

2.2.  $X \sim P(1)$ ,  $Y \sim G(\frac{1}{2})$

$$(a): \text{IP}(X+Y \geq 9) \leq \frac{\text{IE}(X+Y)}{9} = \frac{\text{IE}X + \text{IE}Y}{9}$$

$$\text{IE}X = 1, \quad \text{IE}Y = \frac{1}{\frac{1}{2}} = 2, \quad \text{IE}(X+Y) = 1+2=3$$

$$(b): \text{IP}(X+Y \geq 9) = \text{IP}\left((X+Y) - \text{IE}(X+Y) \geq 6\right)$$

$$\text{IP}(|X - \text{IE}X| \geq a) \leq \frac{\text{Var}X}{a^2}$$

$$\leq \text{IP}\left(|(X+Y) - \text{IE}(X+Y)| \geq 6\right)$$

$$\leq \frac{\text{Var}(X+Y)}{6^2} \stackrel{\text{indep}}{=} \frac{\text{Var}(X) + \text{Var}(Y)}{6^2}$$

2.5:  $X_1, X_2, \dots$  i.i.d.  $\sim P(\lambda)$

$$S_n = X_1 + \dots + X_n$$

$$\text{IP}\left(\frac{S_n}{n} \leq t\right)$$

$$\mathbb{E} S_n = n \cdot \mathbb{E} X_1 = n\lambda, \quad \text{Var}(S_n) = n \cdot \text{Var} X_1 = n\lambda$$

By CLT,

$$\frac{S_n - n\lambda}{\sqrt{n\lambda}} \xrightarrow{d} N(0, 1) \quad (n \rightarrow \infty)$$

$$\text{IP}\left(\frac{S_n}{n} \leq t\right) = \text{IP}(S_n \leq nt) = \text{IP}\left(\frac{S_n - n\lambda}{\sqrt{n\lambda}} \leq \frac{nt - n\lambda}{\sqrt{n\lambda}}\right)$$

$$= \text{IP}\left(\underbrace{\frac{S_n - n\lambda}{\sqrt{n\lambda}}}_{\xrightarrow{d} N(0, 1)} \leq \sqrt{n} \cdot \frac{t - \lambda}{\sqrt{\lambda}}\right)$$

$$= \text{IP}\left(\frac{S_n - n\lambda}{\sqrt{n\lambda}} \cdot \frac{1}{\sqrt{n}} \leq \frac{t - \lambda}{\sqrt{\lambda}}\right) \rightarrow 0 \quad (n \rightarrow \infty)$$

Since  $\frac{S_n - n\lambda}{\sqrt{n\lambda}} \xrightarrow{d} N(0, 1) \quad (n \rightarrow \infty), \quad \frac{1}{\sqrt{n}} \rightarrow 0$

we know  $\frac{S_n - n\lambda}{\sqrt{n\lambda}} \cdot \frac{1}{\sqrt{n}} \xrightarrow{d} 0 \quad (n \rightarrow \infty)$

$$\text{so: } \lim_{n \rightarrow \infty} \text{IP}\left(\frac{S_n}{n} \leq t\right) = \text{IP}\left(0 \leq \frac{t - \lambda}{\sqrt{\lambda}}\right) = \begin{cases} 0 & \lambda > t \\ 1 & \lambda < t \end{cases}$$

$$\underline{\underline{2.3}}: X_1, \dots, X_{997000} \stackrel{i.i.d.}{\sim} B(1, \frac{1}{2})$$

$$S_{997000} = X_1 + \dots + X_{997000}$$

$$P_A = \text{IP}(S_{997000} > 497000)$$

CLT:

$$\text{IE } S_{997000} = 997000 \times \text{IE } X_1 = \frac{997000}{2} = \mu$$

$$\text{Var } S_{997000} = 997000 \times \text{Var } X_1 = 997000 \times \frac{1}{2} \times \frac{1}{2} = 6^2$$

$$\frac{S_{997000} - \text{IE } S_{997000}}{\sqrt{\text{Var } S_{997000}}} \xrightarrow{d} N(0, 1) \quad (n \rightarrow \infty)$$

$$P_A = \text{IP}(S_{997000} > 497000) = \text{IP}\left(\frac{S_{997000} - \mu}{6} > \frac{497000 - \mu}{6}\right)$$

$\underbrace{\qquad}_{\xrightarrow{d} N(0, 1)}$

$$\approx \text{IP}(N(0, 1) > \frac{497000 - \mu}{6})$$

$$= 1 - \text{E}\left(\frac{497000 - \mu}{6}\right)$$

SLLN:

$$\frac{S_n}{n} \xrightarrow{\text{a.s.}} \frac{1}{2} \quad (n \rightarrow \infty)$$

$$\frac{S_n}{n} \xrightarrow{\text{d}} \frac{1}{2} \quad (n \rightarrow \infty)$$

$$\Pr(S_{997000} > 497000) = \Pr\left(\frac{S_{997000}}{997000} > \frac{497000}{997000}\right)$$

close to  $\frac{1}{2}$

$$\approx \Pr\left(\frac{1}{2} > \frac{497000}{997000}\right)$$

(either 0 or 1)

$X_1, \dots, X_{26} \sim \mathcal{P}(5)$  i.i.d.

