UM–SJTU JOINT INSTITUTE

PHYSICS LABORATORY

(VP241)

LABORATORY REPORT

EXERCISE 4

POLARIZATION OF LIGHT

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Date: 19 November 2021

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1 Introduction

The objective of this exercise is to understand some properties of light, in particular to study the polarization phenomenon. Here we will also verify Malus' law, as well as to understand the way half- and quarter-wave plates work in optical systems. Besides, some investigation of elliptically and circularly polarized light will be generated and detected.

1.1 Basic Concepts

- **Polarization:**[1] Polarization (also polarisation) is a property applying to transverse waves that specifies the geometrical orientation of the oscillations. In a transverse wave, the direction of the oscillation is perpendicular to the direction of motion of the wave..

- **Polarizer:**[2] A polarizer or polariser is an optical filter that lets light waves of a specific polarization pass through while blocking light waves of other polarizations.
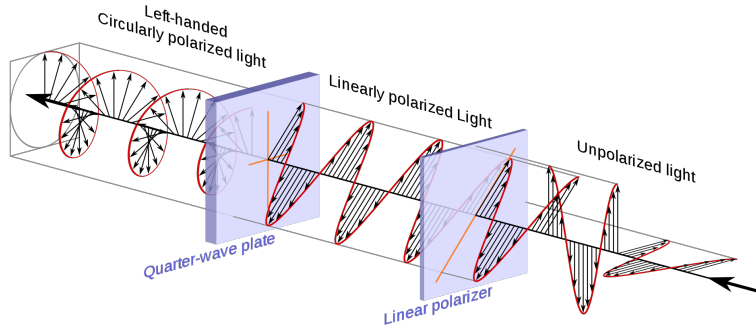
Figure 1: Polarizer

- **Malus' law:**[3] Malus' law states that the intensity of plane-polarized light that passes through an analyzer varies as the square of the cosine of the angle between the plane of the polarizer and the transmission axes of the analyzer.

$$I_{\text{light}} = I_{\text{light },0} \cos^2 \theta \tag{1}$$

where $I_{\text{light },0}$ is the intensity of the linearly polarized light incident on the analyzer.
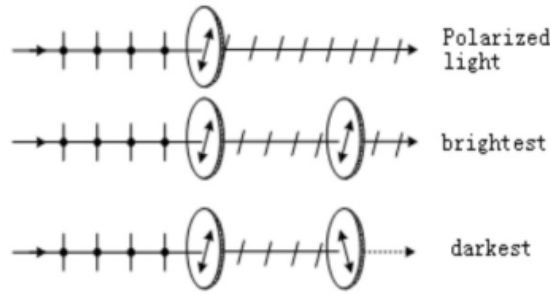
Figure 2: Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

1.2 Theoretical Basis

1.2.1 Polarization of Light[3]

Light in nature is electromagnetic wave. The electric field vector **E** in the electromagnetic wave is referred to as the light vector. Light emitted from ordinary light sources (natural light) is unpolarized. However it can be regarded as a statistical equal-weight mixture of linearly polarized waves with equal amplitudes. There the light may be also partially polarized, which means it can be regarded as a combination of a polarized and the natural (unpolarized) light. The direction corresponding to the maximum amplitude of the light vector of such partially polarized light is the oscillation direction of the polarized component.
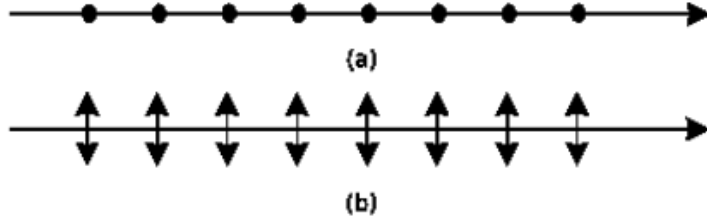
Figure 3: (a) Linearly polarized light with the polarization axis perpendicular to the page plane. (b) Linearly polarized light with the polarization axis parallel to the page plane.

1.2.2 Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

Suppose that linearly polarized light is incident normally on a crystal plate whose surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is α. Then the linearly polarized light is resolved into two waves: an e-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is

$$\Delta = (n_{\mathrm{e}} - n_{\mathrm{o}})\, d \tag{2}$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda}\, (n_{\mathrm{e}} - n_{\mathrm{o}})\, d \tag{3}$$

where λ is the wavelength, n_{e} is the refractive index for the extraordinary axis, and n_{o} is the refractive index for the ordinary axis. In a so-called positive crystal $\delta > 0$, whereas in a negative one $\delta < 0$.
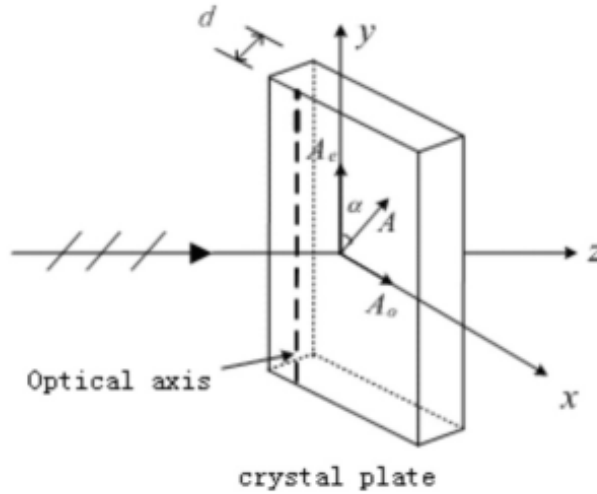
Figure 4: Linearly polarized light passing through a waveplate.

