4.1 A two-class classifier is trained to assign the two patterns into class 1 and 2:

$$\mathbf{y}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad d_1 = 1 \qquad\qquad \mathbf{y}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad d_2 = -1$$

Display the movement of the weight vector on the augmented weight plane starting from the initial weight of $\mathbf{w}^1 = [1 \quad 1]^t$

and follow the intermediate steps until weight falls into the solution region (using forced zero value for TLU neuron and looking maximum three-step updating solutions).

a) Use *c=1* for the standard perceptron learning;

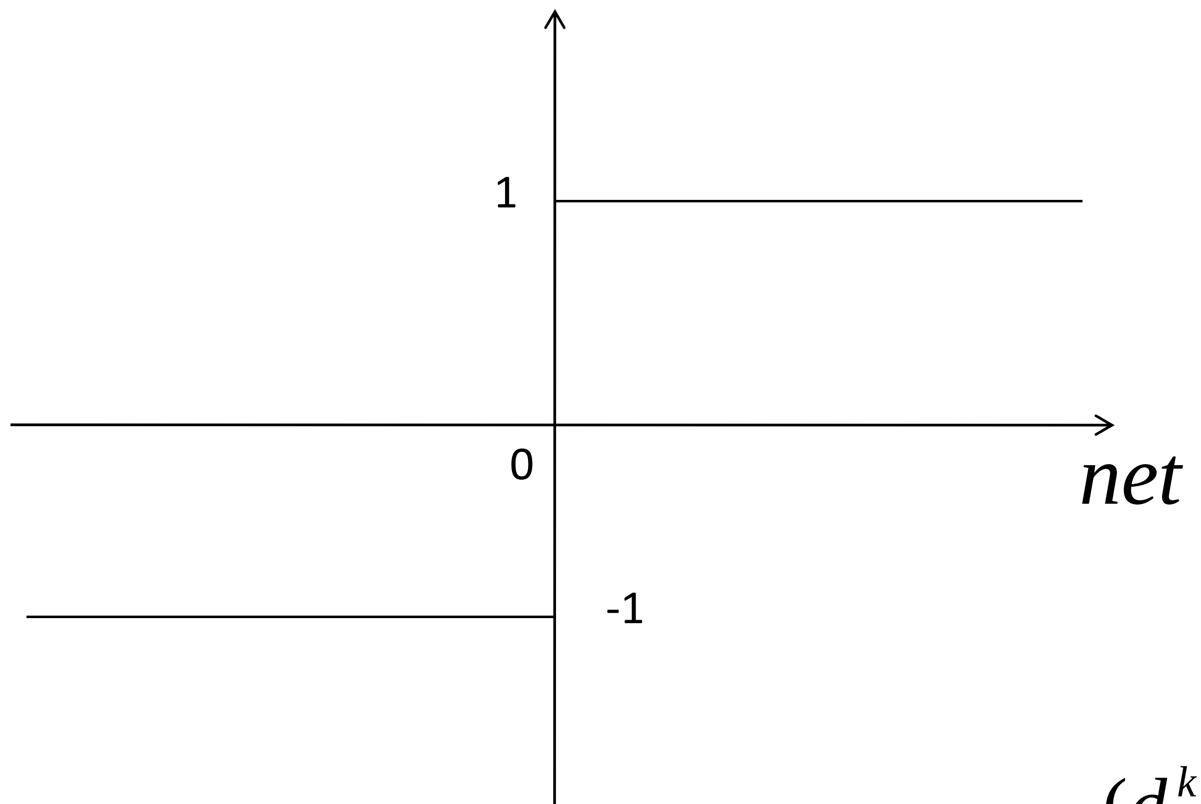
b) Use $\lambda = 1$ for the variable perceptron learning below;

c) Use $\lambda = 2$ for the variable perceptron learning below.

