Dynamics

Introduction
and
Newton's Laws of Motion

Agenda

- Kinematics vs Dynamics
- Force: nature, properties, and mathematical description
- Newton's laws

Dynamics vs Kinematics

KINEMATICS: describes motion quantitatively without studying its cause

DYNAMICS: identifies the cause of motion

What does make particles move?

INTERACTION

Four Fundamental Interactions

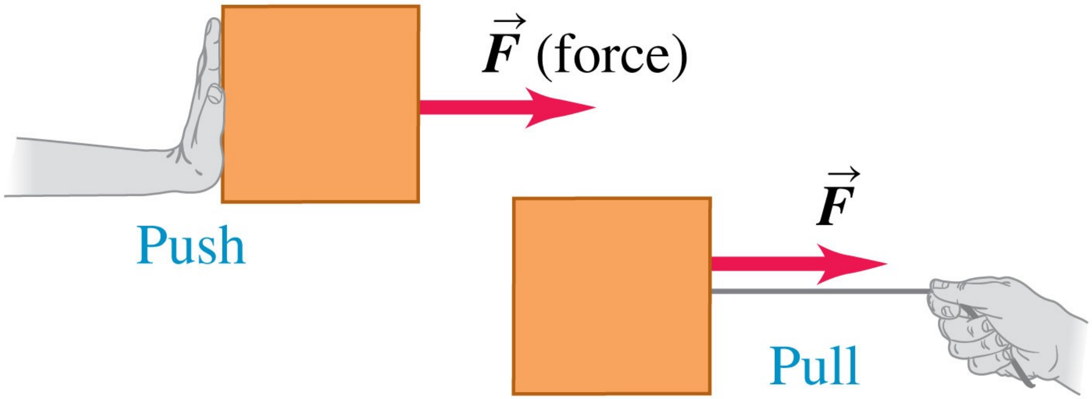
Interaction	Particles Involved	Relative Strength	Range
Gravitational always attractive holds planets in their orbits around Sun	any massive particle	$\sim 10^{-38}$	infinite
Electromagnetic attractive/repulsive fundamental in optics, chemistry, biology; source of friction	electric charge	$\sim 10^{-2}$	infinite
Weak necessary for buildup of heavy nuclei; responsible for radioactive decay (beta decay)	quarks, leptons	$\sim 10^{-6}$	short $\sim 10^{-18}$ m (0.1% of the diameter of the proton)
Strong holds protons and neutrons together in the nucleus	hadrons (protons, neutrons, mesons)	1	short $\sim 10^{-15}$ m (diameter of a medium sized nucleus)

Dynamics

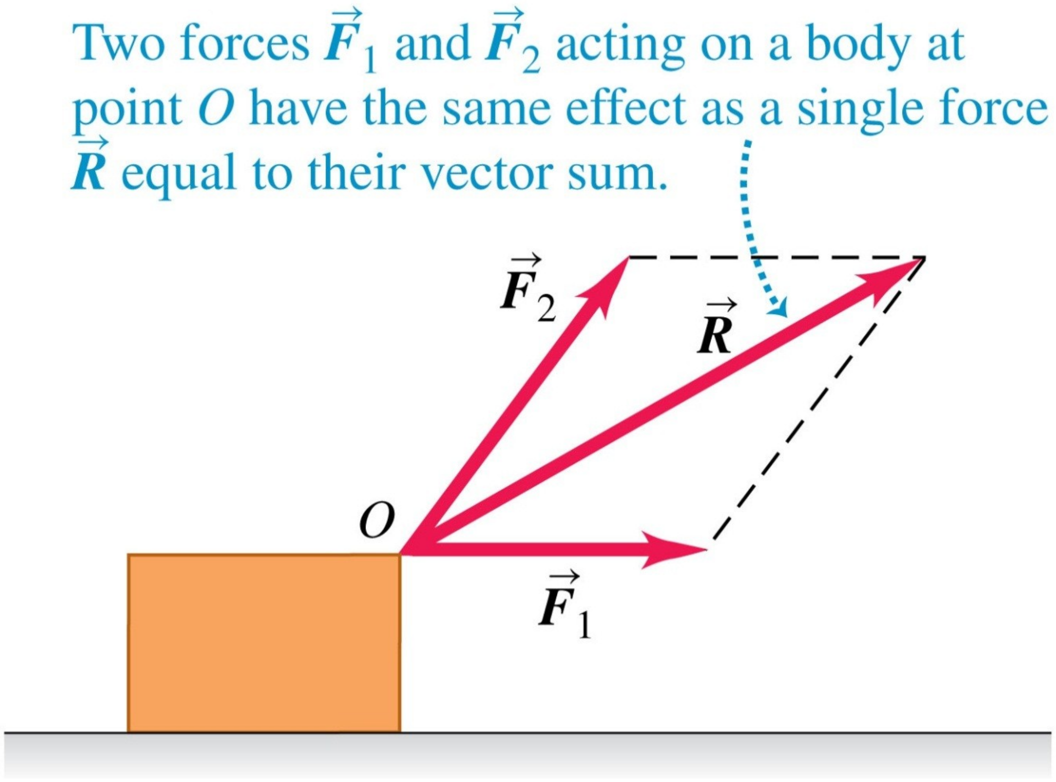
- Causality in mechanics was first understood in the late 17th century by Sir Isaac Newton.
- Newton formulated three laws governing moving objects now known as **Newton's laws of motion**.
- Newton's laws were deduced from huge amounts of **experimental evidence**.
- The laws are simple to state but resulting equations may be intricate to solve.

Key Concept: Force

- Force represents **interaction** between two objects or an object and its environment. Interactions (and hence forces) are of material origin.
- Force is a **vector quantity** (has magnitude and direction)
- **SI unit**: Newton [$1N = 1\ kg \cdot 1\ m/s^2$]



Superposition of Forces



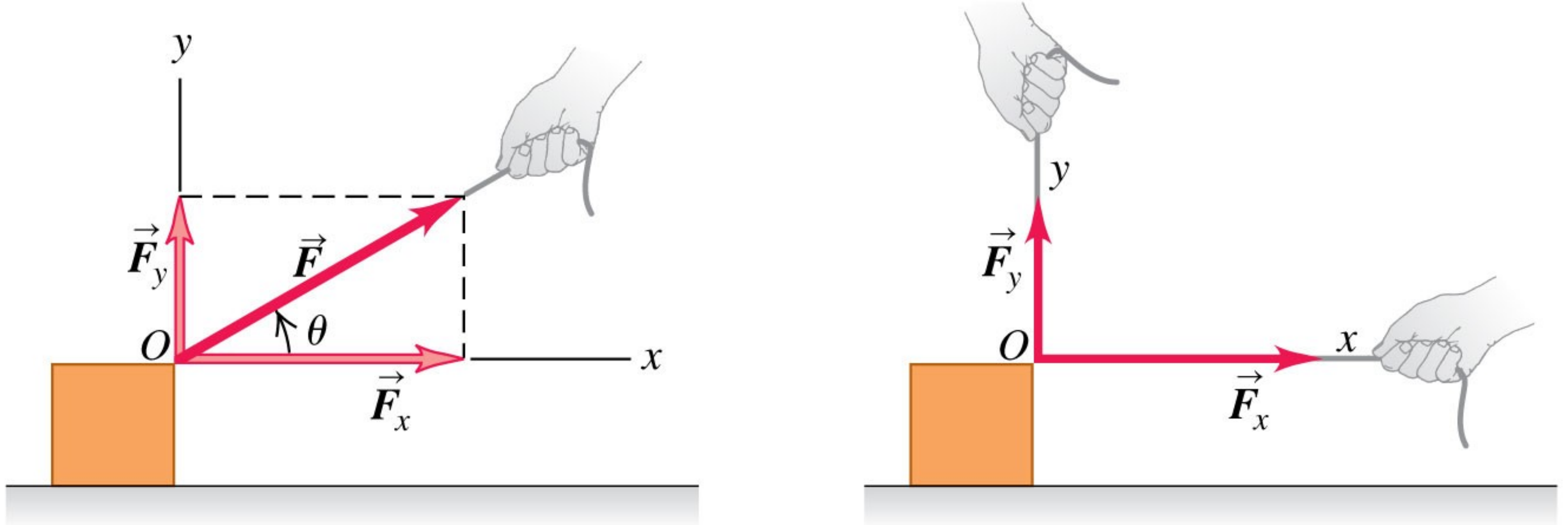
The vector sum of all forces acting upon an object is called the **resultant** of the forces or the **net force**.

