Example 1:

The one-line diagram of a three-phase power system is shown in Figure 1. Impedances are marked in per unit on a 100 MVA, 400 KV base. The load at bus 2 is S_2 = 15.93 MW – j33.4 MVar, and at bus 3 is S_3 = 77 MW + j14 MVar. It is required to maintain the voltage at bus 3 at 400kV. Determine the voltages at buses 2 and 1.

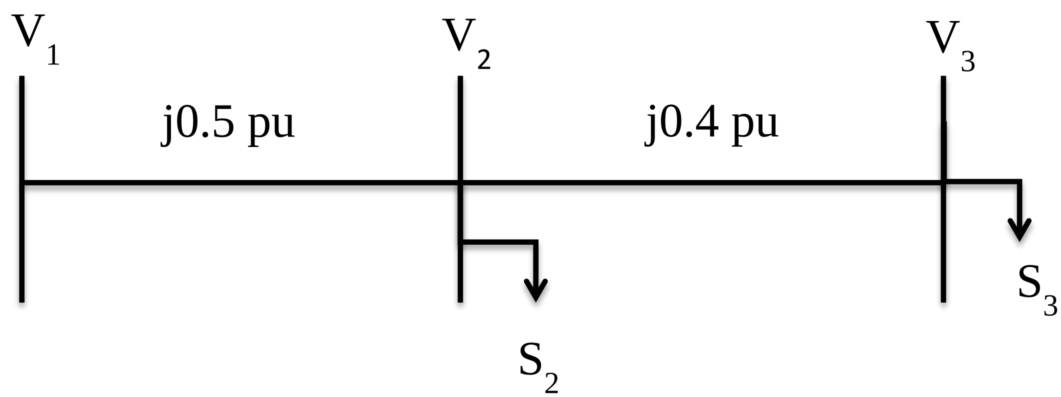

Figure 1 One-line diagram of Example 1

Solution 1: Using per unit to solve the problem

S_{base} = 100 MVA, V_{base} = 400 kV, $V_3 = 1\angle 0^o$ pu

$S_3 = 77 + j14$ MVA $= 78.26\angle 10.3^o$ MVA $= 0.7826\angle 10.3^o$ pu
$S_2 = 15.93 - j33.4$ MVA $= 37\angle -64.5^o$ MVA
$= 0.37\angle -64.5^o$ pu
$X_{23} = 0.4$ pu and $X_{12} = 0.5$ pu
$Q_3 = 0.14$ pu
The voltage at bus 2

$$V_2 = V_3 + \frac{X_{23}Q_3}{V_3} = 1 + \frac{0.4 \times 0.14}{1} = 1.056 \quad pu$$

The line voltage at bus 2
$V_2 = 422.4$ kV

The magnitude of the current at bus 3 is
$I_3 = S_3/V_3 = 0.7826$ pu

The reactive power loss during the power transmission from bus 2 to bus 3 is
$Q_{loss} = I_3^2 \times X_{23} = 0.7826^2 \times 0.4 = 0.245$ pu

The total reactive power at bus 2
Q_2 = local load + power transferred + reactive power loss
$= -0.334 + 0.14 + 0.245 = 0.051$ pu

The voltage at bus 1

$$V_1 = V_2 + \frac{X_{12} Q_2}{V_2}$$

$$= 1.056 + \frac{0.5 \times 0.051}{1.056} = 1.08 \quad pu$$

The line voltage at bus 1

V_1 = 432 kV

Solution 2: Using the actual values (phase values) to do calculation.

S_3 = 78.26 MVA

The phase reactive power and phase voltage at bus 3
Q_3 = 4.667 MVar and V_3 = 230.95 kV

Since Z_{base} = 1600 Ω, X_{23} = 640 Ω and X_{12} = 800 Ω

The voltage at bus 2

$$V_2 = V_3 + \frac{X_{23}Q_3}{V_3} = 230.95 + \frac{640 \times 4.667}{230.95}$$

$$= 243.88 \quad kV \qquad (phase)$$

The line voltage at bus 2
V_2 = 422.4 kV

The magnitude of the current at bus 3 is
$I_3 = S_3/(3V_3) = 78.26/(3 \times 230.95) = 112.95$ A

The reactive power loss during the power transmission from bus 2 to bus 3 is
$Q_{loss} = 3 I_3^2 \times X_{23} = 3 \times 112.95^2 \times 640 = 24.49$ MVar

The per phase reactive power at bus 2
Q_2 = local load + power transferred + reactive power loss
= (–33.4 + 14 + 24.49)/3 = 1.7 MVar

The voltage at bus 1

$$V_1 = V_2 + \frac{X_{12} Q_2}{V_2}$$

$$= 243.88 + \frac{800 \times 1.7}{243.88} = 249.45 \quad kV \qquad (phase)$$

The line voltage at bus 1

V_1 = 432 kV

Example 2:

The one-line diagram of a three-phase power system is shown in Figure 2. The transfer reactance is 20 percent on a base of 100 MVA, 23/115 kV and the line impedance is $Z = j66.125\,\Omega$. The load at bus 2 is S_2 = 184.8 MW + j6.6 MVar, and at bus 3 is S_3 = + j20 MVar. It is required to maintain the voltage at bus 3 at 115 kV. Determine the voltages at buses 2 and 1.

