

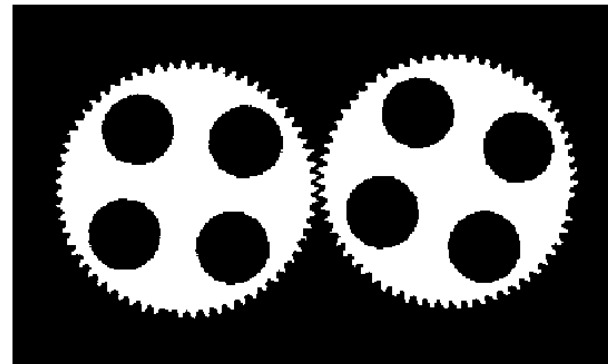
# CS 4495 Computer Vision

## *Binary images and Morphology*

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Computing



# Administrivia

- PS6 – should be working on it! Due Sunday Nov 24<sup>th</sup>.
  - Some issues with reading frames. Resolved?
- Exam: Tues November 26<sup>th</sup>.
  - Short answer and multiple choice (mostly short answer)
  - Study guide is posted in calendar.
  - Bring a pen.
- PS7 – we still hope to have out by 11/26. Will be straight forward implementation of Motion History Images

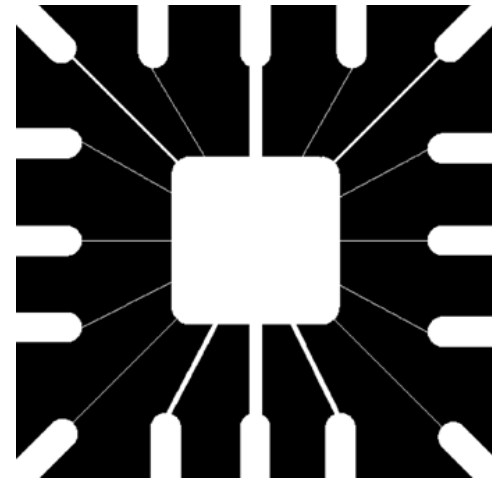
# Binary Image Analysis

## Binary image analysis

- consists of a set of image analysis operations that are used to produce or process binary images, usually images of 0's and 1's.

0 represents the background  
1 represents the foreground

000	1	00	1	000	1	000	
000	1	1	1	1	000	1	000
000	1	00	1	000	1	000	



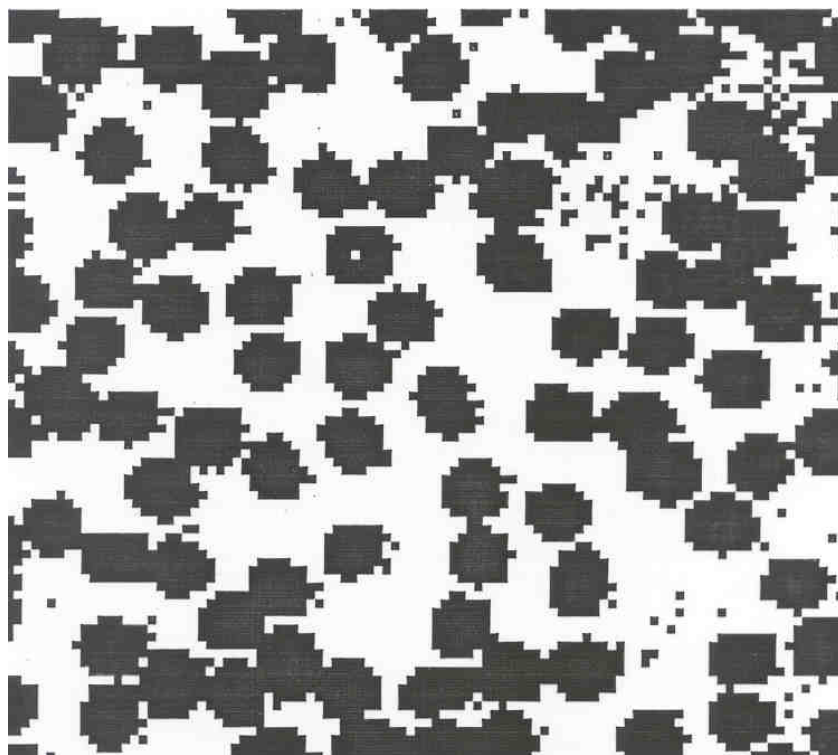
# Binary Image Analysis

- Is used in a number of practical applications
  - Part inspection
  - Manufacturing
  - Document processing

# What kinds of operations?

- Separate objects from background and from one another
- Aggregate pixels for each object
- Compute features for each object

# Example: red blood cell image



- Many blood cells are separate objects
- Many touch – bad!
- Salt and pepper noise from thresholding
- How useable is this data?

# Results of analysis

- 63 separate objects detected
- Single cells have area about 50
- Noise spots
- Gobs of cells

Object	Area	Centroid	Bounding Box	
=====				
1	383	( 8.8 , 20)	[1 22 1 39]	
2	83	( 5.8 , 50)	[1 11 42 55]	
3	11	( 1.5 , 57)	[1 2 55 60]	
4	1	( 1 , 62)	[1 1 62 62]	
5	1048	( 19 , 75)	[1 40 35 100]	gobs
32	45	( 43 , 32)	[40 46 28 35]	cell
33	11	( 44 , 1e+02)	[41 47 98 100]	
34	52	( 45 , 87)	[42 48 83 91]	cell
35	54	( 48 , 53)	[44 52 49 57]	cell
60	44	( 88 , 78)	[85 90 74 82]	
61	1	( 85 , 94)	[85 85 94 94]	
62	8	( 90 , 2.5)	[89 90 1 4]	
63	1	( 90 , 6)	[90 90 6 6]	

