UM–SJTU JOINT INSTITUTE
PHYSICS LABORATORY
(VP241)

LABORATORY REPORT

EXERCISE 2

THE HALL PROBE: CHARACTERISTICS AND APPLICATIONS

Name: Haotian Fu ID: 520021910012 Group: 1

Date: 6 November 2021

[rev. 4.0]

Contents

List of Figures

List of Tables

1 Introduction

The objective of this exercise is basically to use a Hall probe to verify the Hall effect and apply it to measure magnetic field.

1.1 Basic Concepts

- **Hall Effect:**[1] The Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor that is transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current. It was discovered by Edwin Hall in 1879.
- **Lorentz Force:**[2] Lorentz force (or electromagnetic force) is the combination of electric and magnetic force on a point charge due to electromagnetic fields.
- **Integrated Hall Probe:**[3] A device that arranges the Hall probe and the electric circuits together.

1.2 Hall Effect

1.2.1 The Principle of the Hall Effect

For a conducting sheet (made of a metal or a semiconductor) placed in a magnetic field, assume the plane of the sheet is vertical to the direction of the magnetic field **B** and the electric current I passes through the sheet in the direction of shown in Fig.(1). Hence an electric potential difference between the sides a and b of the sheet is generated. The corresponding electric field is perpendicular to both the direction of the current and the direction of the magnetic field. This effect is known as **the Hall effect**, ang we name the electric potential difference as **the Hall voltage** U_H.

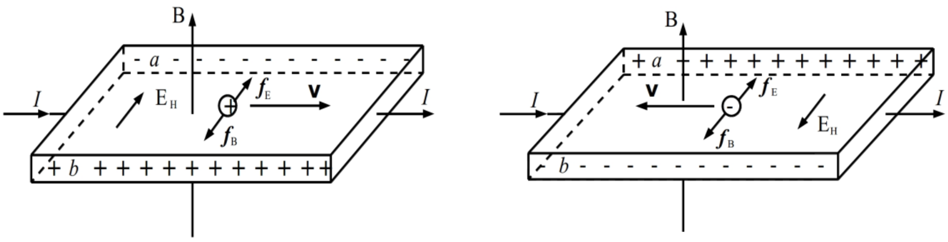


Figure 1: The principle of the Hall effect

1.2.2 Lorentz Force

In a microscopical point of view, the Hall effect is caused by **Lorentz force**. The Lorentz force F_B leads to the deflection of the moving charges, and their accumulation on one side of the sheet, which in turn increases the magnitude of the transverse electric field E_H. Due to this field, there is an electric force F_E acting upon the charges, and since F_B and F_E act in opposite directions, a balance is eventually reached and U_H stabilizes.

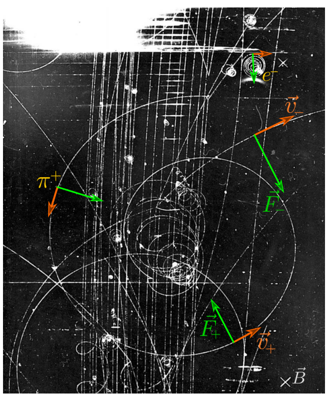


Figure 2: Lorentz force acting on fast-moving charged particles in a bubble chamber[2]

1.2.3 Formula

When the external magnetic field is not too strong, the Hall voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d

$$U_H = R_H \frac{IB}{d} = KIB \tag{1}$$

where R_H is the so-called Hall coefficient and $K = R_H/d = K_H/I$, where K_H is the so-called sensitivity of the Hall element.

1.3 Integrated Hall Probe

1.3.1 Introduction

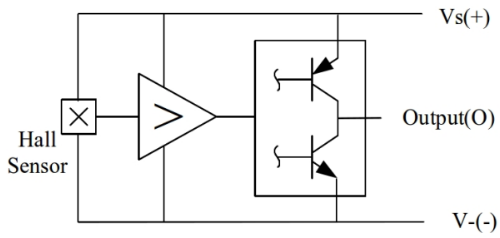
Although the magnitude of the magnetic field can be found by measuring the Hall voltage with a Hall probe when the sensitivity K_H and the current I are fixed, the Hall voltage is usually very small, it should be amplified before the measurement.

Silicon can be used to design both the Hall probe and the integrated circuits, so it is convenient to arrange the Hall probe and the electric circuits into a single device. Such a device is called an **integrated Hall probe**.

1.3.2 Example

The integrated Hall probe SS495A consists of a Hall sensor, an amplifier, and a voltage compensator (Fig.(2)). The output voltage U can be read ignoring the residual voltage. The working voltage $U_S = 5V$, and the output voltage U_0 is approximately $2.5V$ when the magnetic field is zero. The relation between the output voltage U and the magnitude of the magnetic field is

$$B = \frac{U - U_0}{K_H} \tag{2}$$



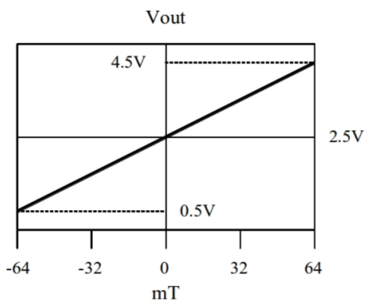


Figure 3: The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

1.4 Magnetic Field Distribution Inside a Solenoid

Solenoid is a typical electromagnetic element. In this exercise we explore the magnetic field distribution of it with the Hall probe. The theoretical value of magnetic field distribution on the axis of a single layer solenoid can be calculated from the following formula.

$$B(x) = \mu_0 \frac{N}{L} I_{\mathrm{M}} \left(\frac{L + 2x}{2[D^2 + (L+2x)^2]^{\frac{1}{2}}} + \frac{L - 2x}{2[D^2 + (L-2x)^2]^{\frac{1}{2}}} \right) = C(x) I_{\mathrm{M}} \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_{M} is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}\mathrm{H/m}$.

