

Zoom polls

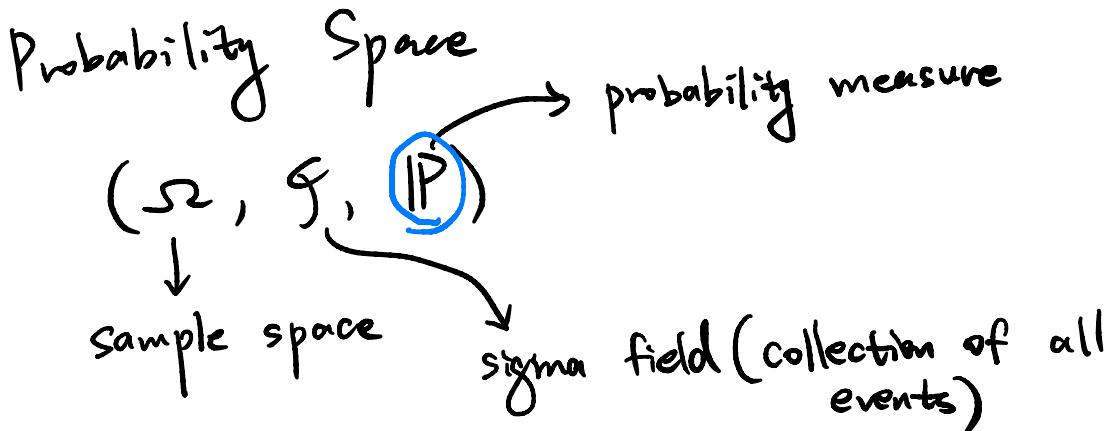
Haosheng

hzhou593@ucsb.edu

Tuesday 7:00-9:00 P.M.

160A Discrete-time stochastic process

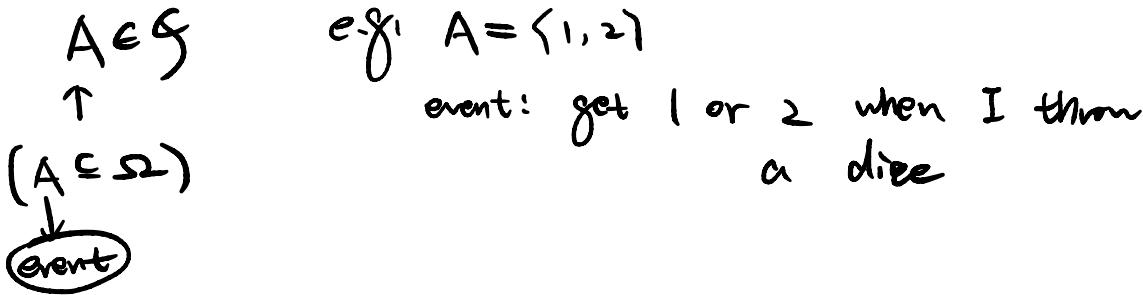
120A - basics in prob & stats



$\omega \in \Omega$

sample points

throw dice $\Omega = \{1, 2, 3, 4, 5, 6\}$



\mathcal{F} properties:

$$P(\Omega) = 1$$

①: $\emptyset \in \mathcal{F}, \Omega \in \mathcal{F}$

②: $\forall A \in \mathcal{F}, A^c \in \mathcal{F}$ (closed under complement)
 $P(A) + P(A^c) = 1$

③: $\forall A_1, A_2, \dots \in \mathcal{F},$

domain of P $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$ (closed under countable union)

$P: \mathcal{F} \rightarrow [0, 1]$
 $A \mapsto P(A)$

\mathcal{F} : closed under complement,
Intersection, Union

$$A \cap B = (A^c \cup B^c)^c$$

$$\text{e.g.: } \Omega = \{1, 2, 3\}$$

$$\mathcal{F} = \left\{ \emptyset, \{1, 2, 3\}, \underline{\underline{\{1\}}} \right\}$$

not closed
under complement

$$\{1\}^c = \{2, 3\} \notin \mathcal{F}$$

X sigma-field

$$\mathcal{F} = \left\{ \emptyset, \{1, 2, 3\}, \underline{\underline{\{1\}}}, \{2\}, \{1, 2\}, \{3\} \right\}$$

$$\{1\}^c = \underline{\underline{\{2, 3\}}} \notin \mathcal{F} \quad X \text{ sigma-field.}$$

