UM–SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP241)

LABORATORY REPORT

EXERCISE 3

SOLAR CELLS: I-V CHRACTERISTICS

Name: Haotian Fu ID: 520021910012 Group: 1

Date: 12 November 2021

[rev. 4.0]

Contents

List of Figures

List of Tables

1 Introduction

The objective of this exercise is to urge students to be familiar with the working principle of a solar cell and study its current-voltage *(I-V)* characteristics.

1.1 Basic Concepts

- **Solar cell:**[1] Solar cells are devices which are able to directly transform solar radiation into electrical energy.

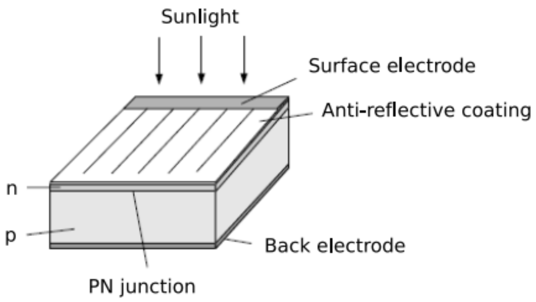

Figure 1: Solar cell

- **P-N junction:**[2] A p–n junction is a boundary or interface between two types of semiconductor materials, p-type and n-type, inside a single crystal of semiconductor.

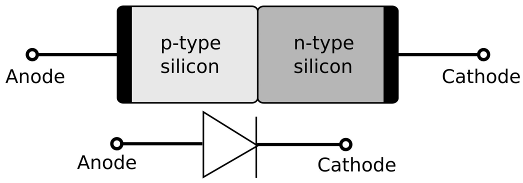

Figure 2: P-N junction

- **Photovoltaic (PV) effect:**[3] Photovoltaic (PV) effect is a process by which PV cell converts the absorbed sunlight energy into electricity.

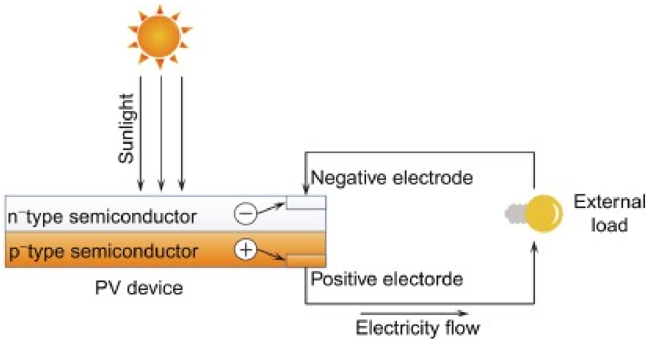

Figure 3: Photovoltaic effect

- **Fill factor:** Fill factor is the available power at the maximum power point (P_{m}) divided by the open circuit voltage (V_{OC}) and the short circuit current (I_{sc}).

$$FF = \frac{P_{\mathrm{m}}}{V_{\mathrm{oc}} I_{\mathrm{sc}}} = \frac{V_{\mathrm{m}} I_{\mathrm{m}}}{V_{\mathrm{oc}} I_{\mathrm{sc}}} \tag{1}$$

- **Solar cell energy conversion efficiency:** The solar cell energy conversion efficiency η is defined as

$$\eta = \frac{P_{\mathrm{m}}}{P_{\mathrm{in}}} \times 100\% \tag{2}$$

where P_{in} denotes the total radiant power incident on the solar cell while P_{m} denotes the maximum output power.

1.2 Theoretical Basis

1.2.1 The Principle of the Photovoltaic Effect[1]

When the light enters the $p-n$ junction near the solar cell surface, and the energy of incident photons is greater than the forbidden bandwidth (energy gap) E_g, the incident photons are absorbed and excite electron-hole pairs. Minority charge carriers in the $n-$ or $p-$type area diffuse due to their density gradient. Some of them are able to diffuse to the region of the $p-n$ junction where a built-in electric field exists. This field is directed from the $n-$type to the $p-$type area. The minority carriers diffusing to the $p-n$ junction zone between the $n-$type area and the $p-$type area are drawn by this electric field to the $p-$type area (in case of the holes), or to the n-type area (in case of the electrons). This results in an increase of positive charge accumulated in the p-type area and negative charge in the $n-$type area. Consequently, a photoelectric potential difference is generated.

