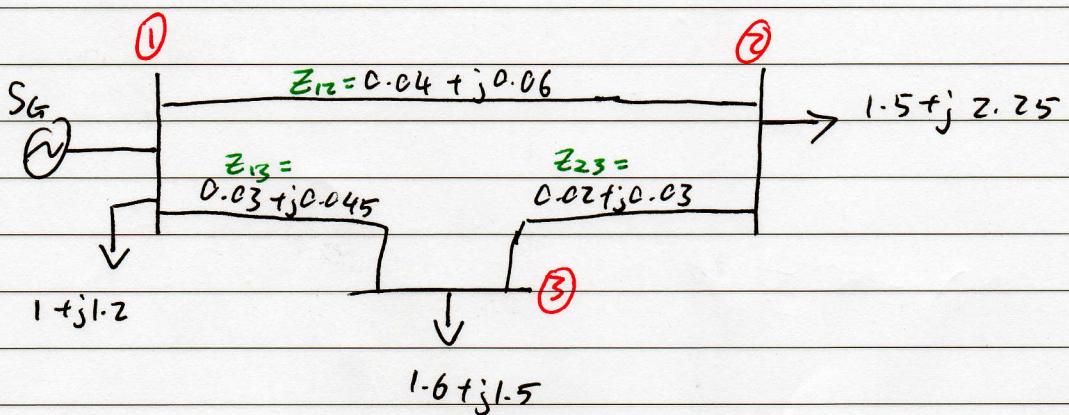


1)



$$\begin{aligned}
 a) Y_{11} &= \frac{1}{Z_{12}} + \frac{1}{Z_{13}} = 32.36 \angle -56.3^\circ \\
 Y_{12} = Y_{21} &= -\frac{1}{Z_{12}} = 13.87 \angle 23.69^\circ \\
 Y_{13} = Y_{31} &= -\frac{1}{Z_{13}} = 18.49 \angle 23.69^\circ \\
 Y_{22} &= \frac{1}{Z_{23}} + \frac{1}{Z_{13}} = 41.6 \angle -56.3^\circ \\
 Y_{23} = Y_{32} &= -\frac{1}{Z_{23}} = 27.74 \angle 23.69^\circ \\
 Y_{33} &= \frac{1}{Z_{13}} + \frac{1}{Z_{23}} = 46.23 \angle -56.3^\circ
 \end{aligned}$$

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 32.36 \angle -56.3^\circ & 13.87 \angle 23.69^\circ & 18.49 \angle 23.69^\circ \\ 13.87 \angle 23.69^\circ & 41.6 \angle -56.3^\circ & 27.74 \angle 23.69^\circ \\ 18.49 \angle 23.69^\circ & 27.74 \angle 23.69^\circ & 46.23 \angle -56.3^\circ \end{bmatrix} \text{X}$$

$$b) V_1 = 1 \angle 0^\circ, \text{ assume } V_2 = V_3 = 1 \angle 0^\circ,$$

$$\begin{aligned}
 V_2' &= \frac{1}{41.6 \angle -56.3^\circ} \left[\frac{(-1.5 - j2.25)^*}{1 \angle 0^\circ} - 13.87 \angle 23.69^\circ \times 1 \angle 0^\circ - 27.74 \angle 23.69^\circ \times 1 \angle 0^\circ \right] \\
 &= 0.9352 \angle -0.01^\circ \text{ X}
 \end{aligned}$$

$$\begin{aligned}
 V_3' &= \frac{1}{46.23 \angle -56.3^\circ} \left[\frac{(-1.6 - j1.5)^*}{1 \angle 0^\circ} - 18.49 \angle 23.69^\circ \times 1 \angle 0^\circ - 27.74 \angle 23.69^\circ \times 0.9352 \angle -0.01^\circ \right] \\
 &= 0.915 \angle -0.69^\circ \text{ X}
 \end{aligned}$$

$$c) V_2 = 0.8026 \angle -0.8^\circ, V_3 = 0.8255 \angle -1.29^\circ$$

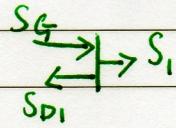
$$S_i = V_i I_i^*, \quad I_i = V_i Y_{11} + V_2 Y_{12} + V_3 Y_{13}$$

