# PHYSICS I Problem Set 6

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## Problem 1 **Solution**

(a) Assume $m = \lambda l$, therefore, for any tiny enough piece of rope, we have

$$W = \int (\mathrm{d}m)gl = \int \lambda gl\mathrm{d} = \frac{1}{2}\lambda gl^2 \tag{1}$$

where $m = \lambda l$. Then we know

$$W = \frac{1}{2}mgl \tag{2}$$

(b) We first calculate the value of $\lambda_0$

$$m = \int \lambda(x)\mathrm{d}x = \lambda_0 \int_0^l (-x^2 + lx)\mathrm{d}x = \frac{1}{6}\lambda_0 l^3 \tag{3}$$

Therefore

$$\lambda_0 = \frac{6m}{l^3} \tag{4}$$

We then try to find the minimium work

$$W = \int (\mathrm{d}m)gl = \int \lambda_0 x(l-x)\mathrm{d}xgx = \frac{1}{12}\lambda_0 gl^4 \tag{5}$$

where equation(4) survives. Then we know

$$W = \frac{1}{2}mgl \tag{6}$$

## Problem 2 **Solution**

We will aplly energy conservation law.

$$W = W_{\text{mg}} - W_{\text{float}} \tag{7}$$

We first try to determine the how deep the cone submerged into the liquid. Suppose the depth is $\alpha H$. Then

$$\frac{1}{3}\pi(\alpha R)^2\alpha H = \frac{2}{3} \times \frac{1}{3}\pi R^2 H \tag{8}$$

$$\Rightarrow \ \alpha = \sqrt[3]{\frac{2}{3}} \tag{9}$$

We then calculate the work done by buoyant force.

$$W_{\text{float}} = \int_0^{\sqrt[3]{\frac{1}{3}}H} \rho V_{\text{sub}} g \mathrm{d}x \tag{10}$$

$$= \int_0^{\sqrt[3]{\frac{1}{3}}H} \rho g \left(\frac{1}{3}\pi\left(\frac{x}{H}R\right)^2 x\right) \mathrm{d}x \tag{11}$$

$$= \frac{\sqrt[3]{\frac{1}{3}}}{36}\rho g \pi R^2 H^2 \tag{12}$$

We now can calculate the work we needed

$$W = mg\sqrt[3]{\frac{1}{3}}H - W_{\text{float}} \tag{13}$$

where

$$mg = \frac{1}{3} \times \frac{1}{3}\rho\pi R^2 H g \tag{14}$$

Therefore

$$W = \frac{\sqrt[3]{\frac{1}{3}}}{12}\rho g \pi R^2 H^2 \tag{15}$$

## Problem 3 Solution

(a) We will apply conservation of energy law.

$$mgx_m = \int_0^{x_m} F \mathrm{d}x = \frac{1}{2}kx_m^2 \tag{16}$$

$$\Rightarrow \; x_m = 0.0327 \text{ (m)} \tag{17}$$

Then we can easily calculate the maximum force $F_m$

$$F_m = kx_m = 98 \text{ (N)} \tag{18}$$

(b) Analogously, we apply conservation of energy law.

$$mgx_m = \int_0^{x_m} F \mathrm{d}x = 120000x_m^4 + 1500x_m^2 \tag{19}$$

$$\Rightarrow \; x_m = 0.0304 \text{ (m)} \tag{20}$$

Then we get

$$F_m = 3 \times 10^3 \times (x_m + 160x_m^3) = 105 \text{ (N)} \tag{21}$$

## Problem 4 Solution

According to conservation of energy law

$$\frac{1}{2}mv_0^2 = mgx\sin\alpha + \int_0^x \mu mg\cos\alpha \mathrm{d}x \tag{22}$$

where

$$\mu = Ax \tag{23}$$

In addition, we need to ensure that

$$\mu mg\cos\alpha \geq mh\sin\alpha \tag{24}$$

Namely

$$x \geq \frac{\sin\alpha}{A\cos\alpha} \tag{25}$$

In order to show we are proving the statement on the track, the process of calculation is followed.

$$\begin{aligned} v_0^2 &= 2gx\sin\alpha + 2\int_0^x Axmg\cos\alpha \mathrm{d}x \\ &= 2gx\sin\alpha + Agx^2\cos\alpha \\ &\geq \frac{2g(\sin\alpha)^2}{A\cos\alpha} + \frac{g(\sin\alpha)^2}{A\cos\alpha} = \frac{3g(\sin\alpha)^2}{A\cos\alpha} \end{aligned}$$

## Problem 5 **Solution**

We first calculate work done by friction

$$W_f = \int_0^{\frac{L_0}{2}} \frac{x}{L_0}\mu mg\mathrm{d}x = \frac{1}{8}\mu mgL_0 \tag{26}$$

Then we calculate the velocity

$$\frac{1}{2}mv_0^2 + W = 0 \tag{27}$$

$$\Rightarrow \ v_0 = \frac{\sqrt{\mu gL_0}}{2} \tag{28}$$

## Problem 6 **Solution**

(a) For (A),

$$\mathbf{F}_1(\mathbf{r}) = (0, 0, 5)$$

Since $\mathbf{F}_1(\mathbf{r})$ is vertical to the x-axis

$$\mathbf{F}_1(\mathbf{r}) \cdot \mathbf{x} = 0$$

Therefore,

$$W_{\mathrm{aA}} = 0 \text{ J} \tag{29}$$

For (B),

$$\mathbf{F}_2(\mathbf{r}) = (-2x, 0, 0)$$
$$\mathbf{F}_2(\mathbf{r})_x = (-2x, 0, 0)$$
$$\mathbf{F}_2(\mathbf{r})_y = (0, 0, 0)$$