As shown in Fig.(4) , when the light propagates through the crystal plate, the two components of the light vector are

$$E_x = A_{\mathrm{o}} \cos \omega t$$
$$E_y = A_{\mathrm{e}} \cos(\omega t + \delta)$$

where $A_{\mathrm{e}} = A \cos \alpha$, $A_{\mathrm{o}} = A \sin \alpha$. Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_{\mathrm{o}}^2} + \frac{E_y^2}{A_{\mathrm{e}}^2} - 2\frac{E_x E_y}{A_{\mathrm{o}} A_{\mathrm{e}}} \cos \delta = \sin^2 \delta \tag{4}$$

Note that is the equation of an ellipse for $\delta = \pm \pi/2$. When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

4

▶ If $\Delta = k\lambda$, where $k = 0, 1, 2, \ldots$, the phase difference $\delta = 0$, and Eq.(4) reduces to

$$E_y = \frac{A_e}{A_o} E_x$$

which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a full-wave plate. The light goes through a full-wave plate without changing its polarization state.

▶ If $\Delta = (2k+1)\lambda/2$, where $k = 0, 1, 2, \ldots$, the phase difference $\delta = \pi$, and Eq.(4) simplifies to

$$E_y = -\frac{A_e}{A_o} E_x$$

The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α. A waveplate that satisfies the condition is called 1/2-wave plate or half-wave plate. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α. If $\alpha = \pi/4$, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

▶ Finally, if $\Delta = (2k+1)\lambda/4$, where $k = 0, 1, 2, \ldots$, the phase difference $\delta = \pm\pi/2$ and Eq. (2) transforms into

$$\frac{E_x^2}{A_o^2} + \frac{E_y^2}{A_e^2} = 1$$

The transmitted light is elliptically polarized. A waveplate that satisfies the above condition is called a 1/4-wave plate or a quarter-waveplate and is an important optical element in many polarization experiments.

If $A_e = A_o = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the o-wave and the e-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle: ▶ if $\alpha = 0$, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate; ▶ if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate; ▶ if $\alpha = \pi/4$, the transmitted light is circularly polarized; ▶ otherwise, the transmitted light is elliptically polarized.

2 Apparatus and Measurement Procedure

2.1 Apparatus

The measurement setup consists of: a semiconductor laser, a tungsten iodine lamp, a silicon photo-cell, a UT51 digital universal meter, as well as two polarizers, 1/2-wave and 1/4-wave plates (the uncertainty of the the angle is 2°) and a lens with a glass sheet. The elements are placed on an optical bench.

The precisions of the devices are shown in Table 1.

Instrument	Uncertainties
Angle	$\pm 2°$
Current	$\pm 0.01~\mu A$

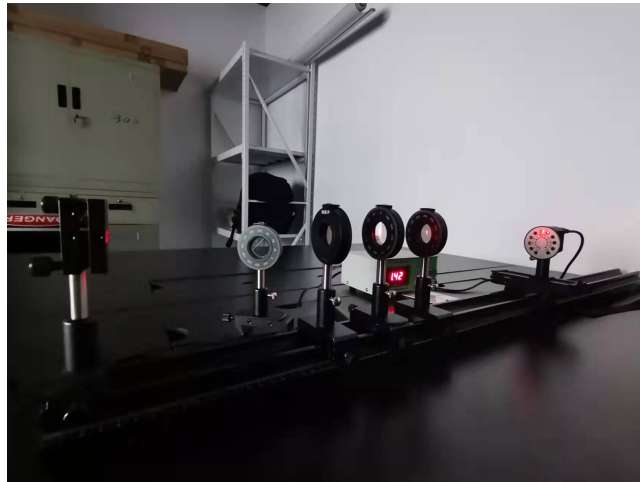Table 1: Information of measurement instruments.

Figure 5: Apparatus

2.2 Measurement Procedure

2.2.1 Apparatus Adjustment

a. Adjust the laser and the photo-cell so that the light can pass through the ϕ 6.0 aperture.

b. Placed the analyzer onto the optical bench, adjust its position so that the current detected is in the range $0.8 \sim 1.8~\mu A$.

2.2.2 Demonstration of Malus' Law

a. Place an analyzer between the polarizer and the photo-cell. Record this value as I_0.

b. Adjust the angle of the analyzer until the electric current reaches 0. Set this position as $\theta = 90°$.

c. Rotate the analyzer from 90° to 0° and record the magnitude of the current I every 5° and record the values.

d. To verify Malus' Law, plot the graph I/I_0 vs. $\cos^2 \theta$ and perform linear fitting.

