PHYSICS I Problem Set 7

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Problem 1 Solution

(a)

$$F = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}\right) = (-y^2 - 2xy, -x^2 - 2xy) \tag{1}$$

Therefore, we can visialize it as

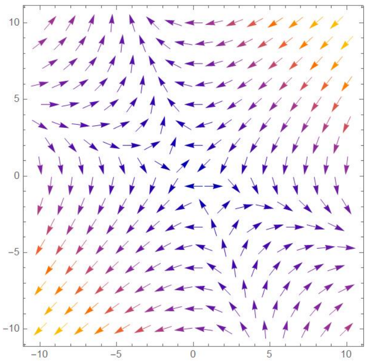


Figure 1: Force in Problem 1

(b)

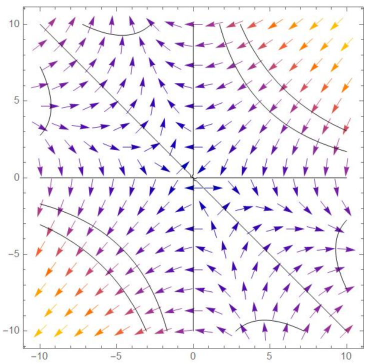


Figure 2: Equipotential Diagram

(c) According to Fig 2, it is trivial that points are equipotential if $x = 0$ or $y = 0$ or $x = -y$ since all these circumstances the potential energy is equal to 0. Meanwhile, the other curves all satisfy the equation $xy^2 + yx^2 = 0$.
(d) We decompose the displacemnet in $x-$axis and $y-$axis. Then we calculate the work

along $x-$axis and $y-$axis separately. Suppose the trajectory is called AB.

$$W_x = \int_{\Gamma_{AB}} F_x \mathrm{d}\bar{r} \tag{2}$$

$$W_y = \int_{\Gamma_{AB}} F_y \mathrm{d}\bar{r} \tag{3}$$

$$\bar{r} = \bar{x} + \bar{y} \tag{4}$$

$$y = x \tag{5}$$

Solving (1)(2)(3)(4)(5), we get

$$W = -2 \quad [\mathrm{J}] \tag{6}$$

(e) Equation (5) changes into equation (7) due to the change of trajectory.

