

PHYSICS I Problem Set 2

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Problem 1 Solution

(a) Since the cable is not elastic, its length is constant. Therefore, based on the constant length, we have following relationships for pulley B

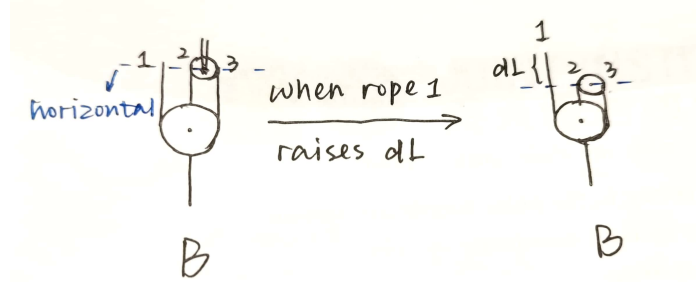


Figure 1: Analysis of pulley B

According to Figure 1, it is evident that cables below the horizontal line (blue line in Figure 1) raise $\frac{dL}{3}$ on average when the rope 1 raises dL .

Analogously, we take the pulley attached to L as our observation.

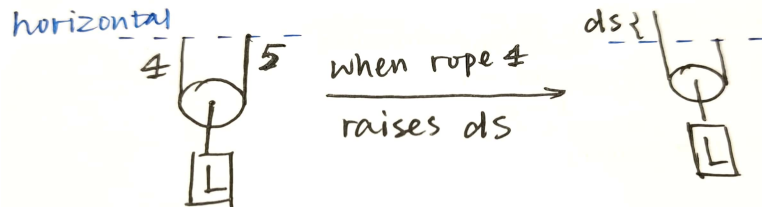


Figure 2: Analysis of pulley attached to L

It is also evident to see that cables below the horizontal line (blue line in Figure 2) raise $\frac{dS}{2}$ when cable 4 raises dS .

Since all these movements occur in the same time period, the speed is directly proportional to the distance correspondingly. Therefore, we have

$$\begin{aligned} v_B &= \frac{1}{3}v_M \\ v_L &= \frac{1}{2}v_B \\ \Rightarrow v_L &= \frac{1}{6}v_M = \frac{50}{3} \approx 16.7(\text{mm/s}) \end{aligned}$$

Since L moves upwards while the cable with $v_M = 100 \text{ mm/s}$ moves downwards, $\bar{v}_L = -16.7 \text{ mm/s}$.

Problem 2 Solution

We first select any instant t and let it be fixed. Then we can analyze both the dynamics and kinematics of this particle as follows.

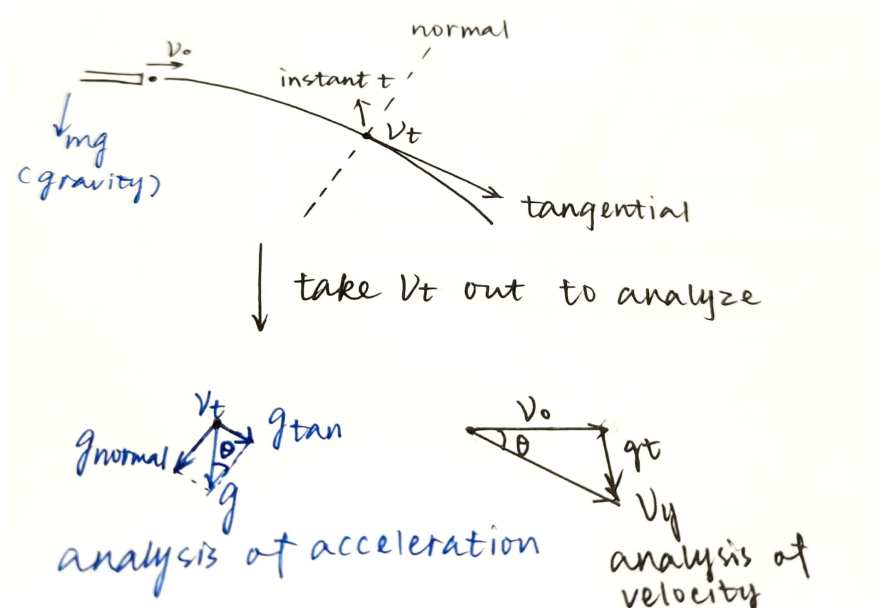


Figure 3: Force analysis and kinematics analysis of the particle

To be more specific, we can conclude equations as follows

$$\begin{cases} \tan \theta = \frac{gt}{v_0} \\ a_t = g \sin \theta \\ a_n = g \cos \theta \end{cases}$$

Solve this system we get

$$\begin{cases} a_t = \frac{g^2 t}{\sqrt{g^2 t^2 + v_0^2}} \hat{a}_t \\ a_n = \frac{g v_0}{\sqrt{g^2 t^2 + v_0^2}} \hat{a}_n \end{cases}$$

Based on geometric triangles, we also know

$$\bar{v}_y = gt = a_t + a_n$$

Problem 3 Solution

(a) $\bar{v} = \bar{v}_1 + \bar{v}_2 = v_1 \hat{n}_y + v_0 \sin\left(\frac{\pi y}{L}\right) \hat{n}_x$

$$|\bar{v}| = \sqrt{v_1^2 + (v_0 \sin \frac{\pi y}{L})^2}$$

(b) Evidently, $y(t) = v_1 t$. Then we apply integral

$$x(t) = \int v_0 \sin \frac{\pi y}{L} = v_0 \int \sin \frac{\pi v_1 t}{L} = -\frac{L}{\pi v_1} \cos \frac{\pi v_1 t}{L}$$

