PHYSICS I Problem Set 11

Name: Haotian Fu

Student ID: 520021910012

Problem 1

Solution

(a) According to the FBD, we have

$$\begin{cases} f \cdot h = F \cdot \frac{1}{2}h \\ \mu_s(T\cos(36.9^\circ) + W) = f \\ F = f + T \cdot \sin(36.9^\circ) \end{cases} \tag{1}$$

Solving Eq. (1), we know

$$\begin{cases} T = \frac{1000}{3}N \\ f = 200N \\ F = 400N \end{cases} \tag{2}$$

(b) Analogously, we list equations as follows

$$\begin{cases} f \cdot h = F \cdot \left(1 - \frac{4}{10}\right) h \\ \mu_s(T\cos(36.9^\circ) + W) = f \\ F = f + T \cdot \sin(36.9^\circ) \end{cases} \tag{3}$$

Solving Eq. (3), we know

$$\begin{cases} T = 750N \\ f = 300N \\ F = 750N \end{cases} \tag{4}$$

(c) Suppose the **critical height** is xh where $x \in (0, 1)$. Then we get

$$\begin{cases} f \cdot h = F \cdot (1 - x)\, h \\ \mu_s(T\cos(36.9^\circ) + W) = f \\ F = f + T \cdot \sin(36.9^\circ) \end{cases} \tag{5}$$

Solving Eq. (5), we get

$$\begin{cases} T = \frac{1000x}{5-7x}N \\ \\ f = \frac{600-600x}{5-7x}N \\ \\ F = \frac{600}{5-7x}N \end{cases} \tag{6}$$

Since at **critical height**, no matter how great the F is, it cannot make the post slip, the denominator of our ideal "static" situation converges to 0. Namely

$$h_{\text{critical}} = \frac{5}{7}h \approx 0.71h \tag{7}$$

Problem 2

Solution

Suppose the **young's modulus** of steel is Y. Then according to the definition of **Young's modulus**

$$Y \stackrel{def}{=\!=} \frac{F/A}{\Delta L/L} \tag{8}$$

We denote the cross-area A as

$$A = \frac{FL}{\Delta L \cdot Y} \tag{9}$$

Moreover, we know

$$A = \pi R^2 = \frac{\pi D^2}{4} \tag{10}$$

where R is radius of the cross-area and D is the diameter of the cross-area.

Then solving Eq. (9) and Eq. (10), we have

$$D = \sqrt{\frac{4FL}{\pi \Delta L Y}} \tag{11}$$

where Y is the **Young's modulus** of steel.

Problem 3

Solution

According to the definition of Young's modulus provided in this problem

$$Y \stackrel{def}{=\!=} \frac{W/A}{\Delta l/l_0} \tag{12}$$

Hooke's law

$$F = -k\Delta l \tag{13}$$

In this problem

$$W = F \tag{14}$$

Therefore, the magnitutde of k is

$$k = \frac{YA}{l_0} \tag{15}$$

Problem 4

Solution

Suppose the desity of water is ρ. According to **Bernoulli's Equation**, for pipe C

$$p_{\text{atm}} + \rho g h_1 = p_C + \frac{1}{2}\rho v_C^2 \tag{16}$$

Since C and E are in the same pipe

$$p_C = p_E \tag{17}$$

For pipe E

$$p_{\text{atm}} = p_E + \rho g h_2 \tag{18}$$

For pipe D

$$p_{\text{atm}} + \frac{1}{2}\rho v_D^2 = p_C + \frac{1}{2}\rho v_C^2 \tag{19}$$

$$A_C v_C = A_D v_D \tag{20}$$

Solving Eq. (16)(17)(18), we get

$$h_2 = 3h_1 \tag{21}$$

Problem 5

Solution

(a) Suppose the desity of water is ρ. According to **Bernoulli's Equation**

$$p_{\text{atm}} + \rho g h = p_{\text{atm}} + \frac{1}{2}\rho v_0^2 + \rho g(h-y) \tag{22}$$

Then we get the initial velocity of the jet of water flowing out of the hole

$$v_0 = \sqrt{2gy} \tag{23}$$

Suppose the jet of water needs t to hit the ground from the hole. List kinematic equations as follows.

$$\begin{cases} \frac{1}{2}gt^2 = h - y \\ v_0 t = D \end{cases} \tag{24}$$

Therefore, we finally get

$$D = \sqrt{4y(h-y)} \tag{25}$$

(b) Suppose the polynomial $\mathcal{P} = y(h-y)$. Apparently, to maximize D is to maximize $\mathcal{P}$.

Since

$$\dot{\mathcal{P}} = -2y + h$$

we deduce that if and only if $y = \frac{1}{2}h$, $\mathcal{P}$ reaches its maximum.

Therefore, the hole should be placed at a depth $y = h/2$ for the jet to cover a maximum horizontal distance