1.2.2 Solar Cell Parameters

The net current I is

$$I = I_{\mathrm{ph}} - I_{\mathrm{D}} = I_{\mathrm{ph}} - I_0 \left[\exp\left(\frac{qV_{\mathrm{D}}}{nk_{\mathrm{B}}T}\right) - 1\right] \tag{3}$$

where I_{ph} is the current from the $n-$type area to the $p-$area when there is light incident on the solar cell and I_D is a forward diode current from the $p-$type to the $n-$type area, opposite to I_{ph}. V_D is the junction voltage, I_0 is the diode inverse saturation current, the coefficient n is a theoretical coefficient, with its values ranging from 1 to 2, that characterizes the $p-n$ junction. Furthermore, q denotes the electron's charge, k_B is the Boltzmann's constant, and T is the temperature in the absolute (Kelvin) scale.

Ignoring the internal series resistance R_s, the voltage V_D equals the terminal voltage V and Eq.(3) can be rewritten as

$$I = I_{\mathrm{ph}} - I_0 \left[\exp\left(\frac{qV}{nk_{\mathrm{B}}T}\right) - 1\right] \tag{4}$$

When the output is short, *i.e.*, $V=0$, the short-circuit current is

$$I_{\mathrm{sc}} = I_{\mathrm{ph}}$$

whereas when the output is open, *i.e.*, $I=0$, the open-circuit voltage is

$$V_{\mathrm{oc}} = \frac{nk_{\mathrm{B}}T}{q} \ln\left(\frac{I_{\mathrm{sc}}}{I_0} + 1\right) \tag{5}$$

1.2.3 Theoretical Graph

Considering Eq.(4) and Eq.(5), the corresponding $I-V$ characteristics curve is shown in Fig.(4)

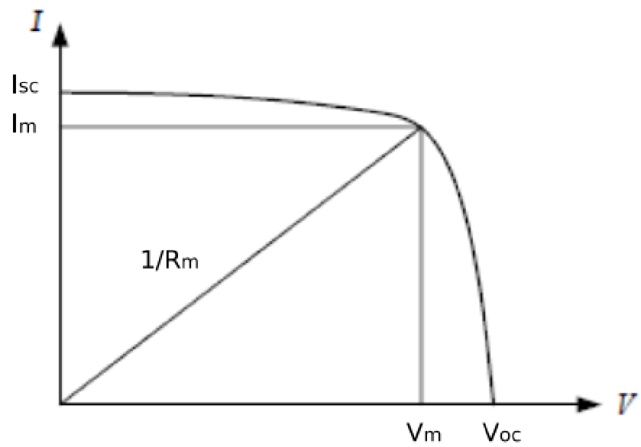

Figure 4: The current-voltage characteristics of a solar cell.

1.2.4 Solar Cell Equivalent Circuit

As shown in Fig.(5) , a solar cell can be thought of as composed of a $p-n$ junction diode D and a constant current source I_{ph}. Along with a series resistance R_{s} due to the electrodes in the solar cell and a parallel resistance R_{sh}, all elements form a circuit equivalent to a p - n junction leak-circuit. For the equivalent circuit one can find the following relationship between the current and the voltage

$$I = I_{\mathrm{ph}} - I_0 \left\{ \exp\left[\frac{q\left(V + R_{\mathrm{s}} I\right)}{n k_{\mathrm{B}} T} \right] - 1 \right\} - \frac{V + R_{\mathrm{s}} I}{R_{\mathrm{sh}}}$$

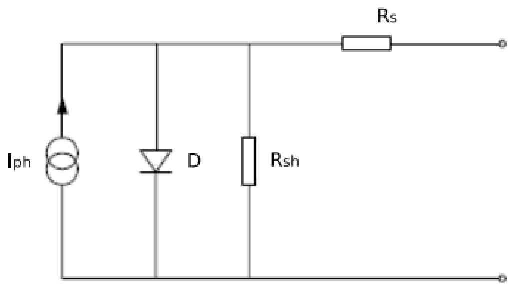

Figure 5: Solar cell equivalent circuit.

2 Apparatus and Measurement Procedure

2.1 Apparatus

The setup consists of a photovoltaic device (5 W), a 300 W tungsten-halogen lamp serving as a radiation source, two digital multimeters, two adjustable resistors, a solar power meter, a wiring board and a measuring tape.

The precisions of the devices are shown in Table 1.

Instrument	Uncertainties
DC Voltage	$\pm(0.5\% + 0.01)\,\mathrm{V}$
DC Current	$\pm(1.5\% + 0.1\,\mathrm{mA}$
Distance	$\pm 0.1\%\,\mathrm{cm}$
Solar power	$\pm 10\,\mathrm{W/cm^2}$

Table 1: Information of measurement instruments.

2.2 Measurement Procedure

2.2.1 Relation Between Sensitivity K_H and Working Voltage U_S

In this part, we explored the relation between sensitivity K_H and working voltage U_S by applying Eq.(**??**) and measuring the corresponding quantities.

