PHYSICS I Problem Set 2

Name: Haotian Fu

Student ID: 520021910012

Problem 1 **Solution**

(a) Since the cable is not elastic, its length is constant. Therefore, based on the constant length, we have following relationships for pulley B

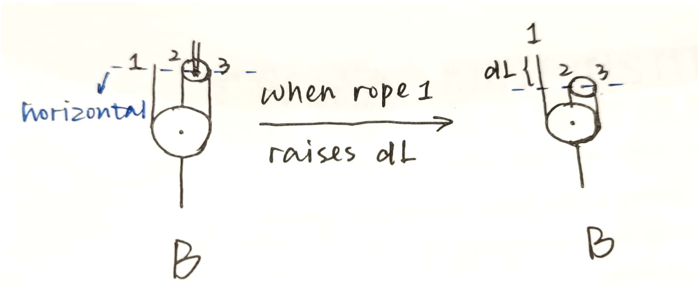


Figure 1: Analysis of pulley B

According to Figure 1, it is evident that cables below the horizontal line (blue line in Figure 1) raise $\frac{dL}{3}$ on average when the rope 1 raises dL.

Analogously, we take the pulley attached to L as our observation.

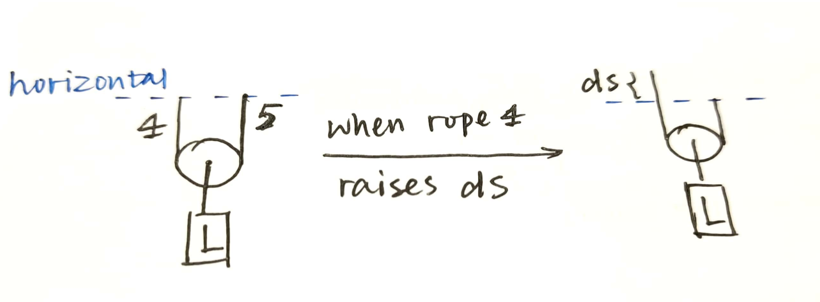


Figure 2: Analysis of pulley attached to L

It is also evident to see that cables below the horizontal line (blue line in Figure 2) raise $\frac{dS}{2}$ when cable 4 raises dS.

Since all these movements occur in the same time period, the speed is directly proportional to the distance correspondingly. Therefore, we have

$$v_B = \frac{1}{3}v_M$$
$$v_L = \frac{1}{2}v_B$$
$$\Rightarrow v_L = \frac{1}{6}v_M = \frac{50}{3} \approx 16.7(\text{mm/s})$$

Since L moves upwards while the cable with $v_M = 100$ mm/s moves downwards, $\bar{v}_L = -16.7$ mm/s.

Problem 2 Solution

We first select any instant t and let it be fixed. Then we can analyze both the dynamics and kinematics of this particle as follows.

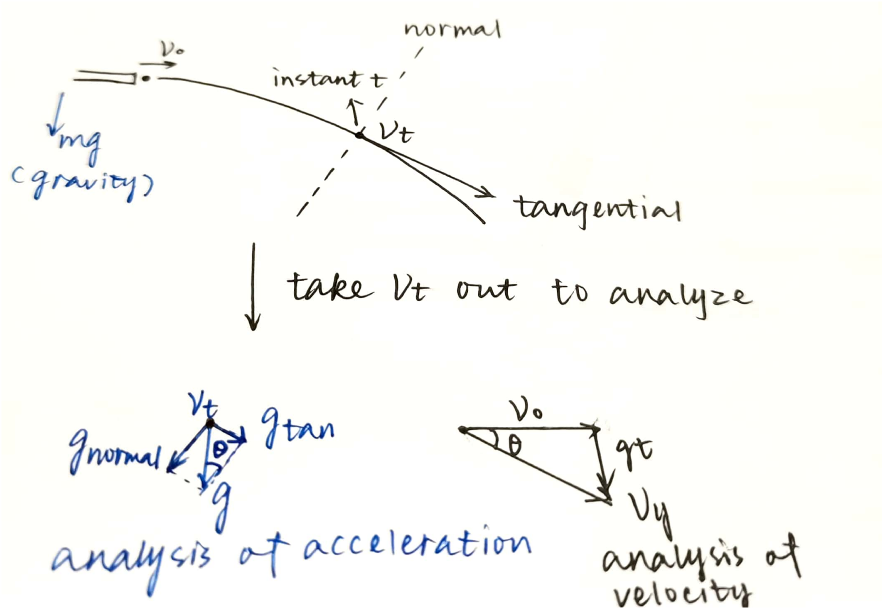


Figure 3: Force analysis and kinematics analysis of the particle

To be more specific, we can conclude equations as follows

$$\begin{cases} \tan\theta = \frac{gt}{v_0} \\ a_t = g\sin\theta \\ a_n = g\cos\theta \end{cases}$$