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_N = \sum_{i=1}^{N} \mathbf{F}_i$$

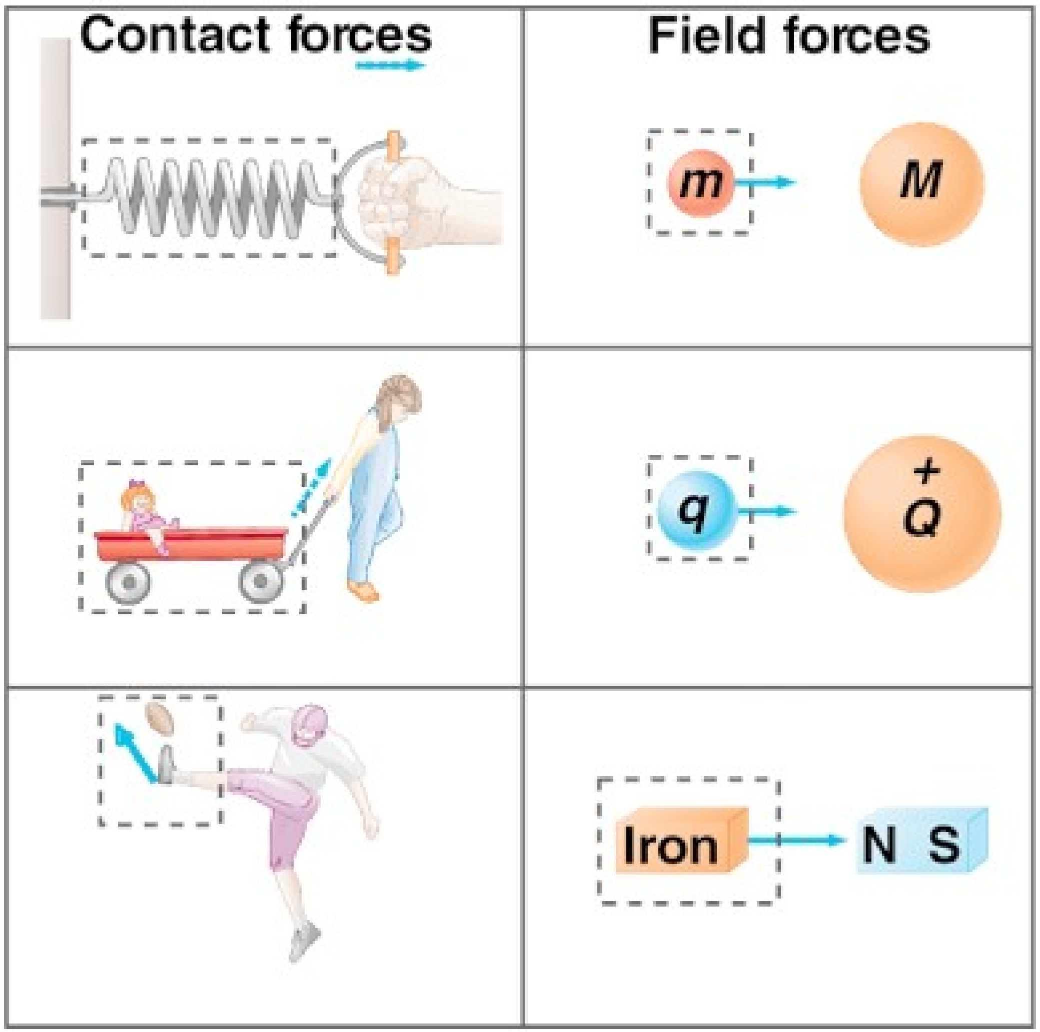
Decomposition of a Force

(a) Component vectors: $\vec{F}_x$ and $\vec{F}_y$
Components: $F_x = F\cos\theta$ and $F_y = F\sin\theta$

(b) Component vectors $\vec{F}_x$ and $\vec{F}_y$ together have the same effect as original force $\vec{F}$.



Nature of the Force: Macroscopic Classification

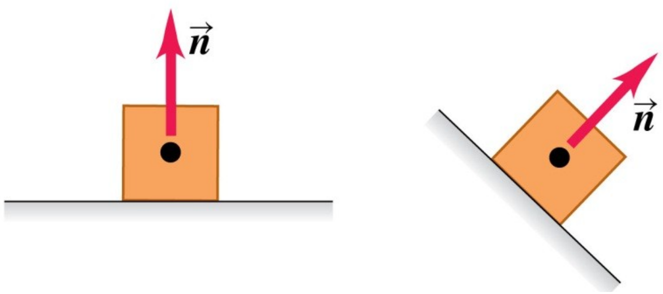


at the atomic scale

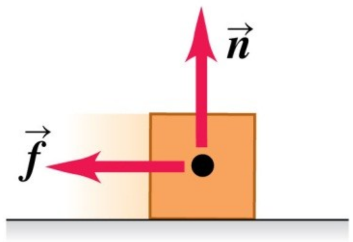
contact forces are field forces (electromagnetic in their nature)

Examples of Common Forces

- **Normal force**: When an object pushes on a surface, the surface pushes back on the object in the direction perpendicular to the surface. It is an example of a contact force.

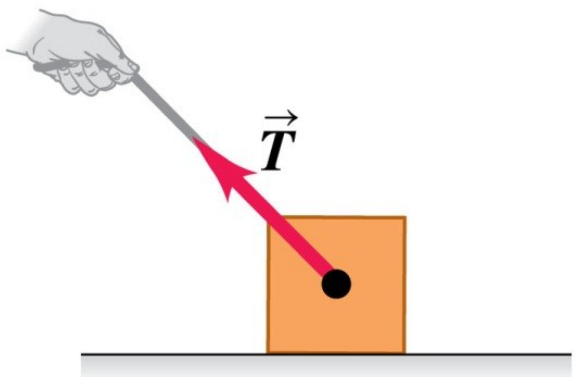


- **Frictional force**: This force occurs when a surface resists sliding of an object and is parallel to the surface. Friction is a contact force.

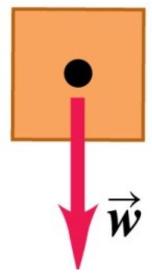


Examples of Common Forces

- **Tension force**: A pulling force exerted on an object by a rope or cord. This is a contact force.



- **Weight**: The pull of gravity on an object. This is a long-range force.



Typical Force Magnitudes

Sun's gravitational force on the earth	3.5×10^{22} N
Thrust of a space shuttle during launch	3.1×10^{7} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250-lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

Alternative Units

British system: mass is measured in *slugs*, distance in *feet*, and force in *pounds*.

cgs system: mass is measured in *grams*, distance in *centimeters*, and force in *dynes*.

System of Units	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
cgs	dyne (dyn)	gram (g)	cm/s^2
British	pound (lb)	slug	ft/s^2

Newton's Laws of Motion

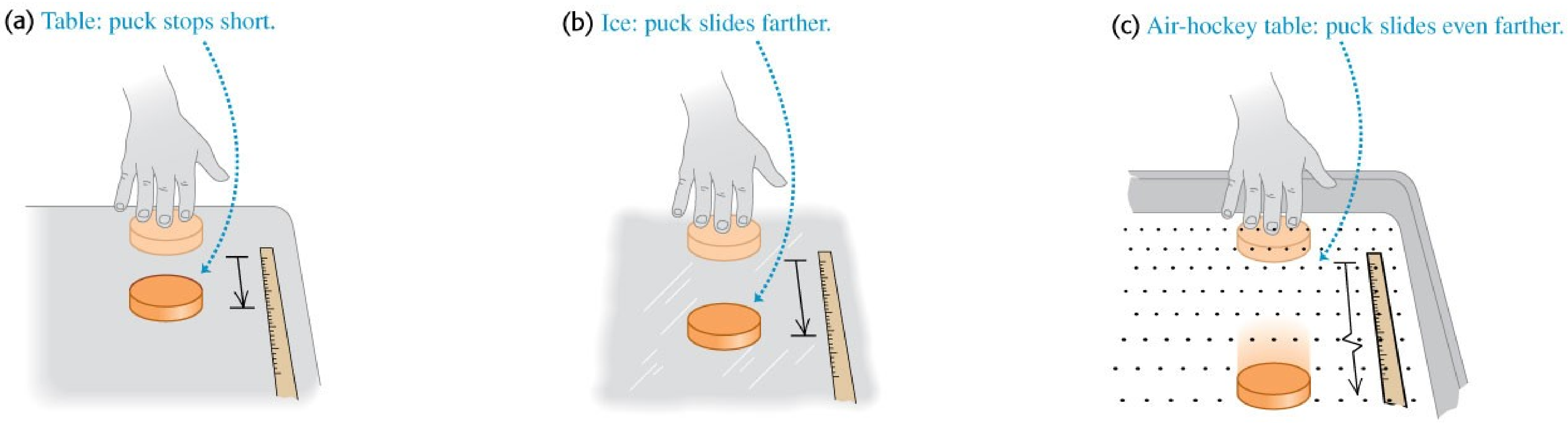
Three laws – called **Newton's Laws of Motion** – are statements about motion of objects. They were first formulated in the 17th century by Sir Isaac Newton (*Philosophiae Naturalis Principia Mathematica*).