1. Place the integrated Hall probe at the center of the solenoid. Set the working voltage at 5 V and measure the output voltage $U_0(I_M = 0)$ and $U(I_M = 250mA)$. Take the theoretical value of $B(x = 0)$ from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).

2. Measure K_H for different values of U_S (from $2.8V$ to $10V$). Calculate K_H/U_S and plot the curve K_H/U_S *vs.* U_S.

2.2.2 Relation Between Output Voltage U and Magnetic Field B

In this part, we explored the relation between the output voltage of the Hall probe and the magnetic field in which it is positioned by adjusting the current in the solenoid.

1. With $B = 0, U_S = 5V$, connect the $2.4 \sim 2.6V$ output terminal of the DC voltage divider and the negative port of the voltmeter. Adjust the voltage until $U_0 = 0$.

2. Place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to $500mA$, with intervals of $50mA$.

3. Explain the relation between $B(x=0)$ and the Hall voltage U_H. Pay attention to the fact that the output voltage U is the amplified signal from U_H. The theoretical value of $B(x=0)$ can be found from Table 1.

4. Plot the curve $U vs. B$ and find the sensitivity K_H by a linear fit (use a computer). Compare the value you obtained with the theoretical value in given in the Apparatus section.

2.2.3 Magnetic Field Distribution Inside the Solenoid

In this part, we explore the magnetic field distribution inside a solenoid by adjusting the position of the Probe Hall and measuring the magnetic field at different positions.

1. Measure the magnetic field distribution along the axis of the solenoid for $I_M = 250mA$, record the output voltage U and the corresponding position x. Then find $B = B(x)$. (Use the value of K_H found in the previous part of the experiment).

2. Use a computer to plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid. Use dots for the data measured and a solid line for the theoretical curve. The origin of the plot should be at the center of the solenoid.

3 Experimental Results

3.1 Experiment 1 - Relationship Between Sensitivity K_H and Working Voltage U_S

The measurement results of U_0 and U when $U_\mathrm{S} = 5.00$ V are shown in Table 2.

U_S [V] $\pm 0.5\%$ [V]	$U_0(I_\mathrm{M}=0)$ [V] $\pm (0.05\% + 6\times 10^{-3})$ [V]	$U(I_\mathrm{M}=250\,\mathrm{mA})$ [V] $\pm (0.05\% + 6\times 10^{-3})$ [V]
5.00 ± 0.03	2.504 ± 0.007	2.621 ± 0.007

Table 2: Data for U_0 and U with $U_S = 5.00$ V.

According to the data in Table **??**, we can obtain that when $I_\mathrm{M} = 100$ mA, $B(x=0, I_\mathrm{M} = 100 \text{ [mA]}) = 1.4366 \times 10^{-3}$ T. Eq. (**??**) implies that B is proportional to current I_M, and therefore,

$$B(x=0, I_\mathrm{M} = 250 \text{ [mA]}) = \frac{250}{100} \times 1.4366 \times 10^{-3} = 3.59 \times 10^{-3} \pm 0.07 \times 10^{-3} \text{ T}.$$

According to Eq. (**??**), the sensitivity of the probe K_H when $U_\mathrm{S} = 5.00$ V is then calculated as

$$K_\mathrm{H} = \frac{U - U_0}{B(x=0, I_\mathrm{M} = 250\,\text{[mA]})} = \frac{2.621 - 2.504}{3.59 \times 10^{-3}} = 33 \pm 3\,[\mathrm{V/T}].$$

The measurement results of U_0 and U for different U_S are shown in Table 3. We calculate $K_\mathrm{H}/U_\mathrm{S}$ for each set of data to explore their relation. That is, for each set of data, we calculate

$$\frac{K_\mathrm{H}}{U_\mathrm{S}} = \frac{U - U_0}{BU_\mathrm{S}}.$$

Taking the first set of data as an example,

$$\frac{K_\mathrm{H}}{U_\mathrm{S}} = \frac{U - U_0}{BU_\mathrm{S}} = \frac{1.4656 - 1.4004}{3.59 \times 10^{-3} \times 2.80} = 6.5 \pm 0.2\,[\mathrm{T}^{-1}].$$

The measurement results and the results of calculation of $K_\mathrm{H}/U_\mathrm{S}$ for each set of data are presented together in Table 3. A plot of the results $K_\mathrm{H}/U_\mathrm{S}$ vs. U_S using Origin is shown in Fig.(**??**). The points in the plot indicates that the ratio of K_H to U_s decreases as U_s increases.

3.2 Relation Between Output Voltage U and Magnetic Field B

In this experiment, we will not take the magnetic field B into consideration when it comes to uncertainty, as requested.

