Newton's Laws

Applications

Agenda

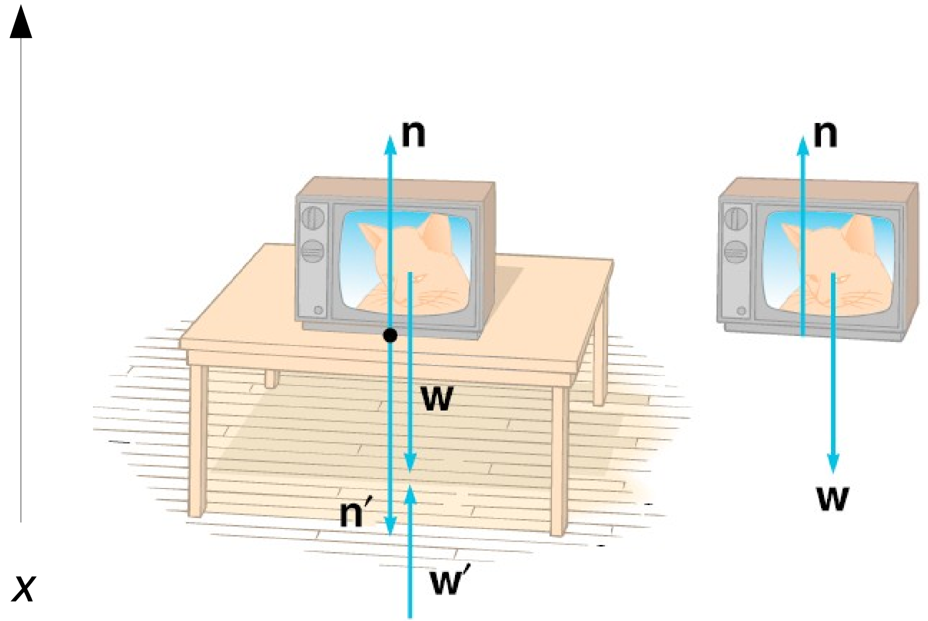
- Particles in equilibrium
- Accelerating particles
- Motion with friction and air/fluid resistance

Particles in Equilibrium

Newton First and Third Laws: Particles in Equilibrium

*A body acted on by **zero net force** moves with **constant velocity**.*

*The mutual forces of **action** and **reaction** between two bodies are equal, opposite and collinear.*



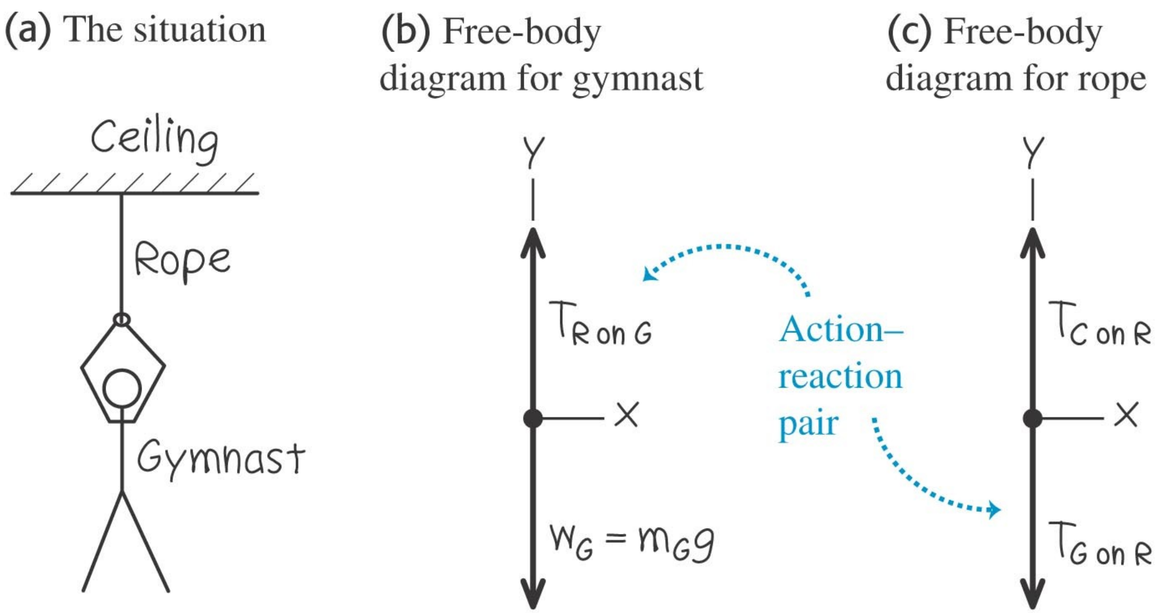
$$\text{net force} = \sum \mathbf{F} = 0$$

$$\mathbf{n} + \mathbf{w} = 0$$

or operating with components and magnitudes (watch the sign!)

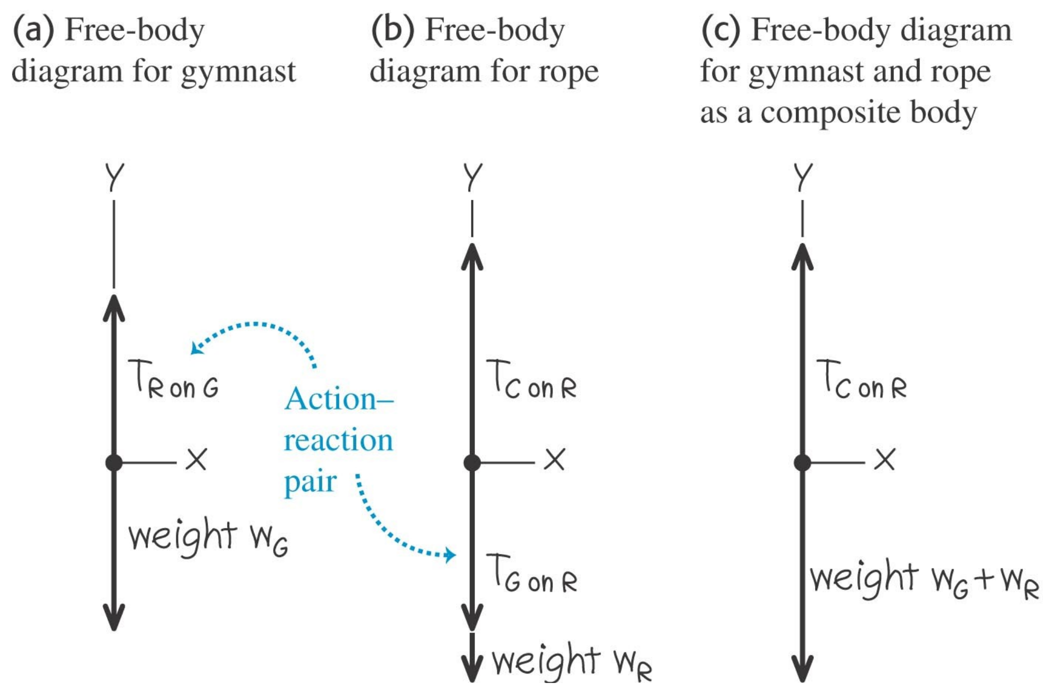
$$n - w = 0$$

Example (equilibrium; collinear forces)