The laws **identify the cause** of motion, and allow to **predict the evolution** of the system.

Newton's Laws were **not derived**, but **deduced** from huge amounts of **experimental evidence**.

Newton's First Law

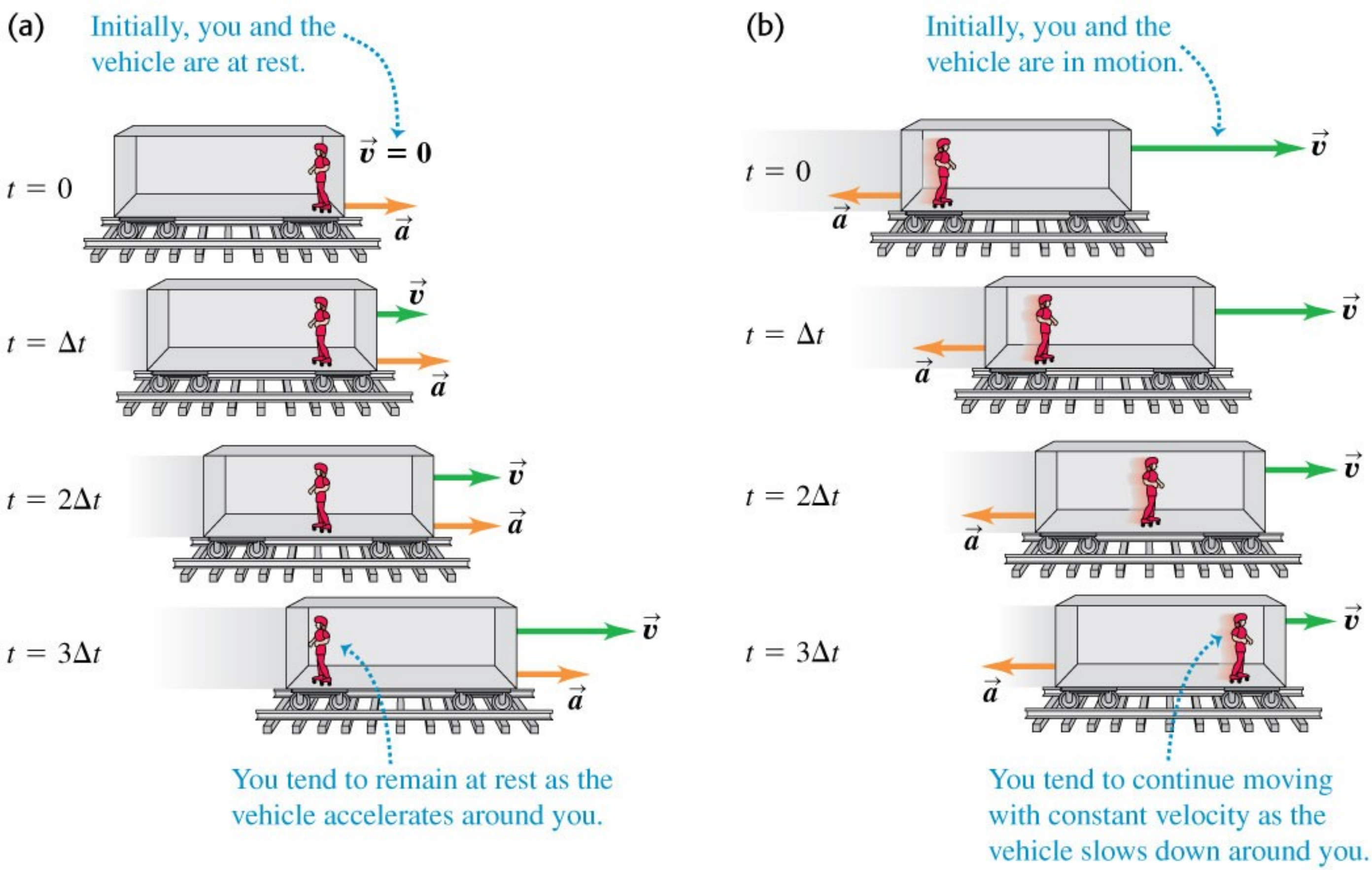
*A particle acted upon by **zero net force** moves with **constant velocity**.*



constant velocity (vector!) ⇔ zero net force

When is Newton's First Law Valid?

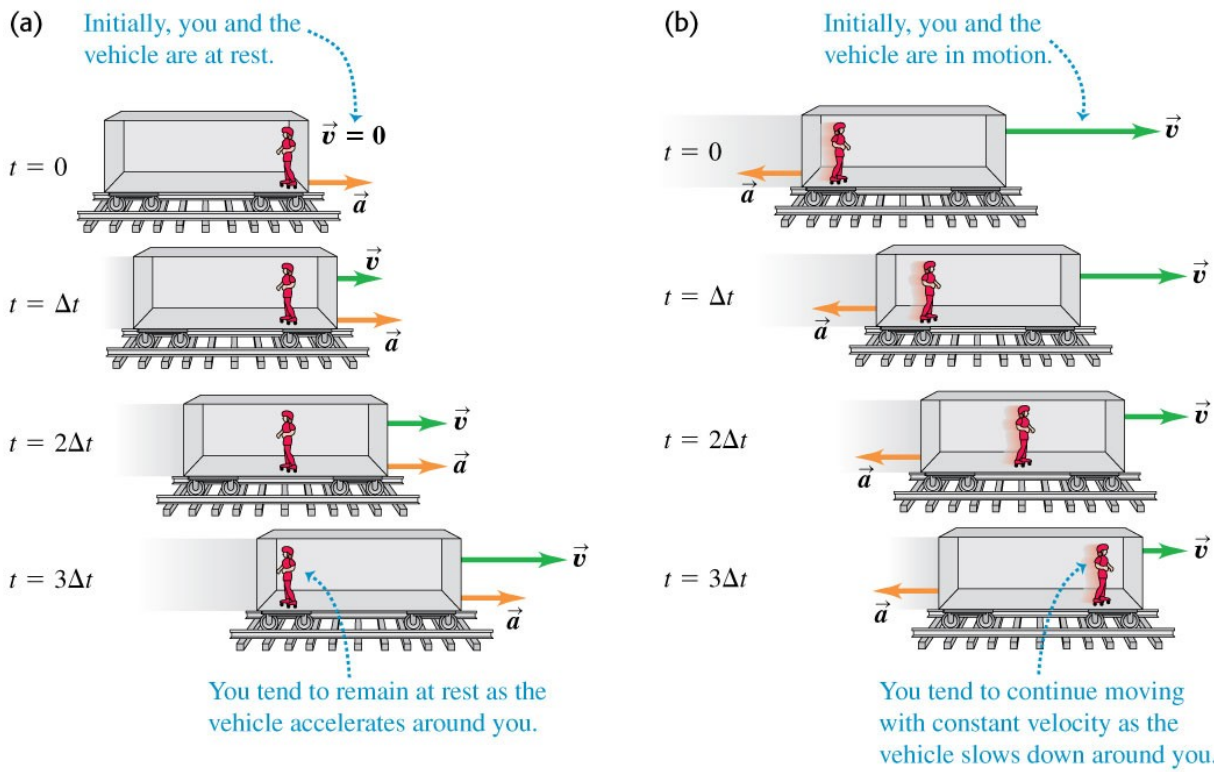
Example: car accelerating along a straight line



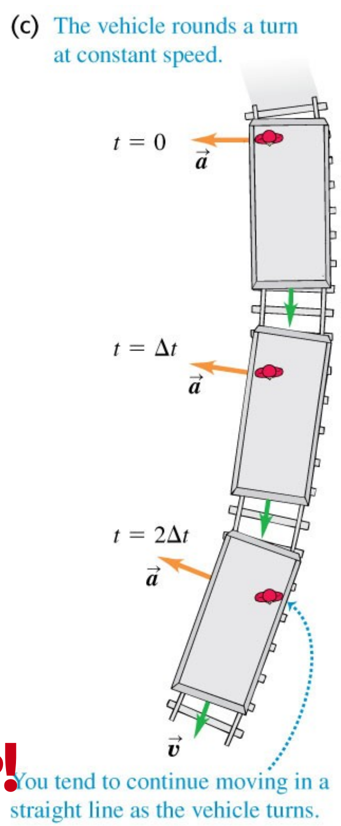
In the frame of reference associated with the rail track no net force acts on the rider, so the rider keeps constant velocity...