$$S_i = 120^\circ \times [120^\circ \times 32.36 \angle -56.3^\circ + 0.8026 \angle -0.8^\circ \times 13.87 \angle 123.69^\circ + 0.8255 \angle -1.29^\circ \times 18.49 \angle 123.69^\circ]^* \\ = 3.731 + j4.687 \cancel{\text{A}} \Rightarrow P_i = 373.1 \text{ MW}, Q_i = 468.7 \text{ MVar}$$

$$S_G = S_i + S_{D1}$$

$$= (3.731 + j4.687) + (1 + j1.2)$$

$$= 4.731 + j5.887 \cancel{\text{A}} \Rightarrow P_G = 473.1 \text{ MW}, Q_G = 588.7 \text{ MVar}$$



$$I_{L12} = (V_i - V_2) / Z_{12} = (120^\circ - 0.8026 \angle -0.8^\circ) / (0.04 + j0.06) \\ = 2.743 \angle -53.1^\circ$$

$$S_{L12} = |I_{L12}|^2 \times Z_{12} = |2.743|^2 \times (0.04 + j0.06) \\ = 0.3 + j0.4514 \cancel{\text{A}} \Rightarrow$$

$$P_{L12} = 30 \text{ MW}, Q_{L12} = 45.14 \text{ MVar}$$

$$2) S_B = 1000 \text{ MVA} \quad f_B = 50 \text{ Hz} \quad B_{\text{new}} = B_{\text{old}} \left[\frac{S_B \text{ old}}{S_B \text{ new}} \right], \quad D_{\text{new}} = D_{\text{old}} \left[\frac{S_B \text{ old}}{S_B \text{ new}} \right]$$

$$\begin{aligned} B_A &= 40 \times \left[\frac{1000}{1000} \right] = 40 & D_A &= 0 \times \left[\frac{1000}{1000} \right] = 0 \\ B_B &= 60 \times \left[\frac{500}{1000} \right] = 30 & D_B &= 2 \times \left[\frac{500}{1000} \right] = 1 \\ B_C &= 25 \times \left[\frac{800}{1000} \right] = 20 & D_C &= 2.5 \times \left[\frac{800}{1000} \right] = 2 \end{aligned}$$

$$\Delta P_{LB} = 93/1000 = 0.093, \quad f_{0 \text{ sys}} = 50 \text{ Hz}$$

a.)

$$\Delta f = \frac{-\Delta P}{B_A + B_B + B_C + D_A + D_B + D_C} = \frac{-0.093}{40 + 30 + 20 + 0 + 1 + 2} = -0.001$$

$$\Rightarrow \Delta f_{\text{act}} = -0.05 \text{ Hz}$$

$$\therefore f_{n \text{ sys}} = \Delta f_{\text{act}} + f_{0 \text{ sys}} = 49.95 \text{ Hz} *$$

ii) $\Delta P_{\text{tie}i} = \Delta P_i - \Delta P_{i_i} - D_i \Delta f$, where $\Delta P_{\text{tie}i}$ is power flowing out of Area i