The solenoid used in this exercise has ten layers, and the magnetic field $B(x)$ for each layer can be calculated using Eq. (3). Then the net magnetic on the axis of the solenoid can be found by adding contributions due to all layers. The theoretical value of the magnetic field inside the solenoid with $I_{\mathrm{M}} = 0.1$ A is given in Table 1.

x [cm]	B [mT]	x [cm]	B [mT]
$\pm$ 0.0	1.4366	$\pm$ 8.0	1.4057
$\pm$ 1.0	1.4363	$\pm$ 9.0	1.3856
$\pm$ 2.0	1.4356	$\pm$ 10.0	1.3478
$\pm$ 3.0	1.4343	$\pm$ 11.0	1.2685
$\pm$ 4.0	1.4323	$\pm$ 11.5	1.1963
$\pm$ 5.0	1.4292	$\pm$ 12.0	1.0863
$\pm$ 6.0	1.4245	$\pm$ 12.5	0.9261
$\pm$ 7.0	1.4173	$\pm$ 13.0	0.7233

Table 1: Theoretical value of the magnetic field inside the solenoid.

2 Apparatus and Measurement Procedure

2.1 Apparatus

The experimental setup (Fig.(4)) consists of an integrated Hall probe SS495A (Fig. (5)) with $K_{\mathrm{H}} = 31.25 \pm 1.25$ V/T (at the working voltage 5 V) or $K_{\mathrm{H}} = 3.125 \pm 0.125$ mV/G, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

Figure 4: Apparatus

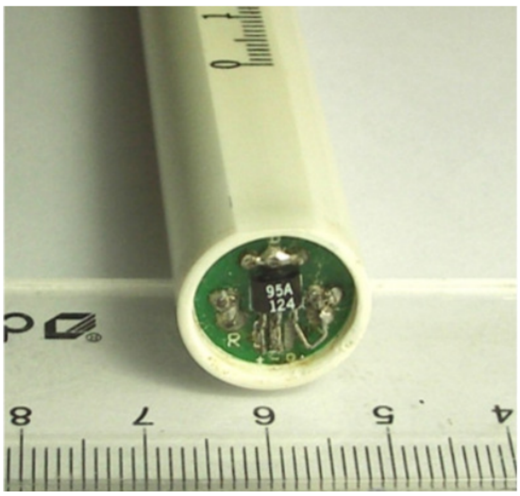


Figure 5: Integrated Hall probe SS495A

The precisions of the devices are shown in Table 2.

Instrument	Measured quantities	Uncertainties
Voltage source	Working voltage U_s	$0.5\%\,\mathrm{V}$
Multimeter	Output voltage U_0, U	$0.05\% + 6 \times 10^{-3}/10^{-4}\,\mathrm{V}$
Current source	Current I_0, I_M	$2\%\,\mathrm{mA}$
Graduated ruler	Distance	$0.05\,\mathrm{cm}$

Table 2: Information of measurement instruments.

2.2 Measurement Procedure

2.2.1 Relation Between Sensitivity K_H and Working Voltage U_S

In this part, we explored the relation between sensitivity K_H and working voltage U_S by applying Eq.(2) and measuring the corresponding quantities.

1. Place the integrated Hall probe at the center of the solenoid. Set the working voltage at 5 V and measure the output voltage $U_0(I_M = 0)$ and $U(I_M = 250mA)$. Take the theoretical value of $B(x = 0)$ from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).

2. Measure K_H for different values of U_S (from $2.8V$ to $10V$). Calculate K_H/U_S and plot the curve K_H/U_S *vs.* U_S.

2.2.2 Relation Between Output Voltage U and Magnetic Field B

In this part, we explored the relation between the output voltage of the Hall probe and the magnetic field in which it is positioned by adjusting the current in the solenoid.

1. With $B = 0, U_S = 5V$, connect the $2.4 \sim 2.6V$ output terminal of the DC voltage divider and the negative port of the voltmeter. Adjust the voltage until $U_0 = 0$.

2. Place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to $500mA$, with intervals of $50mA$.

3. Explain the relation between $B(x = 0)$ and the Hall voltage U_H. Pay attention to the fact that the output voltage U is the amplified signal from U_H. The theoretical value of $B(x = 0)$ can be found from Table 1.

4. Plot the curve $U vs. B$ and find the sensitivity K_H by a linear fit (use a computer). Compare the value you obtained with the theoretical value in given in the Apparatus section.

2.2.3 Magnetic Field Distribution Inside the Solenoid

In this part, we explore the magnetic field distribution inside a solenoid by adjusting the position of the Probe Hall and measuring the magnetic field at different positions.

1. Measure the magnetic field distribution along the axis of the solenoid for $I_M = 250mA$, record the output voltage U and the corresponding position x. Then find $B = B(x)$. (Use the value of K_H found in the previous part of the experiment).

2. Use a computer to plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid. Use dots for the data measured and a solid line for the theoretical curve. The origin of the plot should be at the center of the solenoid.

3 Experimental Results

3.1 Experiment 1 - Relationship Between Sensitivity K_H and Working Voltage U_S

The measurement results of U_0 and U when $U_\mathrm{S} = 5.00$ V are shown in Table 3.

U_S [V] $\pm\,0.5\%$ [V]	$U_0(I_\mathrm{M} = 0)$ [V] $\pm\,(0.05\% + 6 \times 10^{-3})$ [V]	$U(I_\mathrm{M} = 250\,\mathrm{mA})$ [V] $\pm\,(0.05\% + 6 \times 10^{-3})$ [V]
5.00 ± 0.03	2.504 ± 0.007	2.621 ± 0.007

Table 3: Data for U_0 and U with $U_S = 5.00$ V.