When is Newton's First Law Valid (contd)?

...but if seen in the frame of reference of the accelerating vehicle, it **appears** that the rider is **being pushed**.



another example: car moving along a curved trajectory



CONCLUSION: Newton’s first law is valid only in a special class of *frames of reference*!

Newton's First Law: Comment

Newton's first law identifies a particular class of frames of reference: **inertial frames of reference**.

An **inertial frame of reference** is a frame of reference, such that *if the net force on a particle is zero, then its acceleration is zero and vice versa.*

An inertial frame of reference ***exists****.*

equivalent formulation of Newton's first law

What would make a good inertial frame of reference?

Earth?

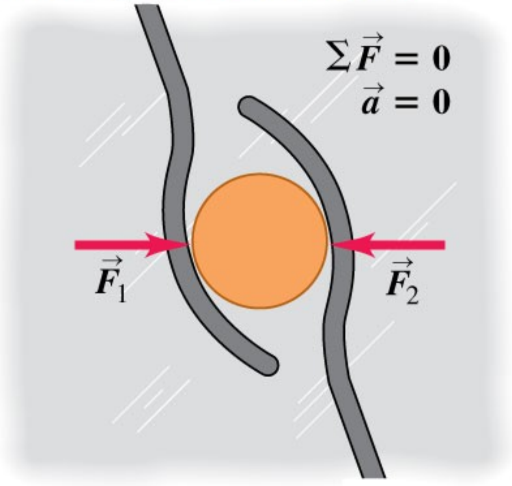
Sun?

distant galaxies?

...?

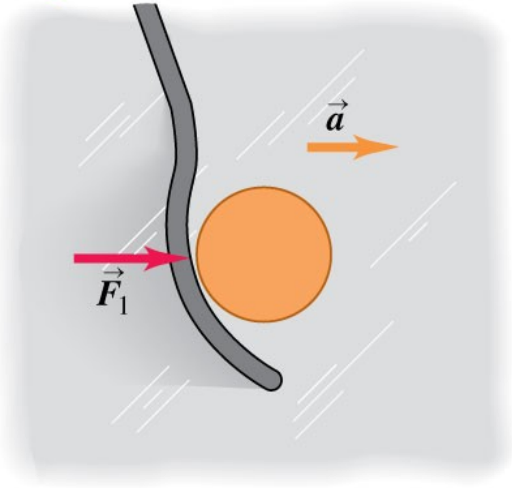
Newton's Second Law

(b) An object acted on by forces whose vector sum is zero behaves as though no forces act on it.



net force is zero,
resulting in no acceleration
(first law)

(a) A puck on a frictionless surface accelerates when acted on by a single horizontal force.

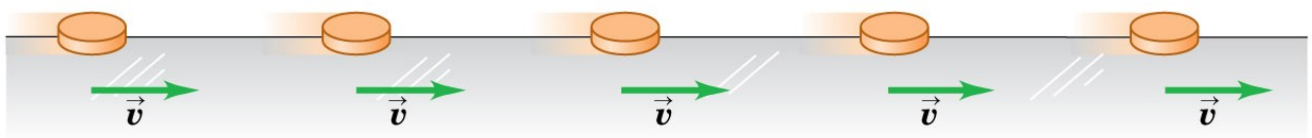


non-zero net force acts,
the object moves
with non-zero acceleration

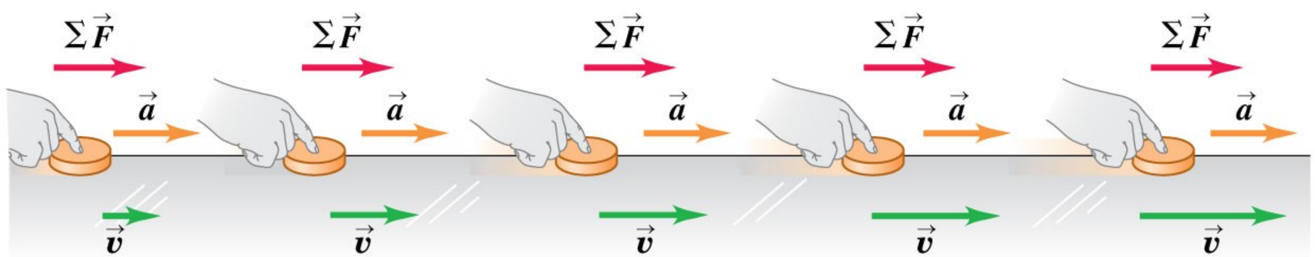
Newton's Second Law

Observation I: If the net force on an object is not zero, it causes the object to accelerate.

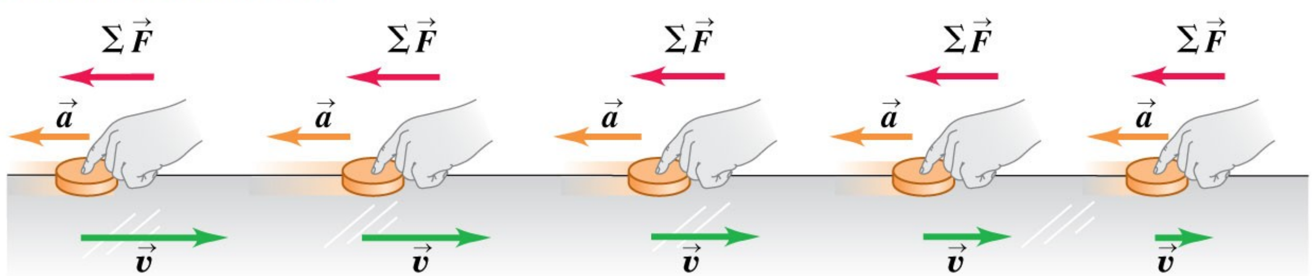
(a) A puck moving with constant velocity (in equilibrium): $\sum \vec{F} = \mathbf{0}$, $\vec{a} = \mathbf{0}$



(b) A constant net force in the direction of motion causes a constant acceleration in the same direction as the net force.



(c) A constant net force opposite the direction of motion causes a constant acceleration in the same direction as the net force.

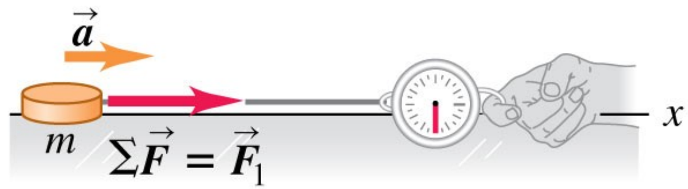


Newton's Second Law

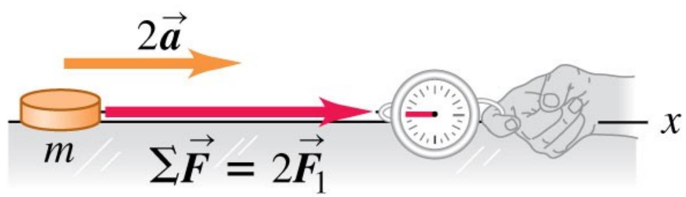
Observation II:

Acceleration of the object is directly proportional to the net force acting upon it.

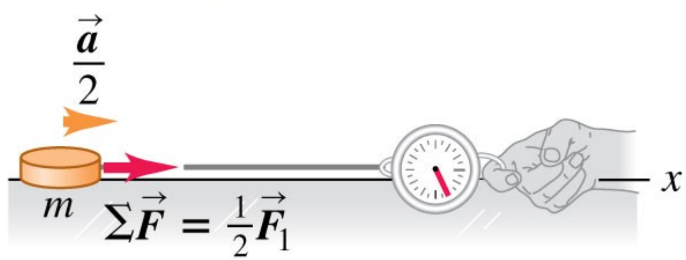
(a) A constant net force $\Sigma\vec{F}$ causes a constant acceleration $\vec{a}$.



(b) Doubling the net force doubles the acceleration.



(c) Halving the force halves the acceleration.

