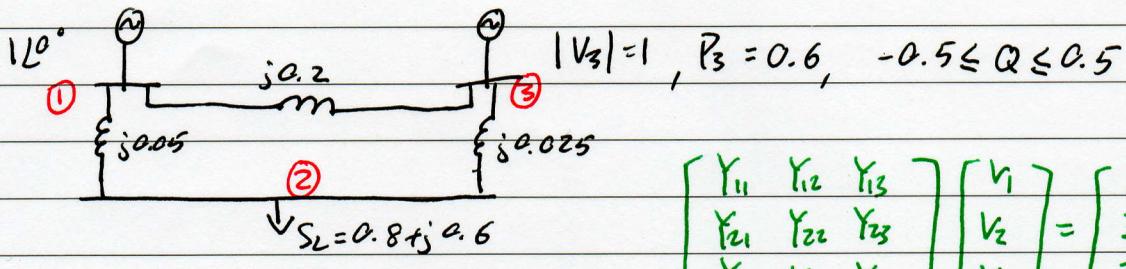


1)



$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

a) $\dot{Y}_{bus} = \begin{bmatrix} -j25 & j20 & j5 \\ j20 & j60 & j40 \\ j5 & j40 & j45 \end{bmatrix}$

$S = VI^* = P + jQ$

$$V_2 = \left[\left(\frac{S_2}{V_2} \right)^* - V_1 Y_{21} - V_3 Y_{23} \right] \frac{1}{Y_{22}}$$

$$V_3 = \left[\left(\frac{S_3}{V_3} \right)^* - V_1 Y_{31} - V_2 Y_{32} \right] \frac{1}{Y_{33}}$$

$$Q_3 = V_3 V_1 Y_{31} \sin(\delta_3 - \delta_1 - Y_{31}) + V_3 V_2 Y_{32} \sin(\delta_3 - \delta_2 - Y_{32}) + V_3 V_3 Y_{33} \sin(\delta_3 - \delta_3 - Y_{33})$$

Using G-S process : $V_2^0 = 1 \angle 0^\circ, \delta_3^0 = 0^\circ, Q_3^0 = 0$

$$V_2 = \left[\left(\frac{S_2}{V_2^k} \right)^* - V_1 Y_{21} - V_3 Y_{23} \right] \frac{1}{Y_{22}}$$

$$Q_3 = V_3 V_1 Y_{31} \sin(\delta_3^k - \delta_1^k - Y_{31}) + V_3 V_2 Y_{32} \sin(\delta_3^k - \delta_2^k - Y_{32}) + V_3 V_3 Y_{33} \sin(\delta_3^k - \delta_3^k - Y_{33})$$

$$V_3 = \left[\left(\frac{P + jQ}{V_3^k} \right)^* - V_1 Y_{31} - V_2 Y_{32} \right] \frac{1}{Y_{33}}$$

$$V_2' = \left[\left(\frac{-0.8 + j0.6}{1 \angle 0^\circ} \right)^* - 1 \angle 0^\circ \times j20 - 1 \angle 0^\circ \times j40 \right] \frac{1}{j60} = 0.99 \angle -0.772^\circ$$

$$Q_3' = 1 \times 1 \times 5 \sin(0 - 0 - 90^\circ) + 1 \times 0.99 \times 40 \sin(0 - (-0.772) - 90^\circ) + 1 \times 1 \times 45 \sin(0 - 0 + 90^\circ) \\ = 0.40359 \text{ (in range)}$$

$$V_3' = \left[\left(\frac{0.6 + j0.40359}{1 \angle 0^\circ} \right)^* - 1 \angle 0^\circ \times j5 - 0.99 \angle 0.772 \times j40 \right] \frac{1}{j45} = 1 \angle 0.0846^\circ$$

$$b) I_{13} = [V_1 - V_3] / j0.2 = 0.007383 \angle 180^\circ$$

$$I_{12} = [V_1 - V_2] / j0.05 = 0.3345 \angle 37.1^\circ$$

$$S_{13} = V_1 I_{13}^* = 0.007383 \angle 180^\circ \text{ } \times$$

$$S_{12} = V_1 I_{12}^* = 0.3345 \angle 37.1^\circ \text{ } \times$$

c) Since the network is pure inductive, sum of all real power supplied by the 2 generator must be equal to load real power.