According to the data in Table 1, we can obtain that when $I_{\mathrm{M}} = 100$ mA, $B(x = 0,\ I_{\mathrm{M}} = 100\ [\mathrm{mA}]) = 1.4366 \times 10^{-3}$ T. Eq. (3) implies that B is proportional to current I_{M}, and therefore,

$$B(x = 0, I_{\mathrm{M}} = 250\ [\mathrm{mA}]) = \frac{250}{100} \times 1.4366 \times 10^{-3} = 3.59 \times 10^{-3} \pm 0.07 \times 10^{-3}\ \mathrm{T}.$$

According to Eq. (2), the sensitivity of the probe K_{H} when $U_{\mathrm{S}} = 5.00$ V is then calculated as

$$K_{\mathrm{H}} = \frac{U - U_0}{B(x = 0, I_{\mathrm{M}} = 250\,[\mathrm{mA}])} = \frac{2.621 - 2.504}{3.59 \times 10^{-3}} = 33 \pm 3\,[\mathrm{V/T}].$$

The measurement results of U_0 and U for different U_{S} are shown in Table 4. We calculate $K_{\mathrm{H}}/U_{\mathrm{S}}$ for each set of data to explore their relation. That is, for each set of data, we calculate

$$\frac{K_{\mathrm{H}}}{U_{\mathrm{S}}} = \frac{U - U_0}{BU_{\mathrm{S}}}.$$

Taking the first set of data as an example,

$$\frac{K_{\mathrm{H}}}{U_{\mathrm{S}}} = \frac{U - U_0}{BU_{\mathrm{S}}} = \frac{1.4656 - 1.4004}{3.59 \times 10^{-3} \times 2.80} = 6.5 \pm 0.2\,[\mathrm{T}^{-1}].$$

The measurement results and the results of calculation of $K_{\mathrm{H}}/U_{\mathrm{S}}$ for each set of data are presented together in Table 4. A plot of the results $K_{\mathrm{H}}/U_{\mathrm{S}}$ vs. U_{S} using Origin is shown in Fig.(6). The points in the plot indicates that the ratio of K_H to U_s decreases as U_s increases.

	U_{S} [V] $\pm 0.5\%$ [V]	U_0 [V] $\pm\,(0.05\% + 6 \times 10^{-3/-4}$ [V]	U [V] $\pm\,(0.05\% + 6 \times 10^{-3/-4}$ [V]	$K_{\mathrm{H}}/U_{\mathrm{S}}$ [T^{-1}]
1	2.80 ± 0.014	1.4004 ± 0.0013	1.4656 ± 0.0014	6.5 ± 0.2
2	3.20 ± 0.016	1.6002 ± 0.0014	1.6760 ± 0.0015	6.6 ± 0.2
3	3.60 ± 0.018	1.8022 ± 0.0015	1.8905 ± 0.0016	6.8 ± 0.2
4	4.00 ± 0.020	2.002 ± 0.007	2.097 ± 0.007	6.6 ± 0.2
5	4.40 ± 0.022	2.201 ± 0.008	2.308 ± 0.008	6.8 ± 0.5
6	4.80 ± 0.024	2.405 ± 0.008	2.518 ± 0.008	6.6 ± 0.6
7	5.20 ± 0.026	2.602 ± 0.008	2.723 ± 0.008	6.5 ± 0.6
8	5.60 ± 0.028	2.800 ± 0.008	2.933 ± 0.008	6.6 ± 0.5
9	6.00 ± 0.030	2.999 ± 0.008	3.136 ± 0.008	6.4 ± 0.5
10	6.40 ± 0.032	3.198 ± 0.008	3.343 ± 0.008	6.3 ± 0.5
11	6.80 ± 0.034	3.394 ± 0.008	3.545 ± 0.008	6.2 ± 0.5
12	7.20 ± 0.036	3.592 ± 0.008	3.748 ± 0.008	6.0 ± 0.4
13	7.60 ± 0.038	3.788 ± 0.008	3.952 ± 0.008	6.0 ± 0.4
14	8.00 ± 0.040	3.988 ± 0.008	4.152 ± 0.008	5.7 ± 0.4
15	8.40 ± 0.042	4.183 ± 0.008	4.351 ± 0.009	5.6 ± 0.4
16	8.80 ± 0.044	4.380 ± 0.009	4.552 ± 0.009	5.4 ± 0.3
17	9.20 ± 0.046	4.573 ± 0.009	4.751 ± 0.009	5.4 ± 0.3
18	10.00 ± 0.050	4.963 ± 0.009	5.143 ± 0.009	5.0 ± 0.3

Table 4: Data for U_0, U and $K_{\mathrm{H}}/U_{\mathrm{S}}$ for different U_{S}.

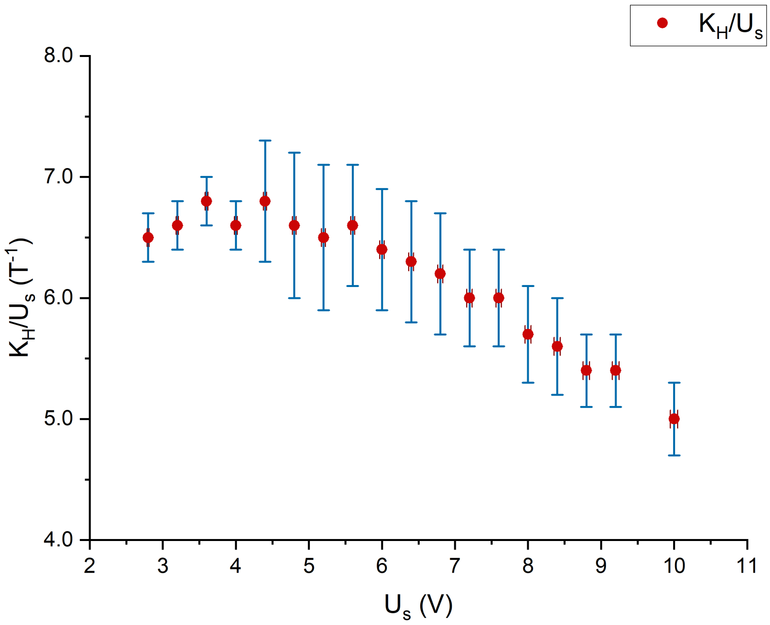


Figure 6: The K_H/U_S vs. U_S relation

3.2 Relation Between Output Voltage U and Magnetic Field B

In this experiment, we will not take the magnetic field B into consideration when it comes to uncertainty, as requested.