massless rope

massive rope

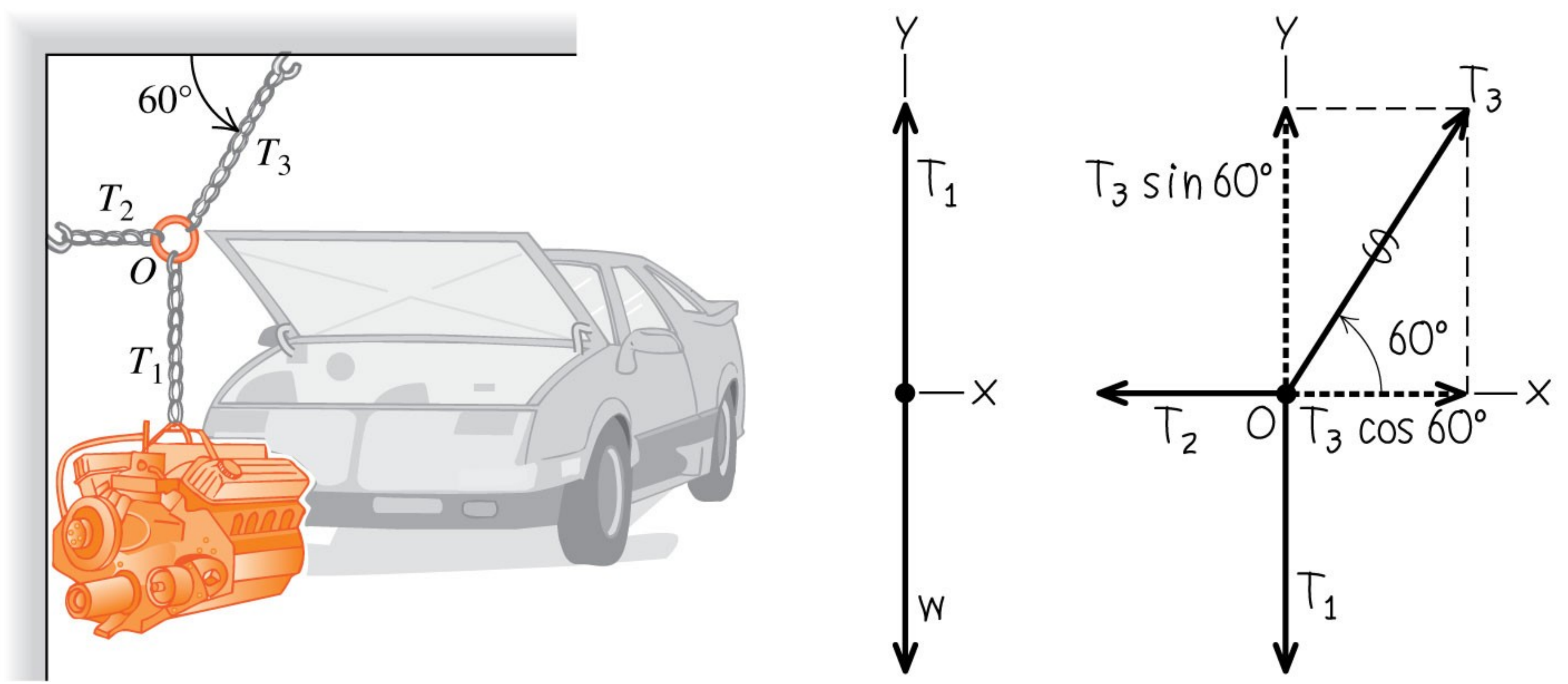


Example
(equilibrium; non-collinear forces)

(a) Engine, chains, and ring

(b) Free-body diagram for engine

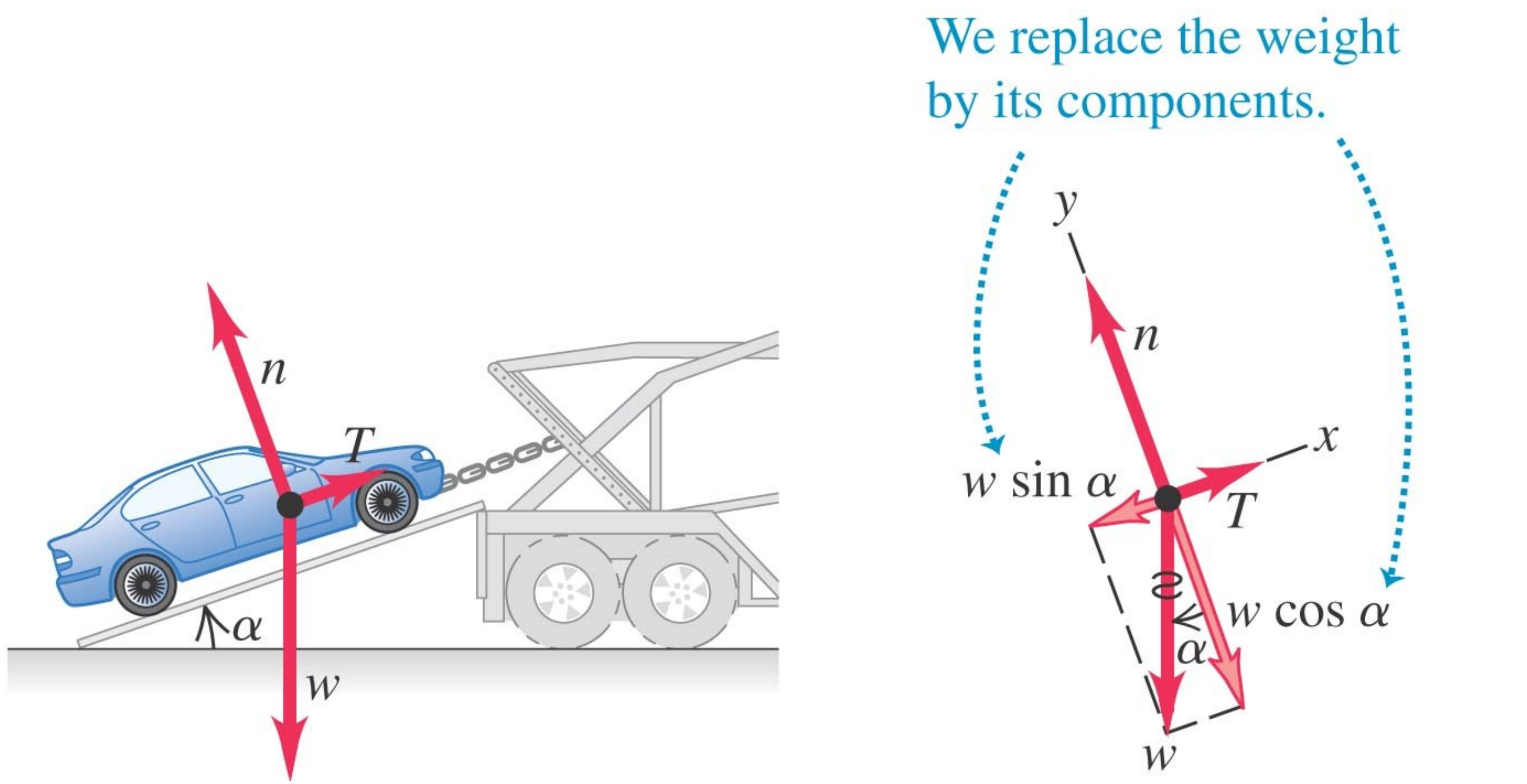
(c) Free-body diagram for ring O



Example
(equilibrium; object on an inclined plane)