2.2.3 Linearly Polarized Light and the Half-wave Plate

a. Place the 1/2-wave plate between the polarizer and the analyzer. Rotate it to make the light extinction appear again and set this position as the initial position.

b. Rotate the 1/2-wave plate for $\alpha = 10°$ from the initial position and rotate the analyzer to make the light extinction appear again, record the angle of rotation $\Delta\theta$.

c. To explore the relationship, plot the graph $\Delta\theta$ vs. θ.

2.2.4 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

a. Replace the 1/2-wave plate by the 1/4-wave plate and rotate it to make the light extinction appear and set this position as the initial position. At this point $\alpha = 0°$. Then rotate the 1/4-wave plate and observe the change in the light intensity.

b. Rotate the analyzer for 360° and record the light intensity (which is indicated by the current I) for every 10° and record the data in a table.

c. Rotate the 1/4-wave plate for 20°, repeat Step 3.

d. Rotate the 1/4-wave plate for 45°, and repeat Step 3.

e. Rotate the 1/4-wave plate for 70°. Rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches a maximum.

f. To find out the relation, plot the relation between the rotation angle of the analyzer and the light amplitude in polar coordinates. Normalize the amplitude by its maximum value. Mark the position recorded in Step 6 and compare it with the data recorded.

g. To compare with the circular polarization, plot a linear fit to the data when the angle is 45°.

2.3 Safety Notice

a. Do not direct the laser beam into the eye.

b. Do not touch the surface of the polarizers or the wave plates.

c. Please leave the equipment in order before leaving.

3 Experimental Results

3.1 Demonstration of Malus' Law

The measurement data are presented in Table 2.

Maximum Electric Current I_0		2.21 ± 0.01 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.01 [μA]	θ [°] ± 2[°]	I [μA] ± 0.01 [μA]
0	2.21	50	0.96
5	2.20	55	0.77
10	2.16	60	0.58
15	2.09	65	0.42
20	1.99	70	0.30
25	1.89	75	0.17
30	1.75	80	0.08
35	1.57	85	0.03
40	1.37	90	0.00
45	1.17		

Table 2: Measurement data for Malus' law demonstration.

To find the relation between $\cos^2 \theta$ and I/I_0, the two set of values need to be calculated first. Take the first set of data as an example.

$$\cos^2 \theta = \cos^2(0 \times \frac{\pi}{180}) = 1 \pm 0,$$

$$\frac{I}{I_0} = \frac{2.21}{2.21} = 1.000 \pm 0.006.$$

Perform similar calculations to each of the other sets of data and the results are shown in Table 3.

θ [°] ± 2 [°]	I [μA] ± 0.01 [μA]	$\cos^2 \theta$	I/I_0
0	2.21	1 ± 0	1.000 ± 0.006
5	2.20	0.992 ± 0.006	0.995 ± 0.006
10	2.16	0.970 ± 0.012	0.977 ± 0.006
15	2.09	0.93 ± 0.02	0.946 ± 0.006
20	1.99	0.88 ± 0.02	0.900 ± 0.006
25	1.89	0.82 ± 0.03	0.855 ± 0.006
30	1.75	0.75 ± 0.03	0.792 ± 0.006
35	1.57	0.67 ± 0.03	0.710 ± 0.006
40	1.37	0.59 ± 0.03	0.620 ± 0.005
45	1.17	0.50 ± 0.03	0.529 ± 0.005
50	0.96	0.41 ± 0.03	0.434 ± 0.005
55	0.77	0.33 ± 0.03	0.348 ± 0.005
60	0.58	0.25 ± 0.03	0.262 ± 0.005
65	0.42	0.18 ± 0.03	0.190 ± 0.005
70	0.30	0.12 ± 0.02	0.136 ± 0.005
75	0.17	0.07 ± 0.02	0.077 ± 0.005
80	0.08	0.030 ± 0.012	0.036 ± 0.005
85	0.03	0.008 ± 0.006	0.014 ± 0.005
90	0.00	0 ± 0	0.000 ± 0.005

Table 3: Result for $\cos^2 \theta$ and I/I_0.

Then, a linear fit of the plot I/I_0 vs. θ are performed (Fig.6). The slope of the linear fitting is 1.009 ± 0.004.
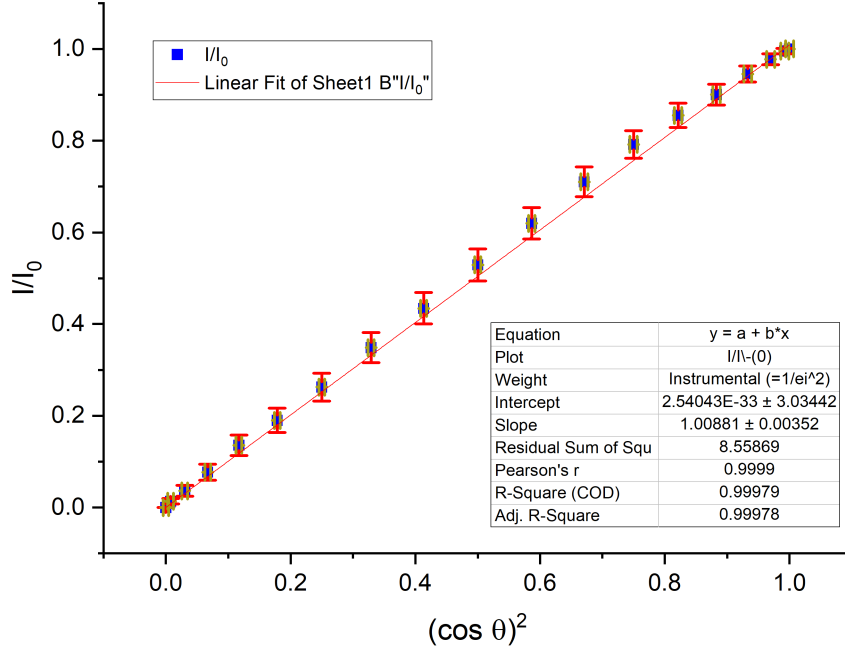
Figure 6: Linear fit of I/I_0 vs. $\cos^2\theta$ relation.