According to Eq.(3), B is proportional to current I_M. We can obtain from Table 1 that when $I_\mathrm{M} = 100$ mA, $B(x = 0, I_\mathrm{M} = 100\,[\mathrm{mA}]) = 1.4366 \times 10^{-3}$ T. Therefore, the theoretical value of the magnetic field is

$$B(x = 0) = \frac{I_\mathrm{M}}{100} \times 1.4366 \times 10^{-3}.$$

Take the last set of data as an example,

$$B(x = 0, I_\mathrm{M} = 0.50\,[\mathrm{A}]) = \frac{1.4366 \times 10^{-3}}{100 \times 10^{-3}} \times I_\mathrm{M} = 7.2 \times 10^{-3}\,[\mathrm{T}].$$

It is noticed that the measured U_out is the amplified output of U_H and we supposed that $U = k \cdot U_H$. Then, theoretically, according to Eq. (2), we can derive that

$$B(x = 0) = \frac{U - U_0}{K_\mathrm{H}} = \frac{U_\mathrm{out}}{K_\mathrm{H}} = k \cdot \frac{U_\mathrm{H}}{K_\mathrm{H}},$$

where k is a constant. Therefore, $B(x = 0)$ is supposed to be proportional to the Hall voltage U_H.

The experimental results are shown in Table 5. By applying linear fit to the I_M vs. U plot (Fig.(7)), the slope of the curve is then the measured sensitivity K_H, which is 33.6 V/T, with uncertainty 0.7 V/T. The measurement result of the sensitivity is then $K_H = 33.6 \pm 0.7\,\mathrm{V/T}$.

	I_M [A] $\pm 2\%$ [A]	U_out [mV] $\pm$ (0.05%+ 6×10^{-3}) [mV]	$B(x = 0)$ [T]
1	0 ± 0	0.83 ± 0.007	0
2	0.05 ± 0.001	30.08 ± 0.03	7.2×10^{-4}
3	0.10 ± 0.002	51.32 ± 0.04	1.4×10^{-3}
4	0.15 ± 0.003	73.38 ± 0.05	2.2×10^{-3}
5	0.20 ± 0.004	97.71 ± 0.06	2.9×10^{-3}
6	0.25 ± 0.005	121.64 ± 0.07	3.6×10^{-3}
7	0.30 ± 0.006	143.67 ± 0.08	4.3×10^{-3}
8	0.35 ± 0.007	166.63 ± 0.09	5.0×10^{-3}
9	0.40 ± 0.008	187.79 ± 0.10	5.7×10^{-3}
10	0.45 ± 0.009	210.10 ± 0.12	6.5×10^{-3}
11	0.50 ± 0.010	231.1 ± 0.13	7.2×10^{-3}

Table 5: Measurement data for the I_M vs. U relation and the calculated data for $B(x = 0)$.

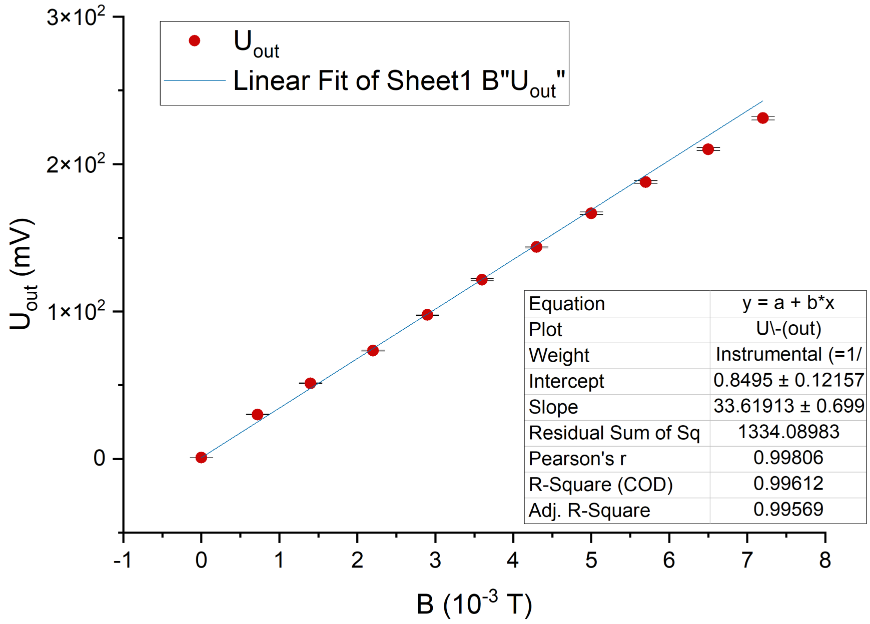


Figure 7: The linear fit of U vs. B relation

3.3 Magnetic Field Distribution Inside the Solenoid

The measurement result of output voltage U and the corresponding position x are shown in Table 6. According to Eq.(2) and the measured value of K_H in section 3.2, $B(x)$ can be obtained from $B(x) = \frac{U}{K_\mathrm{H}} = \frac{U}{33.6}$ Take the first set of data as an example,

$$B(x) = \frac{U}{K_\mathrm{H}} = \frac{12.20 \times 10^{-3}}{33.6} = (0.363 \pm 0.008) \times 10^{-3} \text{ [T]}.$$

The $B(x)$ are calculated for each set of data and the results are shown in Table 7.

