PHYSICS I Problem Set 6

Name: Haotian Fu

Student ID: 520021910012

Problem 1 Solution

(a) Assume $m = \lambda l$, therefore, for any tiny enough piece of rope, we have

$$W = \int (\mathrm{d}m) g l = \int \lambda g l \mathrm{d} = \frac{1}{2}\lambda g l^2 \tag{1}$$

where $m = \lambda l$. Then we know

$$W = \frac{1}{2} m g l \tag{2}$$

(b) We first calculate the value of λ_0

$$m = \int \lambda(x)\mathrm{d}x = \lambda_0 \int_0^l (-x^2 + lx)\mathrm{d}x = \frac{1}{6}\lambda_0 l^3 \tag{3}$$

Therefore

$$\lambda_0 = \frac{6m}{l^3} \tag{4}$$

We then try to find the minimium work

$$W = \int (\mathrm{d}m) g l = \int \lambda_0 x(l-x)\mathrm{d}x g x = \frac{1}{12}\lambda_0 g l^4 \tag{5}$$

where equation(4) survives. Then we know

$$W = \frac{1}{2} m g l \tag{6}$$

Problem 2 Solution

We first try to determine the how deep the cone submerged into the liquid. Suppose the depth is αH. Then

$$\frac{1}{3}\pi(\alpha R)^2 \alpha H = \frac{2}{3} \times \frac{1}{3}\pi R^2 H \tag{7}$$

$$\Rightarrow \ \alpha = \sqrt[3]{\frac{2}{3}} \tag{8}$$

We then try to denote the volume(V) of the part came out of liquid by the length(x) with the force exerting.

$$V(x) = \int \pi \cdot \left(\frac{\sqrt[3]{\frac{2}{3}} + x}{H} \cdot R \right)^2 \mathrm{d}x \tag{9}$$

$$= \pi R^2 \left(\frac{1}{3H^2}x^3 + \frac{\sqrt[3]{\frac{2}{3}}}{H}x^2 + \sqrt[3]{\frac{4}{9}}x \right) \tag{10}$$

We now can express the force F by x

$$F = \rho g V(x) \tag{11}$$

Therefore, we can calculate the minimium work now

$$W = \int_0^{(1-\sqrt[3]{2/3})H} F \mathrm{d}x \tag{12}$$

Through calculation, we then get

$$W = \left(-\frac{5}{36} + \frac{1}{6}\sqrt[3]{\frac{2}{3}}\right) \rho g \pi R^2 H^2 \tag{13}$$

Problem 3 Solution

(a) We will apply conservation of energy law.

$$mgx_m = \int_0^{x_m} F \mathrm{d}x = \frac{1}{2}kx_m^2 \tag{14}$$

$$\Rightarrow \ x_m = 0.0327 \text{ (m)} \tag{15}$$

Then we can easily calculate the maximum force F_m

$$F_m = kx_m = 98 \text{ (N)} \tag{16}$$

(b) Analogously, we apply conservation of energy law.

$$mgx_m = \int_0^{x_m} F \mathrm{d}x = 120000x_m^4 + 1500x_m^2 \tag{17}$$

$$\Rightarrow \ x_m = 0.0304 \text{ (m)} \tag{18}$$

Then we get

$$F_m = 3 \times 10^3 \times (x_m + 160x_m^3) = 105 \text{ (N)} \tag{19}$$

Problem 4 Solution

According to conservation of energy law

$$\frac{1}{2}mv_0^2 = mgx\sin\alpha + \int_0^x \mu mg\cos\alpha \mathrm{d}x \tag{20}$$

where

$$\mu = Ax \tag{21}$$

In addition, we need to ensure that

$$\mu mg\cos\alpha \geq mh\sin\alpha \tag{22}$$

Namely

$$x \geq \frac{\sin\alpha}{A\cos\alpha} \tag{23}$$

In order to show we are proving the statement on the track, the process of calculation is followed.

