PHYSICS I Problem Set 7

Name: Haotian Fu

Student ID: 520021910012

Problem 1 Solution

(a)

$$F = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}\right) = (-y^2, -x^2) \tag{1}$$

Therefore, we can visialize it as

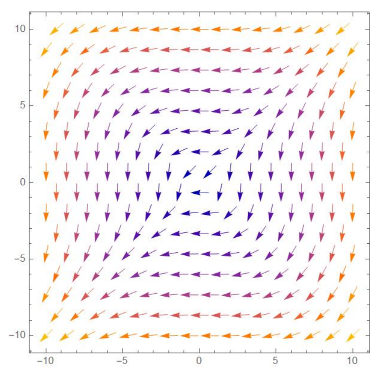


Figure 1: Force in Problem 1

(b)

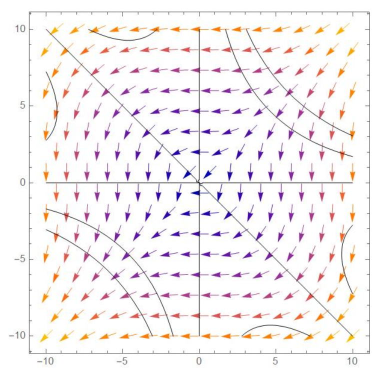


Figure 2: Equipotential Diagram

(c) According to Fig 2, it is trivial that points are equipotential if $x = 0$ or $y = 0$ or $x = -y$ since all these circumstances the potential energy is equal to 0. Meanwhile, the other curves all satisfy the equation $xy^2 + yx^2 = 0$.
(d) We decompose the displacemnet in $x-$axis and $y-$axis. Then we calculate the work

along $x-$axis and $y-$axis separately. Suppose the trajectory is called AB.

$$W_x = \int_{\Gamma_{AB}} F_x \mathrm{d}\bar{r} \tag{2}$$

$$W_y = \int_{\Gamma_{AB}} F_y \mathrm{d}\bar{r} \tag{3}$$

$$\bar{r} = \bar{x} + \bar{y} \tag{4}$$

$$y = x \tag{5}$$

Solving (1)(2)(3)(4)(5), we get

$$W = -\frac{2}{3} \quad [\mathrm{J}] \tag{6}$$

(e) Equation (5) changes into equation (7) due to the change of trajectory.