3.2 Linearly Polarized Light and the Half-wave Plate

The measurement data are presented in Table 4. Note that we have substracted each of the data by the initial angle in the report.

Rotation angle of the 1/2-wave plate $[°] \pm 2[°]$	Rotation angle of the analyzer $[°] \pm 2[°]$
initial	0
10	20
20	42
30	59
40	83
50	100
60	121
70	142
80	159
90	179

Table 4: Measurement data for the 1/2-wave plate.

To find the relation between $\Delta\theta$ and θ, the data are plotted in Figure 7 and linear fit is performed. The slope of the linear fit is 1.992 ± 0.016.
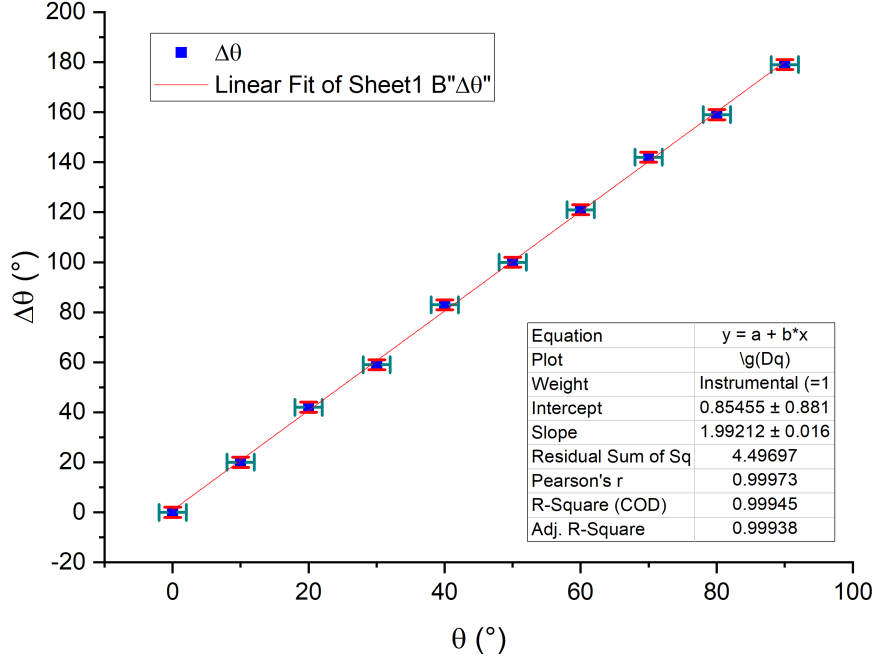
Figure 7: Linear fit of $\Delta\theta$ vs. θ.

Besides, in this part, when the half-wave plate rotates for 360°, **4** times of light extinction are observed. And when the analyzer rotates 360°, **2** times of light extinction during the analyzer rotating 360°. It can be concluded from the above result that **the polarization axis get rotated by twice of the origin angle (2α)** after the light passes through the 1/2-wave plate.

3.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

3.3.1 Rotation Angle: 0°

The measurement data for 0° rotation angle of 1/4-wave plate are presented in Table 5. Note that when filling the data sheet, 0° and 90 ° are mistaken and the mistake is corrected in the report.

Rotation angle of 1/4-wave plate: 0°			
Maximum Electric Current I_0		$1.48 \pm 0.01 \ [\mu A]$	
$\theta \ [°] \pm 2[°]$	$I \ [\mu A] \pm 0.01 \ [\mu A]$	$\theta \ [°] \pm 2[°]$	$I \ [\mu A] \pm 0.01 \ [\mu A]$
0	0.02	180	0.02
10	0.05	190	0.06
20	0.18	200	0.17
30	0.37	210	0.38
40	0.62	220	0.62
50	0.88	230	0.88
60	1.13	240	1.11
70	1.30	250	1.29
80	1.43	260	1.41
90	1.48	270	1.48
100	1.39	280	1.44
110	1.25	290	1.35
120	1.10	300	1.15
130	0.88	310	0.90
140	0.62	320	0.65
150	0.39	330	0.41
160	0.20	340	0.21
170	0.07	350	0.08

Table 5: Measurement data for the 1/4-wave plate (rotation angle 0°).