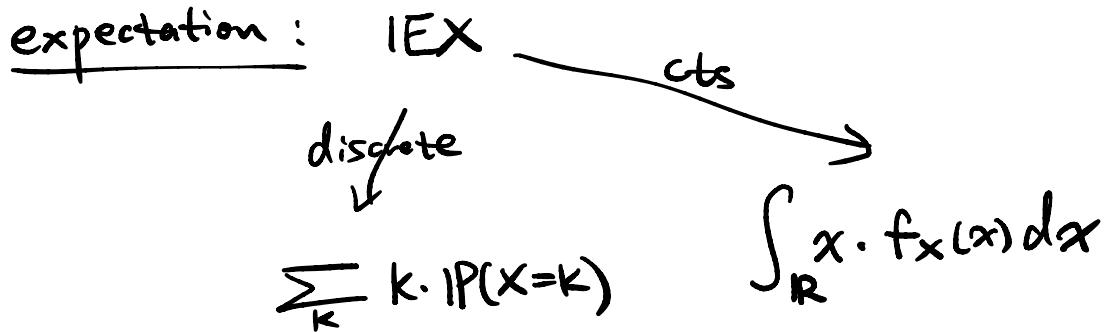
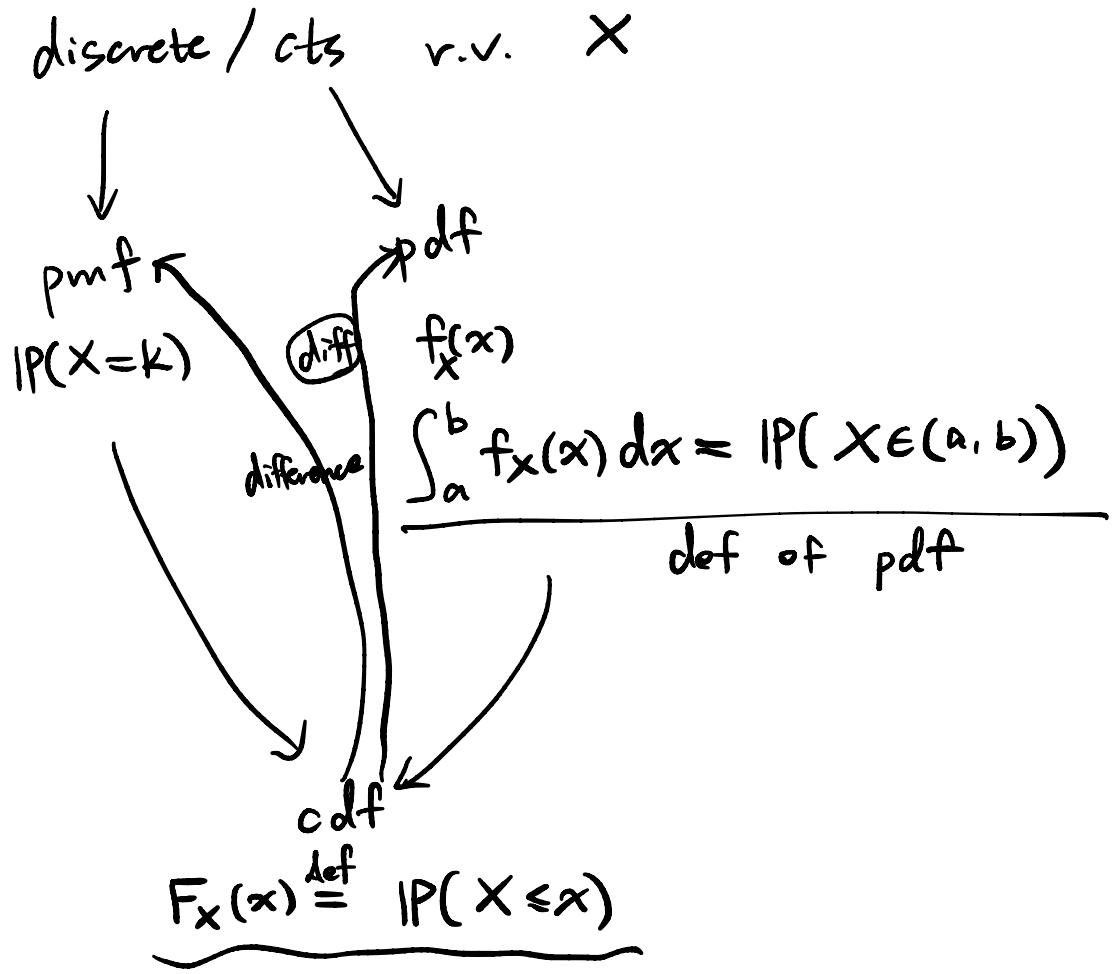
$$\mathcal{F} = \left\{ \emptyset, \{1, 2, 3\}, \{1\}, \{2\}, \{1, 2\}, \{3\}, \{2, 3\}, \{1, 3\} \right\}$$