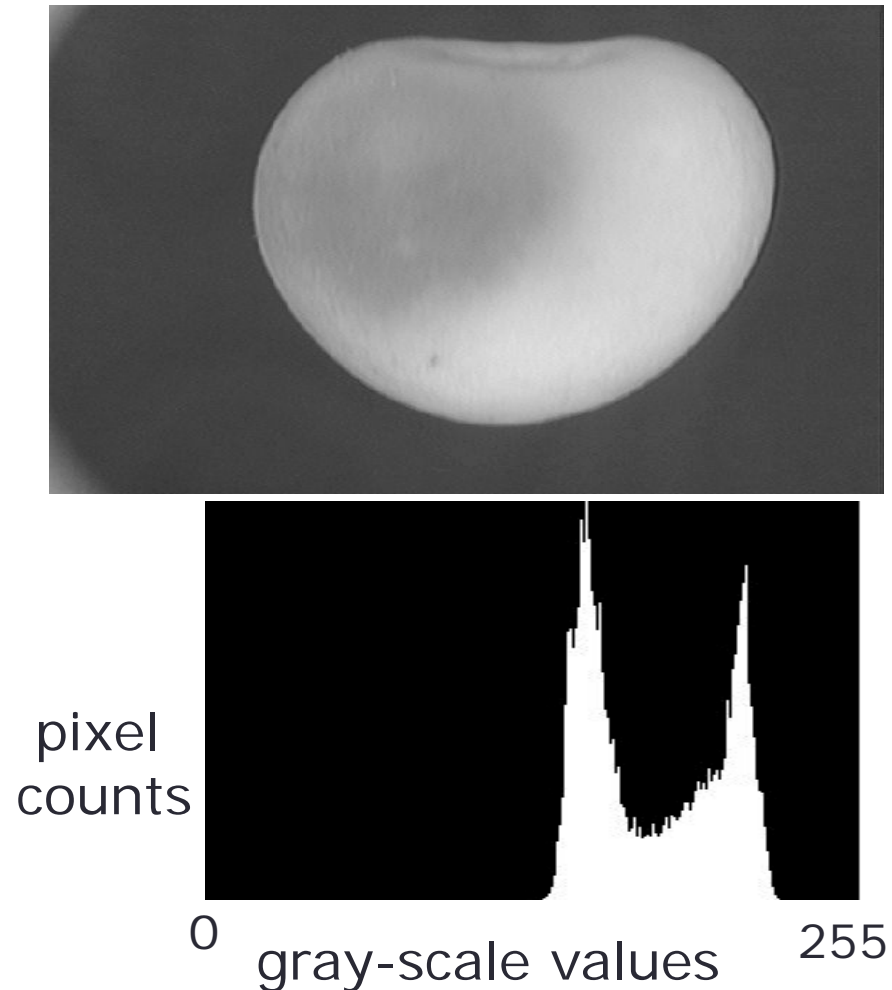
# Useful Operations

- Thresholding a gray-scale image
- Determining good thresholds
- Connected components analysis
- Binary mathematical morphology
- All sorts of feature extractors
  - (area, centroid, circularity, ...)



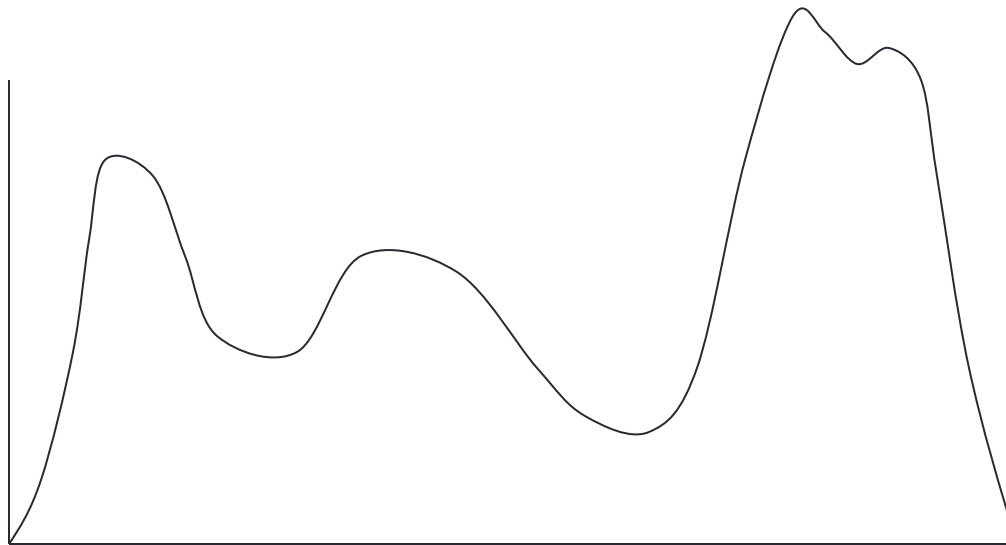
# Thresholding

- Background is black
- Healthy cherry is bright
- Bruise is medium dark
- Histogram shows two cherry regions (black background has been removed)



# Histogram-Directed Thresholding

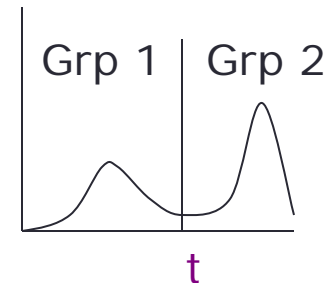
How can we use a histogram to separate an image into 2 (or several) different regions?



**Is there a single clear threshold? 2? 3?**

# Automatic Thresholding: Otsu's Method

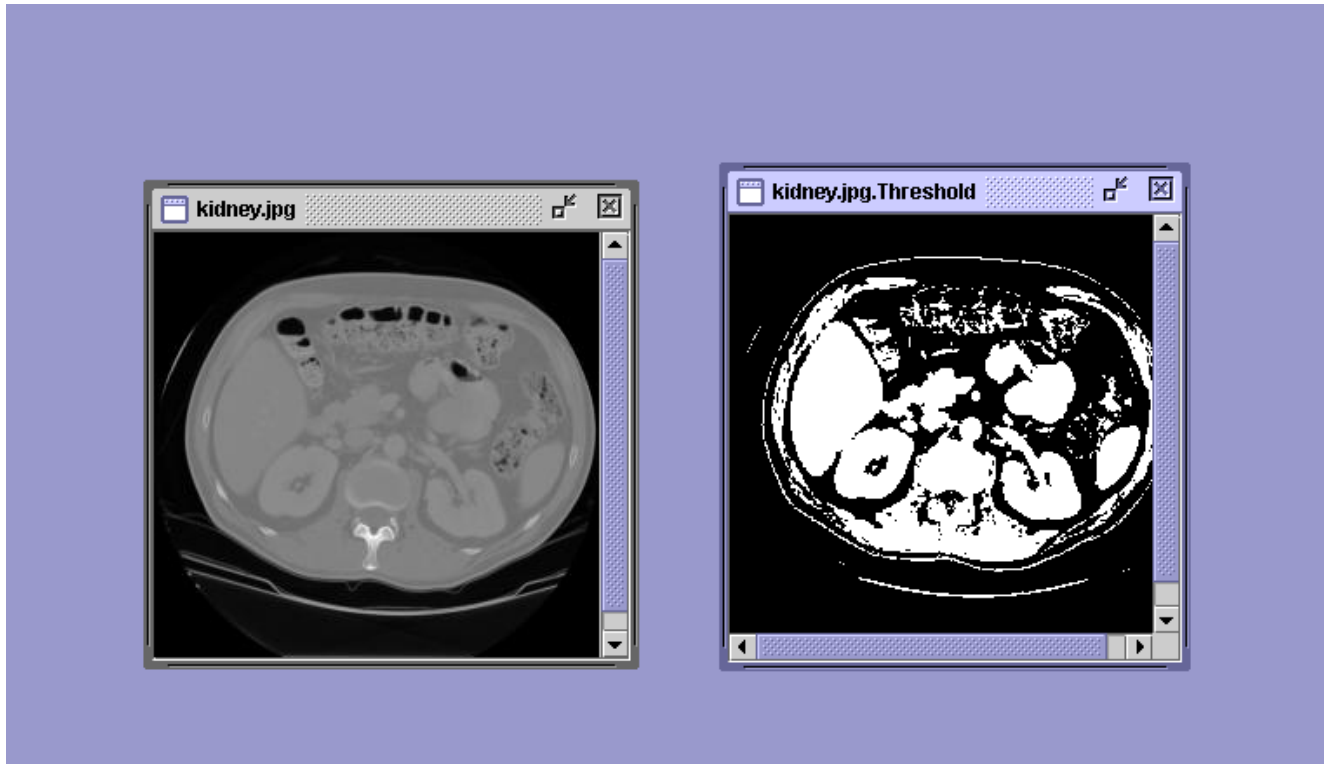
Assumption: the histogram is bimodal



Method: find the threshold  $t$  that minimizes the **weighted sum of within-group variances** for the two groups that result from separating the gray tones at value  $t$ .