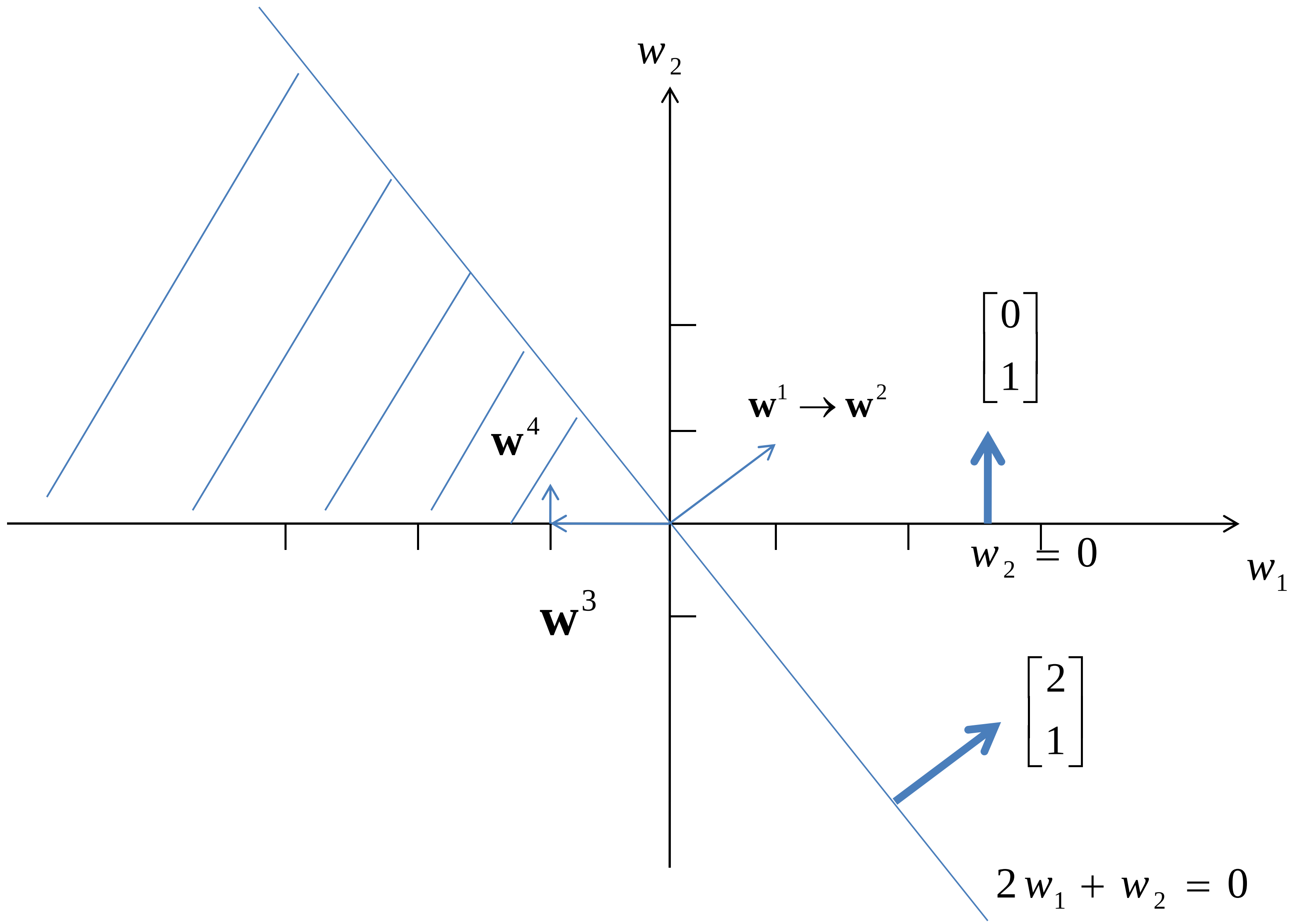
$$\mathbf{w}^{k+1} = \mathbf{w}^k + \lambda \frac{(d^k - o^k)}{2} \frac{\left\| (\mathbf{w}^k)^t \mathbf{y}^k \right\|}{(\mathbf{y}^k)^t \mathbf{y}^k} \mathbf{y}^k$$

TLU: a discrete time neuron activation function

$$f(net) = \mathrm{sgn}(net)$$



$$f(net = 0) = 0 \qquad \text{forced zero} \rightarrow \mathbf{w}^{k+1} = \mathbf{w}^{k} + c\frac{(d^{k} - o^{k})}{2}\mathbf{y}^{k}$$



w_2
w_1
$\mathbf{w}^1 \rightarrow \mathbf{w}^2$
$\mathbf{w}^4$
$\mathbf{w}^3$
$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$w_2 = 0$
$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
$2w_1 + w_2 = 0$

$$\mathbf{w}^1 = \begin{bmatrix}1 & 1\end{bmatrix}^t \qquad o^1 = \operatorname{sgn}\left(\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\right) = 1$$

$$\mathbf{w}^2 = \mathbf{w}^1 \qquad d^1 = o^1$$

$$\mathbf{w}^2 = \begin{bmatrix}1 & 1\end{bmatrix}^t \qquad o^2 = \operatorname{sgn}\left(\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}\right) = 1$$