(a) Car on ramp

(b) Free-body diagram for car



Particles in Motion

Newton's second law: particles in motion

In an inertial frame of reference, the ***acceleration*** *of an object is* ***directly proportional to the net force*** *acting on it, and* ***inversely proportional to the mass*** *of the object.*

$$m\mathbf{a} = \mathbf{F}$$

resulting acceleration

net force

Newton's second law: particles in motion

$$m\mathbf{a} = \mathbf{F}$$

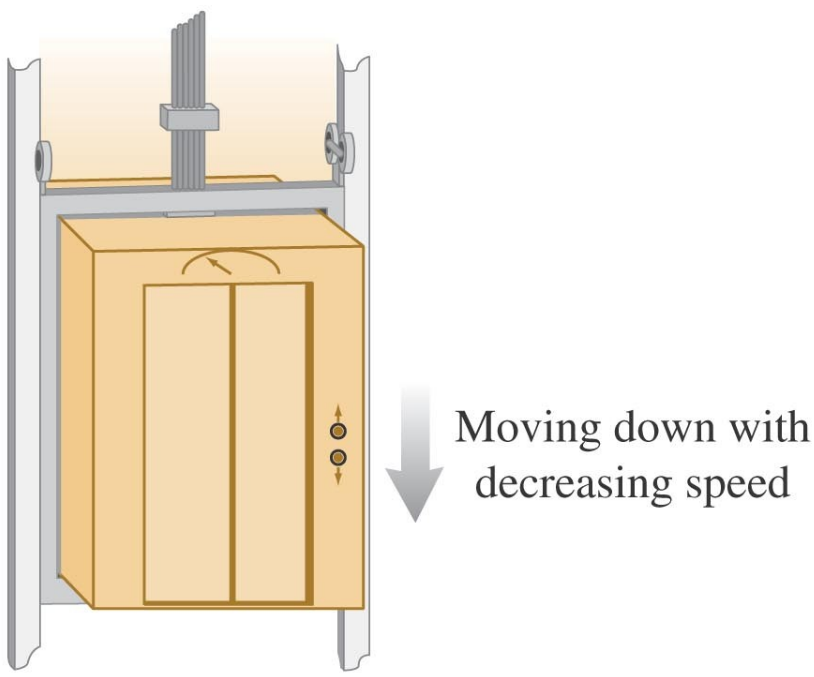
known forces,
find acceleration

known acceleration,
infer about the net force

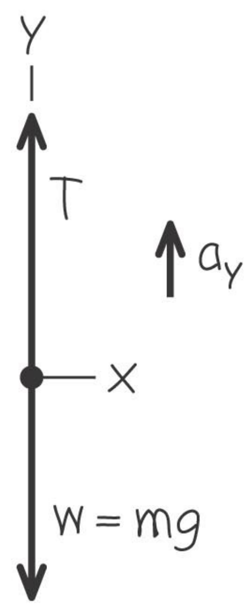
Example:
Elevator (tension in a massless cable)

The elevator is moving downward but slowing to a stop. What is the tension in the supporting massless cable?

(a) Descending elevator



(b) Free-body diagram for elevator



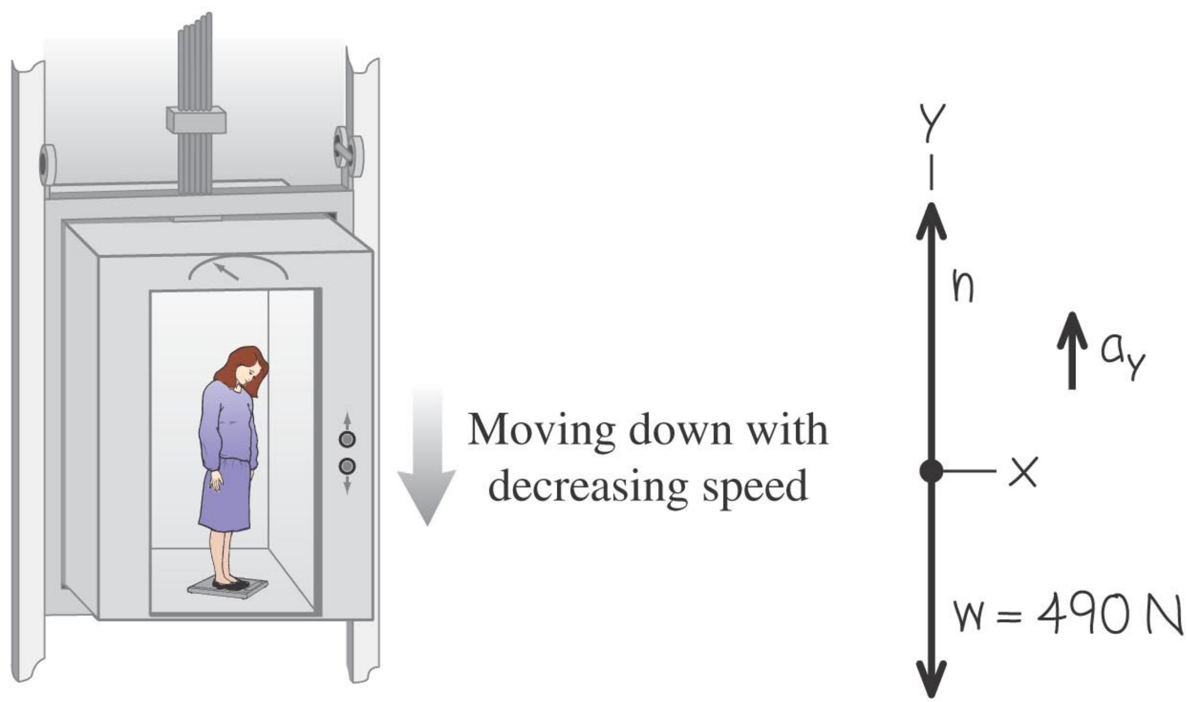
$$ma_y = T - mg \Rightarrow \boxed{T = m(a_y + g)}$$

Example:
Elevator (apparent weight)

A woman inside the elevator of the previous example is standing on a scale. How will the acceleration of the elevator affect the scale reading?

(a) Woman in a descending elevator

(b) Free-body diagram for woman

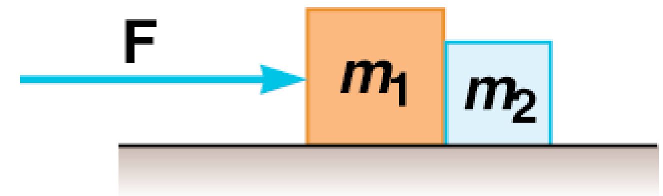


inertial frame of reference
(e.g. the elevator shaft)

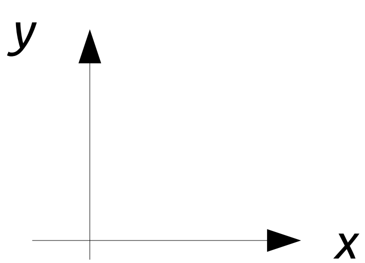
$$ma_y = n - mg \Rightarrow \boxed{n = m(a_y + g)}$$

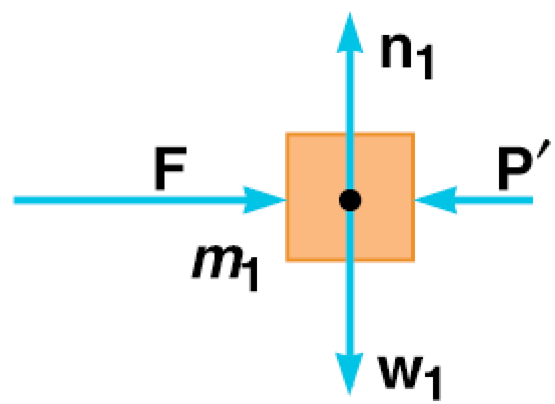
Example: two objects in direct contact

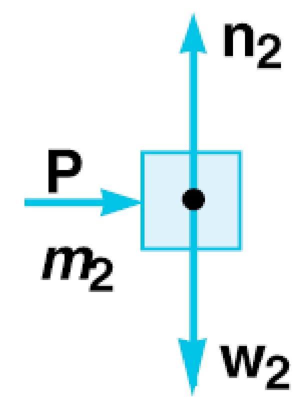
two objects in contact acted upon an external force, frictionless surface



free body diagrams







no motion along *y* direction (net forces have zero *y* component), **Newton's second law for motion along *x* direction** (operate with magnitudes, but watch the signs!)