$$P_{g1} = 0.007383 \cos 180^\circ + 0.3345 \cos 37.1^\circ \\ = 0.2594$$

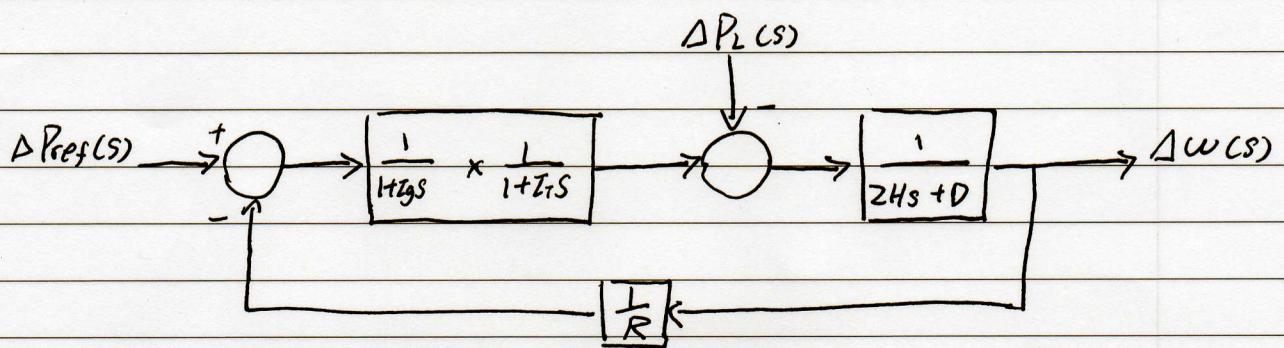
$$P_{g3} = 0.6$$

$$P_L = 0.8$$

$$P_{g1} + P_{g3} = 0.8594 \neq P_L = 0.8$$

∴ We can conclude that the first iteration has yet to reach the correct load flow solution. \times

2)



$$\text{a.i)} \quad \frac{\Delta w(s)}{\Delta P_L(s)} = \frac{-\frac{1}{2Hs+D}}{1 + \frac{1}{2Hs+D} \times \frac{1}{1+I_gs} \times \frac{1}{1+I_ts} \times \frac{1}{R}} = \frac{-(1+I_gs)(1+I_ts)}{(2Hs+D)(1+I_gs)(1+I_ts) + \frac{1}{R}}$$

Characteristic Equation: Let $\frac{1}{R} = k$

$$\begin{aligned}
 & (2Hs+D)(1+I_gs)(1+I_ts) + k \\
 \Rightarrow & (10s+1)(1+0.2s)(1+0.8s) + k \\
 \Rightarrow & (10s + 2s^2 + 1 + 0.2s)(1+0.8s) + k \\
 \Rightarrow & 10s + 8s^2 + 2s^2 + 1 + 0.8s + 0.2s + 0.16s^2 + k \\
 \Rightarrow & [1.6]s^3 + [10.16]s^2 + [11]s + [1+k]
 \end{aligned}$$

Using Routh Array,

s^3	1.6	11	
s^2	10.16	$1+k$	
s^1	$(1.6)(1+k) - (11 \times 10.16)$	0	$\Rightarrow \frac{(-1.6 - 1.6k + 111.76)}{+10.16}$
s^0	-10.16		
	1+k		

For stability, $(-1.6 - 1.6k + 111.76)$ must not be greater than 0

$$\therefore -1.6 - 1.6k + 111.76 > 0$$

$$\Rightarrow 68.85 > k$$

$$\text{Since } k = \frac{1}{R} \quad \therefore R > \frac{1}{68.85} = 0.01452 \text{ **}$$

ii) if $\Delta P_{CS} = \frac{0.1}{S}$, $\Delta w_{ss} < |0.01| \Rightarrow$ magnitude of drop.

