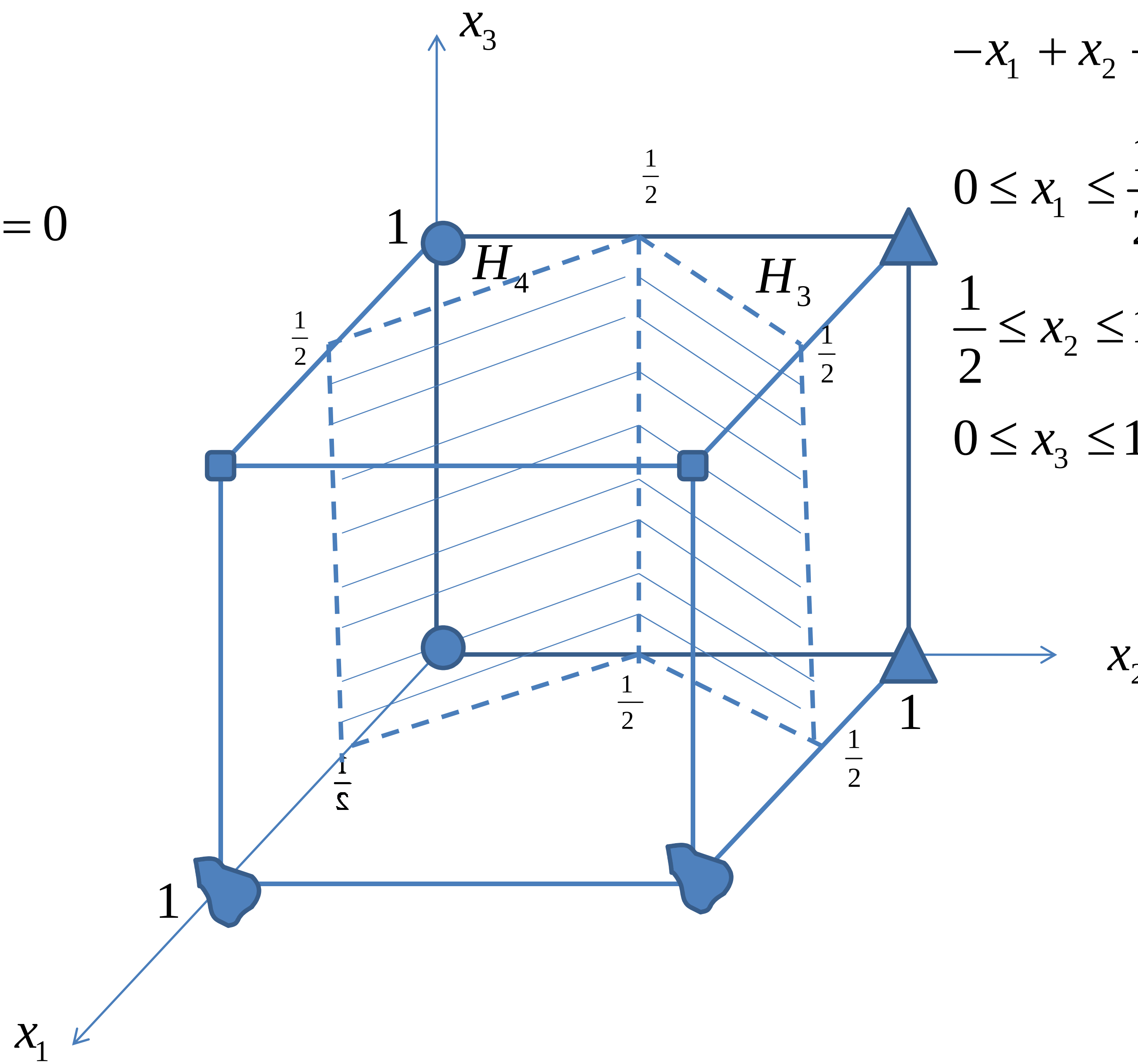


$$-x_1 - x_2 + \frac{1}{2} = 0$$

$$0 \le x_1 \le \frac{1}{2}$$

$$0 \le x_2 \le \frac{1}{2}$$

$$0 \le x_3 \le 1$$



$$-x_1 + x_2 - \frac{1}{2} = 0$$

$$0 \le x_1 \le \frac{1}{2}$$

$$\frac{1}{2} \le x_2 \le 1$$

$$0 \le x_3 \le 1$$

Decision plane H_1

$$x_1 - x_3 - \frac{1}{2} = 0$$

Constraint inequalities
$$\begin{cases} \frac{1}{2} \le x_1 \le 1 \\ 0 \le x_2 \le 1 \\ 0 \le x_3 \le \frac{1}{2} \end{cases}$$

Decision plane H_2

$$x_1 + x_3 - \frac{3}{2} = 0$$

Constraint inequalities
$$\begin{cases} \frac{1}{2} \le x_1 \le 1 \\ 0 \le x_2 \le 1 \\ \frac{1}{2} \le x_3 \le 1 \end{cases}$$

Decision plane H_3

$$-x_1 + x_2 - \frac{1}{2} = 0$$

Constraint inequalities

$$\begin{cases} 0 \le x_1 \le \frac{1}{2} \\ \frac{1}{2} \le x_2 \le 1 \\ 0 \le x_3 \le 1 \end{cases}$$

Decision plane H_4

$$-x_1 - x_2 + \frac{1}{2} = 0$$

Constraint inequalities

$$\begin{cases} 0 \le x_1 \le \frac{1}{2} \\ 0 \le x_2 \le \frac{1}{2} \\ 0 \le x_3 \le 1 \end{cases}$$