$$m_1 a = F - P'$$
$$m_2 a = P$$

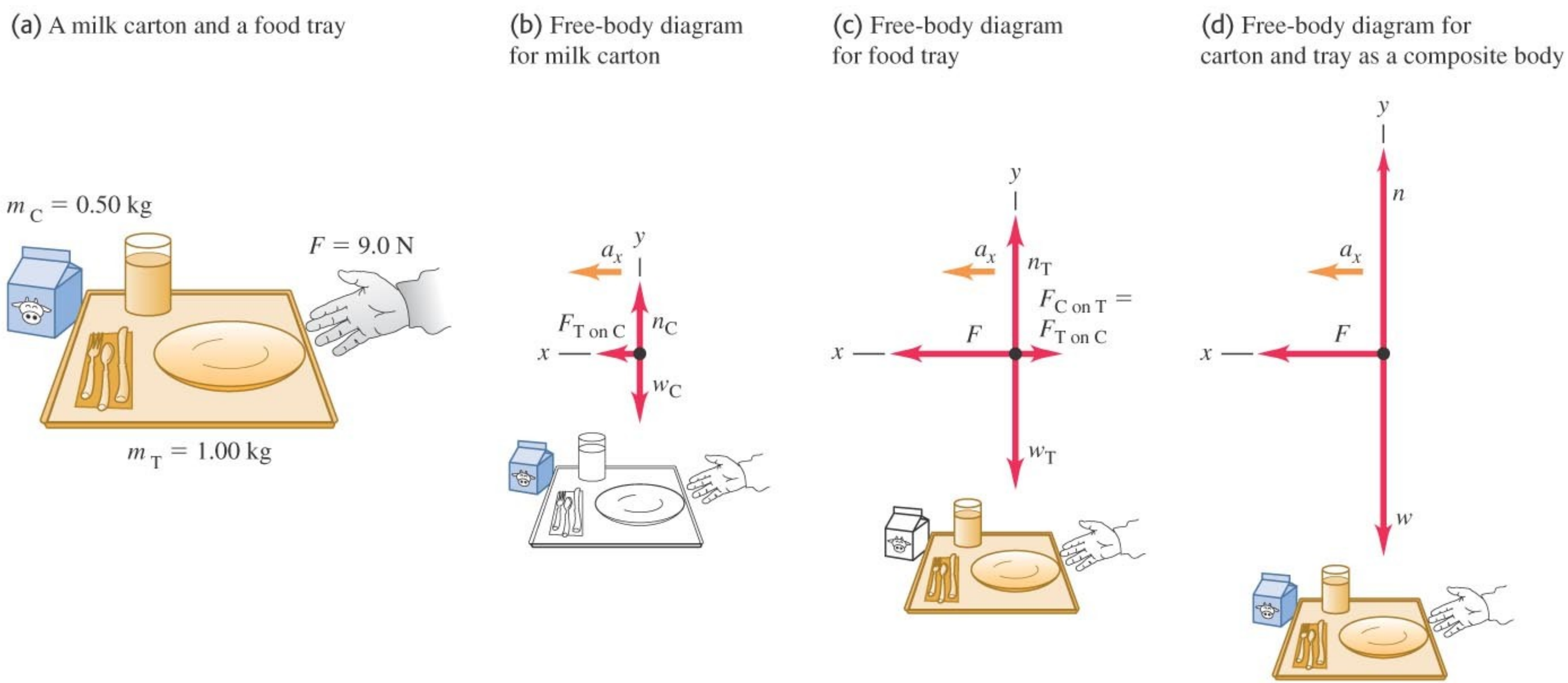
Newton's third law for the pair of forces between the blocks