According to Eq.(**??**), B is proportional to current I_M. We can obtain from Table **??** that when $I_\mathrm{M} = 100$ mA, $B(x=0, I_\mathrm{M} = 100\,\text{[mA]}) = 1.4366 \times 10^{-3}$ T. Therefore, the theoretical value of the magnetic field is

$$B(x=0) = \frac{I_\mathrm{M}}{100} \times 1.4366 \times 10^{-3}.$$

	U_S [V] $\pm 0.5\%$ [V]	U_0 [V] $\pm (0.05\% + 6 \times 10^{-3/-4}$ [V]	U [V] $\pm (0.05\% + 6 \times 10^{-3/-4}$ [V]	K_H/U_S [T^{-1}]
1	2.80 ± 0.014	1.4004 ± 0.0013	1.4656 ± 0.0014	6.5 ± 0.2
2	3.20 ± 0.016	1.6002 ± 0.0014	1.6760 ± 0.0015	6.6 ± 0.2
3	3.60 ± 0.018	1.8022 ± 0.0015	1.8905 ± 0.0016	6.8 ± 0.2
4	4.00 ± 0.020	2.002 ± 0.007	2.097 ± 0.007	6.6 ± 0.2
5	4.40 ± 0.022	2.201 ± 0.008	2.308 ± 0.008	6.8 ± 0.5
6	4.80 ± 0.024	2.405 ± 0.008	2.518 ± 0.008	6.6 ± 0.6
7	5.20 ± 0.026	2.602 ± 0.008	2.723 ± 0.008	6.5 ± 0.6
8	5.60 ± 0.028	2.800 ± 0.008	2.933 ± 0.008	6.6 ± 0.5
9	6.00 ± 0.030	2.999 ± 0.008	3.136 ± 0.008	6.4 ± 0.5
10	6.40 ± 0.032	3.198 ± 0.008	3.343 ± 0.008	6.3 ± 0.5
11	6.80 ± 0.034	3.394 ± 0.008	3.545 ± 0.008	6.2 ± 0.5
12	7.20 ± 0.036	3.592 ± 0.008	3.748 ± 0.008	6.0 ± 0.4
13	7.60 ± 0.038	3.788 ± 0.008	3.952 ± 0.008	6.0 ± 0.4
14	8.00 ± 0.040	3.988 ± 0.008	4.152 ± 0.008	5.7 ± 0.4
15	8.40 ± 0.042	4.183 ± 0.008	4.351 ± 0.009	5.6 ± 0.4
16	8.80 ± 0.044	4.380 ± 0.009	4.552 ± 0.009	5.4 ± 0.3
17	9.20 ± 0.046	4.573 ± 0.009	4.751 ± 0.009	5.4 ± 0.3
18	10.00 ± 0.050	4.963 ± 0.009	5.143 ± 0.009	5.0 ± 0.3

Table 3: Data for U_0, U and K_H/U_S for different U_S.

Take the last set of data as an example,

$$B(x = 0, I_M = 0.50\,[\mathrm{A}]) = \frac{1.4366 \times 10^{-3}}{100 \times 10^{-3}} \times I_M = 7.2 \times 10^{-3}\,[\mathrm{T}].$$

It is noticed that the measured U_{out} is the amplified output of U_H and we supposed that $U = k \cdot U_H$. Then, theoretically, according to Eq. (**??**), we can derive that

$$B(x = 0) = \frac{U - U_0}{K_H} = \frac{U_{out}}{K_H} = k \cdot \frac{U_H}{K_H},$$

where k is a constant. Therefore, $B(x = 0)$ is supposed to be proportional to the Hall voltage U_H.

The experimental results are shown in Table 4. By applying linear fit to the I_M vs. U plot (Fig.(**??**)), the slope of the curve is then the measured sensitivity K_H, which is 33.6 V/T, with uncertainty 0.7 V/T. The measurement result of the sensitivity is then $K_H = 33.6 \pm 0.7\,\mathrm{V/T}$.