$$\lim_{S \rightarrow 0} S \Delta w_{CS} = \lim_{S \rightarrow 0} \frac{-0.1 [1 + I_g S] [1 + I_T S] S}{(2H_S + D) [1 + I_g S] [1 + I_T S] + k} S$$

$$\Rightarrow \frac{0.1}{1+k} < 0.01 \Rightarrow \frac{0.1}{0.01} < 1+k \Rightarrow 9 < k \Rightarrow 9 < \frac{1}{R} \Rightarrow R < 0.111$$

\therefore the highest value of R is 0.111 ~~#~~

b) Loss, $ICR = \frac{\partial CR}{\partial P}$, $P_1 + P_2 + P_3 = P_D + P_{loss}$, $P_D = 500 \text{ MW}$

$$ICR_1 = 5.3 + 0.008P_1 \quad 200 \leq P_1 \leq 450$$

$$ICR_2 = 5.5 + 0.012P_2 \quad 120 \leq P_2 \leq 350$$

$$ICR_3 = 5.8 + 0.018P_3 \quad 90 \leq P_3 \leq 225$$

$$P_{loss} = 0.000228P_2^2$$

$$5.3 + 0.008P_1 = \lambda$$

$$5.5 + 0.012P_2 = \lambda \left(1 - \frac{\partial P_{loss}}{P_2}\right) = \lambda \left(1 - 0.000456P_2\right)$$

$$5.8 + 0.018P_3 = \lambda$$

Method 1: Assume $\lambda = 7.52$

$$5.3 + 0.008P_1 = 7.52 \Rightarrow P_1 = 277.5 \text{ MW}$$

$$5.5 + 0.012P_2 = 7.52 \left(1 - 0.000456P_2\right) \Rightarrow P_2 = 130.9 \text{ MW}$$

$$5.8 + 0.018P_3 = 7.52 \Rightarrow P_3 = 95.6 \text{ MW}$$

$$P_1 + P_2 + P_3 = P_D + P_{loss}$$

$$504 \text{ MW} = 503.9 \text{ MW}$$

Since $P_1 + P_2 + P_3 \approx P_D + P_{loss}$

$$\therefore \lambda \approx \$7.52 / \text{MW h} \text{ (shown)} \quad \#$$

Method 2: Iteration

$$5.3 + 0.008 P_1^{k+1} = \cancel{\lambda^k} - ①$$

$$5.5 + 0.012 P_2^{k+1} = \cancel{\lambda^k} (1 - 0.00456 P_2^k) - ②$$

$$5.8 + 0.018 P_3^{k+1} = \cancel{\lambda^k} - ③$$

$$P_1^{k+1} + P_2^{k+1} + P_3^{k+1} = P_D + P_{loss}^k - ④$$

From ①, $P_1^{k+1} = [\cancel{\lambda^k} - 5.3] / [0.008]$

From ②, $P_2^{k+1} = [\cancel{\lambda^k} - 0.00456 \cancel{\lambda^k} P_2^k - 5.5] / [0.012]$

From ③, $P_3^{k+1} = [\cancel{\lambda^k} - 5.8] / [0.018]$

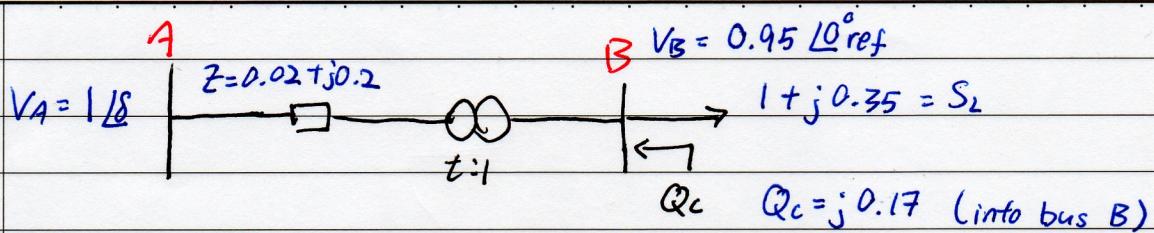
$$\therefore \text{From } ④, [\cancel{\lambda^k} - 5.3] / [0.008] + [\cancel{\lambda^k} - 0.00456 \cancel{\lambda^k} P_2^k - 5.5] / [0.012] + [\cancel{\lambda^k} - 5.8] / [0.018] = P_D + P_{loss}^k$$

Once λ^k is known, sub λ^k into eq. ①, ②, ③ to find $P_1^{k+1}, P_2^{k+1}, P_3^{k+1}$.

