PHYSICS I Problem Set 5

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Problem 1 Solution

(a) To make the amplitude maximize is to make its denominator minimize since its numerator is a constant. Namely, what we need to ensure is the minimium of

$$(\omega_0^2 - \omega_{\mathrm{dr}}^2)^2 + \left(\frac{b\omega_{\mathrm{dr}}}{m}\right)^2 \tag{1}$$

Since equation(1) is equal to

$$\left(\omega_0^2 - \frac{b^2}{2m^2} - \omega_{\mathrm{dr}}^2\right)^2 + \frac{b^4}{4m^4} \tag{2}$$

we can easily conclude that when

$$\omega_{\mathrm{dr}} = \sqrt{\omega_0^2 - b^2/2m^2} \tag{3}$$

the amplitude is at its maximium. Namely

$$\omega_{\mathrm{res}} = \sqrt{\omega_0^2 - b^2/2m^2} \tag{4}$$

(b) we rearrange the formula provided by Problem 1

$$\begin{aligned} A(\omega_{\mathrm{dr}}) &= \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{\mathrm{dr}}^2)^2 + \left(\frac{b\omega_{\mathrm{dr}}}{m}\right)^2}} && (5) \\ &= \frac{F_0}{m\sqrt{\omega_0^4 - 2\omega_0^2\omega_{\mathrm{dr}}^2 + \omega_{\mathrm{dr}}^4 + b^2\omega_{\mathrm{dr}}^2/m^2}} && (6) \end{aligned}$$

According to the description of question(b), we can perceive ω_0 and ω_{dr} as equal by default. Therefore, we then can rearrange equation(6) as follows

$$\begin{aligned} A(\omega_{\mathrm{dr}}) &\approx \frac{F_0}{m\sqrt{4(\omega_0^2 - \omega_{\mathrm{dr}}^2)^2 + \left(\frac{b\omega_{\mathrm{dr}}}{m}\right)^2}} \\ &= \frac{F_0}{2m\sqrt{\omega_0^4 - 2\omega_0^3\omega_{\mathrm{dr}} + \omega^2\omega_{\mathrm{dr}}^2 + b^2\omega_0^2/4m^2}} \\ &= \frac{F_0}{2m\omega_0\sqrt{\omega_0^2 - 2\omega_0\omega_{\mathrm{dr}} + \omega_{\mathrm{dr}}^2 + b^2/4m^2}} \\ &= \frac{F_0}{2m\omega_0\sqrt{(\omega_0 - \omega_{\mathrm{dr}})^2 + b^2/4m^2}} \end{aligned}$$

That is to say

$$A(\omega_{\mathrm{dr}}) \approx \frac{F_0}{2m\omega_0\sqrt{(\omega_0 - \omega_{\mathrm{dr}})^2 + \frac{b^2}{4m^2}}} \tag{7}$$

The figure attached to each problem are is shown below.

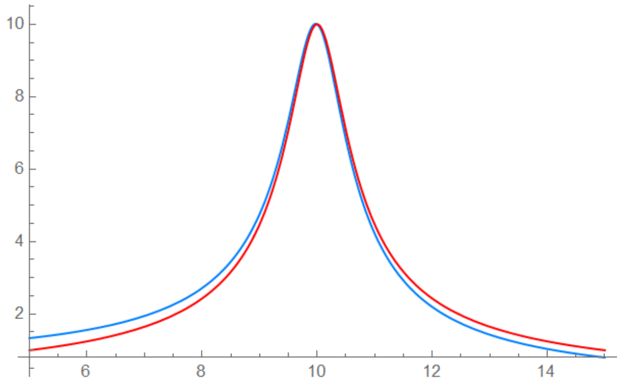


Figure 1: Plots in Problem 1

Problem 2 Solution

(a) Evidently,

$$\phi = -\frac{\pi}{4} \tag{8}$$

$$b/m = 4 \tag{9}$$

Then

$$\tan\phi = \frac{b\omega_{\mathrm{dr}}}{m(\omega_{\mathrm{dr}}^2 - \omega_0^2)} \tag{10}$$

Solving (8)(9)(10), we get

$$\omega_{\mathrm{dr}} = \sqrt{\omega_0^2 + 4} - 2 \tag{11}$$

(b) Since

$$\begin{aligned} A(\Omega_1) &= \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega_1^2)^2 + \left(\frac{b\Omega_1}{m}\right)^2}} \\ = A(\Omega_2) &= \frac{F_0}{m\sqrt{(\omega_0^2 - \Omega_2^2)^2 + \left(\frac{b\Omega_2}{m}\right)^2}} \end{aligned}$$

we get

$$(\omega_0^2 - \Omega_1^2)^2 + \left(\frac{b\Omega_1}{m}\right)^2 = (\omega_0^2 - \Omega_2^2)^2 + \left(\frac{b\Omega_2}{m}\right)^2 \tag{12}$$

Solving equation(12) we get

$$\omega_0 = \sqrt{(\Omega_1^2 + \Omega_2^2)/2 + \frac{b^2}{m^2}} \tag{13}$$

Problem 3 **Solution**

In this problem, we set the car as our frame of reference. Suppose the direction of acceleration is pointing to the right.