	I_M [A] $\pm 2\%$ [A]	U_{out} [mV] $\pm (0.05\%+$ $6 \times 10^{-3})$ [mV]	$B(x = 0)$ [T]
1	0 ± 0	0.83 ± 0.007	0
2	0.05 ± 0.001	30.08 ± 0.03	7.2×10^{-4}
3	0.10 ± 0.002	51.32 ± 0.04	1.4×10^{-3}
4	0.15 ± 0.003	73.38 ± 0.05	2.2×10^{-3}
5	0.20 ± 0.004	97.71 ± 0.06	2.9×10^{-3}
6	0.25 ± 0.005	121.64 ± 0.07	3.6×10^{-3}
7	0.30 ± 0.006	143.67 ± 0.08	4.3×10^{-3}
8	0.35 ± 0.007	166.63 ± 0.09	5.0×10^{-3}
9	0.40 ± 0.008	187.79 ± 0.10	5.7×10^{-3}
10	0.45 ± 0.009	210.10 ± 0.12	6.5×10^{-3}
11	0.50 ± 0.010	231.1 ± 0.13	7.2×10^{-3}

Table 4: Measurement data for the I_M vs. U relation and the calculated data for $B(x = 0)$.

3.3 Magnetic Field Distribution Inside the Solenoid

The measurement result of output voltage U and the corresponding position x are shown in Table 5. According to Eq.(**??**) and the measured value of K_H in section 3.2, $B(x)$ can be obtained from $B(x) = \frac{U}{K_H} = \frac{U}{33.6}$ Take the first set of data as an example,

$$B(x) = \frac{U}{K_H} = \frac{12.20 \times 10^{-3}}{33.6} = (0.363 \pm 0.008) \times 10^{-3}\ [\mathrm{T}].$$

The $B(x)$ are calculated for each set of data and the results are shown in Table 6.

	x[cm]±0.05[cm]	U[mV]±(0.05%+6 × 10^{-3})[mV]		x[cm]±0.05[cm]	U[mV]±(0.05%+6 × 10^{-3})[mV]
1	0.00	12.20 ± 0.012	27	16.00	118.14 ± 0.07
2	0.50	15.27 ± 0.014	28	17.00	118.12 ± 0.07
3	1.00	20.64 ± 0.016	29	18.00	118.04 ± 0.07
4	1.50	27.95 ± 0.02	30	19.00	117.93 ± 0.07
5	2.00	39.47 ± 0.03	31	20.00	117.87 ± 0.07
6	2.50	54.53 ± 0.04	32	21.00	117.48 ± 0.07
7	3.00	71.20 ± 0.05	33	22.00	117.27 ± 0.07
8	3.50	86.03 ± 0.05	34	23.00	116.52 ± 0.07
9	4.00	96.45 ± 0.06	35	24.00	115.35 ± 0.07
10	4.50	103.35 ± 0.06	36	25.00	113.25 ± 0.07
11	5.00	107.76 ± 0.06	37	26.00	109.47 ± 0.06
12	5.50	110.62 ± 0.07	38	27.00	101.05 ± 0.06
13	6.00	112.67 ± 0.07	39	27.20	98.40 ± 0.06
14	6.50	113.90 ± 0.07	40	27.60	91.27 ± 0.06
15	7.00	114.90 ± 0.07	41	27.80	87.47 ± 0.05
16	7.50	115.56 ± 0.07	42	28.00	82.41 ± 0.05
17	8.00	116.18 ± 0.07	43	28.20	75.95 ± 0.05
18	8.50	116.71 ± 0.07	44	28.40	69.49 ± 0.05
19	9.00	117.20 ± 0.07	45	28.60	62.78 ± 0.04
20	9.50	117.28 ± 0.07	46	28.80	56.25 ± 0.04
21	10.00	117.67 ± 0.07	47	29.00	49.60 ± 0.04
22	11.00	117.87 ± 0.07	48	29.20	43.65 ± 0.03
23	12.00	118.02 ± 0.07	49	29.40	37.75 ± 0.03
24	13.00	118.12 ± 0.07	50	29.60	32.68 ± 0.03
25	14.00	118.01 ± 0.07	51	29.80	28.52 ± 0.03
26	15.00	118.19 ± 0.07	52	30.00	25.57 ± 0.019

