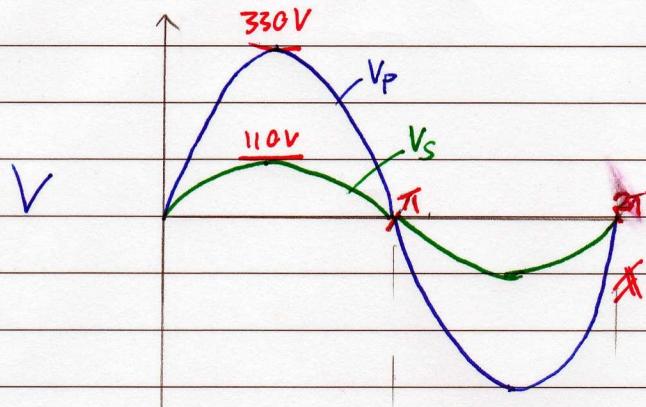


i)

a.i)



$$V_m \sin \alpha - E = 0$$

$$\Rightarrow \sin \alpha = \frac{E}{V_m}$$

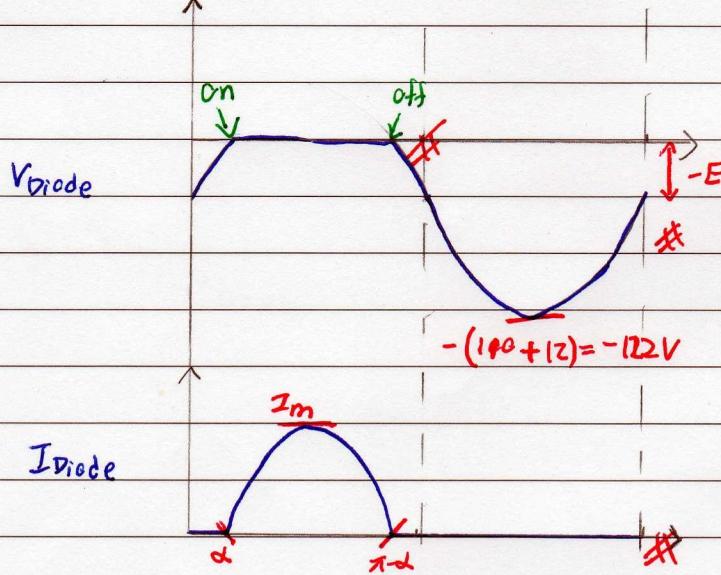
$$\Rightarrow \alpha = \sin^{-1} \left(\frac{E}{V_m} \right) = 6.263^\circ = 0.10931 \text{ rad}$$

$$V_{Ravg} = \frac{1}{2\pi} \int_{\alpha}^{\pi-\alpha} \left(\frac{V_m \sin \omega t - E}{\omega} \right) dt$$

$$= \frac{1}{2\pi \omega} \left[-V_m \cos \omega t - E t \right]_{\alpha}^{\pi-\alpha}$$

$$= 29.22 \text{ V}$$

ii)