In practice, this operator works very well for true bimodal distributions and not too badly for others.

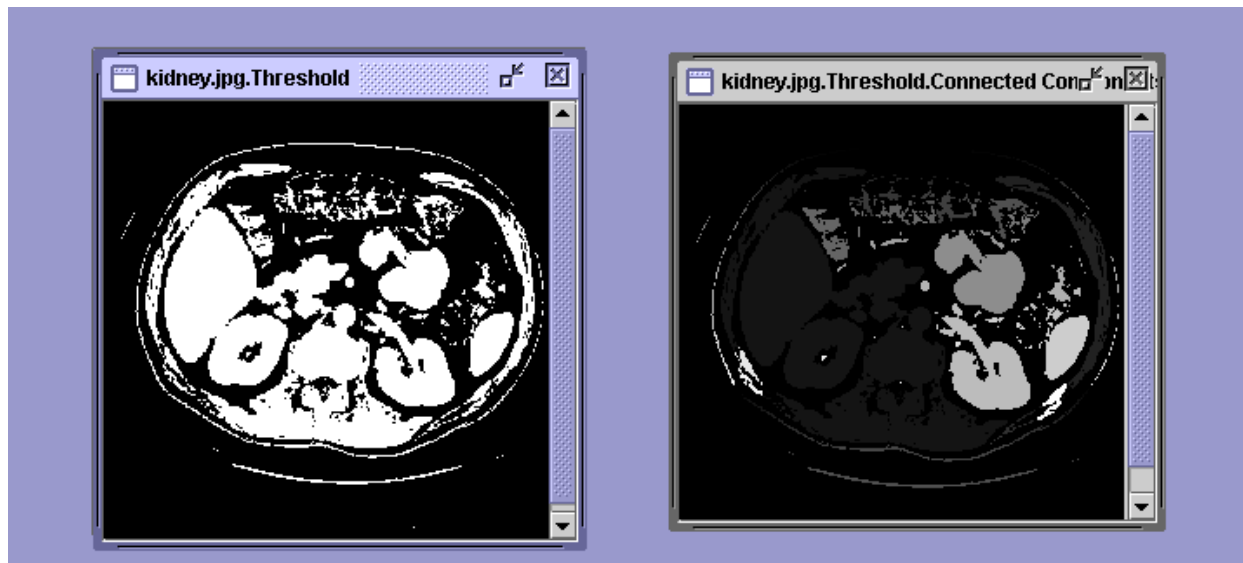
# Thresholding Example



# Connected Components Labeling

Once you have a binary image, you can identify and then analyze each **connected set of pixels**.

The connected components operation takes in a binary image and produces a **labeled image** in which each pixel has the integer label of either the background (0) or a component.



# Methods for CC Analysis

1. Recursive Tracking (almost never used)
2. Parallel Growing (needs parallel hardware)
3. Row-by-Row (most common)
  - Classical Algorithm
  - Efficient Run-Length Algorithm  
(developed for speed in real industrial applications)

# Equivalent Labels

Original Binary Image

0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0	1	1
0	0	0	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1

# Equivalent Labels

CC = 0

Scan across rows:

If 1 and connected,

Propagate lowest label behind or above (4 or 8 connected).

Remember conflicts

If 1 and not connected

CC++ and label CC

If 0, label 0

Relabel based on  
table

0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0	1	1
0	0	0	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1



# Equivalent Labels

CC = 0

Scan across rows:

If 1 and connected,

Propagate lowest label behind or above (4 or 8 connected).

Remember conflicts

If 1 and not connected

CC++ and label CC

If 0, label 0

Relabel based on  
table

0	0	0	1	1	1	0	0	0	0	2	2	2	2	0	0	0	0	3	
0	0	0	1	1	1	1	0	0	0	2	2	2	2	0	0	0	3	3	
0	0	0	1	1	1	1	1	0	0	2	2	2	2	0	0	3	3	3	
0	0	0	1	1	1	1	1	1	0	2	2	2	2	0	0	3	3	3	
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	3	3	3	
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	3	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1

1 ≡ 2

1 ≡ 3

# Equivalent Labels

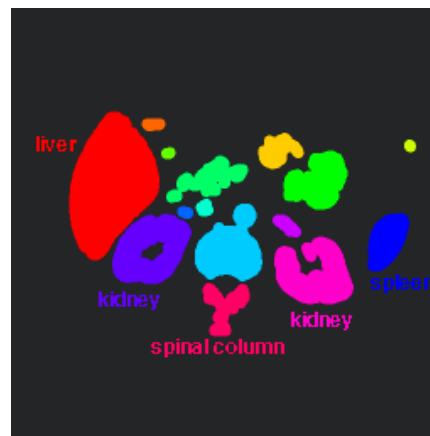
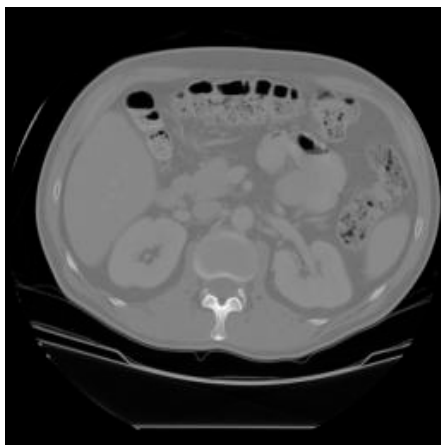
## The Labeling Process

```

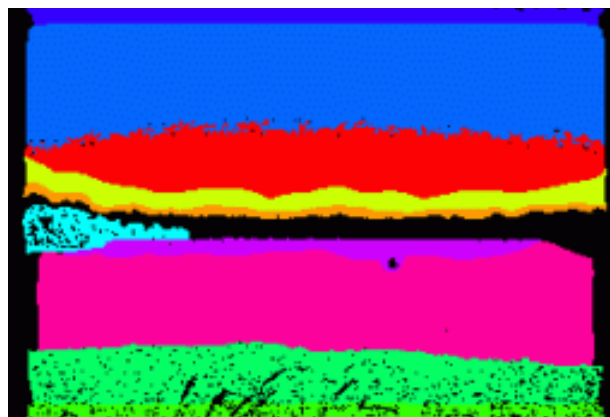
0 0 0 1 1 1 0 0 0 0 2 2 2 2 0 0 0 0 3
0 0 0 1 1 1 1 0 0 0 2 2 2 2 0 0 0 3 3
0 0 0 1 1 1 1 1 0 0 2 2 2 2 0 0 3 3 3
0 0 0 1 1 1 1 1 1 0 2 2 2 2 0 0 3 3 3
0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 3 3 3
0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 3 3 3
0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1
  