As described in the procedure part, $\sqrt{I/I_0}$ is calculated. Take the first set of data as an example,

$$\sqrt{\frac{I}{I_0}} = \sqrt{\frac{0.02}{1.48}} = 0.12 \pm 0.03.$$

Perform similar calculations to each of the other sets of data and the results are presented in Table 6.

θ [°] ± 2[°]	$\sqrt{I/I_0}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$
0	0.12 ± 0.03	180	0.12 ± 0.03
10	0.18 ± 0.02	190	0.20 ± 0.02
20	0.349 ± 0.010	200	0.339 ± 0.010
30	0.500 ± 0.007	210	0.507 ± 0.007
40	0.647 ± 0.006	220	0.647 ± 0.006
50	0.771 ± 0.005	230	0.771 ± 0.005
60	0.874 ± 0.005	240	0.866 ± 0.005
70	0.937 ± 0.005	250	0.934 ± 0.005
80	0.983 ± 0.004	260	0.976 ± 0.004
90	1.000 ± 0.004	270	1.000 ± 0.004
100	0.969 ± 0.004	280	0.986 ± 0.004
110	0.919 ± 0.005	290	0.955 ± 0.004
120	0.862 ± 0.005	300	0.881 ± 0.005
130	0.771 ± 0.005	310	0.780 ± 0.005
140	0.647 ± 0.006	320	0.663 ± 0.006
150	0.513 ± 0.007	330	0.526 ± 0.007
160	0.368 ± 0.010	340	0.377 ± 0.009
170	0.22 ± 0.02	350	0.232 ± 0.015

Table 6: Results for $\sqrt{I/I_0}$ when rotation angle is 0°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Fig.8).
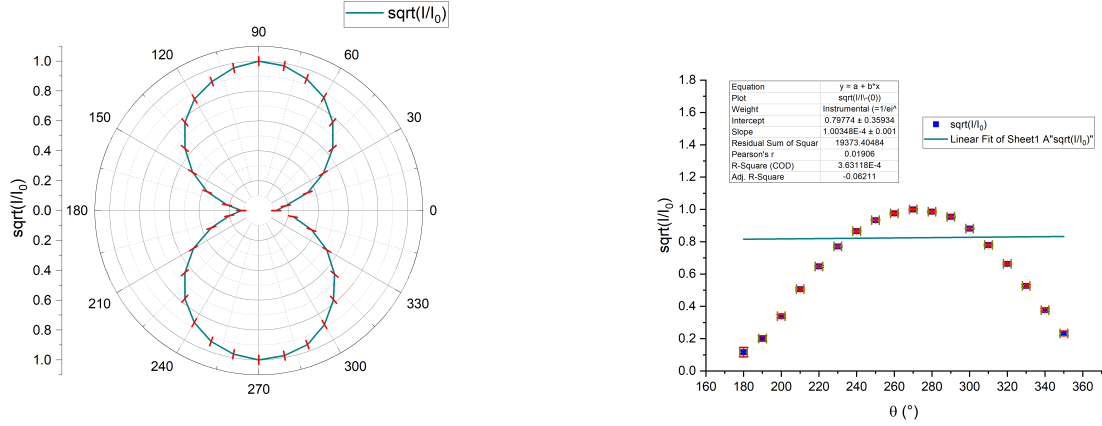
(a) $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 0°.

(b) $\sqrt{I/I_0}$ vs. θ relation in Cartesian coordinate when rotation angle is 0°.

Figure 8: $\sqrt{I/I_0}$ vs. θ relation when rotation angle is 20°.

3.3.2 Rotation Angle: 20°

The measurement data for 20° rotation angle of 1/4-wave plate are shown in Table 7. Note that when filling the data sheet, 0° and 90 ° are mistaken and the mistake is corrected in the report.

Rotation angle of 1/4-wave plate: 20°			
Maximum Electric Current I_0		0.707 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.31	180	0.30
10	0.21	190	0.22
20	0.19	200	0.20
30	0.24	210	0.24
40	0.36	220	0.37
50	0.51	230	0.52
60	0.70	240	0.71
70	0.89	250	0.88
80	1.05	260	1.04
90	1.20	270	1.18
100	1.26	280	1.27
110	1.26	290	1.32
120	1.24	300	1.28
130	1.16	310	1.15
140	0.98	320	0.98
150	0.80	330	0.82
160	0.61	340	0.63
170	0.45	350	0.45

Table 7: Measurement data for the 1/4-wave plate (rotation angle 20°).

Similar to the previous section, $\sqrt{I/I_0}$ is calculated and the results are presented in Table 8 (For sample calculation please refer to section 3.3.1, which will not be repeated here).