$$I_{Ravg} = I_{oavg} = V_{Ravg} / R$$

$$\Rightarrow R = 7.306 \Omega$$

$$I_{m} = (V_m - E) / R$$

$$= 13.41 \text{ A}$$

iii) $PIV = 110 + 12 = 122 \text{ V}$

b.i) Assume $f = 50 \text{ Hz}$, $T = 0.02 \text{ s}$,

$$\therefore t_{on} = T \times (\pi - 2\alpha) / (2\pi) = 9.304 \text{ ms}$$

$$t_{off} = T - t_{on} = 10.695 \text{ ms}$$

ii) $P_{bat.} = I_{oavg} \times E = 48 \text{ W}$

iii) Charging time = Capacity / $P_{bat.} = 1.667 \text{ hr.}$

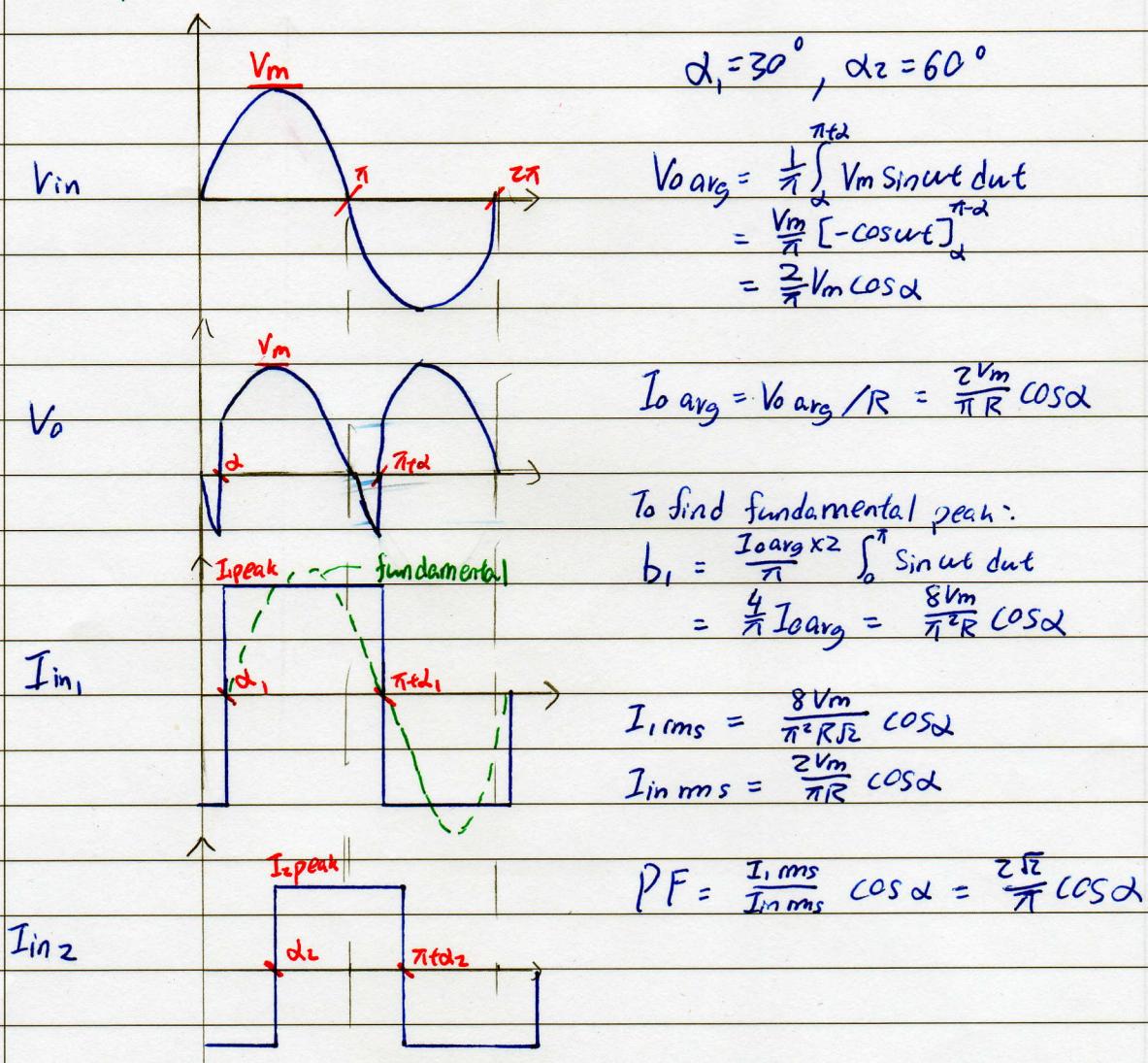
iv) $R = 7.306 \Omega$ to provide average current

$$\begin{aligned}
 V) I_{R\text{ rms}} &= \left[\frac{1}{2\pi R^2} \int_{-\alpha}^{\pi-\alpha} (V_m \sin \omega t - E)^2 dt \right]^{1/2} \\
 &= \left[\frac{1}{2\pi R^2} \int_{-\alpha}^{\pi-\alpha} [V_m^2 \sin^2 \omega t + E^2 - 2E V_m \sin \omega t] dt \right]^{1/2} \\
 &= \left[\frac{1}{2\pi R^2} \int_{-\alpha}^{\pi-\alpha} \left[\frac{V_m^2}{2} - \frac{V_m^2}{2} \cos 2\omega t - 2E V_m \sin \omega t + E^2 \right] dt \right]^{1/2} \\
 &= \left[\frac{1}{2\pi R^2} \left[\left[\frac{V_m^2}{2} + E^2 \right]_{-\alpha}^{\pi-\alpha} + [2E V_m \cos \omega t]_{-\alpha}^{\pi-\alpha} - \left[\frac{V_m^2}{4} \sin 2\omega t \right]_{-\alpha}^{\pi-\alpha} \right] \right]^{1/2} \\
 &= \left[\frac{1}{2\pi R^2} [18105 - 5248 + 1312] \right]^{1/2} \\
 &= 6.499 A
 \end{aligned}$$

$$\begin{aligned}
 n &= \frac{P_{bar}}{P_{bar} + I_{R\text{ rms}}^2 \times R} \times 100\% = \frac{48}{48 + 308.6} \times 100\% \\
 &= 16.03\% \cancel{13.46\%}
 \end{aligned}$$