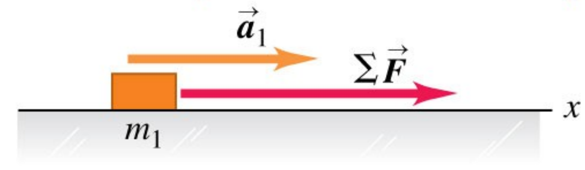


Newton's Second Law

Observation III:

If the net force on an object is fixed, the acceleration of the object is inversely proportional to the object's mass.

(a) A known force $\Sigma\vec{F}$ causes an object with mass m_1 to have an acceleration $\vec{a}_1$.



(b) Applying the same force $\Sigma\vec{F}$ to a second object and noting the acceleration allow us to measure the mass.

(c) When the two objects are fastened together, the same method shows that their composite mass is the sum of their individual masses.

Newton's Second Law: Formulation

*In an inertial frame of reference, **acceleration** of a particle is **directly proportional to the net force** acting upon it, and **inversely proportional to its mass**.*

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

net force

resulting acceleration

object's mass
(measure of the amount of matter in an object, and its inertia)

Newton's Second Law: Do We Still Need the First Law?

Conclusion from Newton's second law:
Zero net force implies zero acceleration.

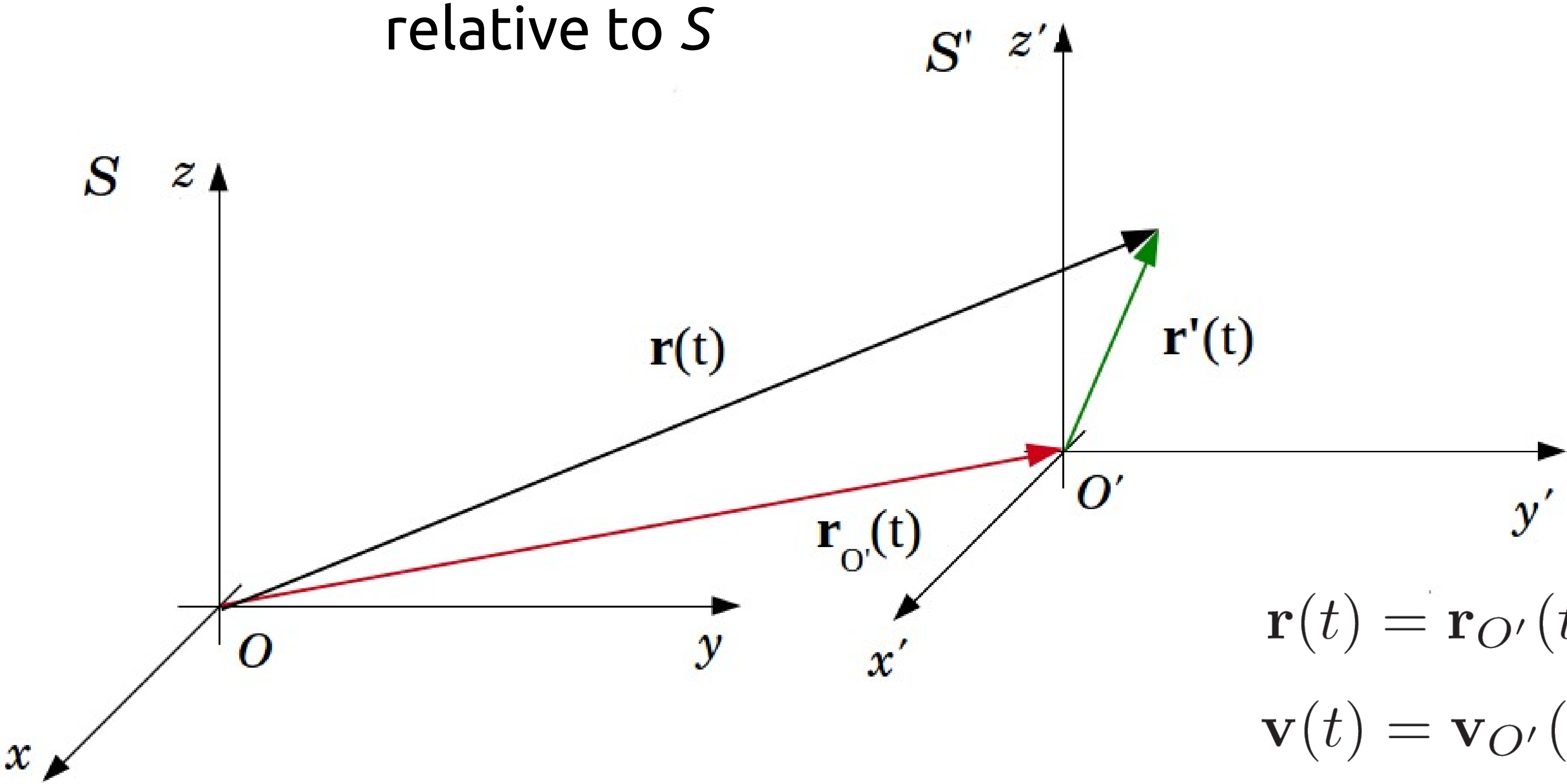
So, isn't the first law included in the second one?! **NO!**
Do we need the first law?! **YES!**

> "*In an <u>inertial frame of reference</u>, the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to the mass of the object.*"

The first law ensures that we have an inertial frame of reference.
Only then we can use the second law (it is not valid in non-inertial frames).

Comment*: Equivalence of All Inertial FoRs

assumptions: *S* is an inertial FoR, *S'* moves with constant velocity relative to *S*



$$\mathbf{r}(t) = \mathbf{r}_{O'}(t) + \mathbf{r}'(t)$$

$$\mathbf{v}(t) = \mathbf{v}_{O'}(t) + \mathbf{v}'(t)$$

If $\mathrm{v}_{O'} = \text{const}$ (i.e. *S'* moves w.r.t. *S* along a straight line with constant speed), then $\mathbf{a}(t) = \mathbf{a}'(t)$ and zero force implies zero acceleration in both FoRs, so *S'* is an inertial FoR, too.

Conclusion: Enough to have **one** inertial FoR

Comment*: Mathematical Structure of Newton's Second Law

Given forces acting on a particle, how to find characteristics of its motion (acceleration, velocity, position)?

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \qquad \mathbf{v} = \frac{d\mathbf{r}}{dt}$$

In general, force may depend on the position and velocity of a particle, as well as on time

$$\mathbf{F} = \mathbf{F}(\mathbf{v}, \mathbf{r}, t)$$

$$\mathbf{a} = \frac{\mathbf{F}}{m} \iff \boxed{\frac{d^2\mathbf{r}}{dt^2} = \frac{\mathbf{F}\left(\frac{d\mathbf{r}}{dt}, \mathbf{r}, t\right)}{m}}$$