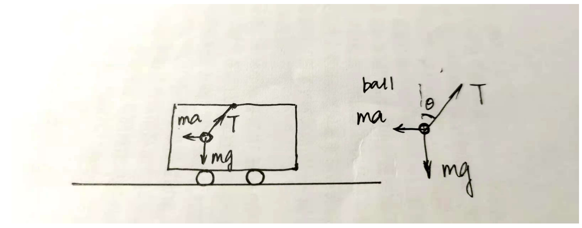
First we consider Problem 2(b) from Problem Set 3. Suppose the magnitude of acceleration is a. According to free body diagram(a), we have

$$\begin{cases} T\cos\theta = mg \\ T\sin\theta = ma \end{cases} \quad \Rightarrow \quad \theta = \arctan(\frac{a}{g})$$

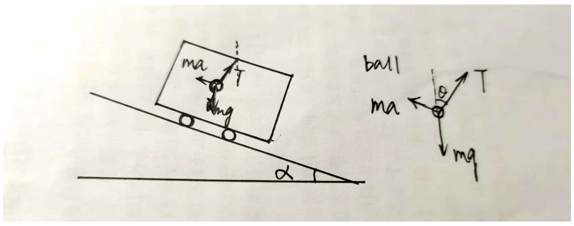
Then we consider Problem 2(c) from Problem Set 3. Suppose the mass of the car is M. Since the ball is attached to the roof of the car, it shares the same acceleration of that car. Then according to free body diagram(b), for the system we have

$$\begin{cases} Ma_{\text{car}} = Mg\sin\alpha \\ a_{\text{car}} = a_{\text{ball}} \\ ma_{\text{ball}} = mg\sin\theta \end{cases} \quad \Rightarrow \quad \theta = \alpha$$

Note that all ma in free body diagrams is the force of inertia and others are real forces.



(a) Free body diagram of Problem 3.1



(b) Free body diagram of Problem 3.2

Figure 2: Free body diagrams in Problem 3

Problem 4 **Solution**

We set the lowerset point fo the parabola as the origin point and denote the coordinate of each point on the parabola by considering the horizontal distance x between itself and the origin point.

Apparently this kind of notation cannot determine a point uniquely, however, due to symmetry of parabola, this failure in uniqueness will not affect our purpose to find all the points remaining rest. In addtion,we set the smaller angle between tangential line through the point on the parabola and the horizontal line through the origin point as θ such that $\tan\theta = \alpha x$.

(a) According to free body diagram, we have

$$\begin{cases} mg\sin\theta \le f_{\text{max}} \\ mg\cos\theta = N \\ f_{\text{max}} = \mu_s N \end{cases} \tag{14}$$

Solving equation system(14) we get

$$x \le \frac{\mu_s}{\alpha} \tag{15}$$

Therefore, all points whose horizontal distance between the lowest point of the parabola remain at rest if the container is at rest.

(b) Since the friction has two cases, we will discuss the cases separately.

Case 1. The friction is vertical to the support force but directs upwards. According to diagram(c), we know

$$\begin{cases} mg\sin\theta \leq f_{\max} + m\omega^2 x\cos\theta \\ N = mg\cos\theta + m\omega^2 x\sin\theta \\ f_{\max} = \mu_s N \end{cases} \tag{16}$$

Namely

$$\mu_s\omega^2\alpha\cdot x^2 + (\omega^2 - \alpha g)\cdot x + \mu_s g \geq 0 \tag{17}$$

Case 2. The friction is vertical to the support force but directs downwards. According to diagram(d), we know

$$\begin{cases} m\omega^2 x\cos\theta \leq f_{\max} + mg\sin\theta \\ N = mg\cos\theta + m\omega^2 x\sin\theta \\ f_{\max} = \mu_s N \end{cases} \tag{18}$$

Namely

$$\mu_s\omega^2\alpha\cdot x^2 - (\omega^2 - \alpha g)\cdot x + \mu_s g \geq 0 \tag{19}$$

We then take equation(17)(19) together. Trivially, $x>0$. Then we try to solve the inequalities.

$$\begin{aligned} \Delta_x &= (\omega^2 - g\alpha)^2 - 4(\mu_s g)(\mu_s\omega^2\alpha) \\ &= \omega^4 - (2g\alpha + 4\mu_s^2\alpha g)\omega^2 + g^2\alpha^2 \\ \Delta_{\omega^2} &= (2g\alpha + 4\mu_s^2\alpha g)^2 - 4g^2\alpha^2 \\ &= 16\mu_s^4 g^2\alpha^2 + 16\mu_s^2 g^2\alpha^2 \\ \sqrt{\Delta_{\omega^2}} &= 4\mu_s\alpha g\sqrt{\mu_s^2+1} \\ \omega_{1,2}^2 &= \frac{2g\alpha + 4\mu_s^2 g \pm 4\mu_s g\sqrt{\mu_s^2+1}}{2} \end{aligned}$$

To make the discussion more clear, we will consider the relative value of ω and αg.

