

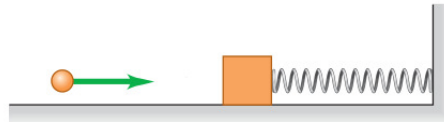
### Problem Set 8

Due: 2 July 2021, 2.30 p.m.

**Problem 1.** One end of a spring with the spring constant  $k$  is attached to the wall and the other to a block with mass  $2m$  resting on a smooth surface. Initially, the spring is neither stretched nor compressed.

A ball with mass  $m$ , moving with speed  $v_0$ , hits the block in an elastic collision. What is the maximum compression of the spring after the collision?

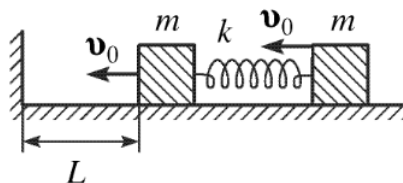
(4 points)



**Problem 2.** Two identical blocks with mass  $m$  are connected by a light spring with spring constant  $k$ . Initially, the spring is neither compressed nor stretched and the system slides with speed  $v_0$  on a frictionless surface towards a wall (see figure below). At  $t = 0$  the left block is at distance  $L$  from the wall. Find the time after which the center of mass of this system will be again at the position it was when  $t = 0$ .

Assume that the block collides with the wall elastically, and the duration of the collision is negligibly short.

(4 points)



**Problem 3.** Consider a system of three particles with masses  $m_1 = 5$  g,  $m_2 = 10$  g,  $m_3 = 15$  g. At the initial instant of time  $t = 0$  the particles were placed at points (3,4,5), (2,4,6), and (0,0,0), respectively (the coordinates are given in centimeters). The particles are acted upon by external forces whose sum is a vector of magnitude  $F = 0.05$  N pointing along the  $x$  axis, in the positive  $x$  direction.

Find the position of the center of mass of the system for  $t = 2$  s.

(2 points)

**Problem 4.** Consider a rod of length  $l$ , mass  $m$ , and cross-sectional area  $A$ . Let us set the origin of the coordinate system at one end of the rod with the positive  $x$ -axis along the symmetry axis of the rod. (a) If the bulk density  $\rho = m/Al$  of the rod is constant, show that the  $x$ -coordinate of the center of mass of the rod is at its half of its length. (b) If the density of the rod varies linearly with  $x$  — that is,  $\rho = \alpha x$ , where  $\alpha$  is a positive constant — calculate the  $x$ -coordinate of the rod's center of mass.

(1 + 2 points)

**Problem 5.** A fisherman (with mass  $m$ ) walks from one end of his boat of mass  $M$  and length  $l$  to the other. Find the distance the boat has moved with respect to the still surface of a lake the boat is on.

(2 points)

**Problem 6.** A toy-rocket with initial mass  $m_0$  is placed at rest on a frictionless horizontal surface at some distance from a wall. At  $t = 0$ , the engine is turned on and the rocket starts moving in the direction perpendicular to the wall colliding with it after time  $T$ . The collision is elastic and the rocket does not change its orientation with respect to the wall. How much time after the collision does it take until the (instantaneous) velocity of the rocket is zero?

Assume that the burned fuel is ejected from the rocket at a constant speed (with respect to the rocket) and the mass of the rocket decreases with time at a constant rate  $\alpha$ .

What is the numerical answer if  $m = 1$  kg,  $\alpha = 0.01$  kg/s, and  $T = 10$  s?

(5 points)