Therefore, trajectory $\bar{r} = (-\frac{L}{\pi v_1} \cos \frac{\pi v_1 t}{L})\hat{n}_x + v_1 t \hat{n}_y$
(c)

$$x(t) = \int_0^t v_0 \sin \frac{\pi y}{L} = v_0 \int_0^t \sin \frac{\pi v_1 t}{L} = -\frac{L}{\pi v_1} \cos \frac{\pi v_1 t}{L} \Big|_0^{L/v_1} = \frac{2v_0}{\pi v_1} L$$

Problem 4 Solution

The units of b and c are m/s^2

(a) Since $x = r \cos \varphi$ and $y = r \sin \varphi$, we have

$$\begin{cases} r \cos \varphi = bt^2 \\ r \sin \varphi = -ct^2 \end{cases}$$

Therefore, we get the parametric equations of the trajectory

$$\begin{cases} r = \sqrt{b^2 + c^2} t^2 \\ \varphi = \arctan(-\frac{c}{b}) \end{cases}$$

(b) Since $x(t) = bt^2$ and $y(t) = -ct^2$, $\frac{x}{y} = -\frac{b}{c} \Rightarrow cx + by = 0$

Therefore, the implicit form in polar coordinate system is $cr \cos \varphi + br \sin \varphi = 0$

(c) In the Cartesian coordinate system, $v(x) = x(t) = 2bt$, $v(y) = y(t) = -2ct$.

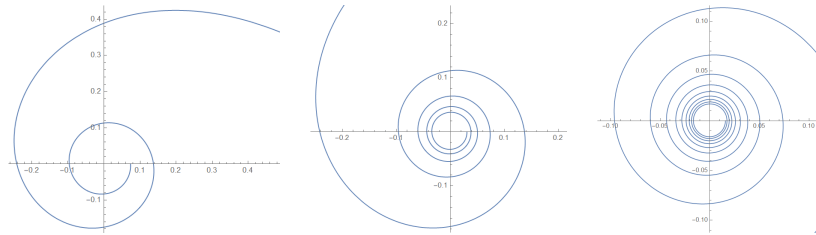
Therefore, $\bar{v} = 2bt\hat{n}_x - 2ct\hat{n}_y$, $\bar{a} = \dot{\bar{v}} = 2b\hat{n}_x - 2c\hat{n}_y$

In polar coordinate system, $\bar{r} = r\hat{n}_r$, $\bar{v} = \dot{\bar{r}} = \dot{r}\hat{n}_r + r\dot{\hat{n}}_r = \dot{r}\hat{n}_r + r\dot{\varphi}\hat{n}_\varphi = 2\sqrt{b^2 + c^2}t\hat{n}_r$, $\bar{a} = (\ddot{r} - r(\dot{\varphi})^2)\hat{n}_r + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{n}_\varphi = 2\sqrt{b^2 + c^2}\hat{n}_r$

Problem 5 Solution

The unit of c is s^{-1} , the unit of r_0 is m .

(a) $\frac{r}{r_0} = 1 - ct \Rightarrow ct = 1 - \frac{r}{r_0}$
 $\Rightarrow \varphi = \frac{1-r/r_0}{r/r_0} = \frac{r_0-r}{r}$



(a) φ ranges from 0 to 4π (b) φ ranges from 0 to 10π (c) φ ranges from 0 to 20π

Figure 4: Trajectory of the particle

(b) $\bar{v} = \dot{\bar{r}} = \dot{r}\hat{n}_r + r\dot{\hat{n}}_r = \dot{r}\hat{n}_r + r\dot{\varphi}\hat{n}_\varphi = -cr_0\hat{n}_r + \frac{cr_0 - c^2r_0t}{(1-ct)^2}\hat{n}_\varphi$

(c) $\bar{a} = (\ddot{r} - r(\dot{\varphi})^2)\hat{n}_r + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{n}_\varphi = (-c^2r_0 + c^3r_0t)/(1-ct)^4\hat{n}_r + (2c^2r_0 - 2r_0c^2)/(1-ct)^3\hat{n}_\varphi$

(d)

Problem 6 Solution

- (a) The unit of $\frac{\pi}{3}$ is radian, the unit of π is s^{-1} , the unit of 6 is cm , the unit of 2 is s^{-1} .
- (b) $\bar{v} = \dot{\bar{r}} = \dot{r} \hat{n}_r + r \dot{\hat{n}}_r = \dot{r} \hat{n}_r + r \dot{\varphi} \hat{n}_\varphi = 12e^{-2} \hat{n}_r + (18e^{-2} - 18) \hat{n}_\varphi$
- (c) $\bar{a} = (\ddot{r} - r(\dot{\varphi})^2) \hat{n}_r + (r \ddot{\varphi} + 2\dot{r} \dot{\varphi}) \hat{n}_\varphi = (54 - 78e^{-2}) \hat{n}_r - 72e^{-2} \hat{n}_\varphi$
- (d) For the rod, $|\bar{r}|$ is constant. Therefore, $a_{collor_to_rod} = \ddot{r} \hat{n}_r + 2\dot{r} \dot{\varphi} \hat{n}_\varphi = -24e^{-2} \hat{n}_r - 72e^{-2} \hat{n}_\varphi$