```

$$\begin{array}{l} 1 \equiv 2 \\ 1 \equiv 3 \end{array}$$

# Labeling shown as Pseudo-Color



connected  
components  
of 1's from  
thresholded  
image



connected  
components  
of cluster  
labels

# Mathematical Morphology

Binary mathematical morphology consists of two basic operations

**dilation and erosion**

and several composite relations

**closing and opening**  
**hit-or-miss transformation**

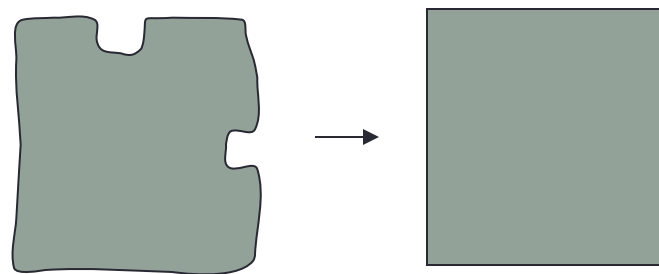
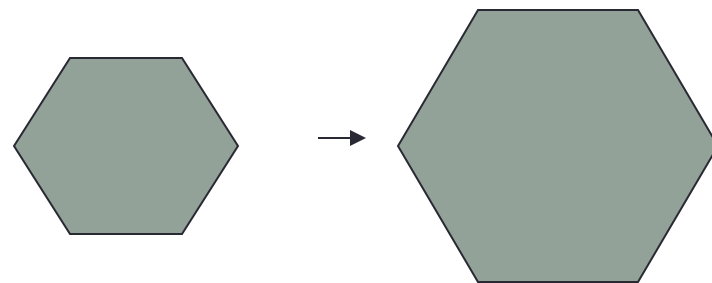
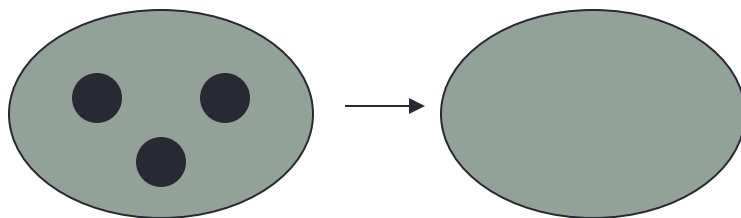
...

# Dilation

Dilation **expands** the connected sets of 1s of a binary image.

It can be used for

1. growing features
2. filling holes and gaps

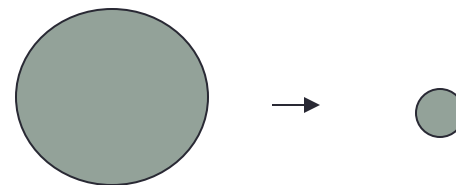


# Erosion

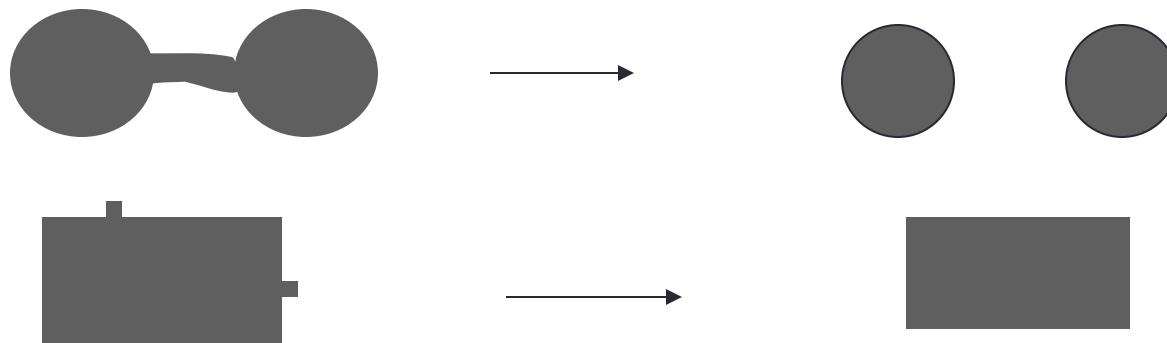
Erosion **shrinks** the connected sets of 1s of a binary image.

It can be used for

1. shrinking features



2. Removing bridges, branches and small protrusions



# Structuring Elements

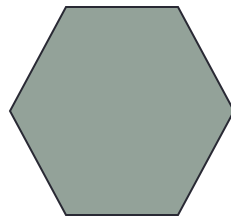
A **structuring element** is a shape mask used in the basic morphological operations.

They can be any shape and size that is digitally representable, and each has an **origin**.

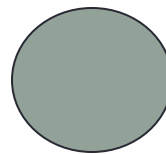


box

`box(length,width)`

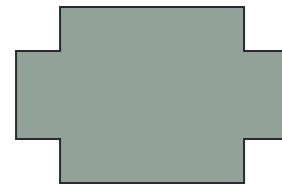


hexagon

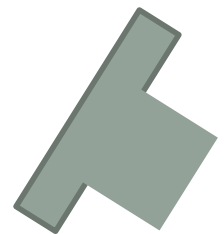


disk

`disk(diameter)`



something

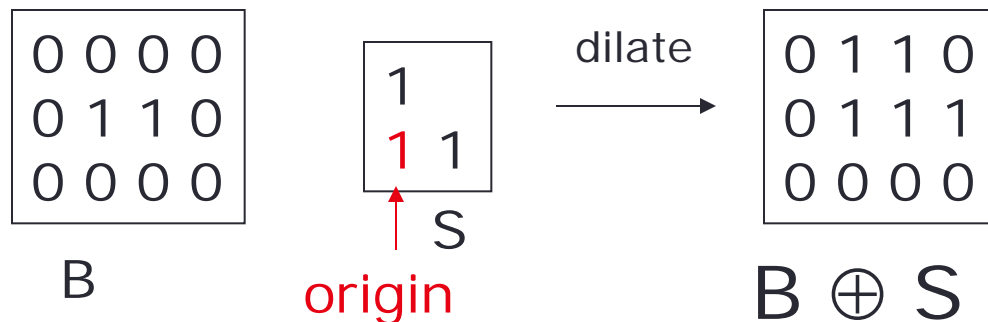


# Dilation with Structuring Elements

The arguments to dilation and erosion are

1. a binary image **B**
2. a structuring element **S**

**dilate(B,S)** takes binary image B, places the origin of structuring element S over each 1-pixel, and ORs the structuring element S into the output image at the corresponding position.