	x[cm]$\pm$0.05[cm]	U[mV]$\pm$(0.05%+6 $\times 10^{-3}$)[mV]		x[cm]$\pm$0.05[cm]	U[mV]$\pm$(0.05%+6 $\times 10^{-3}$)[mV]
1	0.00	12.20 $\pm$ 0.012	27	16.00	118.14 $\pm$ 0.07
2	0.50	15.27 $\pm$ 0.014	28	17.00	118.12 $\pm$ 0.07
3	1.00	20.64 $\pm$ 0.016	29	18.00	118.04 $\pm$ 0.07
4	1.50	27.95 $\pm$ 0.02	30	19.00	117.93 $\pm$ 0.07
5	2.00	39.47 $\pm$ 0.03	31	20.00	117.87 $\pm$ 0.07
6	2.50	54.53 $\pm$ 0.04	32	21.00	117.48 $\pm$ 0.07
7	3.00	71.20 $\pm$ 0.05	33	22.00	117.27 $\pm$ 0.07
8	3.50	86.03 $\pm$ 0.05	34	23.00	116.52 $\pm$ 0.07
9	4.00	96.45 $\pm$ 0.06	35	24.00	115.35 $\pm$ 0.07
10	4.50	103.35 $\pm$ 0.06	36	25.00	113.25 $\pm$ 0.07
11	5.00	107.76 $\pm$ 0.06	37	26.00	109.47 $\pm$ 0.06
12	5.50	110.62 $\pm$ 0.07	38	27.00	101.05 $\pm$ 0.06
13	6.00	112.67 $\pm$ 0.07	39	27.20	98.40 $\pm$ 0.06
14	6.50	113.90 $\pm$ 0.07	40	27.60	91.27 $\pm$ 0.06
15	7.00	114.90 $\pm$ 0.07	41	27.80	87.47 $\pm$ 0.05
16	7.50	115.56 $\pm$ 0.07	42	28.00	82.41 $\pm$ 0.05
17	8.00	116.18 $\pm$ 0.07	43	28.20	75.95 $\pm$ 0.05
18	8.50	116.71 $\pm$ 0.07	44	28.40	69.49 $\pm$ 0.05
19	9.00	117.20 $\pm$ 0.07	45	28.60	62.78 $\pm$ 0.04
20	9.50	117.28 $\pm$ 0.07	46	28.80	56.25 $\pm$ 0.04
21	10.00	117.67 $\pm$ 0.07	47	29.00	49.60 $\pm$ 0.04
22	11.00	117.87 $\pm$ 0.07	48	29.20	43.65 $\pm$ 0.03
23	12.00	118.02 $\pm$ 0.07	49	29.40	37.75 $\pm$ 0.03
24	13.00	118.12 $\pm$ 0.07	50	29.60	32.68 $\pm$ 0.03
25	14.00	118.01 $\pm$ 0.07	51	29.80	28.52 $\pm$ 0.03
26	15.00	118.19 $\pm$ 0.07	52	30.00	25.57 $\pm$ 0.019

Table 6: Data for the U vs. x relation

	x [cm] $\pm$ 0.05 [cm]	$B(x)$ [10^{-3} T]		x [cm] $\pm$ 0.05 [cm]	$B(x)$ [10^{-3} T]
1	0.00	0.363 $\pm$ 0.008	27	16.00	3.47 $\pm$ 0.08
2	0.50	0.449 $\pm$ 0.010	28	17.00	3.47 $\pm$ 0.08
3	1.00	0.607 $\pm$ 0.013	29	18.00	3.47 $\pm$ 0.08
4	1.50	0.822 $\pm$ 0.017	30	19.00	3.47 $\pm$ 0.08
5	2.00	1.16 $\pm$ 0.03	31	20.00	3.47 $\pm$ 0.08
6	2.50	1.60 $\pm$ 0.04	32	21.00	3.46 $\pm$ 0.08
7	3.00	2.09 $\pm$ 0.05	33	22.00	3.45 $\pm$ 0.08
8	3.50	2.53 $\pm$ 0.06	34	23.00	3.43 $\pm$ 0.07
9	4.00	2.83 $\pm$ 0.06	35	24.00	3.39 $\pm$ 0.07
10	4.50	3.04 $\pm$ 0.07	36	25.00	3.33 $\pm$ 0.07
11	5.00	3.17 $\pm$ 0.07	37	26.00	3.22 $\pm$ 0.07
12	5.50	3.25 $\pm$ 0.07	38	27.00	2.97 $\pm$ 0.07
13	6.00	3.31 $\pm$ 0.07	39	27.20	2.89 $\pm$ 0.06
14	6.50	3.35 $\pm$ 0.07	40	27.60	2.68 $\pm$ 0.06
15	7.00	3.38 $\pm$ 0.07	41	27.80	2.57 $\pm$ 0.06
16	7.50	3.40 $\pm$ 0.07	42	28.00	2.42 $\pm$ 0.05
17	8.00	3.42 $\pm$ 0.07	43	28.20	2.23 $\pm$ 0.05
18	8.50	3.43 $\pm$ 0.07	44	28.40	2.04 $\pm$ 0.05
19	9.00	3.45 $\pm$ 0.07	45	28.60	1.85 $\pm$ 0.04
20	9.50	3.45 $\pm$ 0.08	46	28.80	1.65 $\pm$ 0.04
21	10.00	3.46 $\pm$ 0.08	47	29.00	1.46 $\pm$ 0.03
22	11.00	3.47 $\pm$ 0.08	48	29.20	1.28 $\pm$ 0.03
23	12.00	3.47 $\pm$ 0.08	49	29.40	1.11 $\pm$ 0.03
24	13.00	3.47 $\pm$ 0.08	50	29.60	0.96 $\pm$ 0.02
25	14.00	3.47 $\pm$ 0.08	51	29.80	0.839 $\pm$ 0.018
26	15.00	3.48 $\pm$ 0.08	52	30.00	0.752 $\pm$0.016

Table 7: Data for the $B(x)$ vs. x relation.

The theoretical curve of the magnetic field distribution inside the solenoid can be obtained from Eq.(3) and the data in Table 1, by multiplying the data in the table by $\frac{250}{100} = 2.5$, since we have set the current as 250 mA instead of 100 mA.

Then we plot the theoretical curve together with the measured value of the magnetic field distribution in Fig.(8). The origin of the plot is set at the center of the solenoid, thus 15 cm are subtracted from the x in the measurement data.