Table 5: Data for the U vs. x relation

	x [cm] ± 0.05 [cm]	$B(x)$ [10^{-3} T]		x [cm] ± 0.05 [cm]	$B(x)$ [10^{-3} T]
1	0.00	0.363 ± 0.008	27	16.00	3.47 ± 0.08
2	0.50	0.449 ± 0.010	28	17.00	3.47 ± 0.08
3	1.00	0.607 ± 0.013	29	18.00	3.47 ± 0.08
4	1.50	0.822 ± 0.017	30	19.00	3.47 ± 0.08
5	2.00	1.16 ± 0.03	31	20.00	3.47 ± 0.08
6	2.50	1.60 ± 0.04	32	21.00	3.46 ± 0.08
7	3.00	2.09 ± 0.05	33	22.00	3.45 ± 0.08
8	3.50	2.53 ± 0.06	34	23.00	3.43 ± 0.07
9	4.00	2.83 ± 0.06	35	24.00	3.39 ± 0.07
10	4.50	3.04 ± 0.07	36	25.00	3.33 ± 0.07
11	5.00	3.17 ± 0.07	37	26.00	3.22 ± 0.07
12	5.50	3.25 ± 0.07	38	27.00	2.97 ± 0.07
13	6.00	3.31 ± 0.07	39	27.20	2.89 ± 0.06
14	6.50	3.35 ± 0.07	40	27.60	2.68 ± 0.06
15	7.00	3.38 ± 0.07	41	27.80	2.57 ± 0.06
16	7.50	3.40 ± 0.07	42	28.00	2.42 ± 0.05
17	8.00	3.42 ± 0.07	43	28.20	2.23 ± 0.05
18	8.50	3.43 ± 0.07	44	28.40	2.04 ± 0.05
19	9.00	3.45 ± 0.07	45	28.60	1.85 ± 0.04
20	9.50	3.45 ± 0.08	46	28.80	1.65 ± 0.04
21	10.00	3.46 ± 0.08	47	29.00	1.46 ± 0.03
22	11.00	3.47 ± 0.08	48	29.20	1.28 ± 0.03
23	12.00	3.47 ± 0.08	49	29.40	1.11 ± 0.03
24	13.00	3.47 ± 0.08	50	29.60	0.96 ± 0.02
25	14.00	3.47 ± 0.08	51	29.80	0.839 ± 0.018
26	15.00	3.48 ± 0.08	52	30.00	0.752 ±0.016

Table 6: Data for the $B(x)$ vs. x relation.

The theoretical curve of the magnetic field distribution inside the solenoid can be obtained from Eq.(**??**) and the data in Table **??**, by multiplying the data in the table by $\frac{250}{100} = 2.5$, since we have set the current as

250 mA instead of 100 mA.

Then we plot the theoretical curve together with the measured value of the magnetic field distribution in Fig.(**??**). The origin of the plot is set at the center of the solenoid, thus 15 cm are subtracted from the x in the measurement data.

4 Uncertainty Analysis

4.1 Relationship Between Sensitivity K_H and Working Voltage U_S

According to Table 2, the uncertainties are calculated as

$$\begin{aligned} u_{U_\mathrm{S}} &= 5.00 \times 0.5\% = 0.03 \text{ [V]} \\ u_{U_0} &= 2.504 \times 0.05\% + 6 \times 10^{-3} = 0.007 \text{ [V]} \\ u_U &= 2.621 \times 0.05\% + 6 \times 10^{-3} = 0.007 \text{ [V]} \\ u_{I_\mathrm{M2}} &= 250 \times 10^{-3} \times 2\% = 5 \times 10^{-3} \text{ [A]} \end{aligned}$$

while $B(x = 0, I_\mathrm{M} = 250 \text{ [mA]}) = \frac{I_\mathrm{M2}}{I_\mathrm{M1}} \times B(x = 0, I_\mathrm{M} = 100 \text{ [mA]})$

$$u_B = \frac{B(x = 0, I_\mathrm{M} = 100 \text{ [mA]})}{I_\mathrm{M1}} u_{I_\mathrm{M2}} = \frac{1.4366 \times 10^{-3}}{100 \times 10^{-3}} \times 5 \times 10^{-3} = 7 \times 10^{-5} \text{ [T]}$$

Then for $K_\mathrm{H} = \frac{U - U_0}{B}$, its uncertainty is

$$\begin{aligned} u_{K_\mathrm{H}} &= \sqrt{(\frac{\partial K_\mathrm{H}}{\partial U} u_U)^2 + (\frac{\partial K_\mathrm{H}}{\partial U_0} u_{U_0})^2 + (\frac{\partial K_\mathrm{H}}{\partial B} u_B)^2} = \sqrt{(\frac{u_U}{B})^2 + (\frac{-u_{U_0}}{B})^2 + (-\frac{(U - U_0) u_B}{B^2})^2} \\ &= \sqrt{(\frac{0.007}{1.4366 \times 10^{-3} \times 250/100})^2 + (\frac{-0.007}{1.4366 \times 10^{-3} \times 250/100})^2 + (\frac{(2.621 - 2.504) \times 7 \times 10^{-5}}{(1.4366 \times 10^{-3} \times 250/100)^2})^2} \\ &= 3 \text{ [V/T]}. \end{aligned}$$

