PHYSICS I Problem Set 10

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Problem 1

Solution

Suppose the mass of the ball is m with radius r and the mass of the ring is M with radius R.

For the ball

$$\begin{cases} ma_{\text{cm},1} = mg\sin\alpha - f_{s,1} \\ f_{s,1}r = I_1\varepsilon = \frac{2}{5}mr^2\varepsilon \\ a_{\text{cm},1} = \varepsilon_1 r \end{cases} \tag{1}$$

For the ring

$$\begin{cases} Ma_{\text{cm},2} = Mg\sin\alpha - f_{s,2} \\ f_{s,2}R = I_2\varepsilon = MR^2\varepsilon \\ a_{\text{cm},2} = \varepsilon_2 R \end{cases} \tag{2}$$

Then we get

$$\begin{cases} a_{\text{cm},1} = \frac{5}{7}g\sin\alpha \\ a_{\text{cm},2} = \frac{1}{2}g\sin\alpha \end{cases} \tag{3}$$

Since the distance the ball and the ring travel during t period is the same

$$\frac{1}{2}a_{\text{cm},1}t^2 = v_0t + \frac{1}{2}a_{\text{cm},2}t^2 \tag{4}$$

where $a_{\text{cm},1}$ and $a_{\text{cm},2}$ satisfy Equations(3).

Therefore, we get

$$v_0 = \frac{3}{28}gt\sin\alpha \tag{5}$$

which is directing downward along the inclined plane

Problem 2

Solution

(a) According to FBD in Fig. 1, we have

$$\begin{cases} T_1 = m_1a \\ m_2g - T_2 = m_2a \\ T_1 + f_s = T_2 \\ f_sR = I\varepsilon = \frac{1}{2}MR^2\varepsilon \\ a = R\varepsilon \end{cases} \tag{6}$$

Soving Equations(6)

$$\begin{cases} a = \frac{2m_2}{2m_1+2m_2+M} \cdot g \\ T_1 = \frac{2m_1}{2m_1+2m_2+M} \cdot m_2 g \\ T_2 = \frac{2m_1+M}{2m_1+2m_2+M} \cdot m_2 g \end{cases} \tag{7}$$

(b) The acceleration of the box has already been shown in Equations(7). Namely,

$$a = \frac{2m_2}{2m_1+2m_2+M} \cdot g \qquad \text{directing to the right or downwards}$$

(c) For the pulley, according to FBD, the horizontal force exerted on it is T_1 and the vertical force exerted on it is $T_2 + Mg$. Namely,

$$F_{\text{horizontal}} = \frac{2m_1}{2m_1+2m_2+M} \cdot m_2 g \qquad \text{directing to the right} \tag{8}$$

$$F_{\text{vertical}} = \frac{2m_1+M}{2m_1+2m_2+M} \cdot m_2 g + Mg \qquad \text{upwards} \tag{9}$$

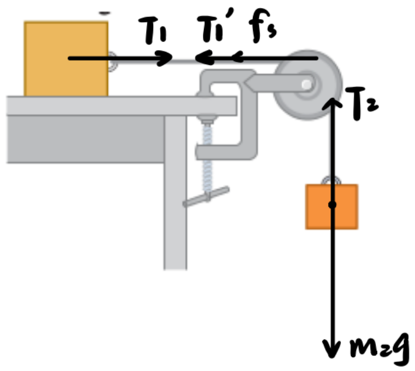

Figure 1: Free Body Diagram in Problem 2

Problem 3

Solution

According to FBD in Fig. 2, we have

$$\begin{cases} (T_1 - T_2)R = I\varepsilon \\ mg - T_1 = ma \\ T_2 = kx \\ \varepsilon = a/R \end{cases} \tag{10}$$

Therefore

$$a = \frac{mg - kx}{m + I/R^2} \tag{11}$$

Since $F = ma \propto x$, the ocsillation is **SHM**.
Therefore, period of this oscillation is

$$T = 2\pi\sqrt{\frac{mR^2 + I}{kR^2}} \tag{12}$$

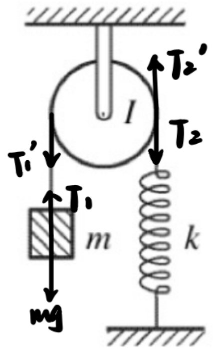

Figure 2: Free Body Diagram in Problem 3

Problem 4

Solution

Definition of impulse

$$J = mv_{\text{cm}} \tag{13}$$

Definition of angular velocity

$$\omega = \frac{v_{\text{cm}}}{d} \tag{14}$$

Second law of dynamics for a rigid body rotating about a fixed axis. For the axis through the end of the bat

$$xF = I\varepsilon \tag{15}$$

Parallel theorem

$$I = I_{\text{cm}} + md^2 \tag{16}$$

Ingegrate Equation(15) by dt from t_1 to t_2

$$\begin{aligned} & x\int_{t_1}^{t_2} F dt = I\int_{t_1}^{t_2} \varepsilon dt \\ \Rightarrow \qquad & xJ = I\omega = I\frac{v_{\text{cm}}}{d} = \frac{J}{md} \end{aligned}$$

Therefore, we get

$$x = 0.71\ m \tag{17}$$