We calculate the work separately corrosponding to $x-$component and $y-$component.

$$W_x = \int_{-1}^{1} \mathbf{F}_2(\mathbf{r})_x \mathrm{d}x \tag{30}$$
$$W_y = \int_{0}^{0} \mathbf{F}_2(\mathbf{r})_y \mathrm{d}y \tag{31}$$
$$W_{\mathrm{aB}} = W_x + W_y \tag{32}$$

Therefore,

$$W_{\mathrm{aB}} = 0 \text{ J} \tag{33}$$

(b) For (A),

$$\mathbf{F}_1(\mathbf{r}) = (0, -xy, 5)$$
$$\mathbf{F}_1(\mathbf{r})_x = (0, 0, 0)$$
$$\mathbf{F}_1(\mathbf{r})_y = (0, xy, 0)$$

Notice that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x \tag{34}$$
$$\Rightarrow \ \mathrm{d}y = 2x\mathrm{d}x \tag{35}$$

We then calculate the work separately corrosponding to $x-$component and $y-$component.

$$W_x = \int_{-1}^{1} \mathbf{F}_1(\mathbf{r})_x \mathrm{d}x \tag{36}$$
$$W_y = \int_{\Gamma_{AB}} \mathbf{F}_1(\mathbf{r})_y \mathrm{d}y = \int_{-1}^{1} -(x^3 - x)2x \ \mathrm{d}x \tag{37}$$
$$W_{\mathrm{aB}} = W_x + W_y \tag{38}$$

Therefore,

$$W_{\mathrm{bA}} = \frac{8}{15} \text{ J} \tag{39}$$

For (B)

$$\mathbf{F}_2(\mathbf{r}) = (-2x, 0, y - xy)$$
$$\mathbf{F}_2(\mathbf{r})_x = (-2x, 0, 0)$$
$$\mathbf{F}_2(\mathbf{r})_y = (0, 0, 0)$$

We calculate the work separately corrosponding to $x-$component and $y-$component.

$$W_x = \int_{-1}^{1} \mathbf{F}_2(\mathbf{r})_x \mathrm{d}x \tag{40}$$

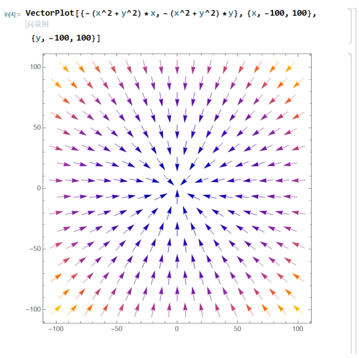
$$W_y = \int_{0}^{0} \mathbf{F}_2(\mathbf{r})_y \mathrm{d}y \tag{41}$$

$$W_{\mathrm{aB}} = W_x + W_y \tag{42}$$

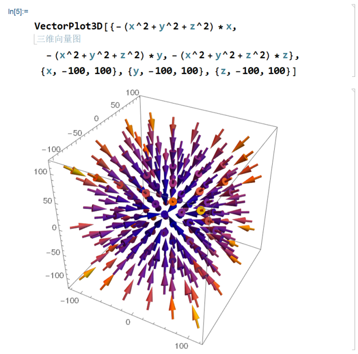
Therefore,

$$W_{\mathrm{aB}} = 0 \text{ J} \tag{43}$$

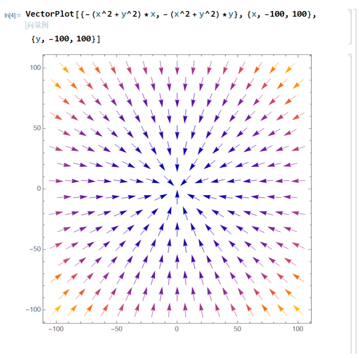
## Problem 7 Solution



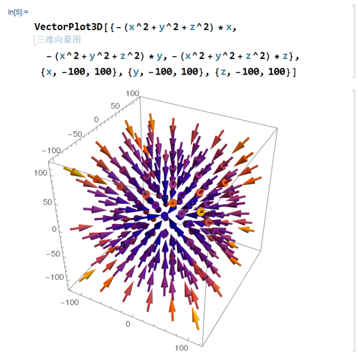

(a) Plot of (a)



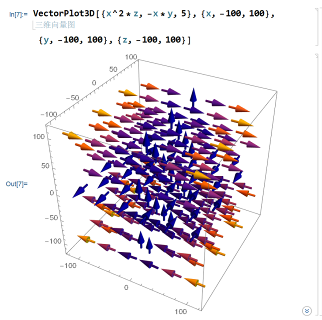

(b) Plot of (b)



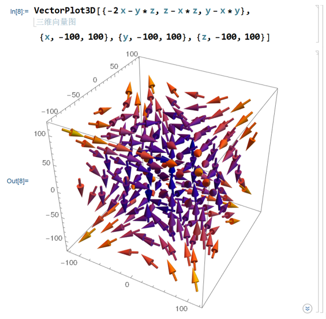

(c) Plot of (c) - 2D




(d) Plot of (c) - 3D



(e) Plot of Problem 6(A)



(f) Plot of Problem 6(B)

Figure 1: Plots in Problem 7