Once λ, P_1, P_2, P_3 & P_{loss} converge, their final value can be known.

k	P1	P2	P3	Ploss	λ
0	100.0	100.0	300.0	2.280	7.480
1	272.4	136.5	93.3	4.250	7.527
2	278.4	129.9	96.0	3.846	7.518
3	277.3	131.1	95.5	3.918	7.520
4	277.5	130.9	95.6	3.905	7.520
5	277.5	130.9	95.5	3.907	7.520
6	277.5	130.9	95.5	3.907	7.520

3)



$$S_B = 1000 \text{ MVA}$$

$$V_{B1} = 400 \text{ kV}$$

$$Z_{B1} = 160 \Omega$$

Base

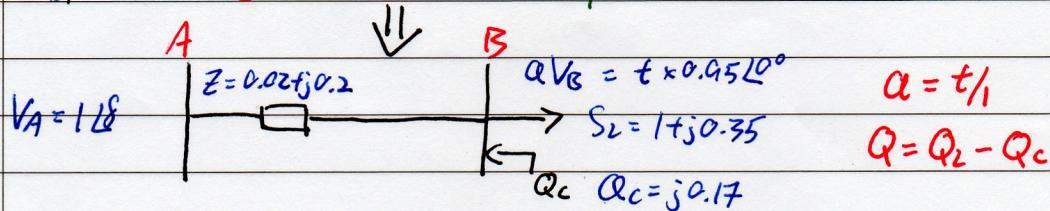
$|V_B| = 95\% \text{ of } |V_A| \text{ when } Q_c \text{ injected.}$

$$V_{\text{pri}} = \alpha V_{\text{sec}}$$

$$Z_{\text{pri}} = \alpha^2 Z_{\text{sec}}$$

, where α is $\frac{\text{Pri}}{\text{Sec}}$ turn ratio.

a)



$$\Delta V = |V_A| - |\alpha V_B| = \frac{PR + QX}{|\alpha V_B|} \Rightarrow 1 - t \cdot 0.95 = \frac{(1 \times 0.02) + [(0.35 - 0.17)(0.2)]}{t \cdot 0.95}$$

$$\Rightarrow t \cdot 0.95 - t^2 \cdot 0.9025 - 0.056 = 0$$

$$\Rightarrow t = 0.0626, 0.9899 \quad (\text{With } Q_c \text{ injected})$$

(reject)

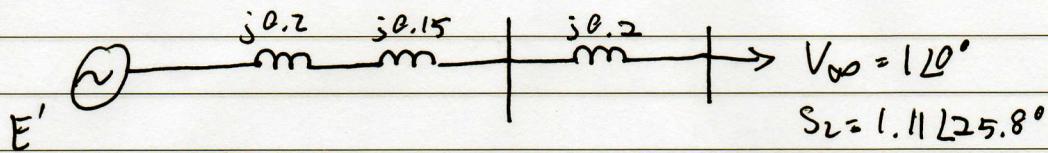
b) $|V_A| = 1, |V_B| = 1, t = 0.95$

$$\Delta V = |V_A| - |tV_B| = \frac{PR + X(Q_L - Q_C)}{|tV_B|}$$

$$\Rightarrow 1 - 0.95 = \frac{0.02 + 0.2(0.35 - Q_C)}{0.95}$$

$$\Rightarrow Q_C = 0.2125 \cancel{\text{A}}$$

4)