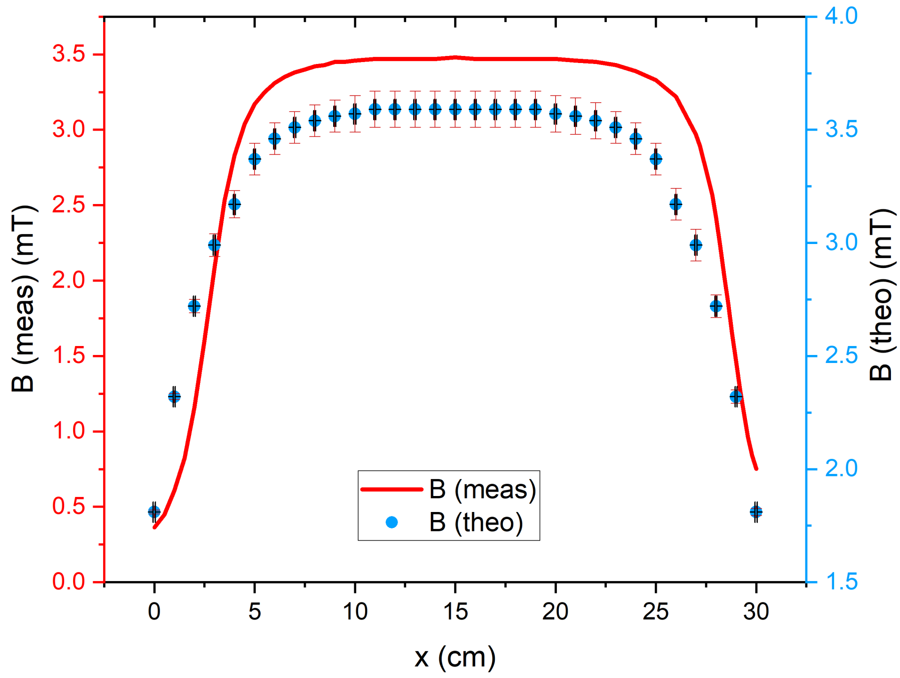


Figure 8: Measured and theoretical magnetic field distribution inside the solenoid.

4 Uncertainty Analysis

4.1 Relationship Between Sensitivity K_H and Working Voltage U_S

According to Table 3, the uncertainties are calculated as

$$\begin{aligned} u_{U_\mathrm{S}} &= 5.00 \times 0.5\% = 0.03 \ [\mathrm{V}] \\ u_{U_0} &= 2.504 \times 0.05\% + 6 \times 10^{-3} = 0.007 \ [\mathrm{V}] \\ u_{U} &= 2.621 \times 0.05\% + 6 \times 10^{-3} = 0.007 \ [\mathrm{V}] \\ u_{I_{\mathrm{M2}}} &= 250 \times 10^{-3} \times 2\% = 5 \times 10^{-3} \ [\mathrm{A}] \end{aligned}$$

while $B(x=0, I_\mathrm{M} = 250\ [\mathrm{mA}]) = \frac{I_{\mathrm{M2}}}{I_{\mathrm{M1}}} \times B(x=0, I_\mathrm{M} = 100\ [\mathrm{mA}])$

$$u_B = \frac{B(x=0, I_\mathrm{M} = 100\ [\mathrm{mA}])}{I_{\mathrm{M1}}} u_{I_{\mathrm{M2}}} = \frac{1.4366 \times 10^{-3}}{100 \times 10^{-3}} \times 5 \times 10^{-3} = 7 \times 10^{-5} \ [\mathrm{T}]$$

Then for $K_\mathrm{H} = \frac{U - U_0}{B}$, its uncertainty is

$$\begin{aligned} u_{K_\mathrm{H}} &= \sqrt{(\frac{\partial K_\mathrm{H}}{\partial U} u_U)^2 + (\frac{\partial K_\mathrm{H}}{\partial U_0} u_{U_0})^2 + (\frac{\partial K_\mathrm{H}}{\partial B} u_B)^2} = \sqrt{(\frac{u_U}{B})^2 + (\frac{-u_{U_0}}{B})^2 + (-\frac{(U-U_0)u_B}{B^2})^2} \\ &= \sqrt{(\frac{0.007}{1.4366 \times 10^{-3} \times 250/100})^2 + (\frac{-0.007}{1.4366 \times 10^{-3} \times 250/100})^2 + (\frac{(2.621 - 2.504) \times 7 \times 10^{-5}}{(1.4366 \times 10^{-3} \times 250/100)^2})^2} \\ &= 3 \ [\mathrm{V/T}]. \end{aligned}$$

For Table 4, the uncertainties of data for voltage measurements are calculated as follows. Take the first set of data as an example,

$$u_{U_\mathrm{S}} = 2.80 \times 0.5\% = 0.014 \ [\mathrm{V}],$$

$$u_{U_0} = 1.4004 \times 0.05\% + 6 \times 10^{-4} = 0.0013 \ [\mathrm{V}],$$

$$u_U = 1.4656 \times 0.05\% + 6 \times 10^{-4} = 0.0014 \text{ [V]}.$$

The uncertainty for $K_{\mathrm{H}}/U_{\mathrm{S}} = \frac{U-U_0}{BU_{\mathrm{S}}}$ is calculated as

$$\begin{aligned}
u_{K_{\mathrm{H}}/U_{\mathrm{S}}} &= \sqrt{(\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial U}u_U)^2 + (\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial U_0}u_{U_0})^2 + (\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial U_{\mathrm{S}}}u_{U_{\mathrm{S}}})^2 + (\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial B}u_B)^2} \\
&= \sqrt{(\frac{u_U}{BU_{\mathrm{S}}})^2 + (\frac{-u_{U_0}}{BU_{\mathrm{S}}})^2 + (-\frac{U-U_0}{BU_{\mathrm{S}}^2}u_{U_{\mathrm{S}}})^2 + (-\frac{U-U_0}{B^2U_{\mathrm{S}}}u_B)^2} \\
&= \sqrt{(\frac{0.0014}{3.59 \times 10^{-3} \times 2.80})^2 + (\frac{-0.0013}{3.59 \times 10^{-3} \times 2.80})^2 + (-\frac{(1.4656-1.4004) \times 0.014}{3.59 \times 10^{-3} \times 2.80^2})^2 + (-\frac{(1.4656-1.4004) \times 7 \times 10}{(3.59 \times 10^{-3})^2 \times 2.80}} \\
&= 0.2 \text{ [T}^{-1}\text{]}.
\end{aligned}$$

The uncertainties of all other data in Table 4 are calculated in this way and the results are presented in Table 8.