θ [°] ± 2[°]	$\sqrt{I/I_0}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$
0	0.485 ± 0.008	180	0.477 ± 0.009
10	0.399 ± 0.010	190	0.408 ± 0.010
20	0.379 ± 0.011	200	0.389 ± 0.010
30	0.426 ± 0.009	210	0.426 ± 0.009
40	0.522 ± 0.008	220	0.529 ± 0.008
50	0.622 ± 0.007	230	0.628 ± 0.007
60	0.728 ± 0.006	240	0.733 ± 0.006
70	0.821 ± 0.006	250	0.816 ± 0.006
80	0.892 ± 0.005	260	0.888 ± 0.005
90	0.953 ± 0.005	270	0.945 ± 0.005
100	0.977 ± 0.005	280	0.981 ± 0.005
110	0.977 ± 0.005	290	1.000 ± 0.005
120	0.969 ± 0.005	300	0.985 ± 0.005
130	0.937 ± 0.005	310	0.933 ± 0.005
140	0.862 ± 0.005	320	0.862 ± 0.005
150	0.778 ± 0.006	330	0.788 ± 0.006
160	0.680 ± 0.006	340	0.691 ± 0.006
170	0.584 ± 0.007	350	0.584 ± 0.007

Table 8: Results for $\sqrt{I/I_0}$ when rotation angle is 20°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Fig.9).
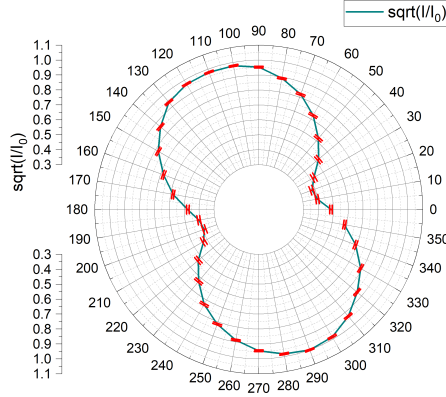
(a) $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 20°.
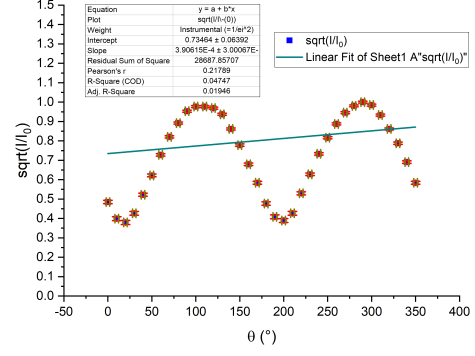
(b) $\sqrt{I/I_0}$ vs. θ relation in Cartesian coordinate when rotation angle is 20°.

Figure 9: $\sqrt{I/I_0}$ vs. θ relation when rotation angle is 20°.

3.3.3 Rotation Angle: 45°

The measurement data for 45° rotation angle of 1/4-wave plate are shown in Table 9. Note that when filling the data sheet, 0° and 90° are mistaken and the mistake is corrected in the report.

Rotation angle of 1/4-wave plate: 45°

Maximum Electric Current I_0		0.395 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.69	180	0.70
10	0.69	190	0.71
20	0.70	200	0.73
30	0.74	210	0.74
40	0.76	220	0.77
50	0.78	230	0.79
60	0.80	240	0.80
70	0.80	250	0.81
80	0.81	260	0.81
90	0.82	270	0.81
100	0.79	280	0.80
110	0.78	290	0.80
120	0.77	300	0.79
130	0.76	310	0.76
140	0.74	320	0.72
150	0.72	330	0.71
160	0.71	340	0.71
170	0.70	350	0.70

Table 9: Measurement data for the 1/4-wave plate (rotation angle 45°).

Similar to the previous section, $\sqrt{I/I_0}$ is calculated and the results are presented in Table 10 (For sample calculation please refer to section 3.3.1, which will not be repeated here).

$\theta\,[°]\pm 2[°]$	$\sqrt{I/I_0}$	$\theta\,[°]\pm 2[°]$	$\sqrt{I/I_0}$
0	0.917 ± 0.009	180	0.924 ± 0.009
10	0.917 ± 0.009	190	0.931 ± 0.009
20	0.924 ± 0.009	200	0.944 ± 0.009
30	0.950 ± 0.009	210	0.950 ± 0.009
40	0.963 ± 0.009	220	0.969 ± 0.009
50	0.975 ± 0.009	230	0.982 ± 0.009
60	0.988 ± 0.009	240	0.988 ± 0.009
70	0.988 ± 0.009	250	0.994 ± 0.009
80	0.994 ± 0.009	260	0.994 ± 0.009
90	1.000 ± 0.009	270	0.994 ± 0.009
100	0.982 ± 0.009	280	0.988 ± 0.009
110	0.975 ± 0.009	290	0.988 ± 0.009
120	0.969 ± 0.009	300	0.982 ± 0.009
130	0.963 ± 0.009	310	0.963 ± 0.009
140	0.950 ± 0.009	320	0.937 ± 0.009
150	0.937 ± 0.009	330	0.931 ± 0.009
160	0.931 ± 0.009	340	0.931 ± 0.009
170	0.924 ± 0.009	350	0.924 ± 0.009