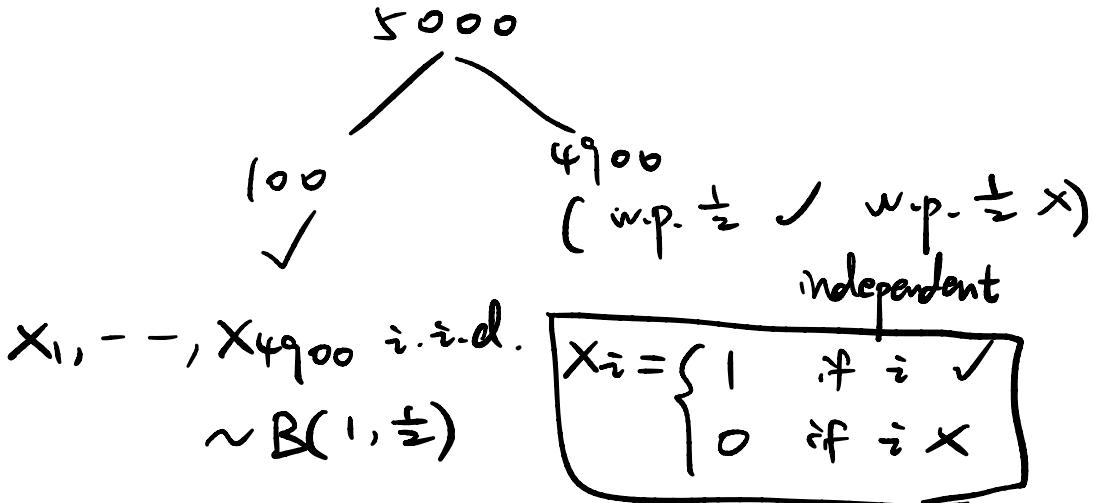
$$S_{26} = X_1 + \dots + X_{26}$$

$$\text{IP}(S_{26} < 120) \quad (\text{CLT}).$$

$$\left\{ \begin{array}{l} \text{IE } S_{26} = 26 \times \text{IE } X_1 = 26 \times 5 \\ \text{Var } S_{26} = 26 \times \text{Var } X_1 = 26 \times 5 \end{array} \right.$$

CLT:  $\frac{S_{26} - 26 \times 5}{\sqrt{26 \times 5}} \xrightarrow{d} N(0, 1)$

$$\begin{aligned} \text{IP}(S_{26} < 120) &= \text{IP}\left(\frac{S_{26} - 26 \times 5}{\sqrt{26 \times 5}} < \frac{120 - 26 \times 5}{\sqrt{26 \times 5}}\right) \\ &= \underline{\Phi}\left(\frac{120 - 26 \times 5}{\sqrt{26 \times 5}}\right) = \underline{\Phi}(-0.88) \end{aligned}$$



$S_{4900} = X_1 + \dots + X_{4900}$  is the num of votes supporting policy

$$P(S_{4900} + 100 > 2500) = P(S_{4900} > 2400)$$

$$\left\{ \begin{array}{l} E S_{4900} = 4900 \times \frac{1}{2} \\ \text{Var } S_{4900} = 4900 \times \text{Var } X_1 = 4900 \times \frac{1}{2} \times \frac{1}{2} \\ \approx N(0, 1) \end{array} \right.$$

$$P\left(\frac{S_{4900} - 4900 \times \frac{1}{2}}{\sqrt{4900 \times \frac{1}{2} \times \frac{1}{2}}} > \frac{2400 - 4900 \times \frac{1}{2}}{\sqrt{4900 \times \frac{1}{2} \times \frac{1}{2}}}\right)$$

$$\approx 1 - \Phi\left(\frac{2400 - 4900 \times \frac{1}{2}}{\sqrt{4900 \times \frac{1}{2} \times \frac{1}{2}}}\right) = \Phi(1.43)$$

$$P(K_1 \leq S_n \leq K_2), \quad S_n = X_1 + \dots + X_{500}$$

$$E S_n = 500 \times E X_1$$

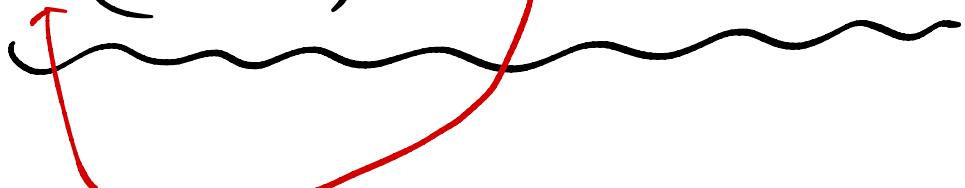
$$\text{Var } S_n = 500 \times \text{Var } X_1$$

part 1

$$\frac{S_n - E S_n}{\sqrt{\text{Var } S_n}} \stackrel{d}{\sim} N(0, 1)$$

$$P\left(\frac{K_1 - E S_n}{\sqrt{\text{Var } S_n}} \leq \frac{S_n - E S_n}{\sqrt{\text{Var } S_n}} \leq \frac{K_2 - E S_n}{\sqrt{\text{Var } S_n}}\right)$$

$$\approx \Phi\left(\frac{K_2 - E S_n}{\sqrt{\text{Var } S_n}}\right) - \Phi\left(\frac{K_1 - E S_n}{\sqrt{\text{Var } S_n}}\right)$$



norm. cdf