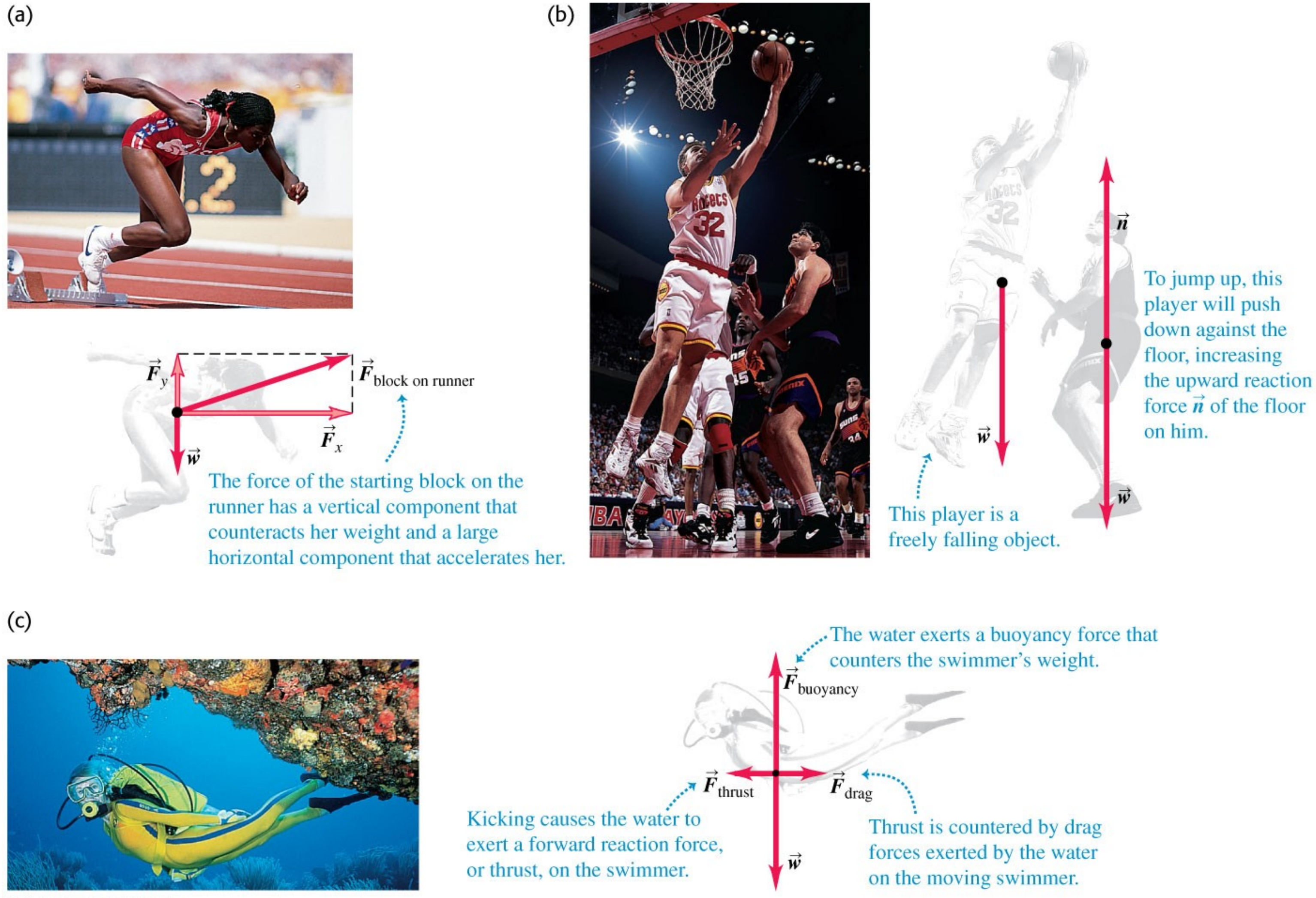
Newton's equation of motion

2^{nd} order ordinary differential equation + initial cond

$\mathbf{v}(t_0) = \mathbf{v}_0$ and $\mathbf{r}(t_0) = \mathbf{r}_0$

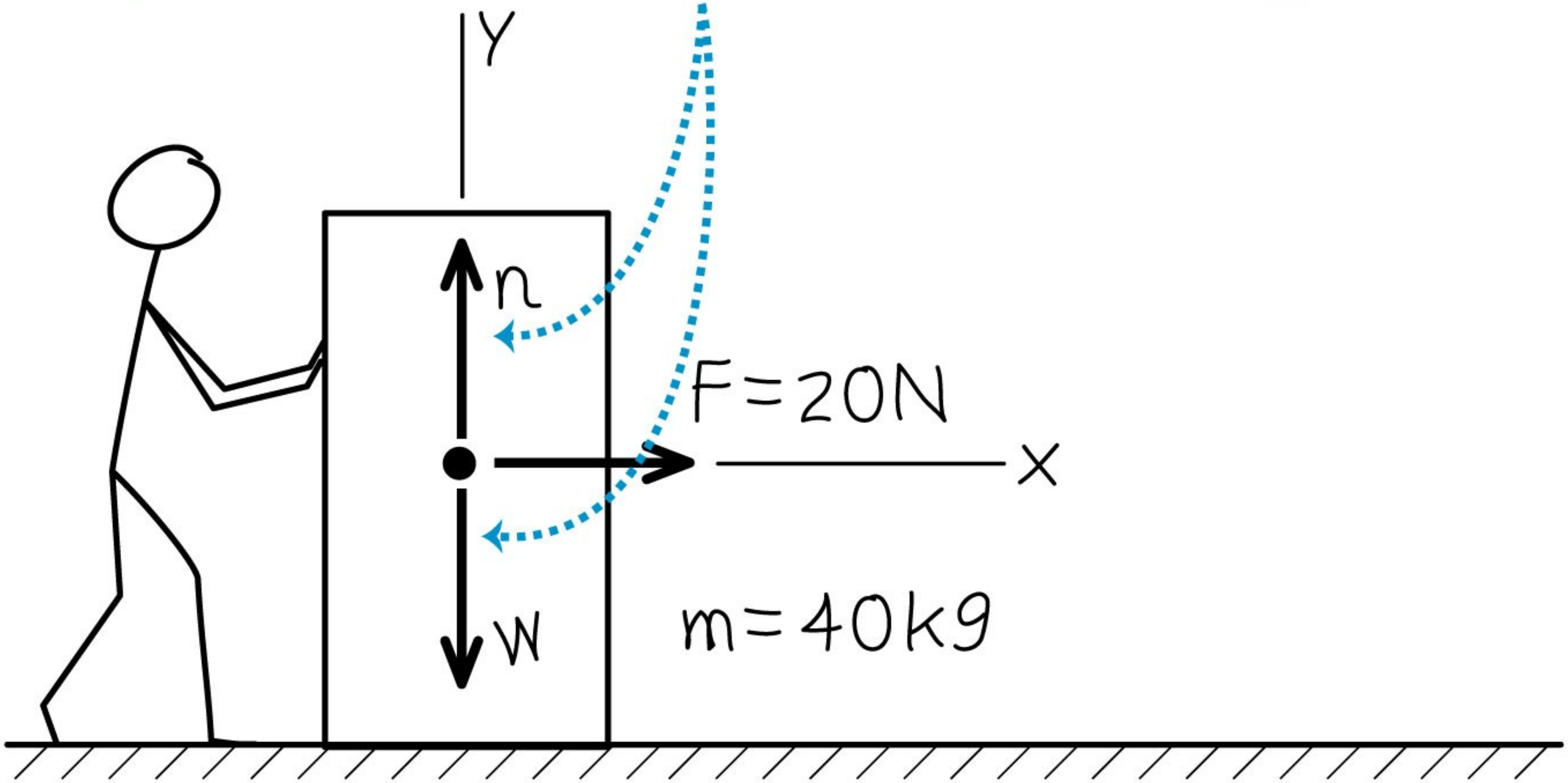
Free Body Diagrams

A *free-body diagram* is a sketch showing all forces acting upon an object.



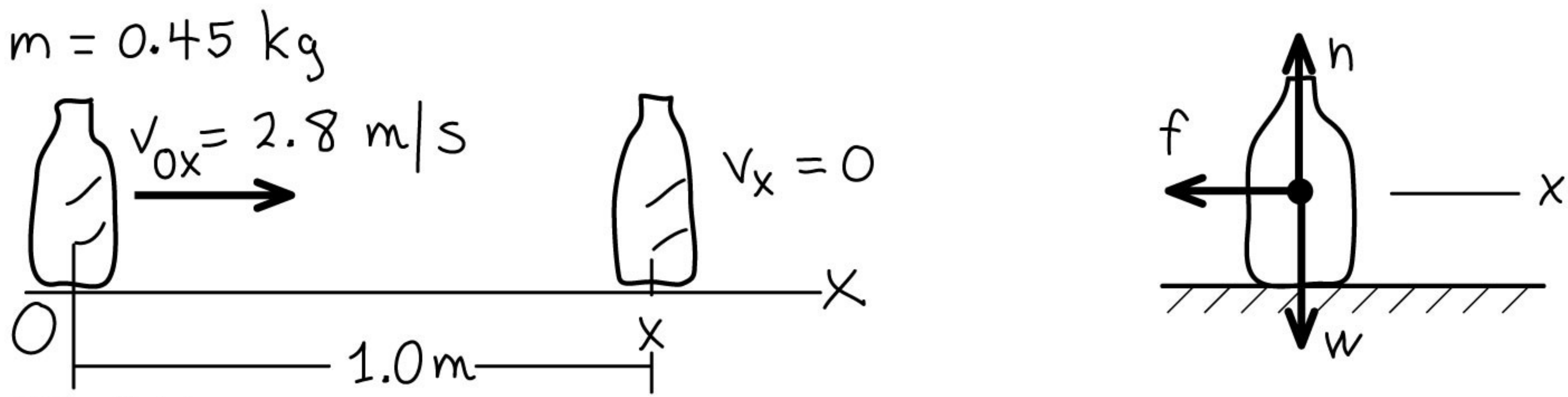
Free Body Diagram: Example (pushing a box)

The box has no vertical acceleration, so the vertical components of the net force sum to zero. Nevertheless, for completeness, we show the vertical forces acting on the box.