	$u_{U_{\mathrm{S}}}$ [V]	u_{U_0} [V]	u_U [V]	$u_{K_{\mathrm{H}}/U_{\mathrm{S}}}$ [T^{-1}]
1	0.014	0.0013	0.0014	0.2
2	0.016	0.0014	0.0015	0.2
3	0.018	0.0015	0.0016	0.2
4	0.020	0.007	0.007	0.2
5	0.022	0.008	0.008	0.5
6	0.024	0.008	0.008	0.6
7	0.026	0.008	0.008	0.6
8	0.028	0.008	0.008	0.5
9	0.030	0.008	0.008	0.5
10	0.032	0.008	0.008	0.5
11	0.034	0.008	0.008	0.5
12	0.036	0.008	0.008	0.4
13	0.038	0.008	0.008	0.4
14	0.040	0.008	0.008	0.4
15	0.042	0.008	0.009	0.4
16	0.044	0.009	0.009	0.3
17	0.046	0.009	0.009	0.3
18	0.050	0.009	0.009	0.3

Table 8: Uncertainties of data in Table 4.

4.2 Uncertainty of Input Current I_{M}, Output Voltage U and Magnetic Field B

Take the last set of data in Table 5 as an example.

The uncertainty for I_{M} is

$$u_{I_{\mathrm{M}}} = 0.50 \times 2\% = 0.010 \text{ [A]}$$

The uncertainty for U is

$$u_U = 231.1 \times 0.05\% + 6 \times 10^{-3} = 0.12 \text{ [mV]}.$$

The uncertainties of all other data in Table 5 are calculated in this way are the results are presented in Table 9.

	$u_{I_{\mathrm{M}}}$ [A]	$u_{U_{\mathrm{out}}}$ [V]
1	0	0.007
2	0.001	0.03
3	0.002	0.04
4	0.003	0.05
5	0.004	0.06
6	0.005	0.07
7	0.006	0.08
8	0.007	0.09
9	0.008	0.10
10	0.009	0.12
11	0.010	0.13

Table 9: Uncertainty of data in Table 5.

4.3 Uncertainty of Magnetic Field Inside the Solenoid Measurement

The uncertainty of position measurement is 0.05 cm.
As for the uncertainty of the output voltage, taking the first set of data as an example,

$$u_U = 12.20 \times 0.05\% + 6 \times 10^{-3} = 0.012 \text{ [mV]}$$

Due to $B(x) = \frac{U}{K_\mathrm{H}}$,

$$u_B = \sqrt{(\frac{\partial B}{\partial U} u_U)^2 + (\frac{\partial B}{\partial K_\mathrm{H}} u_{K_\mathrm{H}})^2} = \sqrt{(\frac{u_U}{K_\mathrm{H}})^2 + (-\frac{U}{K_\mathrm{H}^2} u_{K_\mathrm{H}})^2}.$$

Taking the first set of data as an example,

$$u_B = \sqrt{(\frac{0.012 \times 10^{-3}}{33.6})^2 + (-\frac{12.20 \times 10^{-3}}{33.6^2} \times 0.7)^2} = 0.008 \times 10^{-3} \text{ [T]}.$$

The uncertainties for all other sets of data are calculated and shown in Table 10.

	u_U [mV]	$B(x)$ [10^{-3} T]		u_U [mV]	$B(x)$ [10^{-3} T]
1	0.012	0.008	27	0.07	0.08
2	0.014	0.010	28	0.07	0.08
3	0.016	0.013	29	0.07	0.08
4	0.02	0.017	30	0.07	0.08
5	0.03	0.03	31	0.07	0.08
6	0.04	0.04	32	0.07	0.08
7	0.05	0.05	33	0.07	0.08
8	0.05	0.06	34	0.07	0.07
9	0.06	0.06	35	0.07	0.07
10	0.06	0.07	36	0.07	0.07
11	0.06	0.07	37	0.06	0.07
12	0.07	0.07	38	0.06	0.07
13	0.07	0.07	39	0.06	0.06
14	0.07	0.07	40	0.06	0.06
15	0.07	0.07	41	0.05	0.06
16	0.07	0.07	42	0.05	0.05
17	0.07	0.07	43	0.05	0.05
18	0.07	0.07	44	0.05	0.05
19	0.07	0.07	45	0.04	0.04
20	0.07	0.08	46	0.04	0.04
21	0.07	0.08	47	0.04	0.03
22	0.07	0.08	48	0.03	0.03
23	0.07	0.08	49	0.03	0.03
24	0.07	0.08	50	0.03	0.02
25	0.07	0.08	51	0.03	0.018
26	0.07	0.08	52	0.019	0.016

Table 10: The uncertainties of U and B.

5 Conclusion and Discussion

5.1 Conclusion

In this exercise, we learnt how to use integrated Hall probe and measured the Hall sensitivity of the Hall probe under different working voltages. Besides, we derived two experimental value of K_H in two different ways, where one is direct measurement and the other is linear fit. We also measured the magnetic field inside a solenoid. Among all the steps and operations, we conclude that

- As Fig.(6) suggests, the sensitivity has a decreasing trend with working voltage increasing.
- The output voltage is probably linearly dependent on the magnetic field since the Pearson's r of linear fit in Fig.(7) is 0.998 which is very close to 1.
- The K_H we measure is quite precise since the relative error is only $\frac{33.6-31.25}{31.25} \times 100\% = 7.52\%$.
- The magnitudes of measured magnetic field and theoretical magnetic field differ a bit but with a raltively constant difference throughout the whole curve.

The detailed analysis will be presented in Discussion part.