Solve this system we get

$$\begin{cases} a_t = \frac{g^2 t}{\sqrt{g^2t^2+v_0^2}}\hat{a}_t \\ a_n = \frac{gv_0}{\sqrt{g^2t^2+v_0^2}}\hat{a}_n \end{cases}$$

Based on geometric triangles, we also know

$$\bar{v}_y = gt = a_t + a_n$$

Problem 3 Solution

(a) $\bar{v} = \bar{v}_1 + \bar{v}_2 = v_1\hat{n}_y + v_0\sin(\frac{\pi y}{L})\hat{n}_x$

$|\bar{v}| = \sqrt{v_1^2 + (v_0\sin\frac{\pi y}{L})^2}$

(b) When the canoe started from the southern bank, $y(t) = v_1 t$, and when the canoe started from the northern bank, $y(t) = L - v_1 t$. Then we apply integral

$$x(t) = \int v_0 \sin\frac{\pi y}{L} = v_0 \int \sin\frac{\pi v_1 t}{L} = -\frac{v_0 L}{\pi v_1}\cos\frac{\pi v_1 t}{L}$$

Therefore, trajectory $\bar{r} = (-\frac{v_0 L}{\pi v_1}\cos\frac{\pi v_1 t}{L})\hat{n}_x + v_1 t\hat{n}_y$ or $\bar{r} = (-\frac{v_0 L}{\pi v_1}\cos\frac{\pi v_1 t}{L})\hat{n}_x + (L - v_1 t)\hat{n}_y$

(c)

$$x(t) = \int_0^t v_0 \sin\frac{\pi y}{L} = v_0 \int_0^t \sin\frac{\pi v_1 t}{L} = -\frac{L}{\pi v_1}\cos\frac{\pi v_1 t}{L}|_0^{L/v_1} = \frac{2v_0}{\pi v_1}L$$

Problem 4 Solution

The units of b and c are m

(a) Since $x = r\cos\varphi$ and $y = r\sin\varphi$, we have

$$\begin{cases} r\cos\varphi = bt^2 \\ r\sin\varphi = -ct^2 \end{cases}$$

Therefore, we get the parametric equations of the trajectory

$$\begin{cases} r = \sqrt{b^2 + c^2}t^2 \\ \varphi = \arctan(-\frac{c}{b}) \end{cases}$$

(b) Since $x(t) = bt^2$ and $y(t) = -ct^2$, $\frac{x}{y} = -\frac{b}{c} \Rightarrow cx + by = 0$

Therefore, the implicit form in polar coordinate system is $cr\cos\varphi + br\sin\varphi = 0$

(c) In the Cartesian coordinate system, $v(x) = \dot{x}(t) = 2bt$, $v(y) = \dot{y}(t) = -2ct$.

Therefore, $\bar{v} = 2bt\hat{n}_x - 2ct\hat{n}_y$, $\bar{a} = \dot{\bar{v}} = 2b\hat{n}_x - 2c\hat{n}_y$

In polar coordinate system, $\bar{r} = r\hat{n}_r$, $\bar{v} = \dot{\bar{r}} = \dot{r}\,\hat{n}_r + r\,\dot{\hat{n}}_r = \dot{r}\,\hat{n}_r + r\,\dot{\varphi}\,\hat{n}_\varphi = 2\sqrt{b^2 + c^2}t\hat{n}_r$, $\bar{a} = (\ddot{r} - r(\dot{\varphi})^2)\hat{n}_r + (r\,\ddot{\varphi} + 2\,\dot{r}\,\dot{\varphi})\hat{n}_\varphi = 2\sqrt{b^2 + c^2}\hat{n}_r$

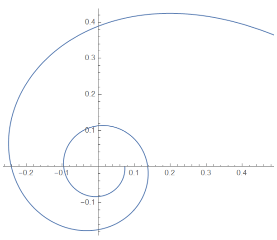
Problem 5 Solution

The unit of c is $\frac{1}{s}$ while the unit of r_0 is m.