$$\mathbf{w}^3 = \mathbf{w}^2 - \mathbf{y}^2 = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}-1\\0\end{bmatrix} \qquad d^2 - o^2 = -1 - 1 = -2$$

$$\mathbf{w}^3 = \begin{bmatrix}-1 & 0\end{bmatrix}^t \qquad o^3 = \operatorname{sgn}\left(\begin{bmatrix}-1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\right) = 0 \qquad \text{forced zero}$$

$$\mathbf{w}^4 = \mathbf{w}^3 + \frac{1}{2}\mathbf{y}^3 = \begin{bmatrix}-1\\0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\1/2\end{bmatrix} \qquad d^3 - o^3 = 1 - 0 = 1$$

$$\lambda = 1 \rightarrow \quad c^k = \frac{\left\| (\mathbf{w}^k)^t \mathbf{y}^k \right\|}{2(\mathbf{y}^k)^t \mathbf{y}^k}$$

$$(\mathbf{y}^1)^t \mathbf{y}^1 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 \quad (\mathbf{y}^2)^t \mathbf{y}^2 = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 5$$

$$\mathbf{w}^1 = \begin{bmatrix} 1 & 1 \end{bmatrix}^t \qquad o^1 = \mathrm{sgn}(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}) = 1$$

$$\mathbf{w}^2 = \mathbf{w}^1 \qquad d^1 = o^1$$

$$\mathbf{w}^2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^t \qquad o^2 = \mathrm{sgn}(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}) = \mathrm{sgn}(3) = 1$$

$$\mathbf{w}^3 = \mathbf{w}^2 - \frac{3}{5} \mathbf{y}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{3}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/5 \\ 2/5 \end{bmatrix} \quad d^2 - o^2 = -1 - 1 = -2$$

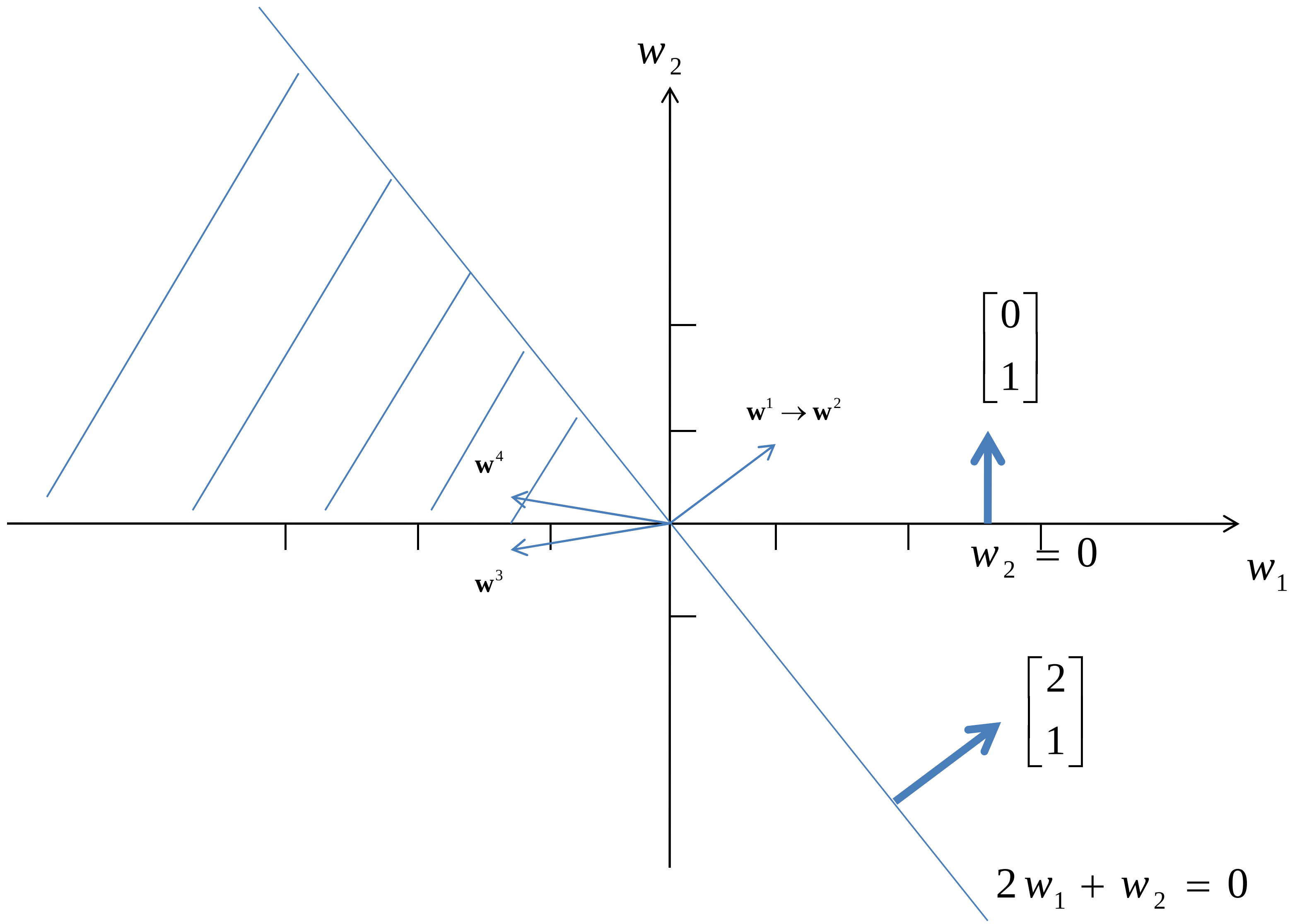
$$\mathbf{w}^3 = \begin{bmatrix} -1/5 & 2/5 \end{bmatrix}^t \qquad o^3 = \mathrm{sgn}(\begin{bmatrix} -1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \mathrm{sgn}(2/5) = 1$$