For Table 3, the uncertainties of data for voltage measurements are calculated as follows. Take the first set of data as an example,

$$u_{U_\mathrm{S}} = 2.80 \times 0.5\% = 0.014 \text{ [V]},$$

$$u_{U_0} = 1.4004 \times 0.05\% + 6 \times 10^{-4} = 0.0013 \text{ [V]},$$

$$u_U = 1.4656 \times 0.05\% + 6 \times 10^{-4} = 0.0014 \text{ [V]}.$$

The uncertainty for $K_\mathrm{H}/U_\mathrm{S} = \frac{U - U_0}{B U_\mathrm{S}}$ is calculated as

$$\begin{aligned} u_{K_\mathrm{H}/U_\mathrm{S}} &= \sqrt{(\frac{\partial K_\mathrm{H}/U_\mathrm{S}}{\partial U} u_U)^2 + (\frac{\partial K_\mathrm{H}/U_\mathrm{S}}{\partial U_0} u_{U_0})^2 + (\frac{\partial K_\mathrm{H}/U_\mathrm{S}}{\partial U_\mathrm{S}} u_{U_\mathrm{S}})^2 + (\frac{\partial K_\mathrm{H}/U_\mathrm{S}}{\partial B} u_B)^2} \\ &= \sqrt{(\frac{u_U}{B U_\mathrm{S}})^2 + (\frac{-u_{U_0}}{B U_\mathrm{S}})^2 + (-\frac{U - U_0}{B U_\mathrm{S}^2} u_{U_\mathrm{S}})^2 + (-\frac{U - U_0}{B^2 U_\mathrm{S}} u_B)^2} \\ &= \sqrt{(\frac{0.0014}{3.59 \times 10^{-3} \times 2.80})^2 + (\frac{-0.0013}{3.59 \times 10^{-3} \times 2.80})^2 + (-\frac{(1.4656 - 1.4004) \times 0.014}{3.59 \times 10^{-3} \times 2.80^2})^2 + (-\frac{(1.4656 - 1.4004) \times 7 \times 1}{(3.59 \times 10^{-3})^2 \times 2.80} \\ &= 0.2 \text{ [T}^{-1}\text{]}. \end{aligned}$$

The uncertainties of all other data in Table 3 are calculated in this way and the results are presented in Table 7.

	u_{U_S} [V]	u_{U_0} [V]	u_U [V]	u_{K_H/U_S} [T^{-1}]
1	0.014	0.0013	0.0014	0.2
2	0.016	0.0014	0.0015	0.2
3	0.018	0.0015	0.0016	0.2
4	0.020	0.007	0.007	0.2
5	0.022	0.008	0.008	0.5
6	0.024	0.008	0.008	0.6
7	0.026	0.008	0.008	0.6
8	0.028	0.008	0.008	0.5
9	0.030	0.008	0.008	0.5
10	0.032	0.008	0.008	0.5
11	0.034	0.008	0.008	0.5
12	0.036	0.008	0.008	0.4
13	0.038	0.008	0.008	0.4
14	0.040	0.008	0.008	0.4
15	0.042	0.008	0.009	0.4
16	0.044	0.009	0.009	0.3
17	0.046	0.009	0.009	0.3
18	0.050	0.009	0.009	0.3

Table 7: Uncertainties of data in Table 3.

4.2 Uncertainty of Input Current I_M, Output Voltage U and Magnetic Field B

Take the last set of data in Table 4 as an example.

The uncertainty for I_M is

$$u_{I_M} = 0.50 \times 2\% = 0.010 \text{ [A]}$$

The uncertainty for U is

$$u_U = 231.1 \times 0.05\% + 6 \times 10^{-3} = 0.12 \text{ [mV]}.$$

The uncertainties of all other data in Table 4 are calculated in this way are the results are presented in Table 8.

	u_{I_M} [A]	$u_{U_{out}}$ [V]
1	0	0.007
2	0.001	0.03
3	0.002	0.04
4	0.003	0.05
5	0.004	0.06
6	0.005	0.07
7	0.006	0.08
8	0.007	0.09
9	0.008	0.10
10	0.009	0.12
11	0.010	0.13

Table 8: Uncertainty of data in Table 4.

4.3 Uncertainty of Magnetic Field Inside the Solenoid Measurement

The uncertainty of position measurement is 0.05 cm.
As for the uncertainty of the output voltage, taking the first set of data as an example,

$$u_U = 12.20 \times 0.05\% + 6 \times 10^{-3} = 0.012 \text{ [mV]}$$

Due to $B(x) = \frac{U}{K_H}$,

$$u_B = \sqrt{(\frac{\partial B}{\partial U} u_U)^2 + (\frac{\partial B}{\partial K_H} u_{K_H})^2} = \sqrt{(\frac{u_U}{K_H})^2 + (-\frac{U}{K_H^2} u_{K_H})^2}.$$