sigma-field

= power set of Ω

$$\mathcal{F} = \left\{ \emptyset, \{1, 2, 3\}, \{1\}, \{2, 3\} \right\} \quad \checkmark$$

sigma-field



$$\text{Var } X = \mathbb{E} X^2 - (\mathbb{E} X)^2$$

e.g.: X cts r.v. pdf $f_X(t) = \begin{cases} C \cdot (4t - 2t^2) & 0 < t < 2 \\ 0 & \text{else} \end{cases}$
 $(C > 0 \text{ constant})$

(a): Find value of C

$$\int_0^2 f_X(t) dt = 1$$

$$C \cdot \int_0^2 4t - 2t^2 dt = 1$$

$$C \cdot \left(2t^2 - \frac{2}{3}t^3 \Big|_{t=0}^2 \right) = 1 \Rightarrow C = \frac{3}{8}.$$

(b): Find cdf of X

$$F_X(x) = \underline{\mathbb{P}(X \leq x)} = \begin{cases} 0 & x < 0 \\ \frac{3}{4}x^2 - \frac{1}{4}x^3 & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

$0 \leq X \leq x$

$$\underline{\int_0^x f_X(t) dt} = C \int_0^x (4t - 2t^2) dt = \underline{\frac{3}{4}x^2 - \frac{1}{4}x^3}$$

$$(c): \mathbb{P}(X=1) = 0$$

$\underset{\downarrow}{X \text{ cts r.v.}}$

$$\mathbb{P}(\underbrace{X > 1}_{X \in (1, 2)}) = \int_1^2 f_X(t) dt$$

$$(d): \text{Var}(X) = \underbrace{\mathbb{E}X^2}_{\text{ }} - (\underbrace{\mathbb{E}X}_{\text{ }})^2$$

$$\left\{ \begin{array}{l} \mathbb{E}X = \int_0^2 t \cdot f_X(t) dt \\ \mathbb{E}X^2 = \int_0^2 t^2 \cdot f_X(t) dt \end{array} \right.$$

Recall: $\mathbb{E} g(x) = \int_{\mathbb{R}} g(x) \cdot f_X(x) dx$

Independence

Def.: A, B are events, they are independent

iff $P(AB) = P(A) \cdot P(B)$
 $\underset{A \cap B}{\approx}$

A, B are { disjoint if $A \cap B = \emptyset$
mutually exclusive

e.g.: If we have A, B exclusive & independent

$$\{ A \cap B = \emptyset \Rightarrow P(AB) = 0$$

$$P(AB) = P(A) \cdot P(B) \rightarrow$$

$$P(A) \cdot P(B) = 0 \Rightarrow \begin{cases} P(A) = 0 \\ \text{or} \\ P(B) = 0 \end{cases}$$

e.g.: $X \sim U(0, 1)$

$$A : \{X = \frac{1}{2}\} \quad B : \{X = \frac{1}{3}\}$$

A, B disj, $P(A) = 0$, $P(B) = 0$

$P(AB) = 0 = P(A) \cdot P(B)$, independent ✓

Conditional Probability:

$$P(A|B) = \frac{P(AB)}{P(B)}$$

↑ ↑
event condition

If A, B independent \rightarrow (throw away condition)

$$P(AB) = P(A) \cdot P(B)$$

$$\frac{P(AB)}{P(A)} = P(B) \quad (\text{assume } P(A) \neq 0)$$

||

$$P(B|A)$$