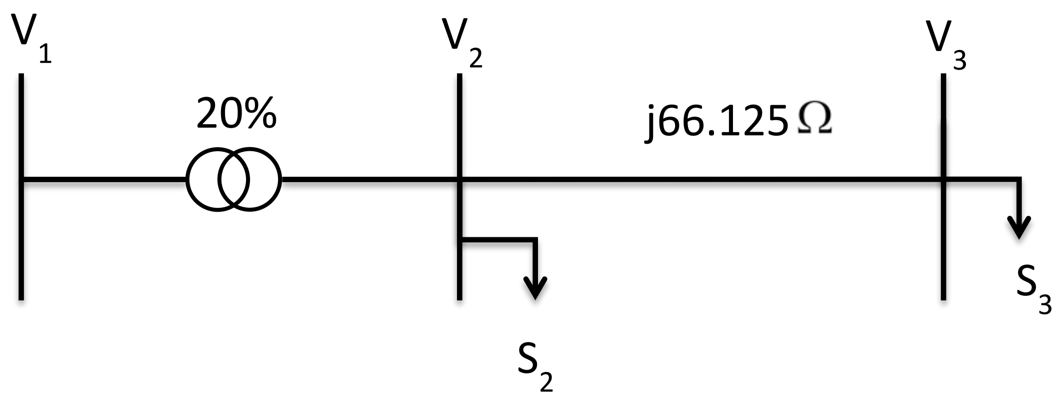

Figure 2 One-line diagram of Example 2

Solution 1: Using per unit to do calculation

S_{base} = 100 MVA, V_{base} = 115 kV, $V_3 = 1\angle 0^o$ pu

S_3 = j20 MVar = $0.2\angle 90^o$ pu

S_2 = 184.8 + j6.6 MVA = $184.92\angle 87.95^o$ MVA

= $1.8492\angle 87.95^o$ pu

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{115^2}{100} = 132.25 \quad \Omega$$

It follows that

$$X = \frac{X}{Z_{base}} = \frac{66.125}{132.25} = 0.5 \quad pu$$

X = 0.5 pu and X_T = 0.2 pu. Q_3 = j20 MVar = 0.2 pu

The voltage at bus 2

$$V_2 = V_3 + \frac{X\ Q_3}{V_3} = 1 + \frac{0.5 \times 0.2}{1} = 1.1 \quad pu$$

V_2 = 126.5 kV.

The magnitude of the current at bus 3 is
$I_3 = S_3/V_3 = 0.2$ pu

The reactive power loss during the power transmission from bus 2 to bus 3 is
$Q_{loss} = I_3^2 \times X_{23} = 0.2^2 \times 0.5 = 0.02$ pu

The total reactive power at bus 2
$Q_2 = 0.066 + 0.2 + 0.02 = 0.286$ pu

The voltage at bus 1

$$V_1 = V_2 + \frac{X_T Q_2}{V_2} = 1.1 + \frac{0.2 \times 0.286}{1.1} = 1.152 \quad pu$$

$V_1 = 26.5$ kV.

Solution 2: Using the actual values (phase values) to do calculation.

S_3 = j20 MVar

The phase reactive power and phase voltage at bus 3
Q_3 = 6.667 MVar and V_3 = 66.4 kV

Since Z_{base} = 132.25 Ω, X_{23} = 66.125 Ω and X_{12} = 26.45 Ω at 115 kV side (or X_{12} = 1.058 Ω at 23 kV side (Z_{base} = 5.29 Ω))

The voltage at bus 2

$$V_2 = V_3 + \frac{X_{23} Q_3}{V_3} = 66.4 + \frac{66.125 \times 6.667}{66.4}$$

$$= 73.04 \quad kV$$

The line voltage at bus 2
V_2 = 126.5 kV

The magnitude of the current at bus 3 is
$I_3 = S_3/(3V_3) = 20/(3 \times 66.4) = 100.4$ A