$$P = P'$$

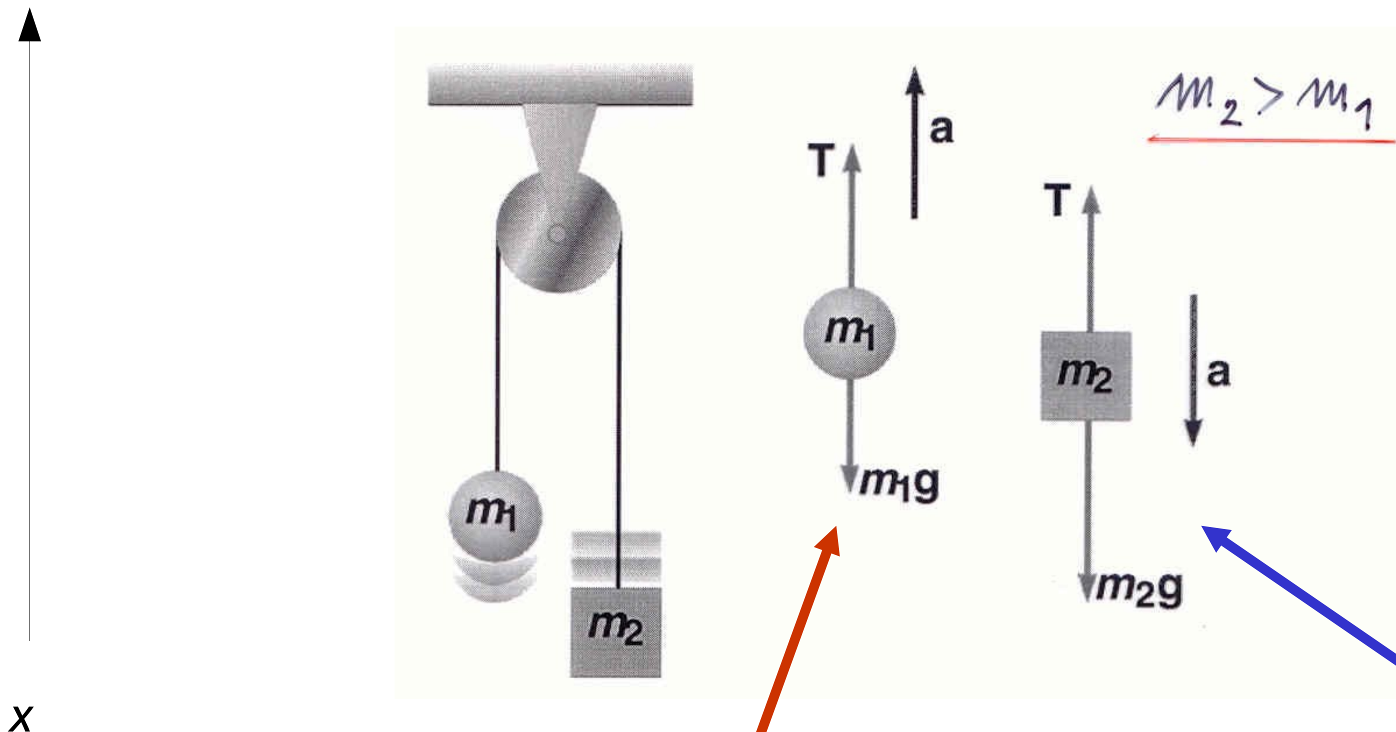
result: $a = \dfrac{F}{m_1 + m_2}$

given force, can find velocity, position

Example: two objects in contact and Newton's third law



Example: Atwood's machine



$$m_1 a = T - m_1 g$$

$$-m_2 a = T - m_2 g$$

solution

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g = \mathrm{const}$$

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

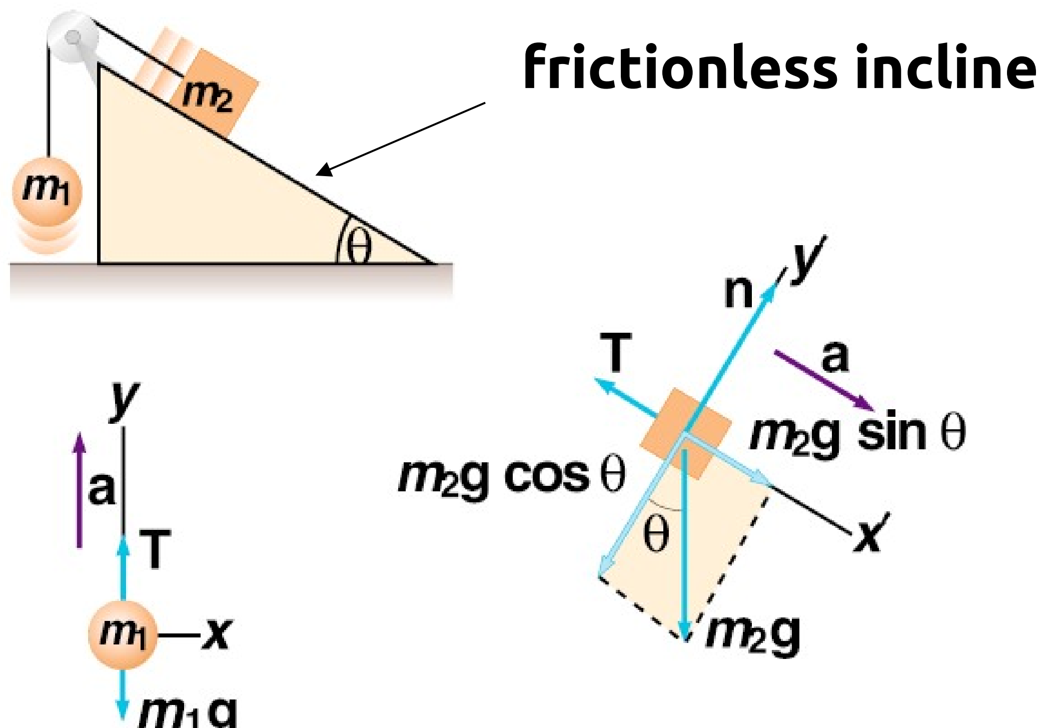
special cases

$$m_1 = m_2 \Rightarrow a = 0,\ T = m_1 g$$

$$m_2 \gg m_1 \Rightarrow a \approx g,\ T = \approx 2 m_1 g$$

Example: incline

Two objects of different masses connected by a massless cord that passes over a frictionless pulley with negligible mass.



solution

$$a = \frac{m_2 \sin\theta - m_1}{m_1 + m_2}\, g = \mathrm{const}$$

$$T = \frac{m_1 m_2}{m_1 + m_2}\,(1 + \sin\theta)\, g$$

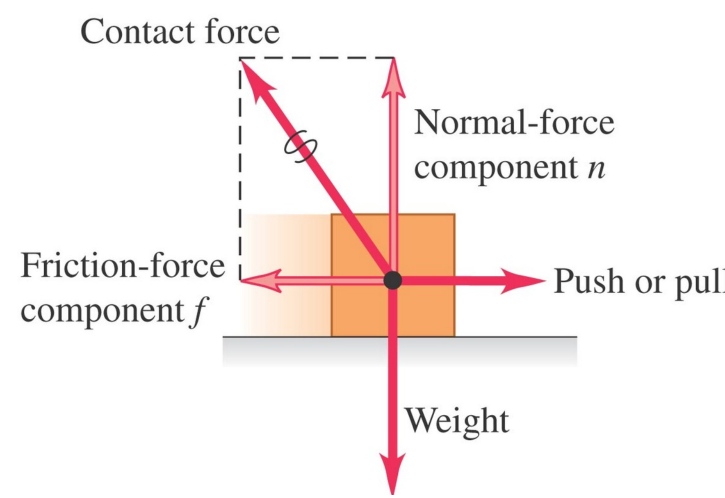
Friction and Fluid/Air Resistance

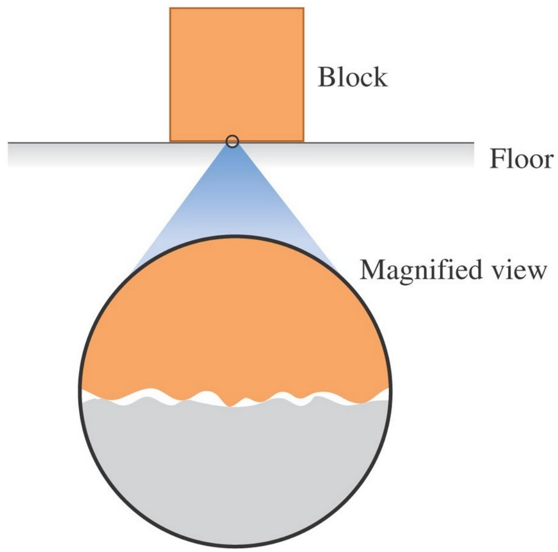
Frictional Forces

When a body rests or slides on a surface, the **friction force** is parallel to the surface.

Friction between two surfaces arises from **interactions between molecules** on the surfaces.

The friction and normal forces are really components of a single contact force.





On a microscopic level, even smooth surfaces are rough; they tend to catch and cling.

Kinetic vs Static Friction

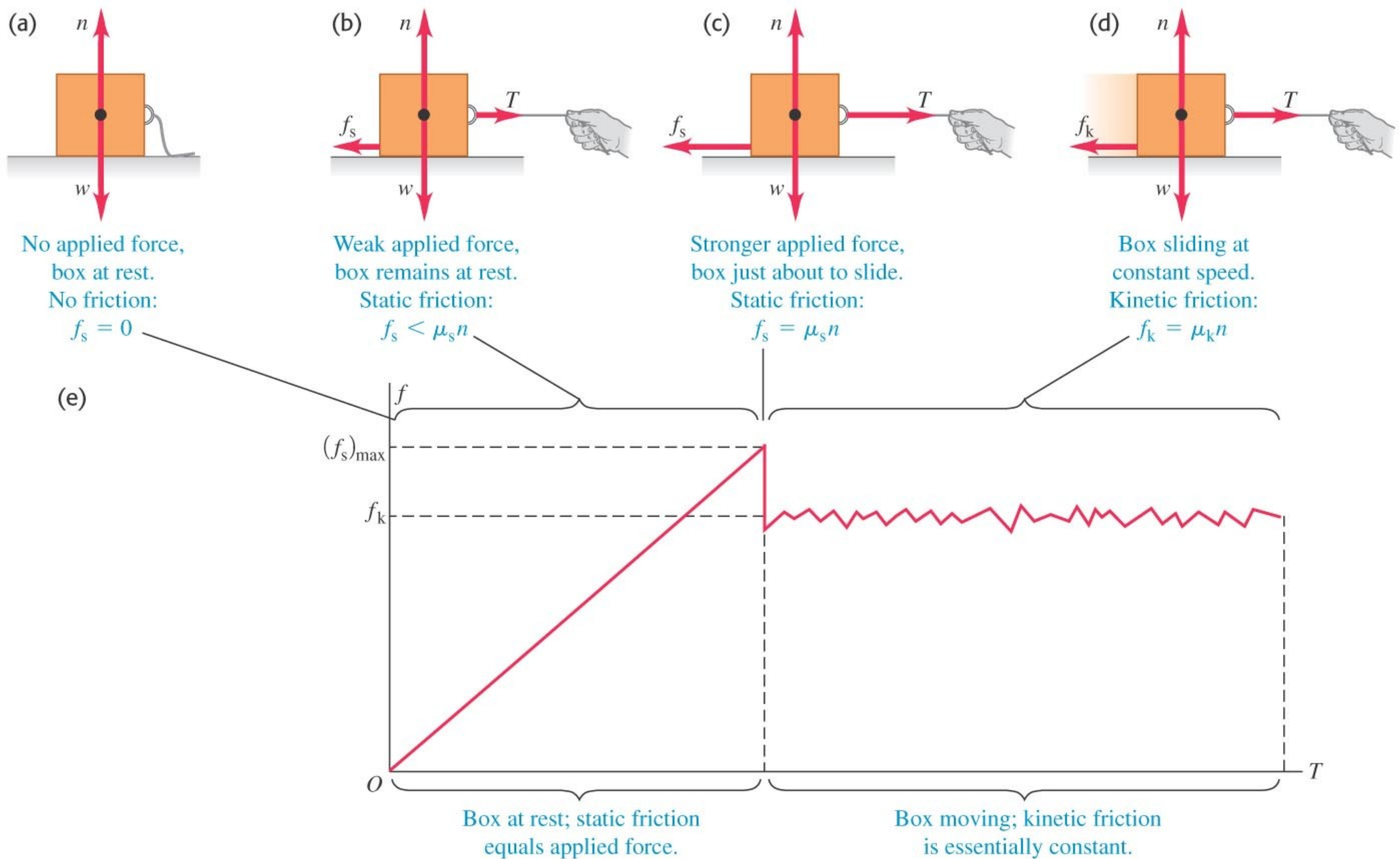
Kinetic friction appears when a body slides over a surface. The magnitude of the **kinetic friction force** is $f_\mathrm{k} = \mu_\mathrm{k} n$.