# Binary text example

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Original

$$\begin{array}{ccc} & 1 & \\ 1 & \boxed{1} & 1 \\ & 1 & \end{array}$$

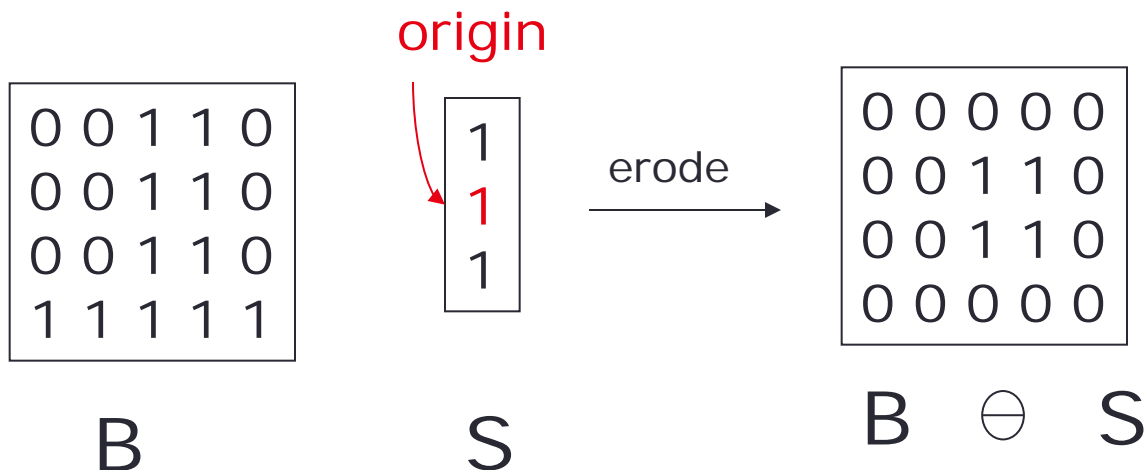
SE S

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

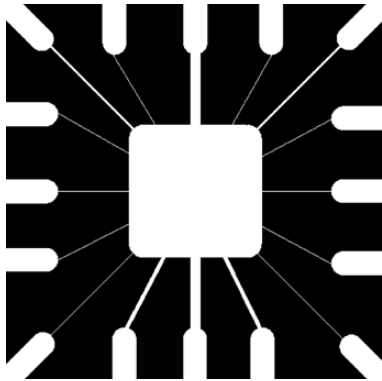
Dilated by S

# Erosion with Structuring Elements

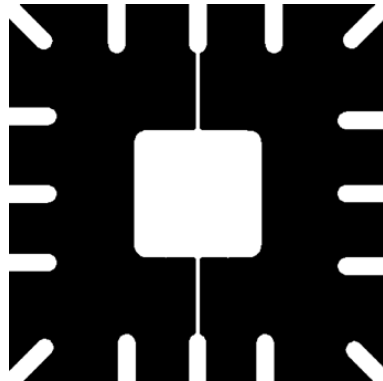
**erode(B,S)** takes a binary image B, places the origin of structuring element S over every pixel position, and ORs a binary 1 into that position of the output image only if every position of S (with a 1) covers a 1 in B.  
(Can also use zeros and “don’t cares”)



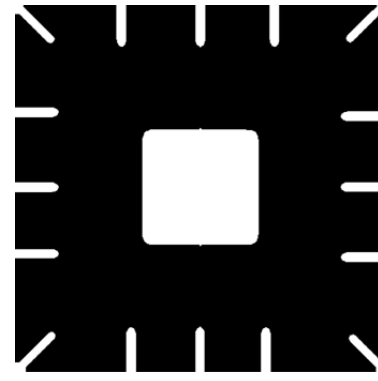
# Effect of disk size on erosion



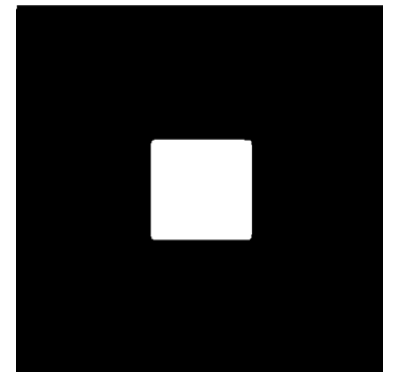
Original image



Erosion  
with a disk  
of radius 5



Erosion  
with a disk  
of radius 10

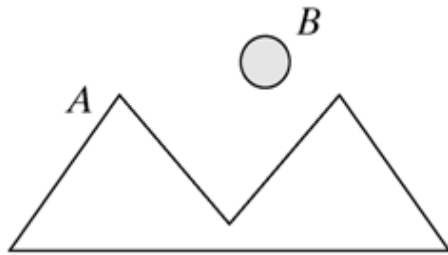


Erosion  
with a disk  
of radius 20

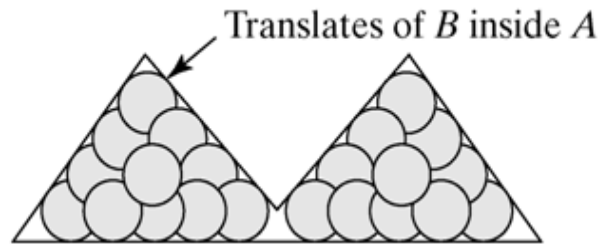
# Opening and Closing

- **Opening** is the compound operation of erosion followed by dilation (with the same structuring element)
  - Can show that the opening of A by B is the union of all translations of B that fit entirely within A.
  - Opening is **idempotent**: Repeated operations has no further effects!
- **Closing** is the compound operation of dilation followed by erosion (with the same structuring element)
  - Can show that the closing of A by B is the complement of union of all translations of B that do not overlap A.
  - Closing is **idempotent**: Repeated operations has no further effects!

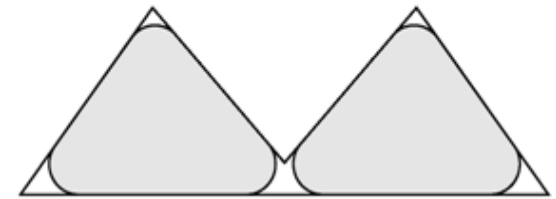
# Opening



Binary image A  
and structuring  
element B.



Translations of  
B that fit  
entirely within  
A.