$$\begin{aligned} v_0^2 &= 2gx\sin\alpha + 2\int_0^x Axmg\cos\alpha \mathrm{d}x \\ &= 2gx\sin\alpha + Agx^2\cos\alpha \\ &\geq \frac{2g(\sin\alpha)^2}{A\cos\alpha} + \frac{g(\sin\alpha)^2}{A\cos\alpha} = \frac{3g(\sin\alpha)^2}{A\cos\alpha} \end{aligned}$$

Problem 5 Solution

We first calculate work done by friction

$$W_f = \int_0^{\frac{L_0}{2}} \frac{x}{L_0}\mu mg\mathrm{d}x = \frac{1}{8}\mu mgL_0 \tag{24}$$

Then we calculate the velocity

$$\frac{1}{2}mv_0^2 + W = 0 \tag{25}$$

$$\Rightarrow \; v_0 = \frac{\sqrt{\mu g L_0}}{2} \tag{26}$$

Problem 6 Solution

(a) For (A),

$$\mathbf{F}_1(\mathbf{r}) = (0, 0, 5)$$

Since $\mathbf{F}_1(\mathbf{r})$ is vertical to the x-axis

$$\mathbf{F}_1(\mathbf{r}) \cdot \mathbf{x} = 0$$

Therefore,

$$W_{\mathrm{aA}} = 0 \text{ J} \tag{27}$$

For (B),

$$\begin{aligned} \mathbf{F}_2(\mathbf{r}) &= (-2x, 0, 0) \\ \mathbf{F}_2(\mathbf{r})_x &= (-2x, 0, 0) \\ \mathbf{F}_2(\mathbf{r})_y &= (0, 0, 0) \end{aligned}$$

We calculate the work separately corrosponding to $x-$component and $y-$component.

$$W_x = \int_{-1}^{1} \mathbf{F}_2(\mathbf{r})_x \mathrm{d}x \tag{28}$$

$$W_y = \int_0^0 \mathbf{F}_2(\mathbf{r})_y \mathrm{d}y \tag{29}$$

$$W_{\mathrm{aB}} = W_x + W_y \tag{30}$$

Therefore,

$$W_{\mathrm{aB}} = 0 \text{ J} \tag{31}$$

(b) For (A),

$$\begin{aligned} \mathbf{F}_1(\mathbf{r}) &= (0, -xy, 5) \\ \mathbf{F}_1(\mathbf{r})_x &= (0, 0, 0) \\ \mathbf{F}_1(\mathbf{r})_y &= (0, xy, 0) \end{aligned}$$

Notice that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x \tag{32}$$

$$\Rightarrow \ \mathrm{d}y = 2x\mathrm{d}x \tag{33}$$

We then calculate the work separately corrosponding to $x-$component and $y-$component.

$$W_x = \int_{-1}^{1} \mathbf{F}_1(\mathbf{r})_x \mathrm{d}x \tag{34}$$

$$W_y = \int_{0}^{0} \mathbf{F}_1(\mathbf{r})_y \mathrm{d}y = \int_{-1}^{1} -(x^3 - x)2x \ \mathrm{d}x \tag{35}$$

$$W_{\mathrm{aB}} = W_x + W_y \tag{36}$$

Therefore,

$$W_{\mathrm{bA}} = \frac{8}{15} \text{ J} \tag{37}$$

For (B)

$$\begin{aligned} \mathbf{F}_2(\mathbf{r}) &= (-2x, 0, y - xy) \\ \mathbf{F}_2(\mathbf{r})_x &= (-2x, 0, 0) \\ \mathbf{F}_2(\mathbf{r})_y &= (0, 0, 0) \end{aligned}$$

We calculate the work separately corrosponding to $x-$component and $y-$component.

$$W_x = \int_{-1}^{1} \mathbf{F}_2(\mathbf{r})_x \mathrm{d}x \tag{38}$$

$$W_y = \int_{0}^{0} \mathbf{F}_2(\mathbf{r})_y \mathrm{d}y \tag{39}$$

$$W_{\mathrm{aB}} = W_x + W_y \tag{40}$$

Therefore,

$$W_{\mathrm{aB}} = 0 \text{ J} \tag{41}$$

Problem 7 **Solution**

(a)