The reactive power loss during the power transmission from bus 2 to bus 3 is
$Q_{loss} = 3 I_3^2 \times X_{23} = 3 \times 100.4^2 \times 66.125 = 2$ MVar

The per phase reactive power at bus 2
Q_2 = local load + power transferred + reactive power loss
= (6.6 + 20 + 2)/3 = 9.533 MVar

The voltage at bus 1

$$V_1 = V_2 + \frac{X_{12}Q_2}{V_2}$$

$$= 73.04 + \frac{26.45 \times 9.533}{73.04} = 76.5 \quad kV$$

The line voltage at bus 1

$$V_1 = \frac{\sqrt{3} \times 76.5 \times 23}{115} = 26.5 \quad kV$$

Example 3:

A 11 kV line is fed through an 11kV/415V transformer from a constant 11 kV supply, as shown in Figure 3. The total impedance of the line and transformers at 11 kV is (1.2 + j 3.6) Ω. The transformer is equipped with tap-changing facility (t:1). If the load on the system is 1.08 MW and 0.81 MV_{AR}, determine the tap ratio of the transformer if the receiving-end voltage is to be maintained at 415 V. Use a base of 1 MVA, 11 kV.

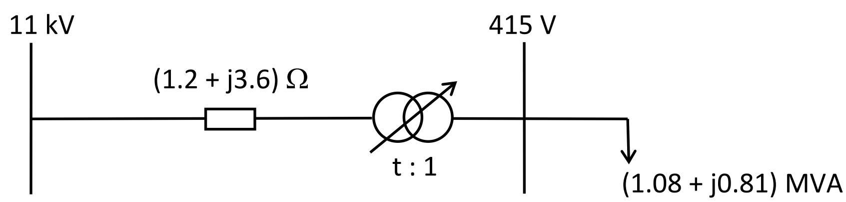

Figure 3The system in Example 3

Solution 1: Using per unit to do calculation

The bases: S_{base} = 1 MVA, V_{base} = 11 kV

It gives that P = 1.08 pu, Q = 0.81 pu and V_S = V_R = 1 pu

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{11^2}{1} = 121 \quad \Omega$$

It follows that

$$R = \frac{R}{Z_{base}} = \frac{1.2}{121} = 0.00992 \quad pu$$

$$X = \frac{X}{Z_{base}} = \frac{3.6}{121} = 0.02975 \quad pu$$

$$\Delta V = \frac{V_S}{t} - V_R = \frac{RP + XQ}{t^2 V_R}$$

It gives that

$$t^2 V_R^2 - tV_S V_R + (RP + XQ) = 0$$

$$t^2 - t + (0.00992 \times 1.08 + 0.02975 \times 0.81) = 0$$

$$\therefore \qquad t = \frac{1 + 0.9278}{2} = 0.964$$

The tap ratio of the transformer t = 0.964.

Solution 2: Using phase values or line values to do calculation

R = 1.2 Ω and X = 3.6 Ω
P = 1.08 MW (3-phase) = 0.36 MW (1-phase)
Q = 0.81 MV_{AR} (3-phase) = 0.27 MW (1-phase)
V_S = 11 kV (line) = 6.35 kV (phase)
V_R = 415 V (line) = 239.61 V (phase)
The turn ratio, *a*, is 11kV/415V

It follows that

$$\Delta V = \frac{V_S}{ta} - V_R = \frac{RP + XQ}{t^2 a^2 V_R}$$

Line values: V_R and V_S should be line values and P and Q are three phase powers.

$$t^2 a^2 V_R^2 - taV_S V_R + (RP + XQ) = 0$$

$$t^2 (11k/415)^2 \times 415^2 - t(11k/415)11k \times 415 + (1.2 \times 1.08 + 3.6 \times 0.81) = 0$$

$$t^2 - t + \frac{(1.2 \times 1.08 + 3.6 \times 0.81)}{(11k)^2} = 0$$

$$\therefore \qquad t = \frac{1 + 0.9278}{2} = 0.964$$

Phase values: V_R and V_S should be phase values and P and Q are single phase powers.

$$t^2a^2V_R^2 - taV_SV_R + (RP + XQ) = 0$$

$$t^2(\frac{11k}{415})^2 \times (239.607)^2 - t(\frac{11k}{415}) \times 6.35k \times 239.61$$
$$+ (1.2 \times 0.36 + 3.6 \times 0.27) = 0$$

$$t^2 - t + \frac{(1.2 \times 0.36 + 3.6 \times 0.27)}{(11k)^2 / (\sqrt{3})^2} = 0$$

$$\therefore \qquad t = \frac{1 + 0.9278}{2} = 0.964$$

The tap ratio of the transformer t = 0.964.