PROBLEM SET 2

Due: 28 September 2021, 2.30 p.m.

Problem 1. Recall that Newton's law of gravitation, i.e. the formula for the gravitational force between two point masses, has exactly the same functional form as Coulomb's law for electric interaction of point charges. By repeating exactly the same logical steps leading to Gauss's law for the electric field, we can argue that the following Gauss's law holds for the gravitational field

$$\oint_{\Sigma} \mathbf{E}_G \, \hat{n} \, dA = -4\pi G M_{\Sigma},$$

where M_{Σ} is the mass enclosed by the Gaussian surface Σ.

(a) Use the form of Gauss's law given above to find the gravitational field at a distance r from a point mass M.

(b) Find the gravitational field at a distance r from the center of a uniform solid ball with mass M and radius R. Consider both cases $r > R$ and $r < R$.

(c) Do the same in the case when the same amount of mass is distributed over the surface of the ball (i.e. the ball is empty inside).

Note. Recall that we obtained the results of parts (b) and (c) in the summer semester using another (more time-consuming) approach.

(*3/2 + 3 + 3/2 points*)

Problem 2. In a certain region of space, the electric field $\mathbf{E}$ is uniform, *i.e.* is constant in both the magnitude and the direction.

(a) Use Gauss's law to argue that this region of space must be electrically neutral, i.e., the volume charge density ρ must be zero.

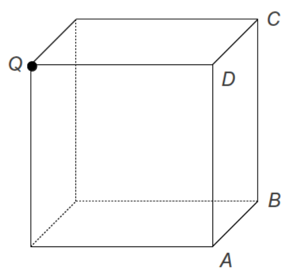
(b) Is the converse true? That is, in a region of space where there is no charge, must $\mathbf{E}$ be uniform? Explain.

(*1 + 1 points*)

Problem 3. A point charge $Q > 0$ is placed at a vertex of a cube as shown in the figure. Find the electric flux through the surface $ABCD$.

Hint. Rather than calculating the flux directly (and tediously) from the definition, solve the problem by using an observation based on symmetry.

(*4 points*)



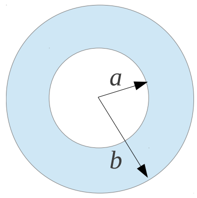
Problem 4. A hollow insulating ball is electrically charged with bulk density $\rho = k/r$, where k is a constant and $a \le r \le b$ (see the figure below).

(a) Find the total electric charge of the ball.

(b) Find the electric field $\mathbf{E}$ in the three regions: (i) $r < a$, (ii) $a < r < b$, (iii) $r > b$.

(c) Plot $\mathbf{E}$ as a function of r.

(2 + (1 + 2 + 1) + 1 points)



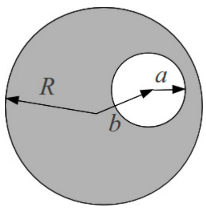
Problem 5. Use Gauss's law to find the electric field due to an infinitely long cylinder with radius R and constant bulk charge density ρ. Sketch a graph of $|\mathbf{E}|$ as a function of the distance r from the axis of the cylinder

(5 points)

Problem 6. An infinite solid insulating cylinder with radius R has a cylindrical hole with radius a bored along its entire length. The axis of the hole is a distance b from the axis of the cylinder, where $a < b < R$. The solid material of the cylinder has a uniform bulk charge density $\rho < 0$. Find the magnitude and direction of the electric field $\mathbf{E}$ inside the hole, and show that $\mathbf{E}$ is uniform over the entire hole.

Hint. Use the results of the previous problem.

(4 points)



Problem 7. Two spherical cavities, of radii r_a and r_b, are hollowed out from the interior of a neutral conducting ball of radius R. At the center of each cavity a point charge is placed: q_a and q_b, respectively.

(a) Find the surface densities of charge σ_a, σ_b on the walls of the cavities as well as on the surface of the ball σ_R.

(b) What is the electric field outside of the conductor?

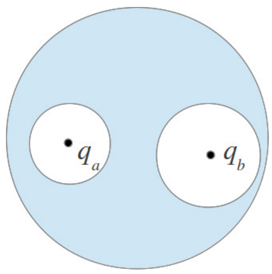
(c) What is the electric field within each cavity?

(d) What is the force on q_a and q_b?

(e) Which of these answers would change if a third charge q_c were brought near the conductor?

<u>Explain</u> your answers.

(5 × 2 points)



(cross-sectional sketch)