2) Highly inductive $V_{in\ rms} = 230V, V_m = \sqrt{2} \times 230V$

a)



Input current is affected by varying α . Higher α will result in lower current as shown in the above derivation.

The PF is not affected by the current but is affected by α . As α increase, PF will decrease. $\cancel{\#}$

$$b) V_{0\arg} = 120V, V_m = \sqrt{2} \times 230V, R = 8\Omega$$

$$i) I_{0\arg} = V_{0\arg}/R = 15A$$

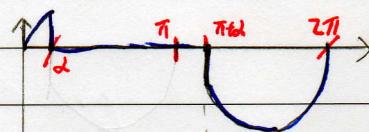
$$\frac{2V_m}{\pi R} \cos \alpha = I_{0\arg} = 15A \Rightarrow \alpha = 54.58^\circ \text{ } \cancel{x}$$

$$ii) V_{0\arg} = 120V \cancel{x}$$

$$I_{0\arg} = 15A \cancel{x}$$

$$I_{0\text{rms}} = 15A \cancel{x}$$

$$iii) V_{D1}, V_{D4}$$

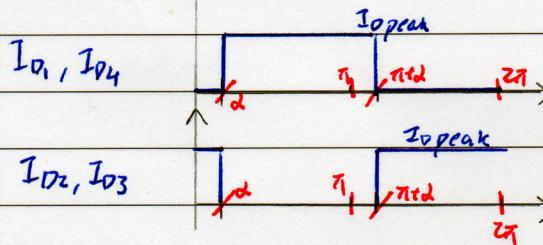
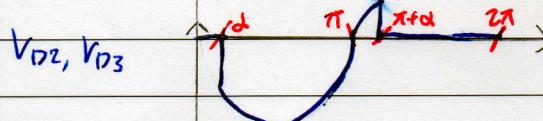


$$PIV = V_m = 325.3V$$

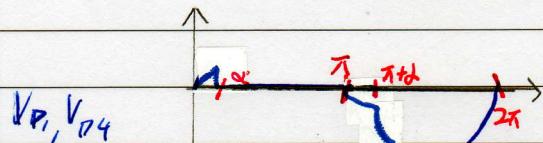
$$I_{D\text{peak}} = 15A$$

$$I_{D\arg} = 15/2 = 7.5A$$

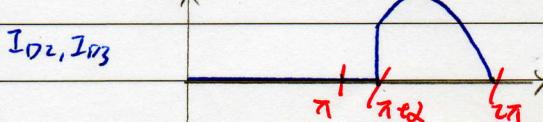
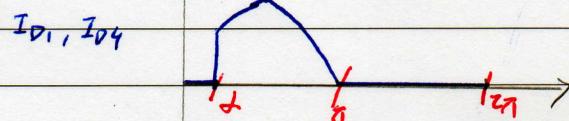
$$I_{D\text{rms}} = 15 \times \sqrt{\frac{1}{2}} = 10.61A$$



Under
inductive
load



Under resistive load



iv) Under inductive:

$$V_{in\ rms} = 230V$$

$$I_{in\ rms} = \frac{2V_m}{\pi R} \cos \alpha = 15A$$

$$\text{Transformer kVA} = V_{in\ rms} \times I_{in\ rms} = 3.450 \text{ kVA} \quad \text{X}$$