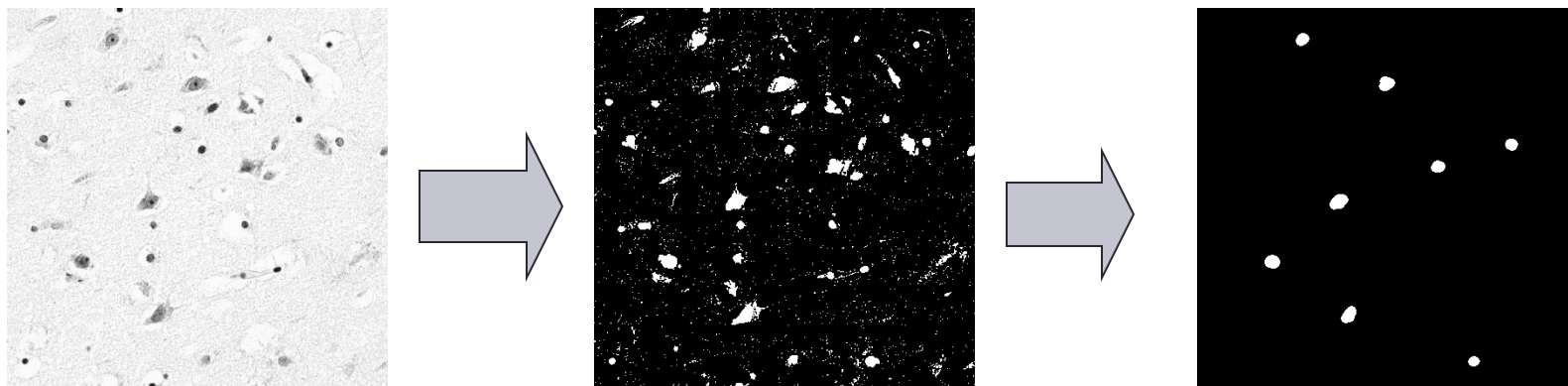


The opening  
of A by B is  
shown shaded.

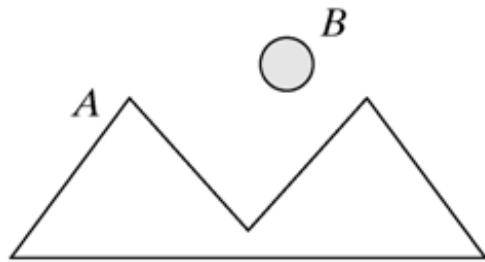
Intuitively, the opening is the area we can paint when the brush has a footprint B and we are not allowed to paint outside A.

# Opening example – cell colony

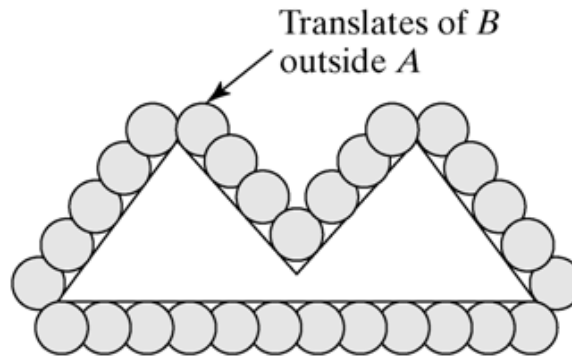
- Use large structuring element that fits into the big objects
- Structuring Element: 11 pixel disc



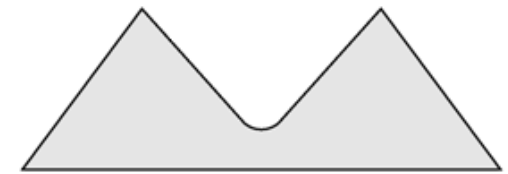
# Closing



Binary image A  
and structuring  
element B.



Translations of  
B that do not  
overlap A.

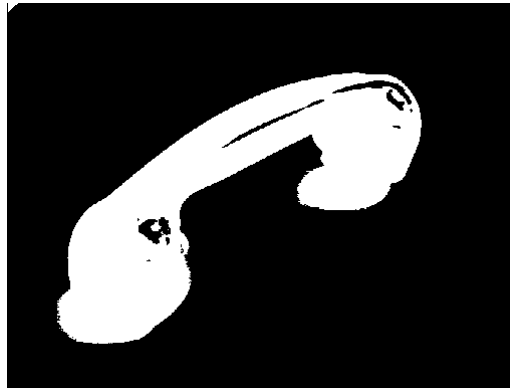


The closing of  
A by B is  
shown shaded.

Intuitively, the closing is the area we can not paint when the brush has a footprint B and we are not allowed to paint inside A.

# Closing Example - Segmentation

- Simple segmentation:
  1. Threshold
  2. Closing with disc of size 20

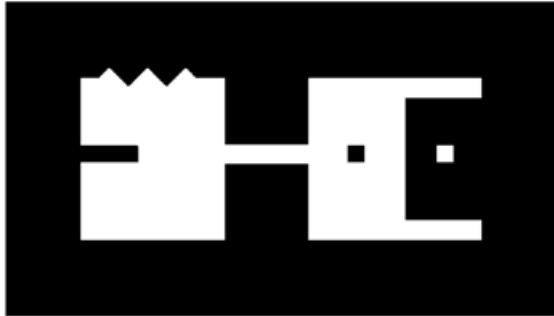




# Applications

- The opening of an image by a disk-like structuring element can be used to smooth contours, break narrow isthmuses, and eliminate small islands and sharp peaks.
- The closing of an image by a disk-like structuring element can be used to smooth contours, fuse narrow breaks and long thin gulfs and eliminate small holes.

# Toy example



Original  
image



Opening



Closing



Opening following by  
closing

A 20x20 square structural element was used.

# Real Example



Original  
image



Opening



Opening  
following by  
closing

A 3x3 square structural element was used.

# The Hit-or-Miss Transformation

- The morphological hit-or-miss transform is a basic tool for *shape detection* or *pattern matching*.
- Let  $B$  denote the set composed of  $X$  and its background.

$$B = (B_1, B_2), \text{ where } B_1 = X, B_2 = W - X.$$

- The match of  $B$  in  $A$ , denoted by  $A \circledast B$ , is

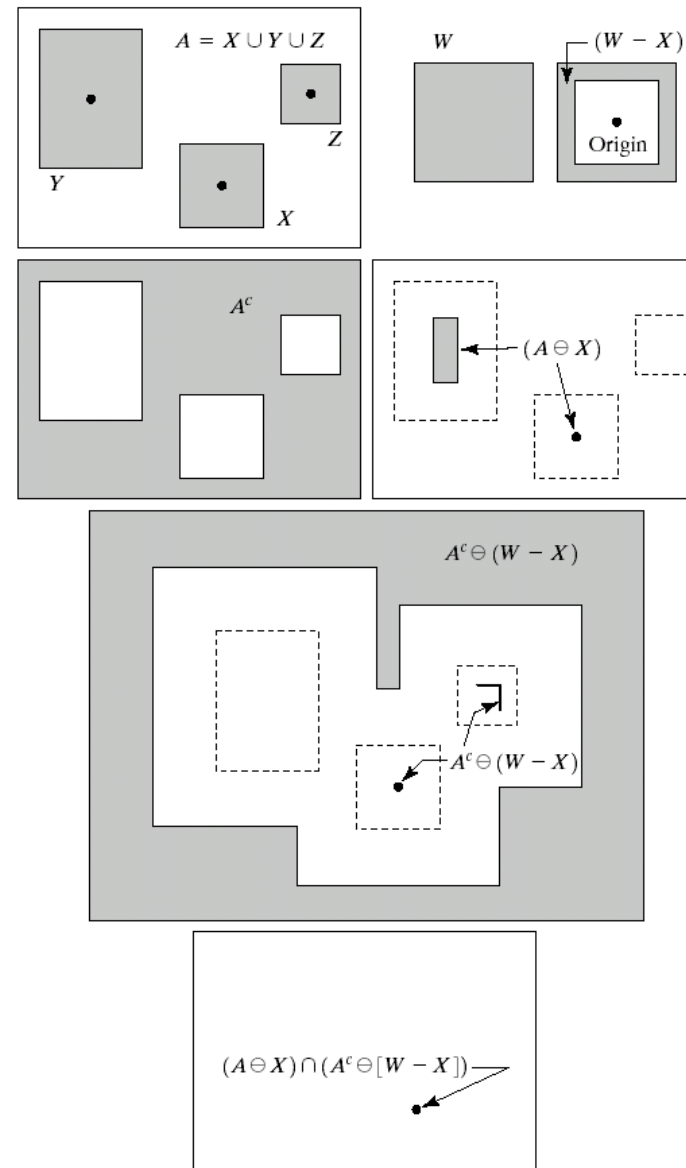
$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