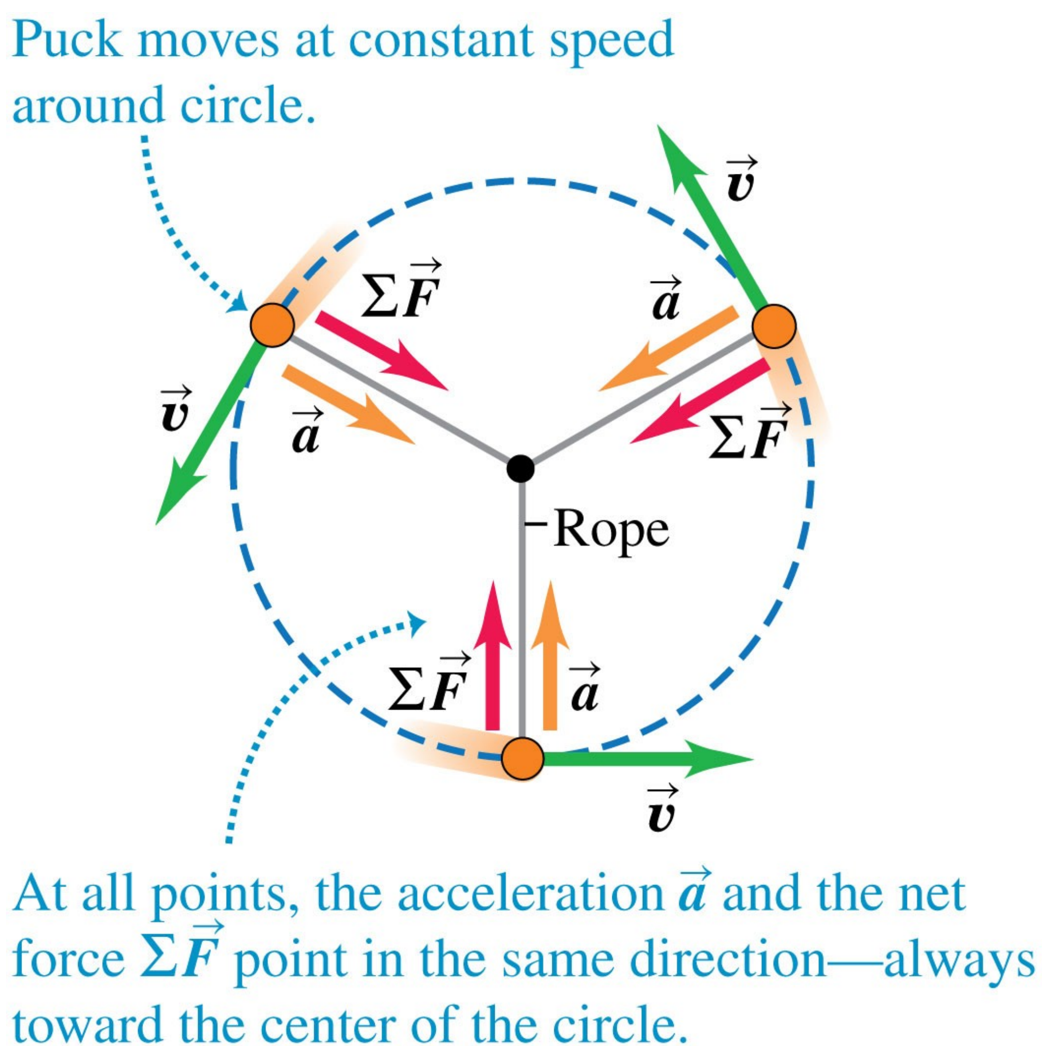
Free Body Diagram: Example (sliding bottle)

We draw one diagram for the bottle's motion and one showing the forces on the bottle.



Free Body Diagram: Example (particle in uniform circular motion)

As we have already seen, an object in uniform circular motion is accelerated towards the center of the circle. Hence the net force on the object must point towards the center of the circle.

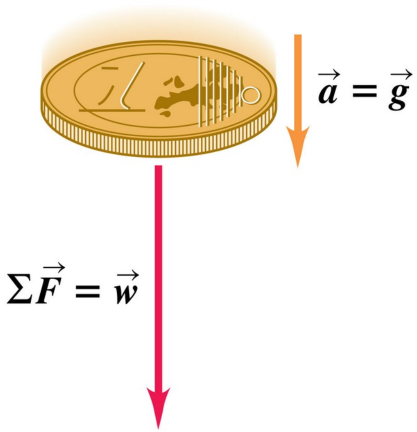


Comment: Mass and Weight

- The *weight* of an object (on the earth) is the gravitational force that Earth exerts on it.

- The magnitude of weight of an object with mass m is

$$W = mg$$



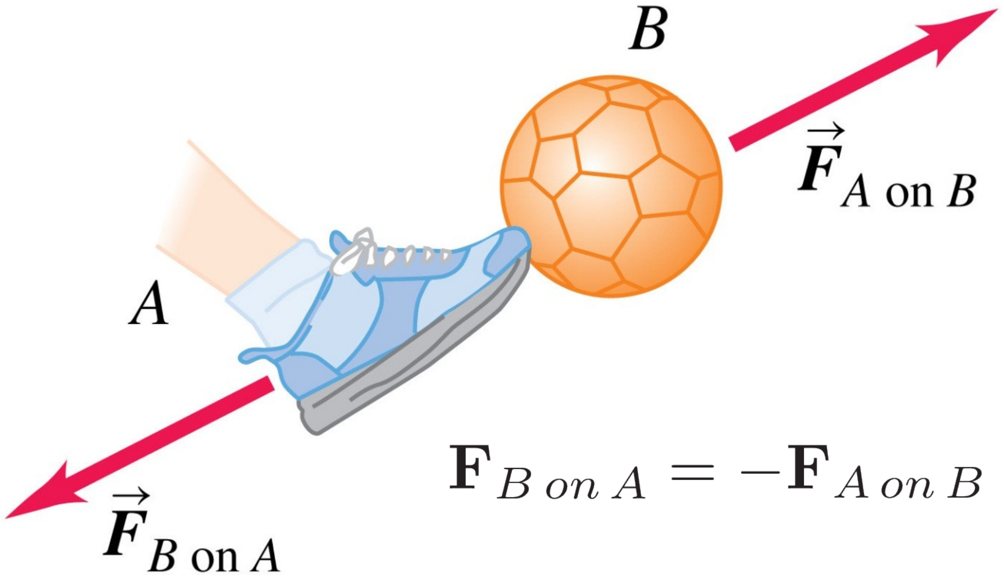
- The value of g depends on altitude.
  Close to Earth's surface it may be taken as constant.

- On other planets, g will have an entirely different value than on Earth.

Newton's Third Law

*The mutual forces of **action** and **reaction** between **two bodies** are **equal in magnitude** and **opposite in direction**.*

If you exert a force (**action**) on another body, that body always exerts a force (**reaction**) back upon you.

