

Problem Set 11

Due: 31 July 2021, 4.00 pm

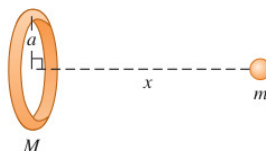
Problem 1. A thin, uniform rod has length L and mass M . Calculate the magnitude of the gravitational force the rod exerts on a particle with mass m that is placed at a point along the axis of the rod a distance x from one end. Discuss the limit $x \gg L$?

(2 points)

Problem 2. Consider a ring-shaped body and a particle with mass m placed a distance x from the center of the ring, along the line through the center of the ring and perpendicular to its plane (see figure below).

- Calculate the gravitational potential energy U of this system. Take the potential energy to be zero when the two objects are far apart.
- Show that your answer to part (a) reduces to the expected result when $x \gg a$.
- Find the magnitude and direction of the force on the particle, and show that your answer reduces to the expected result when $x \gg a$.
- What is the potential energy and the force when $x = 0$?

(2 + 1/2 + 2 + 1/2 points)



Problem 3. Suppose you could drill a tunnel through a uniform planet with radius R and mass M , so that it passes through the planet's center (*i.e.* it is drilled along a diameter). Find the equation of motion of an object of mass m that is dropped into the tunnel (without initial velocity) and show that it describes a harmonic oscillator. Find the period of the oscillations and compare it with a period of a satellite orbiting around the planet close to its surface.

Will the answer change if the tunnel is drilled at an angle to the diameter, so that it does not pass through the center? Explain.

Hint. You may use all the results, derived in the lecture, for the gravitational field due to a spherically symmetric mass distribution.

(5 points)