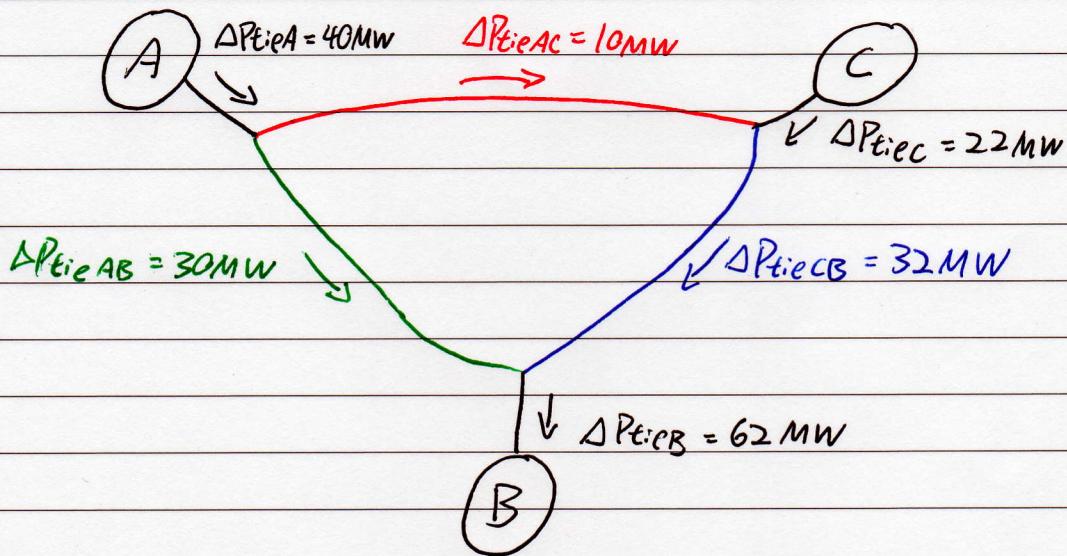
In Per Unit,

$$\Delta P_{\text{tie}A} = (-B_A \Delta f) - 0 - (D_A \Delta f) = 0.04 \Rightarrow \Delta P_{\text{tie}A} = 40 \text{ MW}$$

$$\Delta P_{\text{tie}B} = (-B_B \Delta f) - 0.093 - (D_B \Delta f) = -0.062 \Rightarrow \Delta P_{\text{tie}B} = -62 \text{ MW}$$

$$\Delta P_{\text{tie}C} = (-B_C \Delta f) - 0 - (D_C \Delta f) = 0.022 \Rightarrow \Delta P_{\text{tie}C} = 22 \text{ MW}$$

Power balance $\Rightarrow \text{Total } \Delta P_{\text{tie}} = 0 \quad \therefore \Delta P_{\text{tie}A} + \Delta P_{\text{tie}B} + \Delta P_{\text{tie}C} = 0$



b) lossless, $\lambda = ICR = \frac{dCR}{dP}$, $P_0 = 220 \text{ MW}$

$$ICR_1 = 12 + 0.05P_1 \quad \text{---(1)} \quad 10 \leq P_1 \leq 120 \text{ MW}$$

$$ICR_2 = 17 + 0.03P_2 \quad \text{---(2)} \quad 10 \leq P_2 \leq 60 \text{ MW}$$

$$ICR_3 = 15 + 0.02P_3 \quad \text{---(3)} \quad 10 \leq P_3 \leq 80 \text{ MW}$$

$$P_1 + P_2 + P_3 = 220 \text{ MW}$$

From (1), $P_1 = (\lambda - 12)/0.05$

From (2), $P_2 = (\lambda - 17)/0.03$

From (3), $P_3 = (\lambda - 15)/0.02$

$$\therefore [(\lambda - 12)/0.05] + [(\lambda - 17)/0.03] + [(\lambda - 15)/0.02] = 220$$

$$[20\lambda - 240] + [33.33\lambda - 566.67] + [50\lambda - 750] = 220$$

$$103.33\lambda = 1776.67$$

$$\therefore \lambda = 17.194$$

$$\Rightarrow P_1 = 103.88 \text{ MW}, P_2 = 6.47 \text{ MW}, P_3 = 109.71 \text{ MW}$$