Static friction force acts when there is no relative motion between bodies.

The magnitude of the **static friction force** can vary between zero and its maximum value: $f_\mathrm{s} \leq \mu_\mathrm{s} n$.

Kinetic vs Static Friction

Before the box slides, static friction acts.
But once it starts to slide, it turns into kinetic friction.

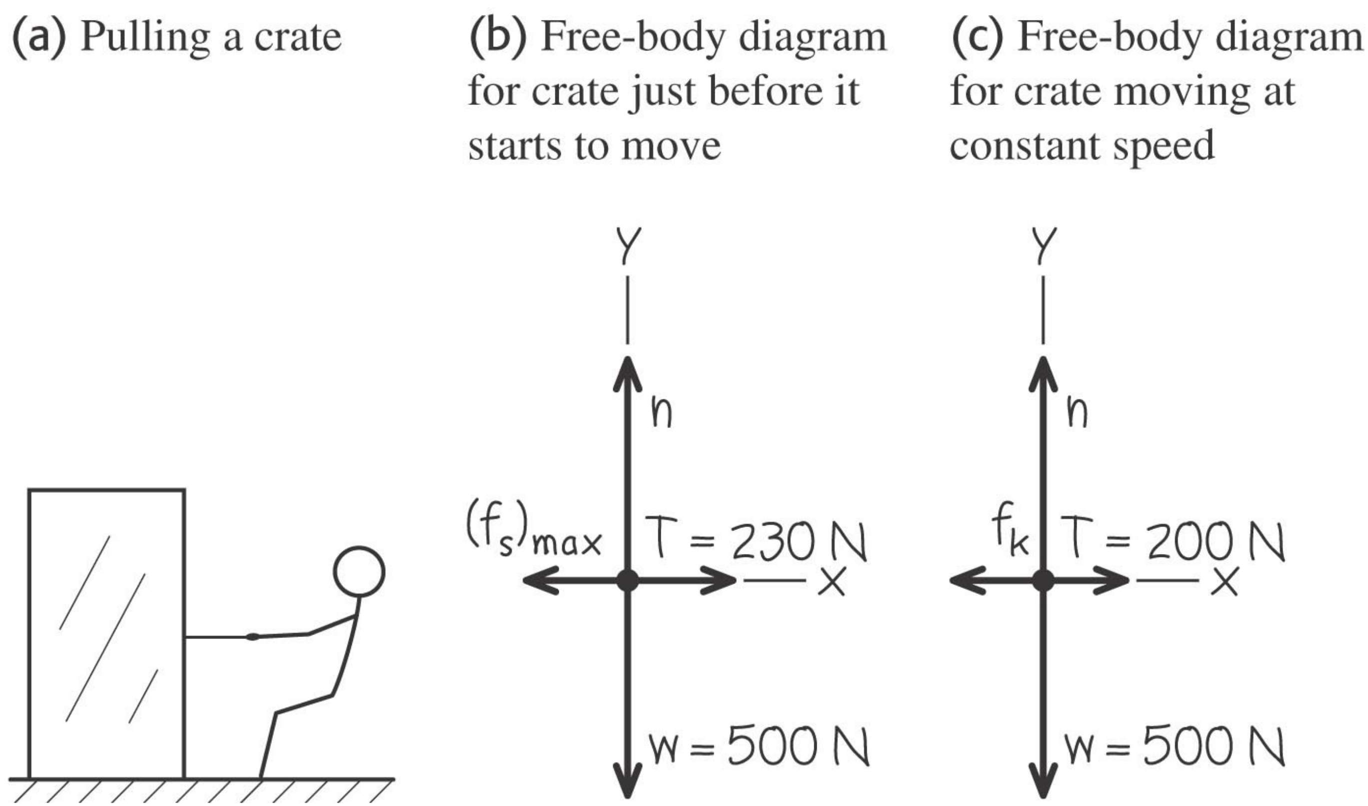


Values of Coefficients of Friction

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

Example (static vs kinetic friction)

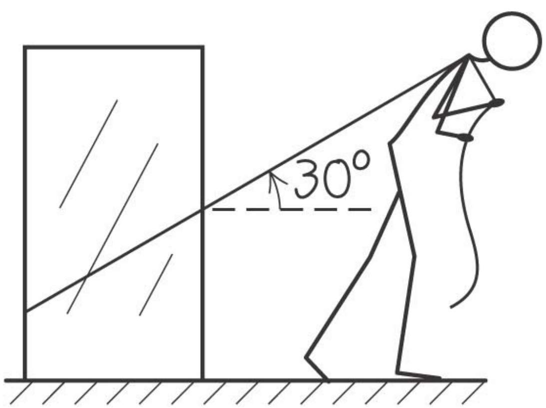
Before the crate moves, static friction acts on it.
After it starts to move, kinetic friction acts.



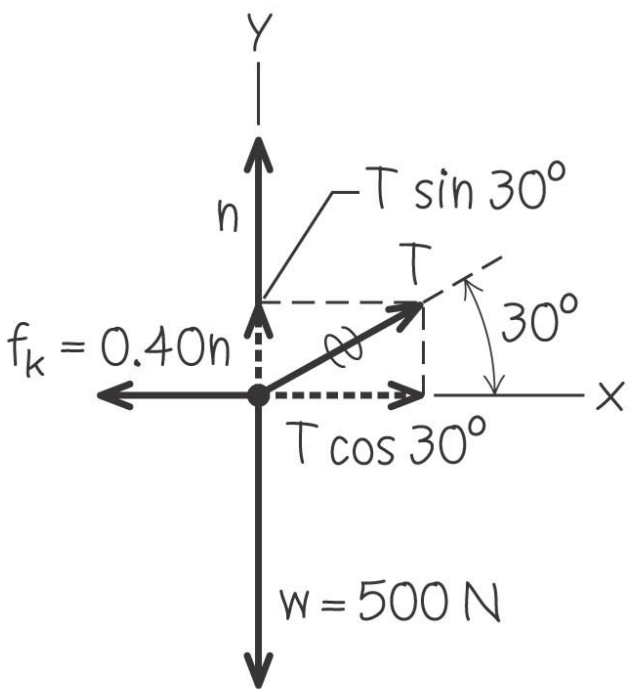
Example (inclined pull)

The angle of the pull affects the normal force, which in turn affects the friction force.