Table 10: Results for $\sqrt{I/I_0}$ when rotation angle is $45°$.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 10).
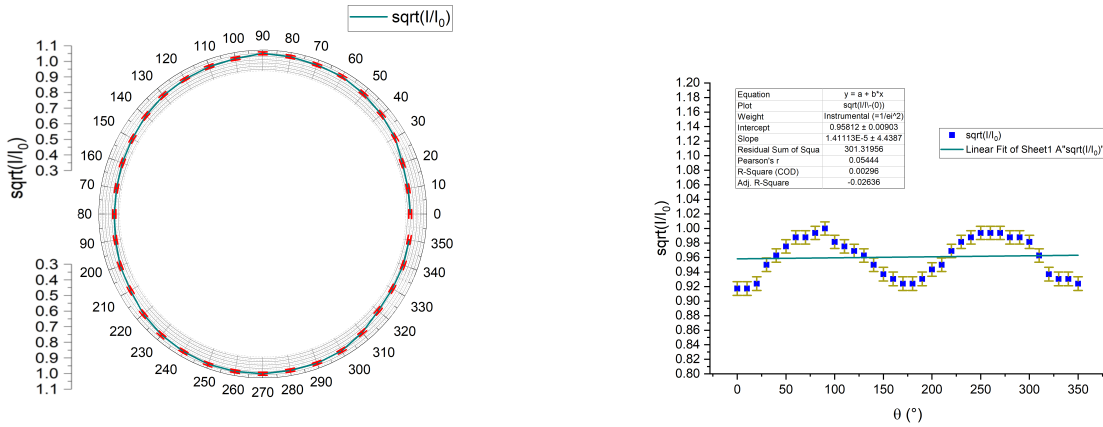
(a) $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is $45°$.

(b) $\sqrt{I/I_0}$ vs. θ relation in Cartesian coordinate when rotation angle is $45°$.

Figure 10: $\sqrt{I/I_0}$ vs. θ relation when rotation angle is $45°$.

To compare the result with circular polarization, as described in the procedure part, the data is also plotted in Cartesian coordinate and linear fit is performed (Fig.10(b)). The slope of the linear fitting is 1.4×10^{-5}.

4 Uncertainty Analysis

4.1 Uncertainty for Data in Demonstration of Malus' Law

The uncertainty of $\cos^2\theta$ is

$$
\begin{aligned}
u_{\cos^2\theta} &= \sqrt{\left(\frac{\partial \cos^2\theta}{\partial \theta}u_\theta\right)^2} \\
&= |-2\cos\theta\sin\theta u_\theta| \\
&= |\sin 2\theta u_\theta|,
\end{aligned}
$$

where $u_\theta = 2[°] = \pi/90$. Take $\theta = 5°$ as an example,

$$
u_{\cos^2\theta} = |\sin 2\theta u_\theta| = \left|\sin(2\times 5°)\times \frac{\pi}{90}\right| = 0.006,
$$

All the results are shown in Table 11.

The uncertainty of I/I_0 is

$$u_{I/I_0} = \sqrt{(\frac{\partial I/I_0}{\partial I}u_I)^2 + (\frac{\partial I/I_0}{\partial I_0}u_{I_0})^2}$$

$$= \sqrt{(\frac{u_I}{I_0})^2 + (-\frac{I}{I_0^2}u_{I_0})^2},$$

where $u_I = u_{I_0} = 0.01$ [μA], $I_0 = 2.21 \pm 0.01$ [μA]. Take the second set of data as an example,

$$u_{I/I_0} = \sqrt{(\frac{u_I}{I_0})^2 + (-\frac{I}{I_0^2}u_{I_0})^2} = \sqrt{(\frac{0.01}{2.21})^2 + (-\frac{2.20}{2.21^2}\times 0.01)^2} = 0.011,$$

All the results are shown in Table 11.

$u_{\cos^2\theta}$	u_{I/I_0}	$u_{\cos^2\theta}$	u_{I/I_0}
0	0.011	0.03	0.006
0.006	0.011	0.03	0.006
0.012	0.011	0.03	0.005
0.02	0.010	0.03	0.005
0.02	0.010	0.02	0.005
0.03	0.010	0.02	0.005
0.03	0.009	0.012	0.005
0.03	0.008	0.006	0.005
0.03	0.008	0	0.005
0.03	0.007		

Table 11: Uncertainty of $\cos^2\theta$ and I/I_0.

4.2 Uncertainty for Linearly Polarized Light and the Half-wave Plate

In this part, the uncertainty of the measurement depends on the device, which is

$$u_\theta = 2°.$$

4.3 Uncertainty for Circularly and Elliptically Polarized Light and the 1/4-wave Plate

The uncertainty of $\sqrt{I/I_0}$ is

$$u_{\sqrt{I/I_0}} = \sqrt{(\frac{\partial \sqrt{I/I_0}}{\partial I}u_I)^2 + (\frac{\partial \sqrt{I/I_0}}{\partial I_0}u_{I_0})^2}$$

$$= \sqrt{\frac{1}{4II_0}u_I^2 + \frac{I}{4I_0^3}u_{I_0}^2},$$

where $u_I = u_{I_0} = 0.01$ [μA], $I_0 = 1.48 \pm 0.01$ [μA] for rotation angle of $0°$, $I_0 = 1.32 \pm 0.01$ [μA] for rotation angle of $20°$, $I_0 = 0.82 \pm 0.01$ [μA] for rotation angle of $45°$. Take the first set of data as an example,

$$u_{\sqrt{I/I_0}} = \sqrt{\frac{1}{4\times 0.02 \times 1.48}\times 0.01^2 + \frac{0.02}{4 \times 1.48^3}\times 0.01^2} = 0.03.$$

All the results are shown in Table 12.