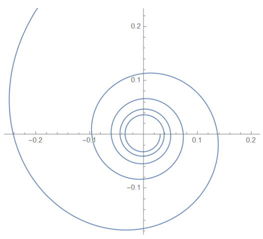
(a) $\frac{r}{r_0} = 1 - ct \Rightarrow ct = 1 - \frac{r}{r_0}$

$\Rightarrow \varphi = \frac{1 - r/r_0}{r/r_0} = \frac{r_0 - r}{r}$

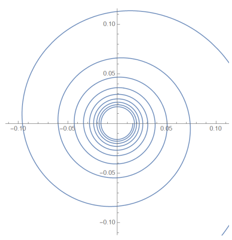
In order to show helix trajectory completely, I limited the range of φ in three ranges. Since the figure will become concentric circles when φ is really large, I only drew the figure in a limited range.



(a) φ ranges from 0 to 4π



(b) φ ranges from 0 to 10π



(c) φ ranges from 0 to 20π

Figure 4: Trajectory of the particle

(b) $\bar{v} = \dot{\bar{r}} = \dot{r}\,\hat{n}_r + r\,\dot{\hat{n}}_r = \dot{r}\,\hat{n}_r + r\,\dot{\varphi}\,\hat{n}_\varphi = -cr_0\hat{n}_r + \frac{cr_0 - c^2 r_0 t}{(1-ct)^2}\hat{n}_\varphi = -cr_0\hat{n}_r + \frac{cr_0}{1-ct}\hat{n}_\varphi$

$v = \sqrt{(-cr_0)^2 + (cr_0/(1-ct))^2} = |cr_0|\sqrt{1 + (1/(1-ct)^2)}$

(c) $\bar{a} = (\ddot{r} - r(\dot{\varphi})^2)\hat{n}_r + (r\,\ddot{\varphi} + 2\,\dot{r}\,\dot{\varphi})\hat{n}_\varphi = (-c^2 r_0 + c^3 r_0 t)/(1-ct)^4\hat{n}_r = -\frac{c^2 r_0}{(1-ct)^3}\hat{n}_r$

(d) When $c > 0$, $\bar{a}$>0 as time goes by and $\bar{v}_r$<0. Therefore, the particle moves closer and closer, faster and faster to the origin and the acceleration increases.

When $c < 0$, $\bar{a}$<0 as time goes by and $\bar{v}_r$>0. Therefore, the particle moves farther and farther, slower and slower to the origin and the acceleration decreases.

Problem 6 Solution

(a) The unit of $\frac{3}{\pi}$ is radian, the unit of π is s^{-1}, the unit of 6 is cm, the unit of 2 is s^{-1}.

(b) $\bar{v} = \dot{\bar{r}} = \dot{r}\,\hat{n}_r + r\,\dot{\hat{n}}_r = \dot{r}\,\hat{n}_r + r\,\dot{\varphi}\,\hat{n}_\varphi = 12e^{-2}\hat{n}_r + (18e^{-2} - 18)\hat{n}_\varphi$
radial: $v_r = \frac{12}{e^2}\hat{n}_r$ transverse: $v_\varphi = \frac{18}{e^2} - 18\hat{n}_\varphi$

(c) $\bar{a} = (\ddot{r} - r(\dot{\varphi})^2)\hat{n}_r + (r\,\ddot{\varphi} + 2\,\dot{r}\,\dot{\varphi})\hat{n}_\varphi = (30e^{-2} - 54)\hat{n}_r$ - $72e^{-2}\hat{n}_\varphi$
ratial: $a_r = (30e^{-2} - 54)\hat{n}_r$ tranverse: $a_\varphi =$ - $72e^{-2}\hat{n}_\varphi$

(d) For the rod system, the collor moves along the rod so there is no φ change. Therefore, $a = a_{\text{ratial}} = (30e^{-2} - 54)\hat{n}_r$