Problem Set 6

Due: 18 June 2021, 2.30 p.m.

In Problems 1–5, assume that the acceleration due to gravity is g.

Problem 1. A rope with length l and mass m rests on a flat surface. Find minimum work needed to be done to lift it up, by holding one of its ends, in the case when

(a) the rope is uniform,

(b) the (linear) mass density of the rope is $\lambda(x) = \lambda_0 x(l - x)$ (what is the value of λ_0?), where x is the distance, measured along the rope, from the end that is being lifted up.

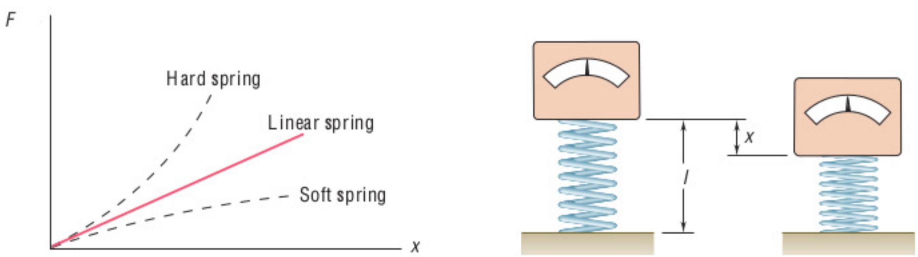
(1 + 2 points)

Problem 2. A right circular uniform cone with base radius R and height H floats base-up in a liquid of density ρ, so that one third of its volume is submerged. What is minimum work needed to pull the cone above the liquid?

(5 points)

Problem 3. Nonlinear springs are classified as hard or soft, depending upon the curvature of their force-deflection curve (see figure). If a delicate instrument having a mass of $m = 5$ kg is placed on a spring of length l so that its base is just touching the undeformed spring and then inadvertently released from that position, determine the maximum deflection x_m of the spring and the maximum force F_m exerted by the spring, assuming (a) a linear spring of constant $k = 3$ kN/m, (b) a hard, nonlinear spring, for which $F = (3\ [\text{kN/m}])(x + 160x^3)$.

(1 + 2 points)



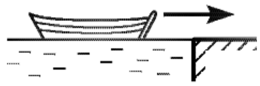
Problem 4. On a winter day in Maine, a warehouse worker is shoving boxes up a rough plank inclined at an angle a above the horizontal. The plank is partially covered with ice, with more ice near the bottom of the plank than near the top, so that the coefficient of friction increases with the distance x along the plank: $\mu = Ax$, where A is a positive constant and the bottom of the plank is at $x = 0$. (For this plank the

coefficients of kinetic and static friction are equal: $\mu_k = \mu_s = \mu$.) The worker shoves a box up the plank so that it leaves the bottom of the plank moving at speed v_0. Show that when the box first comes to rest, it will remain at rest if

$$v_0^2 \geq \frac{3g\sin^2\alpha}{A\cos\alpha}.$$

(*4 points*)

Problem 5. A boat of length L_0, with its engine turned off, reaches the beach and stops with half of its length on the sand. The kinetic coefficient of friction between the boat and the sand is μ. What was the initial speed of the boat?



(*3 points*)

Problem 6. Calculate work done by the forces

(A) $\mathbf{F}_1(\mathbf{r}) = (x^2 z, -xy, 5)$,
(B) $\mathbf{F}_2(\mathbf{r}) = (-2x - yz, z - xz, y - xy)$.

acting on a particle that moves from $(-1, 0, 0)$ to $(1, 0, 0)$ along

(a) the x axis,
(b) an arc of the parabola $y = x^2 - 1$.

2 $\times$ (*1 + 3/2 points*)

Problem 7. Use a computer (e.g. with *Wolfram's Mathematica*; commands `VectorPlot` and `VectorPlot3D`) to visualize the following force fields, attach the graphs to your homework.

(a) $\mathbf{F}(\mathbf{r}) = -y^3\hat{n}_x + x^3\hat{n}_y$ in 2D,
(b) $\mathbf{F}(\mathbf{r}) = -(x^2+1)\hat{n}_x - (y^2-1)\hat{n}_y$ in 2D,
(c) $\mathbf{F}(\mathbf{r}) = -r^2\,\mathbf{r}$ in 2D and 3D,
(d) one of the two force fields from the previous problem.

(*1/2 + 1/2 + 1 + 1 points*)