Taking the first set of data as an example,

$$u_B = \sqrt{(\frac{0.012 \times 10^{-3}}{33.6})^2 + (-\frac{12.20 \times 10^{-3}}{33.6^2} \times 0.7)^2} = 0.008 \times 10^{-3} \text{ [T]}.$$

The uncertainties for all other sets of data are calculated and shown in Table 9.

	u_U [mV]	$B(x)$ [10^{-3} T]		u_U [mV]	$B(x)$ [10^{-3} T]
1	0.012	0.008	27	0.07	0.08
2	0.014	0.010	28	0.07	0.08
3	0.016	0.013	29	0.07	0.08
4	0.02	0.017	30	0.07	0.08
5	0.03	0.03	31	0.07	0.08
6	0.04	0.04	32	0.07	0.08
7	0.05	0.05	33	0.07	0.08
8	0.05	0.06	34	0.07	0.07
9	0.06	0.06	35	0.07	0.07
10	0.06	0.07	36	0.07	0.07
11	0.06	0.07	37	0.06	0.07
12	0.07	0.07	38	0.06	0.07
13	0.07	0.07	39	0.06	0.06
14	0.07	0.07	40	0.06	0.06
15	0.07	0.07	41	0.05	0.06
16	0.07	0.07	42	0.05	0.05
17	0.07	0.07	43	0.05	0.05
18	0.07	0.07	44	0.05	0.05
19	0.07	0.07	45	0.04	0.04
20	0.07	0.08	46	0.04	0.04
21	0.07	0.08	47	0.04	0.03
22	0.07	0.08	48	0.03	0.03
23	0.07	0.08	49	0.03	0.03
24	0.07	0.08	50	0.03	0.02
25	0.07	0.08	51	0.03	0.018
26	0.07	0.08	52	0.019	0.016

Table 9: The uncertainties of U and B.

5 Conclusion and Discussion

5.1 Conclusion

In this exercise, we learnt how to use integrated Hall probe and measured the Hall sensitivity of the Hall probe under different working voltages. Besides, we derived two experimental value of K_H in two different ways, where one is direct measurement and the other is linear fit. We also measured the magnetic field inside a solenoid. Among all the steps and operations, we conclude that

- As Fig.(**??**) suggests, the sensitivity has a decreasing trend with working voltage increasing.
- The output voltage is probably linearly dependent on the magnetic field since the Pearson's r of linear fit in Fig.(**??**) is 0.998 which is very close to 1.
- The K_H we measure is quite precise since the relative error is only $\frac{33.6-31.25}{31.25} \times 100\% = 7.52\%$.
- The magnitudes of measured magnetic field and theoretical magnetic field differ a bit but with a raltively constant difference throughout the whole curve.

The detailed analysis will be presented in Discussion part.

5.2 Discussion

5.2.1 Problems

1. As for the relationship between sensitivity of SS495A and the working voltage applied on it, it is positively proportional in fact[4], which contradicts the data we derived in this exercise.
2. Measured data has relatively great deviation of theoretical value as Fig.(**??**) suggests.

5.2.2 Potential errors

Here we conclude some potential errors occuring in this exercise.

1. Uneven Hall Probe

The materials consisting of the integrated Hall probe may not be so even that there will also trigger electric field even if there is no magnetic field applied. This magnetic filed triggered by the material itself may be opposite to the magnetic filed applied, which will leesen the magnitude of magnetic field we detected.

2. Ettingshausen effect and Nernst effect

   Due to Hall effect, electrons are forced to move perpendicular to the applied current. However, the accumulation of electrons on one side of the sample enables the number of collisions to increase and a heating of the material occurs, which will trigger a heated electromotive force. This emf will decrease the the effect of origin current in converse.

3. Reading Process

   The sensibility of the experiment device is so low that we can hardly grasp a more precise number about two digits after the decimal point. The reading process may cause huge deviation due to the unstability of the data displayed.

According to the above discussion, we may explain the difference between data we measured and the theoretical value. In general, the results we derived are still credible and worthy. Here we also provide some suggestions to impove.

5.2.3 Imporvements

- Use more precise experiment devices.
- Control the experimental temperature better to avoid temperarture deviation.
- Cool down the integrated Hall probe as we measure the data.

6 Reference

[1] M. Krzyzosiak (2021). Exercise 3 - lab manual [rev 5]. (UMJI-SJTU, Shanghai).
[2] `https://en.wikipedia.org/wiki/P-n_junction`
[3] `https://www.sciencedirect.com/topics/engineering/photovoltaic-effect`
[4] Sinocomopto Technologies Ltd. Company, *SS495A Product Manual*, Archived from the original on 2021-11-11.

APPENDIX - DATA SHEET