$$\mathbf{w}^4 = \mathbf{w}^3 \qquad d^3 - o^3 = 0$$

$$\mathbf{w}^4 = \begin{bmatrix} -1/5 & 2/5 \end{bmatrix}^t \qquad o^4 = \mathrm{sgn}(\begin{bmatrix} -1/5 & 2/5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}) = \mathrm{sgn}(0) = 0$$

$$\mathbf{w}^5 = \mathbf{w}^4 - \frac{0}{2*5}\mathbf{y}^2 = \mathbf{w}^4 \qquad d^4 - o^4 = -1 - 0 = -1$$

Since there are two subsequent weight updating steps with non-updating, the learning is stopped and non-solution weight region has been reached due to the variable learning rate is not good enough.



w_2
w_1
$\mathbf{w}^1 \rightarrow \mathbf{w}^2$
$\mathbf{w}^4$
$\mathbf{w}^3$
$\begin{bmatrix}0\\1\end{bmatrix}$
$w_2 = 0$
$\begin{bmatrix}2\\1\end{bmatrix}$
$2w_1 + w_2 = 0$

$$\lambda = 2 \rightarrow \quad c^k = 2\frac{\left\| (\mathbf{w}^k)^t \mathbf{y}^k \right\|}{(\mathbf{y}^k)^t \mathbf{y}^k}$$

$$(\mathbf{y}^1)^t \mathbf{y}^1 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 \quad (\mathbf{y}^2)^t \mathbf{y}^2 = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 5$$

$$\mathbf{w}^1 = \begin{bmatrix} 1 & 1 \end{bmatrix}^t \qquad o^1 = \operatorname{sgn}(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}) = 1$$

$$\mathbf{w}^2 = \mathbf{w}^1 \qquad d^1 = o^1$$

$$\mathbf{w}^2 = \begin{bmatrix} 1 & 1 \end{bmatrix}^t \qquad o^2 = \operatorname{sgn}(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}) = \operatorname{sgn}(3) = 1$$

$$\mathbf{w}^3 = \mathbf{w}^2 - 2 * \frac{3}{5} \mathbf{y}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2 * \frac{3}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -7/5 \\ -1/5 \end{bmatrix} \quad d^2 - o^2 = -1 - 1 = -2$$

$$\mathbf{w}^3 = \begin{bmatrix} -7/5 & -1/5 \end{bmatrix}^t \qquad o^3 = \operatorname{sgn}(\begin{bmatrix} -7/5 & -1/5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \operatorname{sgn}(-1/5) = -1$$

$$\mathbf{w}^4 = \mathbf{w}^3 + 2 * \frac{1/5}{1} \mathbf{y}^3 = \begin{bmatrix} -7/5 \\ -1/5 \end{bmatrix} + 2 * \frac{1}{5} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -7/5 \\ 1/5 \end{bmatrix} \qquad d^3 - o^3 = 1 + 1 = 2$$