(a) Pulling a crate at an angle



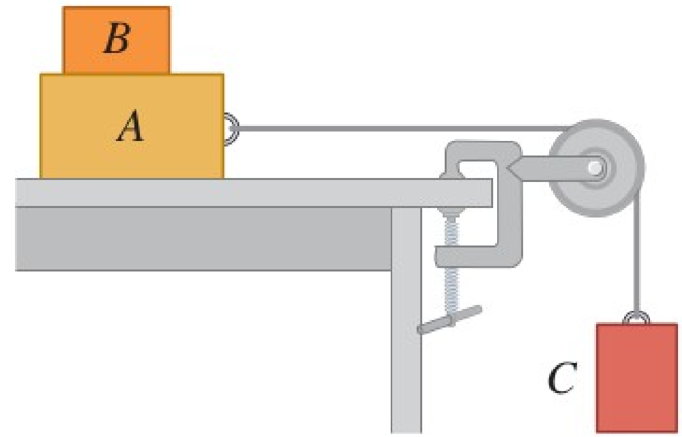
(b) Free-body diagram for moving crate



Example (static friction)

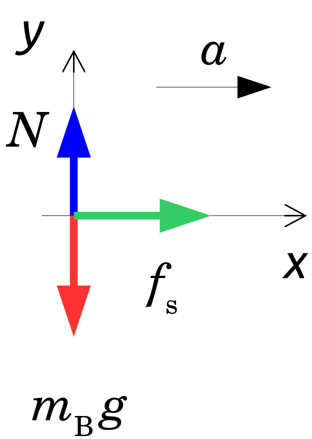
There is no friction between block A and the tabletop, but the coefficient of static friction between block A and block B is $\mu_s \neq 0$. A light string attached to block A passes over a frictionless, massless pulley, and block C is suspended from the other end of the string. Masses of blocks A and B are given.

What is the maximum mass that block C can have, so that blocks A and B still slide together when the system is released from rest?

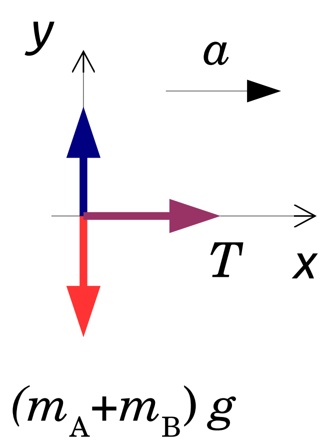


If A and B move together, all three blocks have the same acceleration.

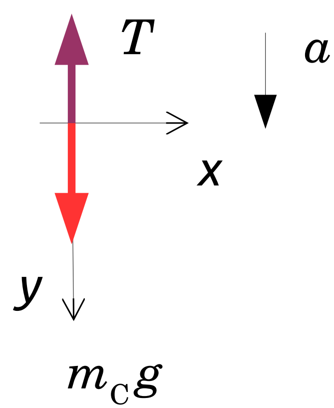
block B



blocks A & B



block C



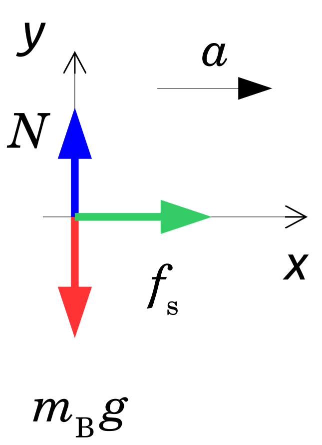
$$(m_A + m_B)a = T$$

$$m_C a = m_C g - T$$

$$\boxed{a = \frac{m_C}{m_A + m_B + m_C} g}$$

Example (static friction, contd)

block B



Block B moves acted upon the (static) frictional force.

$$m_B a = f_s \qquad f_s \leq \mu_s N = \mu_s m_B g = f_{s,\max}$$

Hence it will not slide as long as $a \leq \mu_s g$

case $\mu_s \geq 1$

Since a < g for this system, the inequality a < $\mu_s g$ always holds, irrespective of the value of mass m_C.

(Maximum frictional force that can be provided is greater than that required for the block to move with acceleration a.)

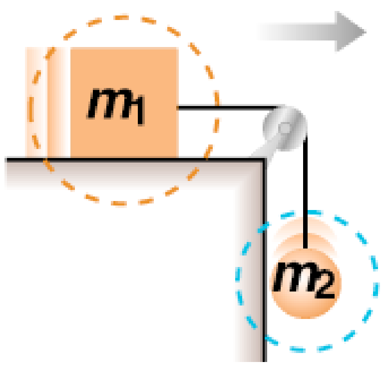
For any mass m_C block B moves together with block A.

case $\mu_s < 1$

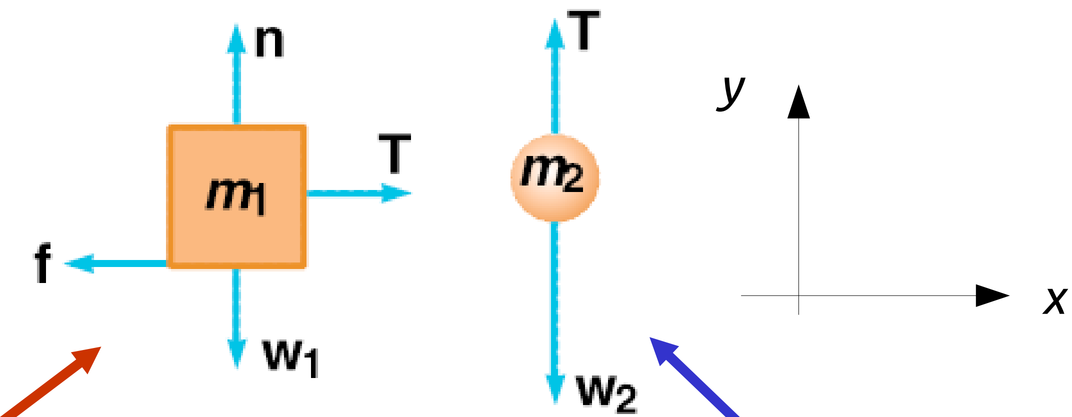
$$\frac{m_C}{m_A + m_B + m_C} g \leq \mu_s g$$

$$m_C \leq \frac{(m_A + m_B)\mu_s}{1 - \mu_s}$$

Example (two objects on a rough surface, connected by a massless cord)



Two masses connected by a light cord. The surface is rough and the pulley is frictionless



$$m_1 a = T - f$$
$$n - w_1 = 0$$
$$f = \mu n$$

$$-m_2 a = T - w_2$$

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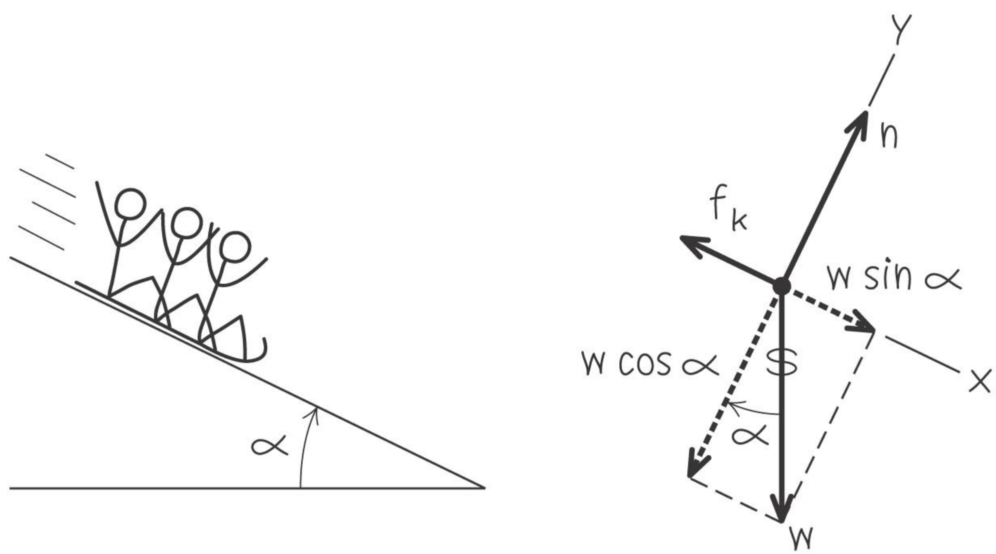
solution

$$a = \frac{m_2 - \mu m_1}{m_1 + m_2} g = \text{const}$$
$$T = \frac{m_1 m_2}{m_1 + m_2} (1 + \mu) g$$

Example (motion on a rough incline)