Since $P_3 > P_{3\max}$, \therefore Set $P_3 = 80 \text{ MW}$

$$P_1 + P_2 + 80 = 220 \Rightarrow P_2 = 140 - P_1 \quad \text{---(4)}$$

$$12 + 0.05P_1 = 17 + 0.03P_2 \quad \text{---(5)}$$

Sub (4) into (5),

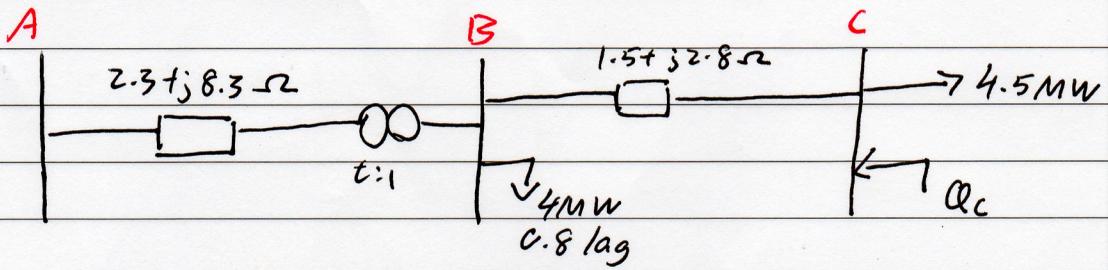
$$12 + 0.05P_1 = 17 + 0.03[140 - P_1]$$

$$\Rightarrow P_1 = 115 \text{ MW} \quad \text{#}$$

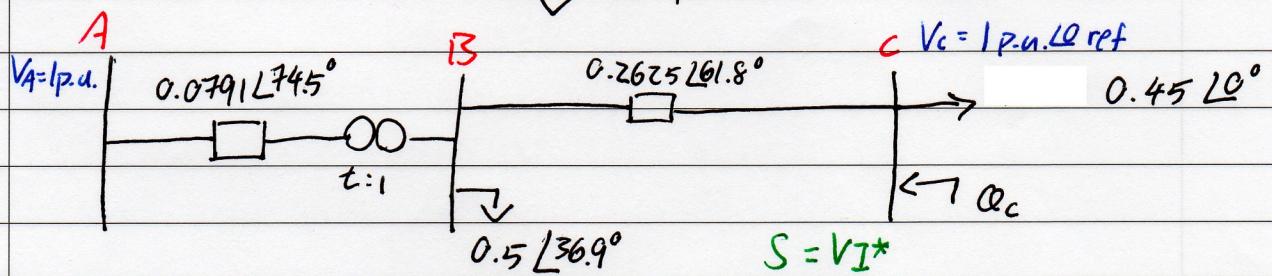
$$\Rightarrow P_2 = 25 \text{ MW} \quad \text{#}$$

$$\& P_3 = 80 \text{ MW} \quad \text{#}$$

3) Under nominal transformer turn ratio:



↓ to per unit



$$S_B = 10 \text{ MVA}$$

$$V_{B1} = 33 \text{ kV}$$

$$Z_{B1} = 108.9 \Omega$$

$$V_{B2} = 11 \text{ kV}$$

$$Z_{B2} = 12.1 \Omega$$

$$V_{pri} = \alpha V_{sec}$$

$$Z_{pri} = \alpha^2 Z_{sec}$$

$$I_{pri} = \frac{1}{\alpha} I_{sec}$$

where α is
pri/sec turn ratio

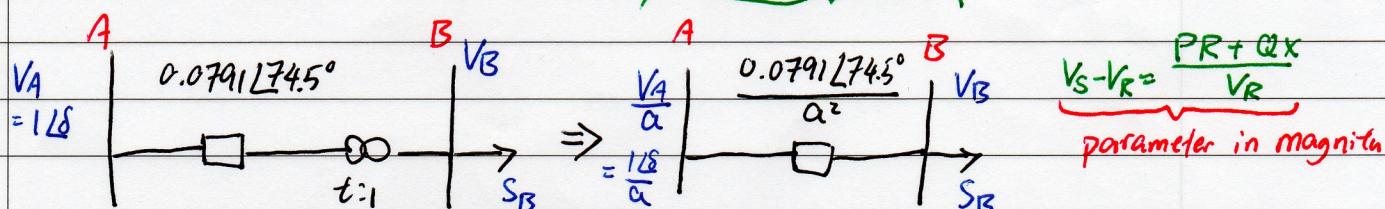
$$I_C = \left(\frac{S_B}{V_C} \right)^* = 0.45 L^\circ$$

$$V_B = V_C + I_C (0.2625 L 61.8^\circ) = 1.061 L 5.63^\circ$$

$$S_B = 0.5 L 36.9^\circ + 0.45 L^\circ + |I_C|^2 (0.2625 L 61.8^\circ)$$

$$= 0.9413 L 21.6^\circ$$

refer to sec., $\alpha = t/1$



$$\Delta V = \frac{V_A}{t} - V_B = \frac{PR + QX}{t^2 V_B} \Rightarrow t V_A V_B - t^2 V_B^2 = PR + QX$$

