Recitation Notes for PSTAT 170

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Sept, 2022

This note is based on the contents I have taught on the recitation class of PSTAT 170. The notes may be subject to typos, and you are welcome to provide you advice or ask questions at hzhou593@ucsb.edu.

Week 1

Mean Return and Volatility

For a given period of time, keep track of the stock price. The price at the beginning of the period is denoted P_s and the price at the end of the period is denoted P_e. The return rate during this period is defined as

$$r = \frac{P_e - P_s}{P_s} \tag{1}$$

Let's assume that we have kept track of the stock price for n periods, naturally, we would have $r_1, r_2, ..., r_n$ as the return rates for each period calculated. The **mean return** would be defined as

$$r_{mean} = \frac{\sum_{i=1}^{n} r_i}{n} \tag{2}$$

and the **volatility** would be defined as the standard deviation of the sequence $r_1, ..., r_n$.

$$\sigma = sd(r_1, ..., r_n) \tag{3}$$

Remark. *There's many kinds of averages and one can also apply the geometric average instead of the arithmetic average mentioned above, i.e.*

$$r_{mean} = \left(\prod_{i=1}^{n} (1 + r_i) \right)^{\frac{1}{n}} - 1 \tag{4}$$

The geometric average also has a natural interpretation as the accumulation of interest. If we have 1 dollar at the very beginning, after n periods the amount would accumulate to $\prod_{i=1}^{n}(1 + r_i)$. On the other hand, if r_{mean} is adopted for each period, the amount would accumulate to $(1 + r_{mean})^n$. The geometric mean return is the return rate such that these two amounts are the same.

Remark. *There's many selections for the time period of the return rate. For example, it can be on a daily basis, or on a yearly basis etc. In practice, we would not use something like "annualized daily mean return" since the error would be huge. One may refer to the two real datasets I have posted to observe that the close price on one trading day is generally not equal to the open price on the next trading day because of after-hour tradings. That's why in practice we typically choose the time period to be 3 months, 1 year etc.*

In this course, you should be more likely to get in touch with the return rate on a yearly basis and the arithmetic mean return.

Remark. *You can run the program* ***stock.py*** *to see some of the plots and calculations I have made (it's written in Python). Note that there should be two datasets, one called* ***AMC.csv****, the other called* ***GS.csv****. You can change the file name in the ninth line of the code, i.e.* ***price = pd.read_csv('AMC.csv')*** *to run it on different datasets.*

You should observe that AMC's stock price has much more fluctuation and results in a much higher volatility. Also note the difference from daily mean return and yearly mean return. (The daily mean return here is not annualized, you have to annualize it in order to compare with the yearly mean return)

Stock Indices

The stock indices tells you what is happening on the whole stock market. The two main stock indices of consideration would be the DJIA (Dow Jones Industrial Average) and the SP500 (Standard & Poor's 500). The main difference in these two constructions is that DJIA is **dollar-weighted** but SP500 is **market capitalization weighted**.

The construction of DJIA only depends on the stock price of the component companies (30 large companies selected including Apple, Microsoft, Goldman Sachs etc.):

$$DJIA = \frac{\sum_i P_i}{Dow\ Index} \tag{5}$$

where P_i stands for the stock price of a component company and the Dow index is a fixed constant (currently 0.152 approximately). As a result, if the stock price of a component company rise 1, then $DJIA$ is going to rise $\frac{1}{0.152} = 6.59$ points.

The construction of SP500, however, takes market capitalization into consideration.

$$SP500 = \frac{\sum_i P_i Q_i}{Divisor} \tag{6}$$

where P_i stands for the stock price of a company and Q_i stands for the number of shares publicly available of a company and the divisor is a fixed constant.

One fact to notice is that we can use market captalization weights to simplify our calculations. Since the market capitalization weights are proportional to Q_i (actually the market capitalization weights are formed as $\frac{Q_i}{\sum_j Q_j} \propto Q_i$), if we know that a company has market capitalization weight w_i and its stock price increases by α, then the SP 500 index should increase by $w_i\alpha$ (the percentage of increase). By doing so, it's possible to compute the SP 500 index without knowing the value of Q_i and the value of the divisor.

Example

Let's use an example to illustrate these points (provided by Professor Michael). Now Microsoft is having market capitalization weight 5.72% and Goldman Sachs is having market capitalization weight 0.33%, and the SP 500 index now is 3655.

It's known that the stock price of Microsoft is changing from 237 to 239 with the stock prices of all the other companies fixed. Since Microsoft is one of the DJIA component companies, the DJIA will rise $2 \times 6.59 = 13.18$ points.

As stated above, since the stock price is rising by $\frac{2}{237}$, the SP 500 index will increase by $5.72\% \times \frac{2}{237} = 0.048\%$, thus resulting the SP 500 index to increase $3655 \times 0.048\% = 1.76$ points.

It's known that the stock price of Goldman Sachs is changing from 294 to 296 with the stock prices of all the other companies fixed. Since Goldman Sachs is one of the DJIA component companies, the DJIA will also rise $2 \times 6.59 = 13.18$ points. As stated above, since the stock price is rising by $\frac{2}{294}$, the SP 500 index will increase $0.33\% \times \frac{2}{294} \times 3655 = 0.08$ points.

As we can see, although the DJIA index is having the same amount of change, the stock price of Microsoft has a much larger impact on SP 500 index than Goldman Sachs. Actually, Microsoft has about 1.77 trillion dollars market capitalization and Goldman Sachs only has about 100 billion dollars market capitalization. It also tells us that the stock price does not necessarily reflect the value of the company. In this case, Goldman Sachs is having a higher stock price but Microsoft is a more valuable company.