5.2 Discussion

5.2.1 Problems

1. As for the relationship between sensitivity of SS495A and the working voltage applied on it, it is positively proportional in fact[4], which contradicts the data we derived in this exercise.

2. Measured data has relatively great deviation of theoretical value as Fig.(8) suggests.

5.2.2 Potential errors

Here we conclude some potential errors occuring in this exercise.

1. Uneven Hall Probe

   The materials consisting of the integrated Hall probe may not be so even that there will also trigger electric field even if there is no magnetic field applied. This magnetic filed triggered by the material itself may be opposite to the magnetic filed applied, which will leesen the magnitude of magnetic field we detected.

2. Ettingshausen effect and Nernst effect

   Due to Hall effect, electrons are forced to move perpendicular to the applied current. However, the accumulation of electrons on one side of the sample enables the number of collisions to increase and a heating of the material occurs, which will trigger a heated electromotive force. This emf will decrease the the effect of origin current in converse.

3. Reading Process

   The sensibility of the experiment device is so low that we can hardly grasp a more precise number about two digits after the decimal point. The reading process may cause huge deviation due to the unstability of the data displayed.

According to the above discussion, we may explain the difference between data we measured and the theoretical value. In general, the results we derived are still credible and worthy. Here we also provide some suggestions to impove.

5.2.3 Imporvements

- Use more precise experiment devices.
- Control the experimental temperature better to avoid temperarture deviation.
- Cool down the integrated Hall probe as we measure the data.

6 Reference

[1] Edwin Hall (1879). "On a New Action of the Magnet on Electric Currents". *American Journal of Mathematics.* **2** (3): 287–92. doi:10.2307/2369245. JSTOR 2369245. Archived from the original on 2011-07-27. Retrieved 2008-02-28.
[2] Per F. Dahl, *Flash of the Cathode Rays: A History of J J Thomson's Electron*, CRC Press, 1997, p. 10.
[3] M. Krzyzosiak (2021). Exercise 2 - lab manual [rev 3.9]. (UMJI-SJTU, Shanghai).
[4] Sinocomopto Technologies Ltd. Company, *SS495A Product Manual*, Archived from the original on 2021-11-11.

APPENDIX - DATA SHEET

UM-SJTU Physics Laboratory Vp241
Data Sheet (Exercise 2)

Name: 付昊天 Student ID: 520021910012

Name: Haotian Fu Student ID: 520021910012

Group: 1 Date: 2021.11.5

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with a pencil or modified with a correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used.
You are required to hand in the original data with your lab report, so please keep the data sheet properly.

U_S [V] ± 0.5% [V]	$U_0(I_M = 0)$ [V] ± $0.05\% + 6\times10^{-3}$ [V]	$U(I_M = 250\text{ mA})$ [V] ± $0.05\% + 6\times10^{-3}$ [V]
5.00	2.504	2.621

Table 1. Data for U_0 and U with $U_S = 5$ V.

	U_S [V] ± 0.5% [V]	U_0 [V] ± $0.05\% + 6\times10^{-3}/6\times10^{-4}$ [V]	U [V] ± $0.05\% + 6\times10^{-3}/6\times10^{-4}$ [V]
1	2.80	1.4004	1.4656
2	3.20	1.6002	1.6760
3	3.60	1.8022	1.8905
4	4.00	2.002	2.097
5	4.40	2.201	2.308
6	4.80	2.405	2.518
7	5.20	2.602	2.723
8	5.60	2.800	2.933
9	6.00	2.999	3.136
10	6.40	3.198	3.343
11	6.80	3.394	3.545
12	7.20	3.592	3.748
13	7.60	3.788	3.952
14	8.00	3.988	4.152
15	8.40	4.183	4.351
16	8.80	4.380	4.552
17	9.20	4.573	4.751
18	10.00	4.963	5.143

Table 2. Data for U_0 and U with different U_S.

Instructor's signature: Tang

	I_M [A] ± 2% [A]	U [mV] ± $0.05\% + 6\times10^{-3}$ [mV]
1	0	0.83
2	0.05	30.08
3	0.10	51.32
4	0.15	73.38
5	0.20	97.71
6	0.25	121.64
7	0.30	143.67
8	0.35	166.63
9	0.40	187.79
10	0.45	210.10
11	0.50	231.10

Table 3. Measurement data for the I_M vs. U relation.

Instructor's signature: Tang

	x [cm] ± 0.05 [cm]	U [mV] ± 0.05%+6×10^{-3} [mV]		x [cm] ± 0.05 [cm]	U [mV] ± 0.05%+6×10^{-3} [mV]
1	0.00	12.20	27	16.00	118.14
2	0.50	15.27	28	17.00	118.12
3	1.00	20.64	29	18.00	118.04
4	1.50	27.95	30	19.00	117.93
5	2.00	39.47	31	20.00	117.87
6	2.50	54.53	32	21.00	117.48
7	3.00	71.20	33	22.00	117.27
8	3.50	86.03	34	23.00	116.52
9	4.00	96.45	35	24.00	115.35
10	4.50	103.35	36	25.00	113.25
11	5.00	107.76	37	26.00	109.47
12	5.50	110.62	38	27.00	101.05
13	6.00	112.67	39	27.20	98.40
14	6.50	113.96	40	27.60	~~94.84~~ 91.27
15	7.00	114.90	41	~~28~~ 27.80	87.47
16	7.50	115.56	42	28.00	82.41
17	8.00	116.18	43	28.20	75.95
18	8.50	116.71	44	28.40	69.49
19	9.00	117.20	45	28.60	62.78
20	9.50	117.28	46	28.80	56.25
21	10.00	117.67	47	29.00	49.60
22	11.00	117.87	48	29.20	43.65
23	12.00	118.03	49	29.40	37.75
24	13.00	118.12	50	29.60	32.68
25	14.00	118.01	51	29.80	28.52
26	15.00	118.17	52	30.00	25.57

Table 4. Data for the U vs. x relation.

Instructor's signature: Tang

rev. 3-1