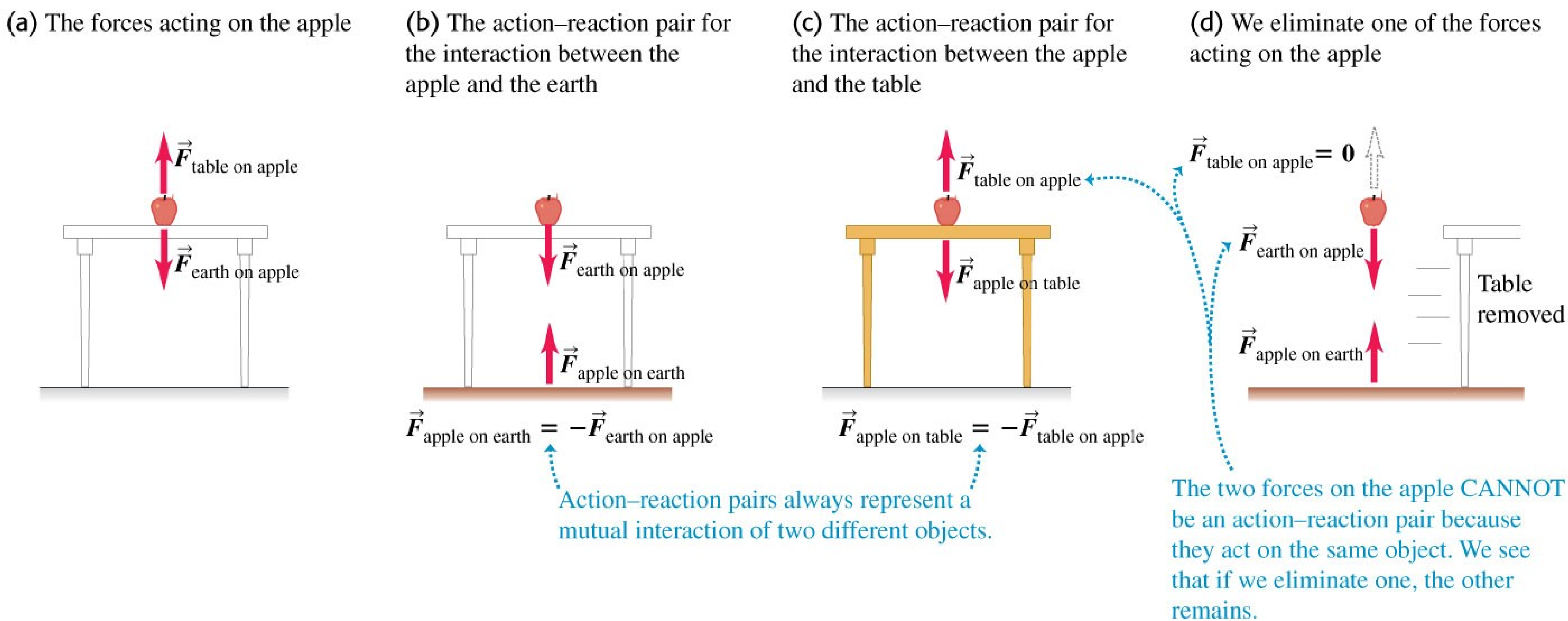


$$\mathbf{F}_{B\ on\ A} = -\mathbf{F}_{A\ on\ B}$$

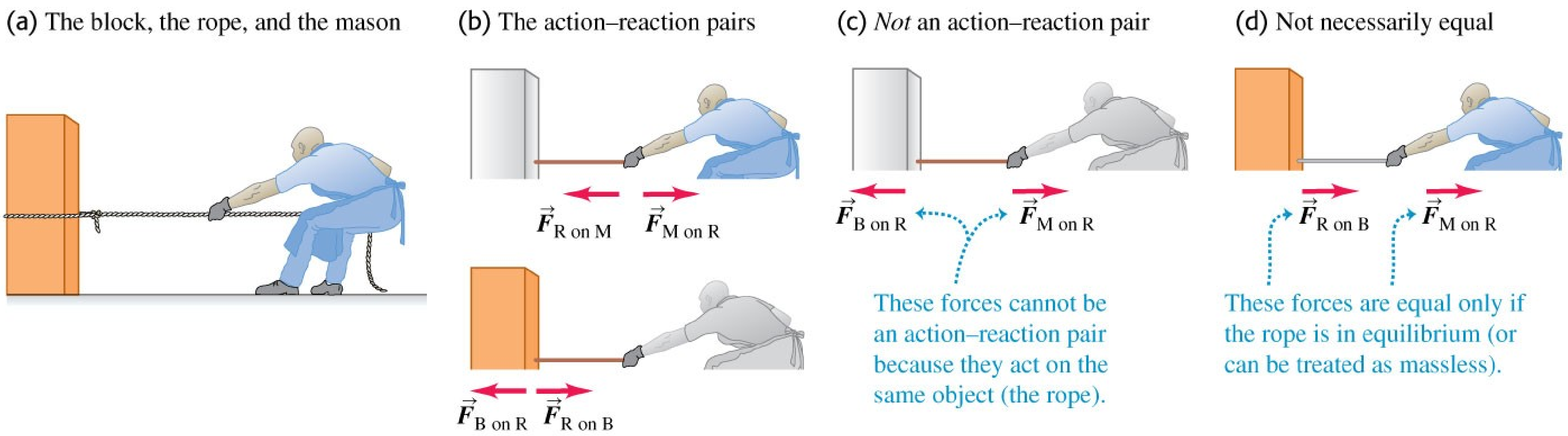
A force and its reaction force have the **same magnitude** but point in **opposite directions**.

These forces act on **different bodies!**

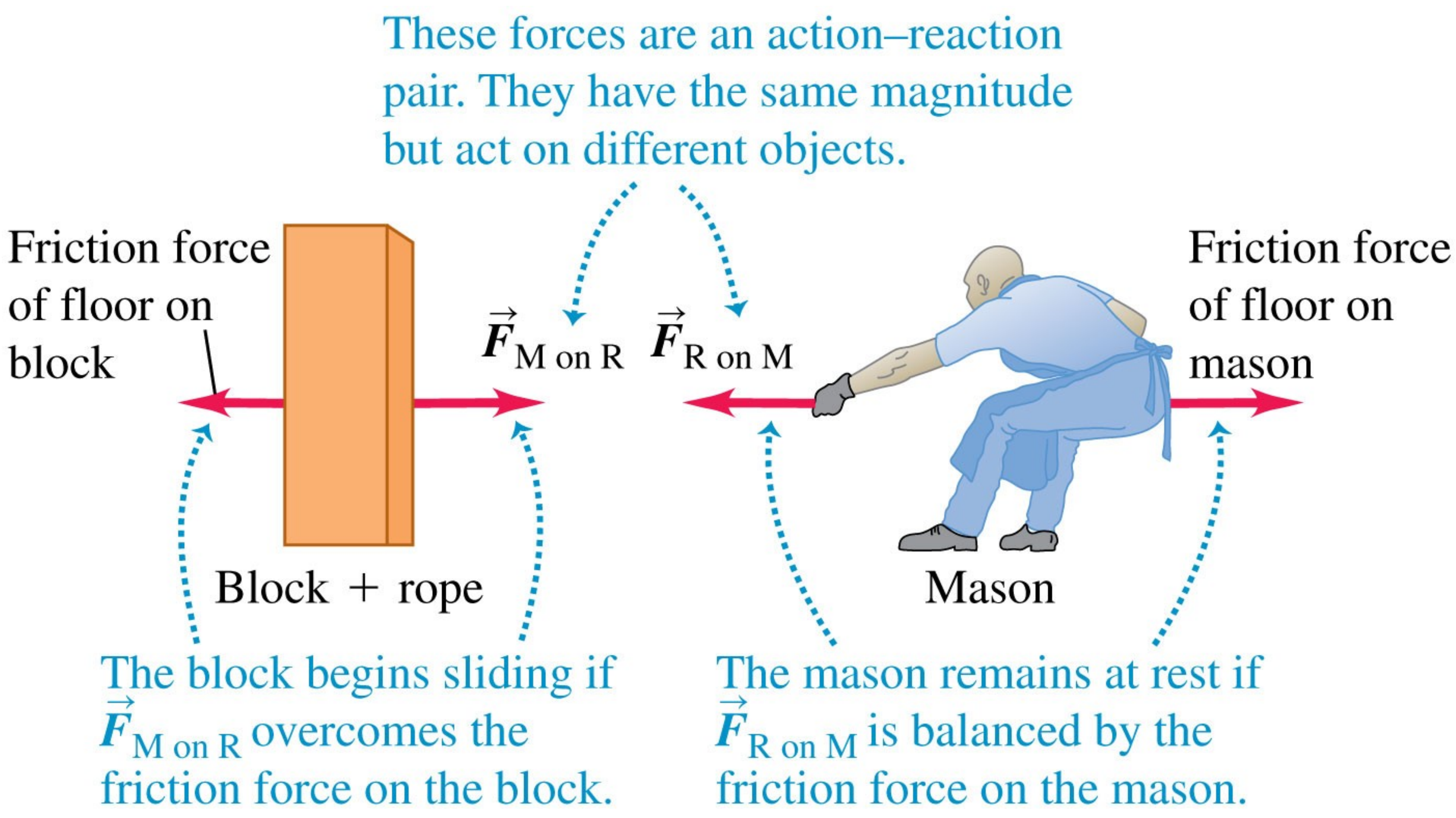
Newton's Third Law: Example (normal force)



Newton's Third Law: Example (pulling an object)



Newton's Third Law: Example (pulling an object, contd)



Newton's Laws: Summary

*An **inertial frame** of reference exists.*

*In an inertial frame of reference, the **acceleration** of a body is **directly proportional to the net force** acting on it, and **inversely proportional to the mass** of the body.*

*The mutual forces of **action** and **reaction** between two bodies are **equal in magnitude**, and **opposite in direction**.*