$$y = x^2 \tag{7}$$

Solving (1)(2)(3)(4)(7), we get

$$W = -\frac{7}{10} \quad [\mathrm{J}] \tag{8}$$

Problem 2 Solution

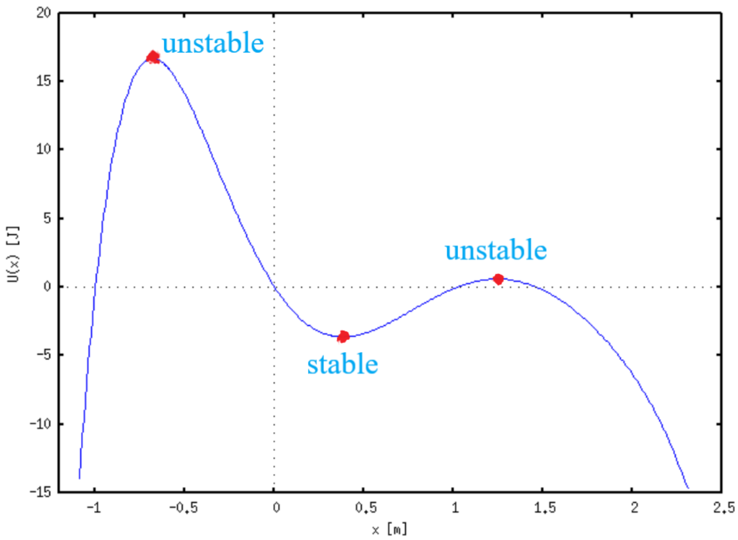


Figure 3: Energy Diagram

Problem 3 Solution

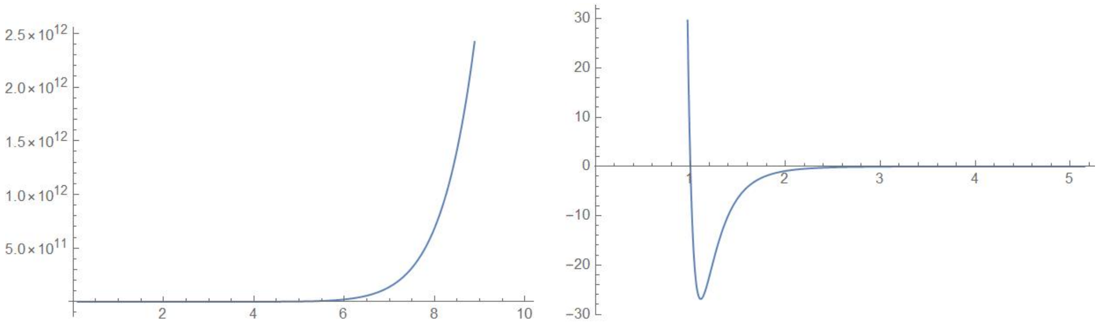
(a) Solving

$$F = -\frac{\mathrm{d}}{\mathrm{d}\bar{r}} U \tag{9}$$

we get

$$F = 12U_0R_0^6\left(\frac{1}{r^{13}}\cdot R_0^6 - \frac{1}{r^7}\right) \tag{10}$$

Then we plot



(a) Graph of U(r) (b) Graph pf F(r)

Figure 4: Graphs in Problem 3(a)

(b) We rewrite the Lennard-Jones potential energy equation.

$$U = U_0\left(\left(\left(\frac{R_0}{r}\right)^6 - 1\right)^2 - 1\right) \tag{11}$$

Thus when $\left(\frac{R_0}{r}\right) = 1$, the potential energy reaches its minimum. Namely, the whole system is at its equilibrium. Therefore, R_0 refers to the distance between the pair of neutral atoms or molecules at equilibrium. Meanwhile, U_0 refers to the magnitude of the lowest energy of the whole system.

(c) We may use $r = R_0 + x$ to replace r so that we can have less variables to deal with.

$$\begin{aligned} F(x) &= -R_0^6\cdot\frac{1}{(R_0+x)^7} + R_0^{12}\cdot\frac{1}{(R_0+x)^13} \\ &= -\frac{1}{R_0}\cdot\frac{1}{(1+x/R_0)^7} + \frac{1}{R_0}\cdot\frac{1}{(1+x/R_0)^{13}} \end{aligned}$$

Since $(1+x)^n \doteq 1+nx$ when $x \to 0$

$$\begin{aligned} F(x) &\doteq -\frac{1}{R_0+7x} + \frac{1}{R_0+13x} \\ &= -\frac{6x}{(R_0+7x)(R_0+13x)} \end{aligned}$$

Since $x \ll R_0$

$$F(x) \doteq -\frac{6}{R_0^2}\cdot x \tag{12}$$

For SHM

$$F(x) = -kx \tag{13}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{14}$$

$$T = \frac{2\pi}{\omega} \tag{15}$$

Solving (12)(13)(14)(15)

$$k = \frac{6}{R_0^2} \tag{16}$$

$$T = 2\pi\sqrt{\frac{mR_0^2}{6}} \tag{17}$$

(d) The chemical bond is oscillating here.

Problem 4 Solution

Suppose the equilibrium position is at x_0.

$$U(x) \doteq U(x_0) + \frac{1}{2}\ddot{U}(x_0)(x - x_0)^2 \tag{18}$$

$$F(x) = -\frac{\mathrm{d}}{\mathrm{d}x}U(x) = -\ddot{U}(x_0)(x - x_0) \tag{19}$$

Meanwhile

$$\dot{U}(x) = \frac{2U_0\alpha \sin \alpha x}{\cos^3 \alpha x} \tag{20}$$

$$\dot{U}(x_0) = 0 \tag{21}$$

$$\Rightarrow \quad x_0 = 0 \tag{22}$$

In addition

$$\ddot{U}(x) = \frac{2\alpha^2 U_0 + 4\alpha^2 U_0 \sin^2 \alpha x}{\cos^4 \alpha x} \tag{23}$$

Solving (22)(23)

$$\ddot{U}(x_0) = 2\alpha^2 U_0 \tag{24}$$

For SHM

$$F = -k(x - x_0) \tag{25}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{26}$$

$$T = \frac{2\pi}{\omega} \tag{27}$$

Solving (19)(24)(25)(26)(27)

$$T = 2\pi\sqrt{\frac{m}{2\alpha^2 U_0}} \tag{28}$$