Under resistive:

$$\begin{aligned} I_{in\ rms} &= \left[\frac{1}{\pi} \int_0^{\pi} \left(\frac{V_m \sin \omega t}{R} \right)^2 d\omega t \right]^{1/2} = \left[\frac{V_m^2}{\pi R^2} \int_0^{\pi} \sin^2 \omega t d\omega t \right]^{1/2} \\ &= \left[\frac{V_m^2}{\pi R^2} \int_0^{\pi} \left[\frac{1}{2} - \frac{1}{2} \cos 2\omega t \right] d\omega t \right]^{1/2} \\ &= \left[\frac{V_m^2}{2\pi R^2} \left[\omega t - \frac{1}{2} \sin 2\omega t \right]_0^{\pi} \right]^{1/2} = 26.46A \end{aligned}$$

$$\text{Transformer kVA} = 26.46 \times 230 = 6.0858 \text{ kVA} \quad \text{X}$$

v) $\text{PF} = \frac{2\sqrt{2}}{\pi} \cos \alpha = 0.52179$ (inductive)

$$\begin{aligned} V_{rms} &= \left[\frac{1}{\pi} \int_0^{\pi} V_m^2 \sin^2 \omega t d\omega t \right]^{1/2} = \left[\frac{V_m^2}{2\pi} \int_0^{\pi} [1 - \cos 2\omega t] d\omega t \right]^{1/2} \\ &= \left[\frac{V_m^2}{2\pi} \left[\omega t - \frac{1}{2} \sin 2\omega t \right]_0^{\pi} \right]^{1/2} \\ &= 211.7V \end{aligned}$$

$$P_{o\ rms} = V_{rms}^2 / R = 5.601 \text{ kVA}$$

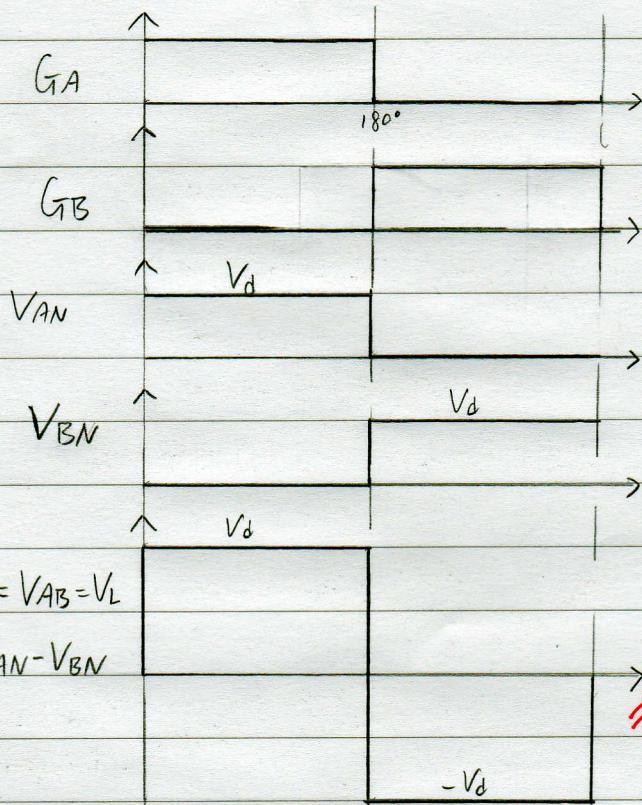
$$\begin{aligned} \text{PF} &= P_{o\ rms} / \text{transformer kVA} = \frac{5.601}{6.0858} \\ &= 0.92 \quad \text{(resistive)} \end{aligned}$$

\therefore Resistive load PF is higher than inductive load. X

FINISH STRONG!

May/June 13

3)



a)

$$V_o = V_{AB} = V_L \\ = V_{AN} - V_{BN}$$

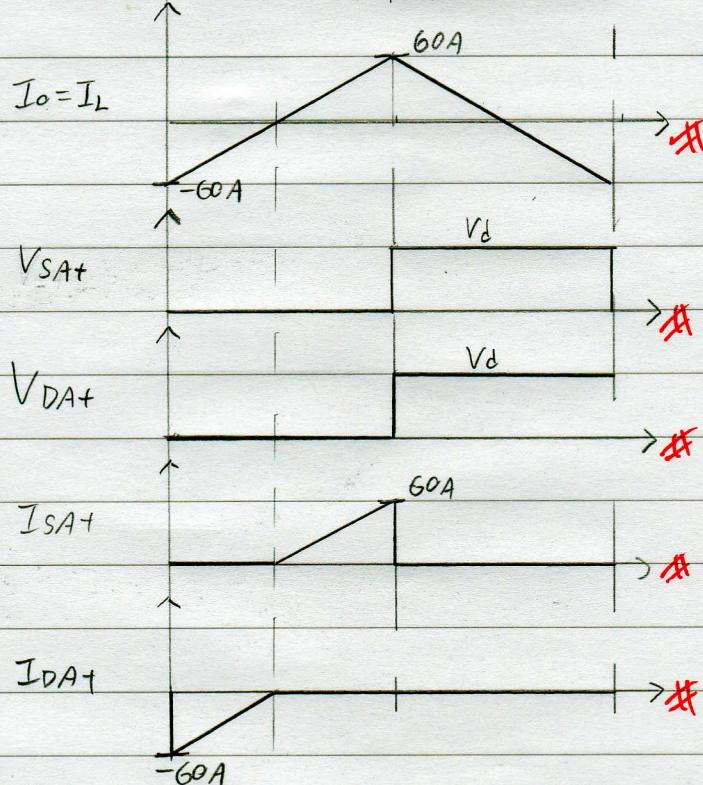
$$I_{Lpeak} = \frac{1}{2} \Delta I_L, L = 10mH$$