From what we know in high scholl, $x_1+x_2 = -(\omega^2-\alpha g)/(\mu_s\omega^2\alpha)$ for equation(17) and $x_1+x_2 = (\omega^2-\alpha g)/(\mu_s\omega^2\alpha)$ for equation(19). Then we can finally solve equation(17)(19).

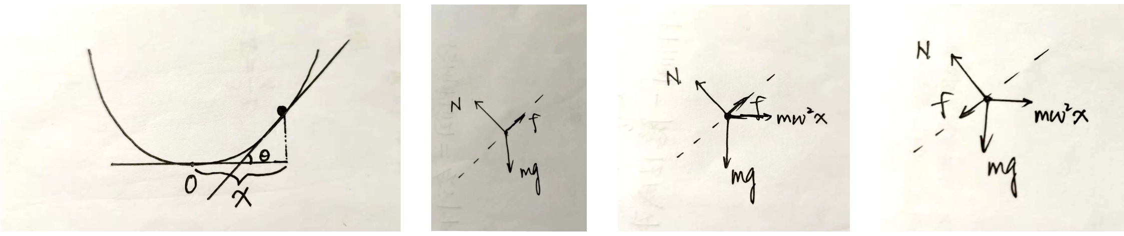
When $\omega^2 = \frac{2g\alpha+4\mu_s^2 g-4\mu_s g\sqrt{\mu_s^2+1}}{2} < \alpha g$, equation(17) works.

$$0 \leq x \leq \frac{\alpha g - \omega^2 - 4\sqrt{\Delta_x}}{2\mu_s\omega^2\alpha} \text{ or } x \geq \frac{\alpha g - \omega^2 + 4\sqrt{\Delta_x}}{2\mu_s\omega^2\alpha}$$

When $\omega^2 = \frac{2g\alpha+4\mu_s^2 g+4\mu_s g\sqrt{\mu_s^2+1}}{2} > \alpha g$, equation(19) works.

$$0 \leq x \leq \frac{\omega^2 - \alpha g - 4\sqrt{\Delta_x}}{2\mu_s\omega^2\alpha} \text{ or } x \geq \frac{\omega^2 - \alpha g + 4\sqrt{\Delta_x}}{2\mu_s\omega^2\alpha}$$

where $\sqrt{\Delta_x} = \omega^4 - (2g\alpha + 4\mu_s^2\alpha g)\omega^2 + g^2\alpha^2$



(a) sketch map of Problem 4

(b) free body diagram of Problem4(a)

(c) free body diagram of Problem4(b) case 1

(d) free body diagram of Problem4(b) case 2

Figure 3: Free body diagrams in Problem 4

Problem 5 Solution

The direction of the angular acceleration is pointing to the south pole, with no angular accelerations at the poles.

The most common phenomenon is release the water in a sink. Before the Earth started to stopp its rotational motion, the person at a pole may see the water current forms a clockwise or counterclockwise swirl while the person on the equator would not see. However, during the Earth stopping its rotational motion, the person at the pole would observe the water current forming a counterclockwise or clockwise swirl corresponding to the previous swirls, however, the one on the equator would observe nothing.

Problem 6 Solution

Based on our common sense, they obviously cannot blame their nonproficiency to Coriolis force. We now try to prove this.

For the one shooting at a wolf to the west of him, $\langle \omega, v_0 \rangle = 90°$ since the west and north is vertical to each other.

$$\begin{aligned}\overline{a_c} &= -2\overline{\omega} \times \overline{v_0} \\ &= 2 \times |\overline{\omega}| \times |\overline{v_0}| \times \sin 90° \\ &= 0.042 \text{ (m/s)}\end{aligned}$$

Then we can calculate the displacement caused by Coriolis force.

$$\Delta x = \frac{1}{2} a_c t^2 = 0.021 \text{ (m)}$$

For the one shooting at the wolf to the north of him, $\langle \omega, v_0 \rangle = 131°$.

$$\begin{aligned}\overline{a_c} &= -2\overline{\omega} \times \overline{v_0} \\ &= 2 \times |\overline{\omega}| \times |\overline{v_0}| \times \sin 131° \\ &= 0.032 \text{ (m/s)}\end{aligned}$$

Then we can calculate the displacement caused by Coriolis force.

$$\Delta x = \frac{1}{2} a_c t^2 = 0.016 \text{ (m)}$$