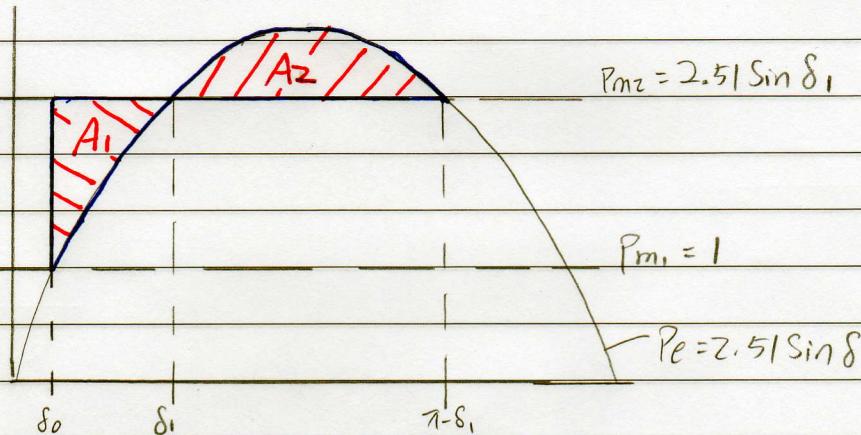


a) $I_L = (S_2/V_{oo})^* = 1.11 L 25.8^\circ$
 $Z_T = j0.2 + j0.15 + j0.20 = j0.55$

$$E/\delta = V_{oo} + I_L Z_T = 1.38 L 23.5^\circ$$

$$P_e = \frac{1.38 \times 1}{0.55} \sin \delta = 2.51 \sin \delta$$

b)



$$\delta_0 = 23.5^\circ = 0.4117 \text{ rad}, \quad A_1 = A_2$$

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m2} - P_e) d\delta = \int_{\delta_0}^{\delta_1} (2.51 \sin \delta_1 - 2.51 \sin \delta) d\delta$$

$$= \left[2.51 \delta_1 \sin \delta_1 + 2.51 \cos \delta_1 \right]_{\delta_0}^{\delta_1} = [2.51 \delta_1 \sin \delta_1 + 2.51 \cos \delta_1 - 2.51 \delta_0 \sin \delta_1 - 2.51 \cos \delta_0]$$

$$= 2.51 \delta_1 \sin \delta_1 + 2.51 \cos \delta_1 - 1.109 \sin \delta_1 - 2.269$$

$$A_2 = \int_{\delta_1}^{\pi - \delta_1} (P_e - P_{m2}) d\delta = \int_{\delta_1}^{\pi - \delta_1} (2.51 \sin \delta - 2.51 \sin \delta_1) d\delta$$

$$= \left[2.51 \cos \delta - 2.51 \sin \delta_1 \right]_{\delta_1}^{\pi - \delta_1}$$

$$= \left[-2.51 \cos(\pi - \delta_1) - (\pi - \delta_1) \times 2.51 \sin \delta_1 + 2.51 \cos \delta_1 + \delta_1 \cdot 2.51 \sin \delta_1 \right]$$

$$= 2.51 \cos \delta_1 - 7.885 \sin \delta_1 + 2.51 \delta_1 \sin \delta_1 + 2.51 \cos \delta_1 + 2.51 \delta_1 \sin \delta_1$$

$$\Rightarrow A_1 = A_2$$

$$-1.109 \sin \delta_1 - 2.269 = -7.885 \sin \delta_1 + 2.51 \delta_1 \sin \delta_1 + 2.51 \cos \delta_1$$

$$\Rightarrow 2.51 \cos \delta_1 = -2.269 + 6.776 \sin \delta_1 - 2.51 \delta_1 \sin \delta_1$$

$$\delta_1 = \cos^{-1} \left[\frac{6.776 \sin \delta_1 - 2.51 \delta_1 \sin \delta_1 - 2.269}{2.51} \right]$$

Using iteration, $\delta_1^0 = 45^\circ = 0.7854 \text{ rad}$ (initial assumption)

k ~~δ_1^{k+1}~~ δ_1^k (calculation in Radian)

0) 45°

1) 63.3°

2) 58.6°

3) 58.2°

4) 58.2°

$$\therefore \delta_1 = 58.2^\circ$$

$$P_{m2} = 2.51 \sin \delta_1 = 2.133$$

$$\Delta P = P_{m2} - P_{m1} = 1.133$$

\therefore The maximum permissible sudden increase of the generator power output for maintaining the system stability is 1.133 p.u.