$$\Delta I_L = \frac{V_d}{L} t_{on}$$

$$= \frac{V_d}{L} \times \frac{T_s}{2}$$

$$= 120A$$

$$\therefore I_{Lpeak} = 120/2 = 60A$$



b) Voltage across $RL = V_o - V_{ac}$,

$$R = 1\Omega, L = 10 \text{ mH}, f = 50 \text{ Hz}$$

For fundamental, $n = 1$

$$b_1 = \frac{1}{\pi} \int_0^{2\pi} [V_o - V_{ac}] \sin(\omega t) dt$$

$$= \frac{1}{\pi} \int_0^{\pi} [V_o \sin \omega t - 50 \sin^2 \omega t] + \frac{1}{\pi} \int_{\pi}^{2\pi} [-V_o \sin \omega t - 50 \sin^2 \omega t] dt$$

$$= \frac{V_o}{\pi} [-\cos \omega t]_0^{\pi} - \frac{50}{2\pi} \int_0^{\pi} (1 - \cos 2\omega t) dt - \frac{V_o}{\pi} [-\cos \omega t]_{\pi}^{2\pi} - \frac{50}{2\pi} \int_{\pi}^{2\pi} (1 - \cos 2\omega t) dt$$

$$= \frac{2}{\pi} V_o + \frac{2}{\pi} V_o - \frac{50}{2\pi} [\pi] + \left[\frac{50}{4\pi} \sin 2\omega t \right]_0^{\pi} - \frac{50}{2\pi} [\pi] + \left[\frac{50}{4\pi} \sin 2\omega t \right]_{\pi}^{2\pi}$$

$$= \frac{4}{\pi} V_o - 50$$

$$\Rightarrow V_{RL(1)} = \left(\frac{4}{\pi} V_o - 50 \right) \sin \omega t \quad \text{Same as } V_{ac} - V_{ac}$$

$$\therefore V_{RL(1), rms} = \left[\frac{4}{\pi} V_o - 50 \right] / \sqrt{2} = 72.68 \text{ V}$$

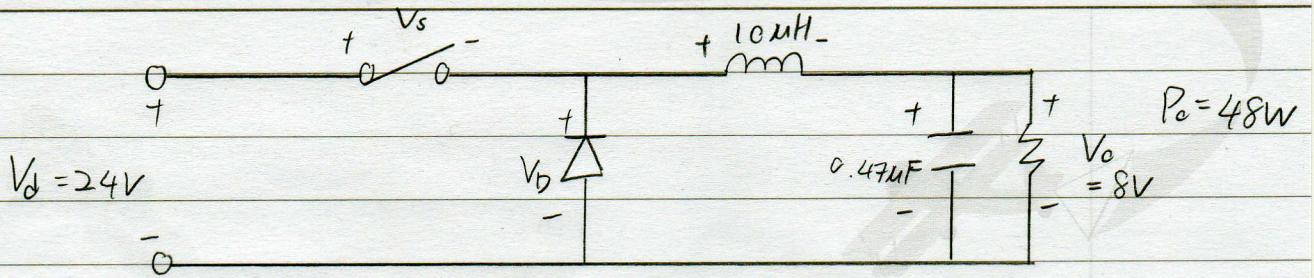
$$I_{o(1), rms} = V_{RL(1), rms} / Z = 72.68 / \left[\sqrt{R^2 + (2\pi f_1 L)^2} \right]$$