Rotation angle: 0°	Rotation angle: 20°	Rotation angle: 45°
$u_{\sqrt{I/I_0}}$	$u_{\sqrt{I/I_0}}$	$u_{\sqrt{I/I_0}}$
0.03	0.008	0.009
0.02	0.010	0.009
0.010	0.011	0.009
0.007	0.009	0.009
0.006	0.008	0.009
0.005	0.007	0.009
0.005	0.006	0.009
0.005	0.006	0.009
0.004	0.005	0.009
0.004	0.005	0.009
0.004	0.005	0.009
0.005	0.005	0.009
0.005	0.005	0.009
0.005	0.005	0.009
0.006	0.005	0.009
0.007	0.006	0.009
0.010	0.006	0.009
0.02	0.007	0.009
0.03	0.009	0.009
0.02	0.010	0.009
0.010	0.010	0.009
0.007	0.009	0.009
0.006	0.008	0.009
0.005	0.007	0.009
0.005	0.006	0.009
0.005	0.006	0.009
0.004	0.005	0.009
0.004	0.005	0.009
0.004	0.005	0.009
0.004	0.005	0.009
0.005	0.005	0.009
0.005	0.005	0.009
0.006	0.005	0.009
0.007	0.006	0.009
0.009	0.006	0.009
0.015	0.007	0.009

Table 12: Uncertainty for $\sqrt{I/I_0}$ when the rotation angle is 0°, 20° and 45°.

5 Conclusion and Discussion

5.1 Conclusion

5.2 Demonstration of Malus' Law

The slope of the fitting is 1.009 ± 0.004 and the Pearson's r is 0.9999, which is very close to 1. This suggests that the value of I/I_0 is proportional to the value of $\cos^2 \theta$ with the coefficient to be about 1. Theoretically, by Eq. (1), the slope is

$$\frac{I/I_0}{\cos^2 \theta} = 1,$$

Therefore, the relative error is

$$\varepsilon = \frac{1.009 - 1}{1} \times 100\% = 0.9\%.$$

This in an acceptable range of error verifies the Malus' Law.

5.3 Linearly Polarized Light and the Half-wave Plate

The slope of the linear fit is 1.992 ± 0.016. Theoretically, as introduced in the Introduction part, the rotation angle of the polarization axis is twice of the origin angle for a half-wave plate. Therefore the theoretical value

of the slope of our linear fitting is 2. The relative error is therefore

$$\varepsilon = \frac{1.992 - 2}{2} \times 100\% = -0.4\%.$$

This conforms to the theoretical fact.

5.4   Circularly and Elliptically Polarized Light and the 1/4-wave Plate

Here we conclude several points in this exercise.

- From Table 5 and Figure 8, it can be seen that the maximum of light intensity occurs at about $\theta = 0°$. This suggests that polarizing axis is parallel to the optical axis of the plate, which is the theoretical conclusion stated in the Introduction part. The shape of the plot also indicates that it is linearly polarized.

- From Table 7 and Figure 9, it can be seen that the maximum of light intensity occurs at about $\theta = 20°$. This suggests that polarizing axis forms $20°$ to the optical axis of the plate, which conforms to the fact stated in [4]. The shape of the plot also indicates that it is elliptically polarized.

- From Figure 10, it can be seen that the value of $\sqrt{I/I_0}$ changes little when θ changes. Besides, the slop of linear fit of the plot of $\sqrt{I/I_0}$ vs. θ in the Cartesian coordinate in Figure 10 is 1.4×10^{-5} but the Pearson's coefficient is only 0.0544. All the results suggest that $\sqrt{I/I_0}$ is a constant, with no relation with the value of θ. Therefore it can be concluded that when rotation angle is $45°$, the light is circularly polarized. This conforms to the fact stated in the Introduction part.

Detailed analysis and discussion about experimental results are left to DISCUSSION part.

5.5   Discussion

Here we will discuss the problems in this exercise, some potential errors which may lead to huge variation, and some suggestions.

5.5.1   Problems

- **Photocurrent:** Photocurrent meter reading varies continuously during experiments, which may cause huge errors.

- **Apparatus:** Photocurrent meter reading does not change significantly even if the rotation angle reaches $10°$ or $20°$.

- **Light path:** The apparatus may not be exactly horizontal, which may affect the ligh absorption.

5.5.2   Potential errors

- Huge reading errors due to variation of photocurrent meter.

- Inaccurate measurement due to not sensitive apparatus at the boundary point.

- Constant variation due to non-horizontal light path.

5.5.3   Imporvements

- Measure the data for more times and record the mean value of them.

- Apply more advanced experimental device, which can adjust the incident angle more flexible and precise[6].

- Adjust the apparatus and make sure it is properly set before conducting the experiment.

- Add one more polarizer into the apparatus[5].

6 Reference

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APPENDIX - DATA SHEET