(a) The situation (b) Free-body diagram for toboggan



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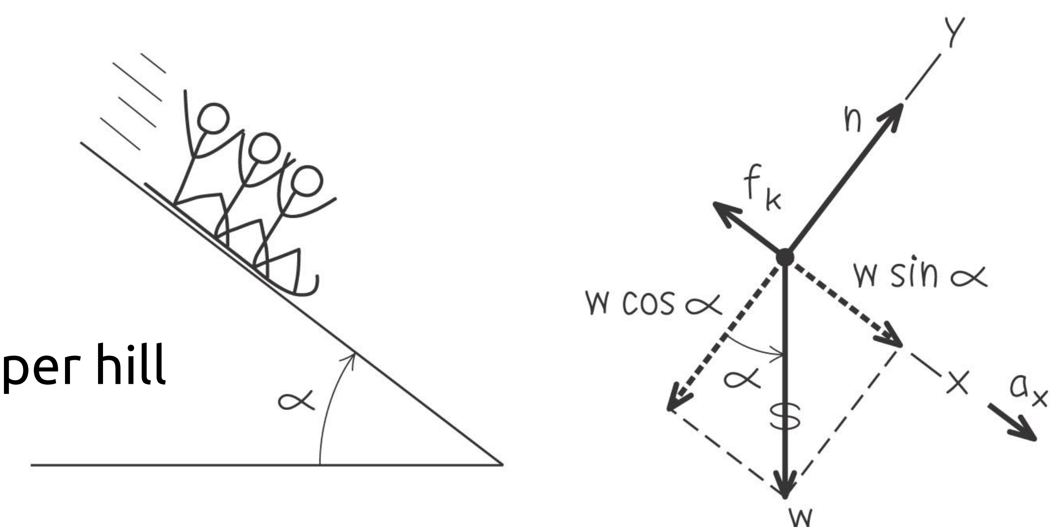
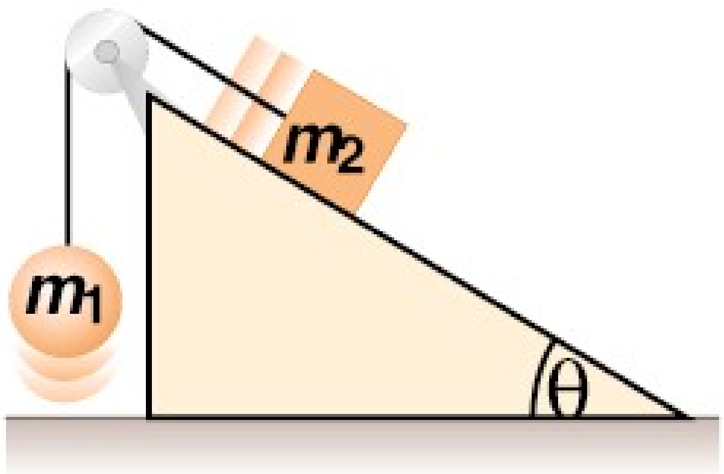


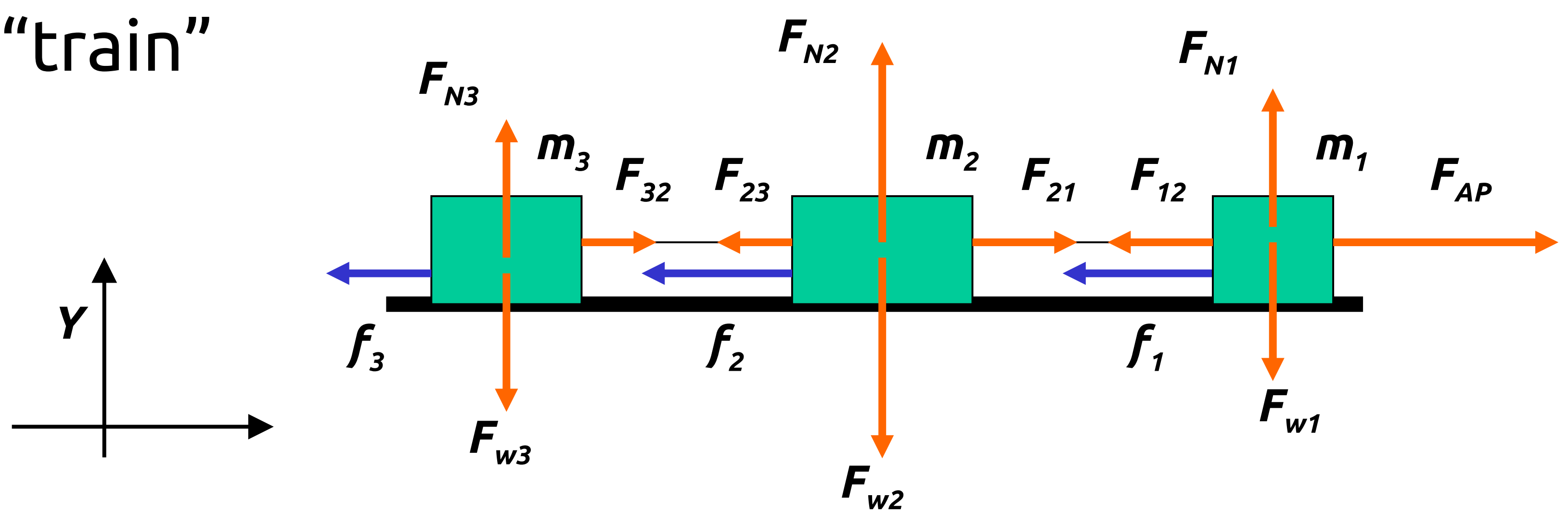
steeper hill

More Examples (DIY)...

- two objects connected by a massles cord, rough incline, frictionless pulley



- "train"



Fluid/Air Resistance

Fluid/Air Resistance (Drag)

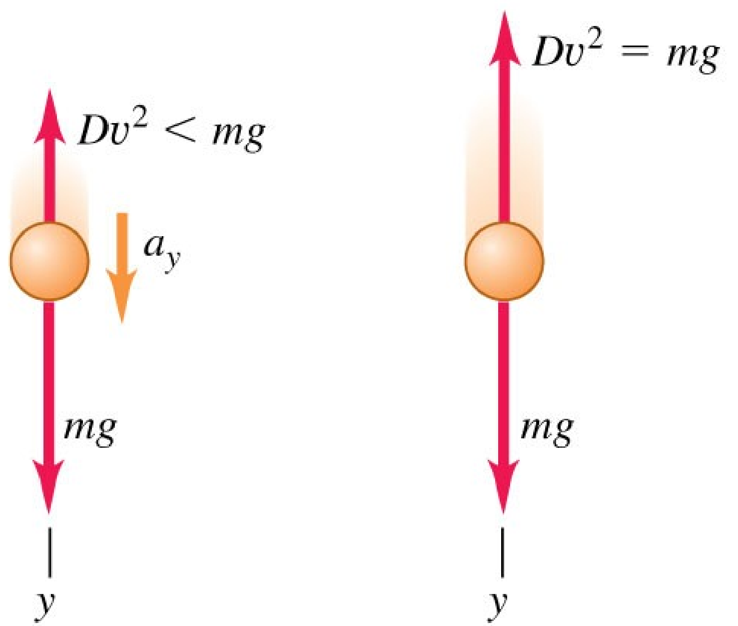
The **fluid resistance (drag) force** f on a body depends on the speed of the body.
Usually $\mathbf{f} \propto -v^p \frac{\mathbf{v}}{v}$

unit vector in the direction of **v**

with $p = 1$ or 2.

A falling body reaches its **terminal speed** when the resisting force equals the weight of the body.

(a) Free-body diagrams for falling with air drag



Before terminal speed: Object accelerating, drag force less than weight.

At terminal speed v_t: Object in equilibrium, drag force equals weight.

Example: fall with linear air drag $\quad \mathbf{f} = -k\mathbf{v}$

(see the blackboard)