$$= 22.045 A$$

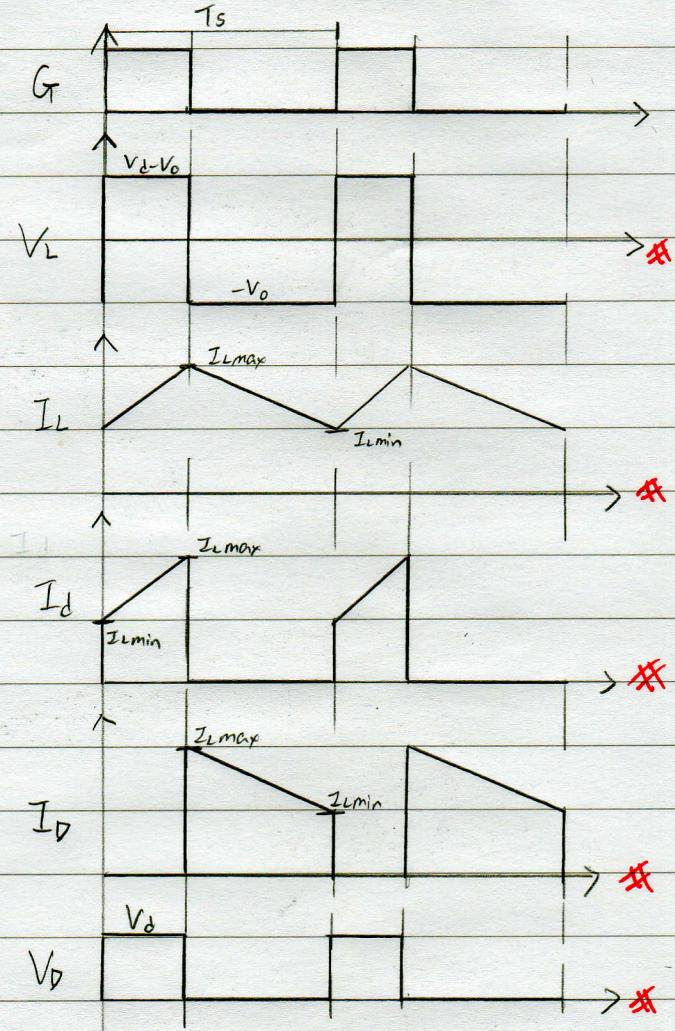
Average output power = 0W, since $V_o \text{ avg} = 0V$.

FINISH STRONG!

4)



a) $f_s = 50 \text{ kHz}$, $T_s = 0.02 \text{ ms}$, $D = \frac{8}{24} = \frac{1}{3}$



b) $\frac{V_o}{V_d} = D \Rightarrow D = \frac{8}{24} = \frac{1}{3} \text{ X}$

$$P_o = V_o I_o = V_d I_d \Rightarrow I_o = \frac{P_o}{V_o} = 6A \text{ X}$$

$$I_d = \frac{P_o}{V_d} = 2A \text{ X}$$

FINISH STRONG!

c) Find $I_{L\max}$ & $I_{L\min}$, $I_{L\max} = I_0 + \frac{1}{2} \Delta I_L$, $I_{L\min} = I_0 - \frac{1}{2} \Delta I_L$

$$\Delta I_L = \frac{V_d - V_o}{L} T_S = \frac{V_d - V_o}{L} D T_S = \frac{24 - 8}{10 \times 10^{-6}} \times \frac{1}{3} \times 0.02 \times 10^{-3} = 10.667 A$$

$$\therefore I_{L\max} = [10.667]/2 + 6 = 11.333 A \text{ } \cancel{\#}$$

$$I_{L\min} = 6 - [10.667]/2 = 0.6665 A \text{ } \cancel{\#}$$

Under BCM, $I_{oB} = I_{LB}$

$$I_{oB} = [\frac{1}{2} \times T_S \times \Delta I_L] / T_S = \frac{1}{2} \Delta I_L = 10.667 / 2 = 5.3335 A$$

$$I_{oB} = \frac{V_o}{R_B} \Rightarrow R_B = V_o / I_{oB} = 1.5 \cancel{A} - 2 \cancel{\Omega}$$

d) When $P_o = 5W$, $V_o = 8V$,

$$I_0 = \frac{P_o}{V_o} = 0.625 A$$

Since $I_0 < I_{oB}$, $\therefore \text{DCM}$ $\cancel{\#}$