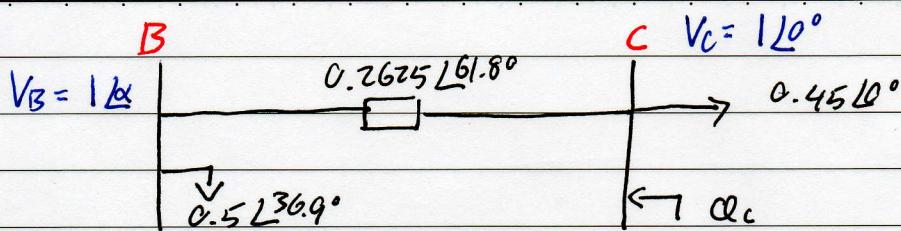
$$\text{Since } V_A = 1, \quad P = 0.8752, \quad Q = 0.3465, \quad R = 0.02114, \quad X = 0.07622$$

$$\Rightarrow \therefore t = \frac{0.898}{0.0444} \quad (\text{rejected})$$

$$(1.061)^2 t^2 - (1.061)t + 0.04491 = 0$$

* range typically $\pm 10\%$ for
of nominal value.

b)



$$\Delta V = V_B - V_C = \frac{PR + QX}{V_C} \quad \left. \right\} \text{in Per Unit Magnitude}$$

$$V_B = 1, V_C = 1, P = 0.45, Q = -Q_C, R = 0.124, X = 0.2313,$$

$$\Rightarrow PR + QX = 0 \Rightarrow (0.45)(0.124) = Q_C (0.2313)$$

$$\Rightarrow Q_C = 0.2412 \Rightarrow Q_{C\text{act}} = 2.412 \text{ MVar into bus C} \quad \text{X}$$

b) Method 2: with angle

$$I_C = \frac{V_B \angle \alpha - V_C \angle \theta}{Z \angle \beta}, \quad S_C = V_C I_C^* = V_C \angle \theta \left[\frac{V_B \angle \alpha - V_C \angle \theta}{Z \angle \beta} \right]$$

$$S_C = \frac{V_C V_B}{Z} \angle -\alpha + \beta - \frac{V_C^2}{Z} \angle \beta$$

$$= \underbrace{\left[\frac{V_C V_B}{Z} \cos(-\alpha + \beta) - \frac{V_C^2}{Z} \cos(\beta) \right]}_{P_R} + j \underbrace{\left[\frac{V_C V_B}{Z} \sin(-\alpha + \beta) - \frac{V_C^2}{Z} \sin(\beta) \right]}_{Q_R}$$

Since P_R, V_C, V_B, Z, β are known, \therefore find α . where $\alpha \neq 90^\circ$

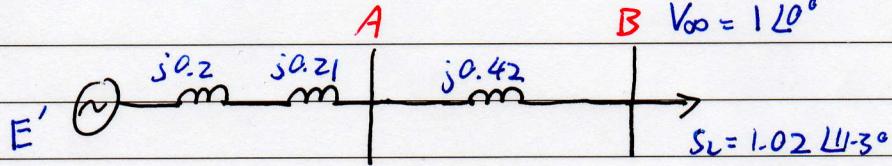
$$\Rightarrow \alpha = 8^\circ$$

Since α is known, use Q_R equation to find Q_R

$$\Rightarrow Q_R = -0.2832 \therefore \text{capacitive compensator.}$$

$$Q_C = 0.2832 \Rightarrow Q_{C\text{act}} = 2.832 \text{ MVar into bus C.} \quad \text{X}$$

4)



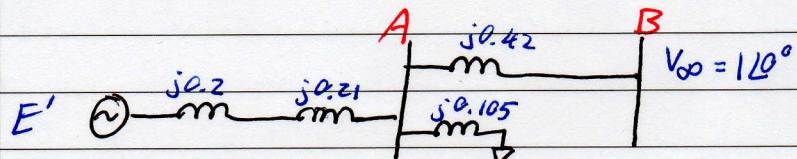
$$a) I_L = \left(\frac{S_L}{V_{00}} \right)^* = 1.02 L - 11.3^\circ$$

$$Z_{T_1} = j0.2 + j0.21 + j0.42 = j0.83$$

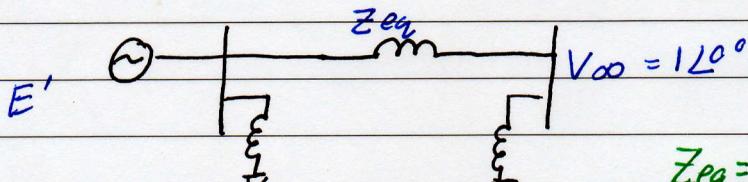
$$E' \angle \delta = V_{00} + Z_{T_1} I_L = 1.4313 L 35.45^\circ \quad (\text{Pre})$$

$$P_{e1} = \left| \frac{E' V_{00}}{Z_{T_1}} \right| \sin \delta = 1.7245 \sin \delta \quad (\text{Pre})$$

b)

