PHYSICS I Problem Set 8

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Problem 1 Solution

Due to elastic collision, the kinetic energy and momentum remain constant. Suppose the velocity of the ball is V_1 and the velocity of the block is V_2. Then

Conservation of kinetic energy

$$\frac{1}{2}mV_0^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}(2m)V_2^2 \tag{1}$$

Conservation of momentum

$$m\bar{V_0} = m\bar{V_1} + (2m)\bar{V_2} \tag{2}$$

Solving Eq.(1)(2)

$$\begin{cases} V_1 = -\frac{1}{3}V_0 \\ V_2 = \frac{2}{3}V_0 \end{cases} \tag{3}$$

Since V_1 changes the sign, namely, opposite to its initial direction, the ball will never hit the block again thus no longer having no effect on the spring. Therefore, the energy of system of the block and spring is conserved.

Conservation of energy

$$\frac{1}{2}(2m)V_2^2 = \frac{1}{2}kx_{\max}^2 \tag{4}$$

Therefore

$$x_{\max} = \frac{2}{3}\sqrt{\frac{2m}{k}}V_0 \tag{5}$$

Problem 2 Solution

The block system moves in four statges: 1) uniform linear motion; 2) simple harmonic motion; 3) uniform linear motion. Suppose the times for each stage is t_1, t_2, t_3, t_4 correspondingly.

Apparently

$$t_1 = t_3 = \frac{L}{V_0} \tag{6}$$

Then for the SHO.

$$\omega = \sqrt{\frac{2k}{m}} \tag{7}$$

$$T = \frac{2\pi}{\omega} \tag{8}$$

Solving Eq.(7)(8)

$$t_2 = \frac{T}{2} = \pi\sqrt{\frac{m}{2k}} \tag{9}$$

Therefore

$$t_{\text{total}} = t_1 + t_2 + t_3 = \frac{2L}{V_0} + \pi\sqrt{\frac{m}{2k}} \tag{10}$$

Problem 3 Solution

Center of mass

$$\bar{r}_{\text{cm}} = \frac{\sum\limits_{i=1}^{3} m_i \bar{r}_i}{\sum\limits_{i=1}^{3} m_i} = \left(\frac{7}{6}, 2, \frac{17}{6}\right) \tag{11}$$

Newton's second law

$$F = \sum_{i=1}^{3} m_i a \tag{12}$$

After $t = 2s$

$$\bar{r}_{\text{cm}_{\text{final}}} = \left(\frac{669}{2}, 2, \frac{17}{6}\right) \tag{13}$$

unit: cm

Problem 4 Solution

(a)

$$m = Al\rho \tag{14}$$

$$\Rightarrow \qquad \mathrm{d}m = A\rho\mathrm{d}l \tag{15}$$

Suppose for every ith dl, its position is x.
Center of mass ($x-$coordinate)

$$\bar{r}_{\text{cm}} = \frac{\int_0^l A\rho \mathrm{d}l \cdot x}{\int_0^l A\rho \mathrm{d}l} = \frac{\int_0^l A\rho l \mathrm{d}x}{A\rho l} = \frac{\frac{1}{2}A\rho l^2}{A\rho l} = \frac{1}{2}l \tag{16}$$

(b) Analogously, for every ith dl, its position is x.

$$m = \int_0^l A\alpha x \mathrm{d}x = \frac{1}{2}A\alpha l^2 \tag{17}$$

$$\Rightarrow \qquad \mathrm{d}m = A\alpha x \mathrm{d}x \tag{18}$$

Center of mass ($x-$coordinate)

$$\bar{r}_{\text{cm}} = \frac{\int\limits_{\text{rod}} x \mathrm{d}m}{m} = \frac{\int_0^l x \cdot A\alpha x \mathrm{d}x}{m} = \frac{\frac{1}{3}A\alpha l^3}{\frac{1}{2}A\alpha l^2} = \frac{2}{3}l \tag{19}$$

Problem 5 Solution

Method 1

Suppose the velocity of the fisherman is v_1 and the velocity of the boat is v_2. The distance the boat has moved is x.

Conservation of momentum

$$mv_1 + Mv_2 = 0 \tag{20}$$

Multiple time t on both sides of Eq.(22)

$$mS_1 + MS_2 = 0 \tag{21}$$

where the distance the fisherman travels $S_1 = l - x$ and the distance the boat moves $S_2 = -x$.

Therefore

$$x = \frac{m}{m+M}\, l \tag{22}$$

Method 2

Suppose the original position of the fisherman is the origin and the direction the man moves is the positive $x-$axis.

In the beginning Center of mass

$$\bar{r}_{\text{cm1}} = \frac{M \times \frac{l}{2}}{m+M} \tag{23}$$

In the end Center of mass

$$\bar{r}_{\text{cm2}} = \frac{m \times (l-x) + M \times (\frac{l}{2} - x)}{m+M} \tag{24}$$

Due to conservation of momentum, the center of mass does not move. Thus

$$\bar{r}_{\text{cm1}} = \bar{r}_{\text{cm2}} \tag{25}$$

Therefore

$$x = \frac{m}{m+M}\, l \tag{26}$$

Problem 6 Solution

At time t, the mass of the rocket

$$M = m_0 - \alpha t \tag{27}$$

During time peirod $\mathrm{d}t$, the mass of gas ejected

$$\mathrm{d}m = \alpha \mathrm{d}t \tag{28}$$

Suppose the velocity of the rocket when it ejects $\mathrm{d}m$ gas is $\mathrm{d}v$, the constant speed of gas ejected is u.

$$u\mathrm{d}m = M\mathrm{d}v \tag{29}$$

Solving Eq.(27)(28)(29)

$$\mathrm{d}v = \frac{\alpha}{m_0 - \alpha t} u \mathrm{d}t \tag{30}$$

Integrate Eq.(30)

$$v(t) = u \ln \frac{m_0}{m_0 - \alpha t} \tag{31}$$

After collision, the velocity of the rockert changes its direction, Thus when the rocket stops at the first time

$$|v(t)| = 2|v(T)| \tag{32}$$

Therefore

$$t = 2T - \frac{\alpha T^2}{m_0} \tag{33}$$

$$\Delta t = T - \frac{\alpha T^2}{m_0} \tag{34}$$

Plug the data provided in the problem

$$\Delta t = 9 \text{ [s]} \tag{35}$$