# The Hit-or-Miss Transformation

Other interpretation:

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2]$$

$$A \circledast B = (A \ominus B_1) - (A \ominus \hat{B}_2)$$



a	b
c	d
e	
f	

**FIGURE 9.12**

(a) Set  $A$ . (b) A window,  $W$ , and the local background of  $X$  with respect to  $W$ ,  $(W - X)$ . (c) Complement of  $A$ . (d) Erosion of  $A$  by  $X$ . (e) Erosion of  $A^c$  by  $(W - X)$ . (f) Intersection of (d) and (e), showing the location of the origin of  $X$ , as desired.

# Some Basic Morphological Algorithms

- Boundary extraction
- Region filling
- Extraction of connected components
- Convex Hull
- Thinning
- Skeletons
- Pruning

# Boundary extraction

Let  $A \oplus B$  denotes the dilation of  $A$  by  $B$  and let  $A \ominus B$  denotes the erosion of  $A$  by  $B$ .

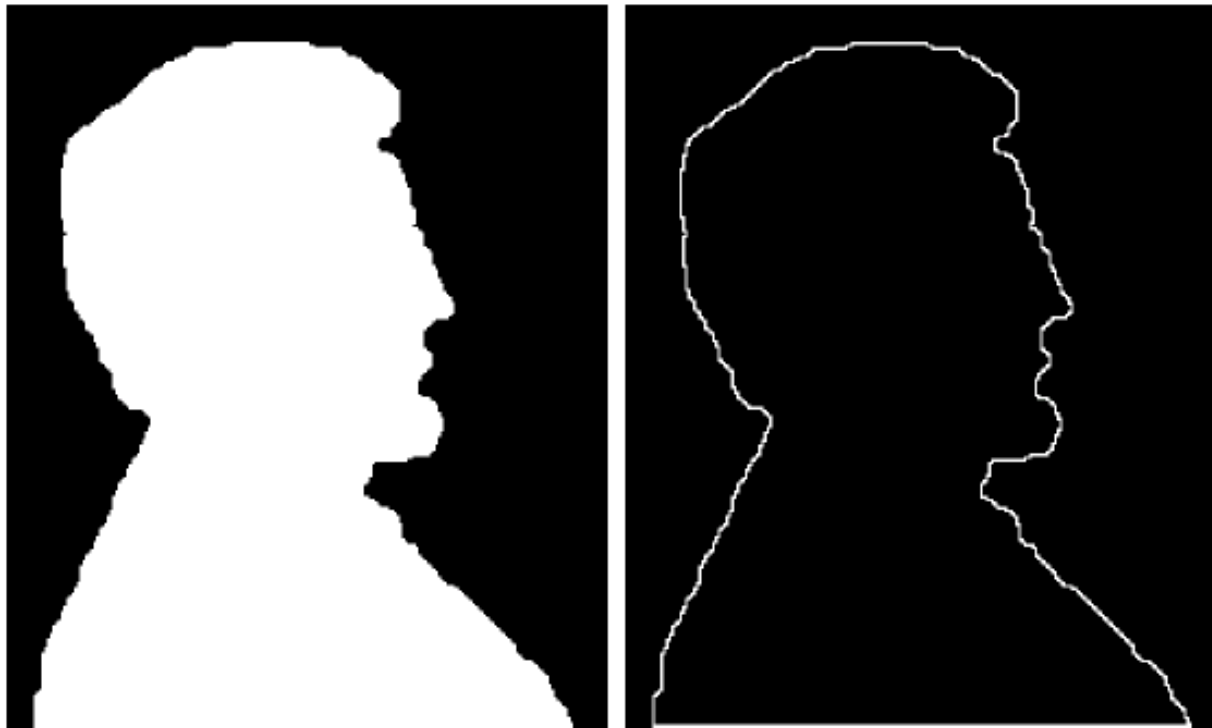
The boundary of  $A$  can be computed as

$$A - (A \ominus B)$$

where  $B$  is a 3x3 square structuring element.

That is, we subtract from  $A$  an erosion of it to obtain its boundary.

# Example of boundary extraction



a b

**FIGURE 9.14**

(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

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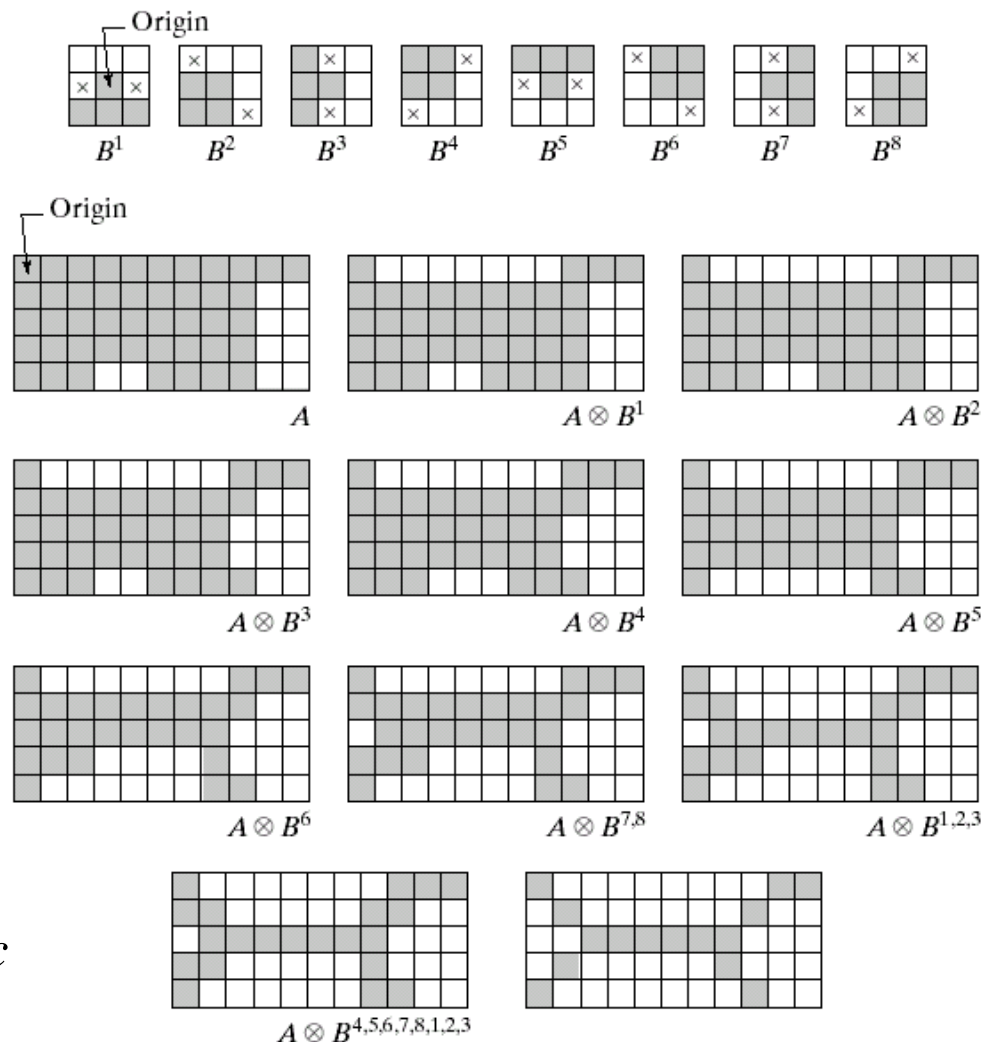