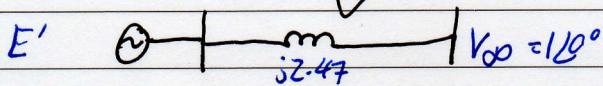


\Downarrow Y-Δ transformation



$$Z_{eq} = j0.2 + j0.21 + j0.42 + \frac{(j0.2 + j0.21)(j0.42)}{j0.105}$$

$$= j2.47$$

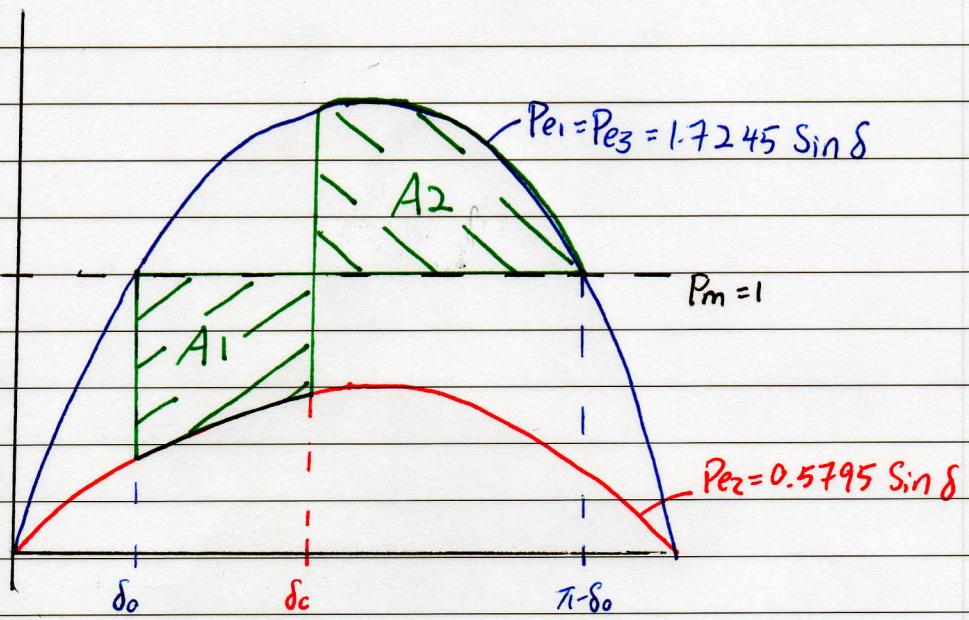


$$P_{e2} = \left| \frac{E' V_{00}}{Z_{eq}} \right| \sin \delta = 0.5795 \sin \delta \quad (\text{During})$$

$$P_{e3} = P_{e1} = 1.7245 \sin \delta \quad (\text{Post})$$

$$\delta_0 = 35.45^\circ = 0.6187 \text{ rad}$$

$$\delta_C = 82^\circ = 1.4312 \text{ rad}$$



$$A_1 = \int_{\delta_0}^{\delta_c} (P_m - P_{e2}) d\delta = \int_{\delta_0}^{\delta_c} (1 - 0.5795 \sin \delta) d\delta = [\delta + 0.5795 \cos \delta]_{\delta_0}^{\delta_c}$$

$$= [\delta_c + 0.5795 \cos \delta_c - \delta_0 - 0.5795 \cos \delta_0] = 0.4211$$

$$A_2 = \int_{\delta_c}^{\pi - \delta_0} (P_{e3} - P_m) d\delta = \int_{\delta_c}^{\pi - \delta_0} (1.7245 \sin \delta - 1) d\delta = [-1.7245 \cos \delta - \delta]_{\delta_c}^{\pi - \delta_0}$$

$$= [-1.7245 \cos(\pi - \delta_0) - \pi + \delta_0 + 1.7245 \cos \delta_c + \delta_c] = 0.5531$$

Since $A_2 > A_1 \therefore$ stable system \cancel{x}