$$y = x^2 \tag{7}$$

Solving (1)(2)(3)(4)(7), we get

$$W = -2 \quad [\mathrm{J}] \tag{8}$$

Problem 2 Solution

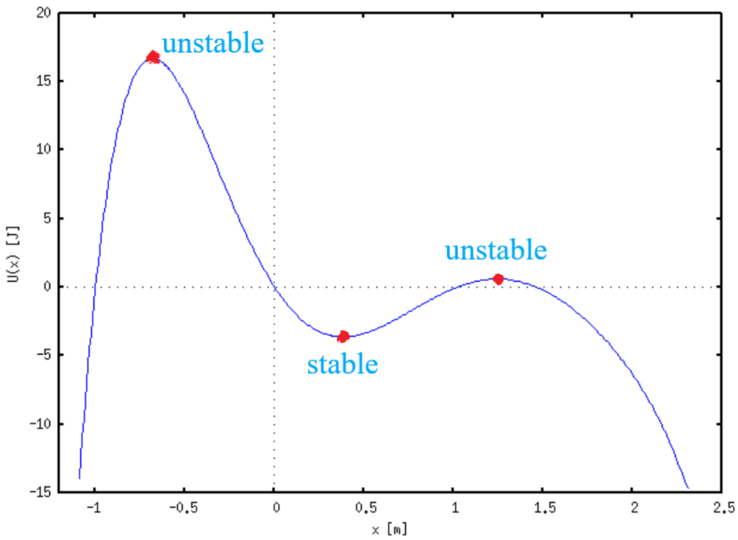


Figure 3: Energy Diagram

Problem 3 Solution

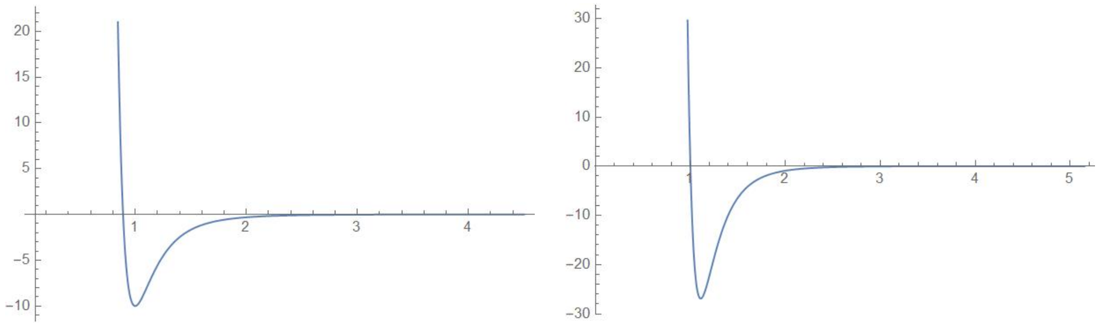
(a) Solving

$$F = -\frac{\mathrm{d}}{\mathrm{d}\bar{r}} U \tag{9}$$

we get

$$F = 12U_0R_0^6 \left(\frac{1}{r^{13}} \cdot R_0^6 - \frac{1}{r^7} \right) \tag{10}$$

Then we plot



(a) Graph of U(r)

(b) Graph pf F(r)

Figure 4: Graphs in Problem 3(a)

$12U_0R_0^{12}/r^{13}$ is responsible for repulsion while $-12U_0R_0^6/r^7$ is responsible for attraction.

(b) We rewrite the Lennard-Jones potential energy equation.

$$U = U_0\left(\left(\left(\frac{R_0}{r}\right)^6 - 1\right)^2 - 1\right) \tag{11}$$

Thus when $\left(\frac{R_0}{r}\right) = 1$, the potential energy reaches its minimum. Namely, the whole system is at its equilibrium. Therefore, R_0 refers to the distance between the pair of neutral atoms or molecules at equilibrium. Meanwhile, U_0 refers to the magnitude of the lowest energy of the whole system.

(c) We may use harmonic approximation to address this problem. Since $r = R_0$ is the equilibrium point

$$U(r) \doteq U(R_0) + \frac{1}{2}\ddot{U}(R_0)(r - R_0)^2 \tag{12}$$

Then

$$F(r) = -\frac{\mathrm{d}U(r)}{\mathrm{d}r} = \ddot{U}(R_0)r - \ddot{U}(R_0)R_0 = \ddot{U}(R_0)(r - R_0) \tag{13}$$

For SHM

$$F(x) = -kx \tag{14}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{15}$$

$$T = \frac{2\pi}{\omega} \tag{16}$$

Solving (13)(14)(15)(16)

$$k = \ddot{U}(R_0) = -\frac{72U_0}{R_0^2} \tag{17}$$

$$T = \frac{\pi}{3}\sqrt{\frac{mR_0^2}{2U_0}} \tag{18}$$

(d) The chemical interaction(like Van der Waals force) is oscillating here.

Problem 4 **Solution**

The unit of U_0 is J while the unit of α is $\frac{1}{m}$
Suppose the equilibrium position is at x_0.

$$U(x) \doteq U(x_0) + \frac{1}{2}\ddot{U}(x_0)(x - x_0)^2 \tag{19}$$

$$F(x) = -\frac{\mathrm{d}}{\mathrm{d}x}U(x) = -\ddot{U}(x_0)(x - x_0) \tag{20}$$

Meanwhile

$$\dot{U}(x) = \frac{2U_0\alpha \sin \alpha x}{\cos^3 \alpha x} \tag{21}$$

$$\dot{U}(x_0) = 0 \tag{22}$$

$$\Rightarrow \qquad x_0 = 0 \tag{23}$$

In addition

$$\ddot{U}(x) = \frac{2\alpha^2 U_0 + 4\alpha^2 U_0 \sin^2 \alpha x}{\cos^4 \alpha x} \tag{24}$$

Solving (23)(24)

$$\ddot{U}(x_0) = 2\alpha^2 U_0 \tag{25}$$

For SHM

$$F = -k(x - x_0) \tag{26}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{27}$$

$$T = \frac{2\pi}{\omega} \tag{28}$$

Solving (20)(25)(26)(27)(28)

$$T = 2\pi\sqrt{\frac{m}{2\alpha^2 U_0}} \tag{29}$$