# Thinning

$$A \otimes B = A - (A * B)$$

$$= A \cap (A * B)^c$$

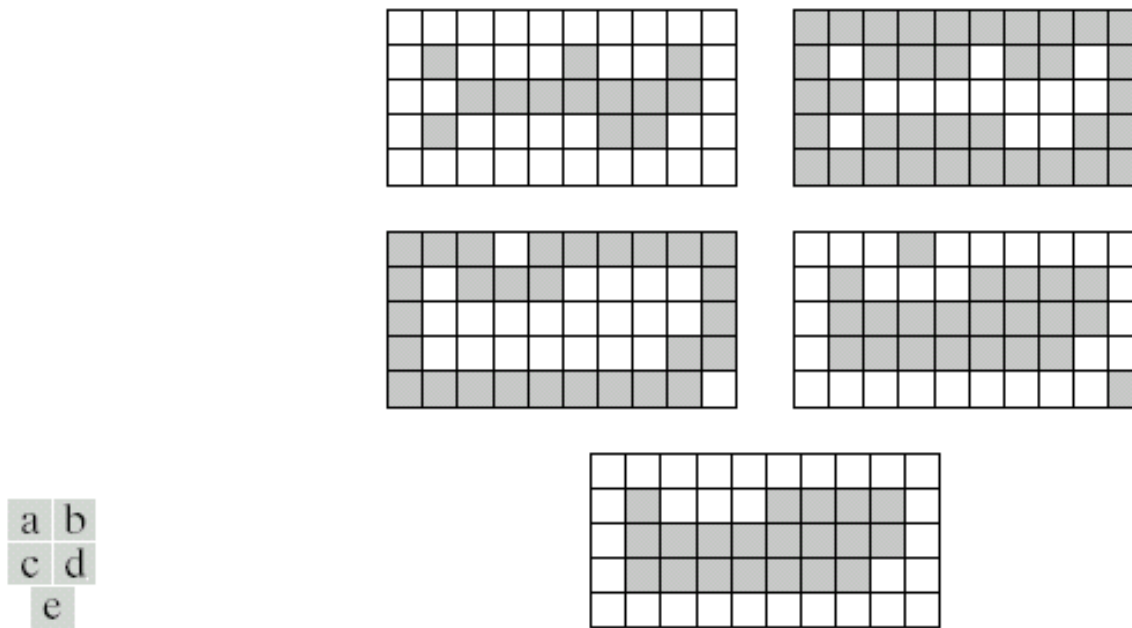
a
b c d
e f g
h i j
k l



**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set  $A$ . (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to  $m$ -connectivity.

# Thickening

$$A \odot B = A \cup (A \otimes B)$$

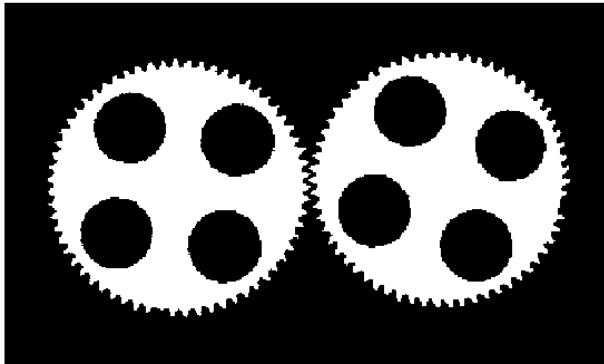


**FIGURE 9.22** (a) Set  $A$ . (b) Complement of  $A$ . (c) Result of thinning the complement of  $A$ . (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

# How powerful is morphology

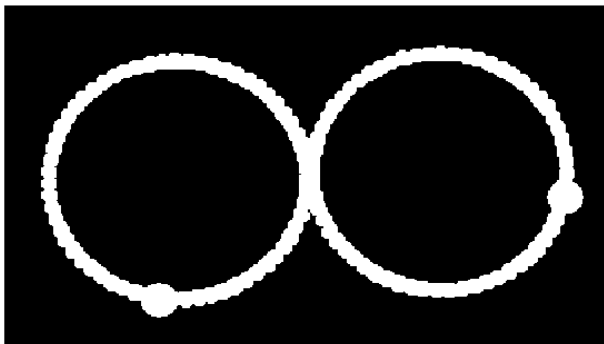
- If depends...
- If almost “clean” binary images then very powerful to both clean up images and to detect variations from desired image.
- Example...

# Gear Tooth Inspection

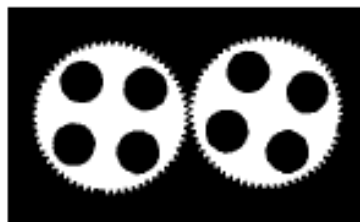


original  
binary  
image

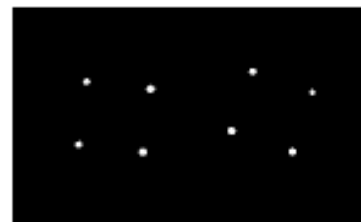
How did  
they do it?



detected  
defects



a) original image B

b)  $B1 = B \ominus \text{hole\_ring}$ c)  $B2 = B1 \oplus \text{hole\_mask}$ d)  $B3 = B \text{ OR } B2$ 

e) B7 (see text)

f)  $B8 = B \text{ AND } B7$ g)  $B9 = B8 \oplus \text{tip\_spacing}$ h)  $\text{RESULT} = ((B7 - B9) \oplus \text{defect\_cue}) \text{ OR } B9$

# Geometric and Shape Properties

- area
- centroid
- perimeter
- perimeter length
- circularity
- elongation
- mean and standard deviation of radial distance
- bounding box
- extremal axis length from bounding box
- second order moments (row, column, mixed)
- lengths and orientations of axes of best-fit ellipse