# PS8_Zhang

Haotian Zhang

March 2025

## 1 Answer for question 5

Table 1: Comparison of True and Estimated $\beta$ Values using the closed-form solution

| **Coefficient** | **True** $\beta$ | **Estimated** $\hat{\beta}_{OLS}$ |
|---|---|---|
| $\beta_1$ | 1.50 | 1.5010518 |
| $\beta_2$ | -1.00 | -1.0008296 |
| $\beta_3$ | -0.25 | -0.2516480 |
| $\beta_4$ | 0.75 | 0.7490406 |
| $\beta_5$ | 3.50 | 3.5005531 |
| $\beta_6$ | -2.00 | -2.0008185 |
| $\beta_7$ | 0.50 | 0.4987148 |
| $\beta_8$ | 1.00 | 1.0028269 |
| $\beta_9$ | 1.25 | 1.2465102 |
| $\beta_{10}$ | 2.00 | 2.0010012 |

The OLS estimate $\hat{\beta}_{OLS}$, computed using the closed-form solution, is very close to the true value of $\beta$. Each estimated coefficient differs from its corresponding true value by only a very small amount, typically less than 0.01.

Because the sample size is large ($N$ = 100,000), the law of large numbers ensures that the OLS estimator converges closely to the true parameter values.

## 2 Answer for question 7

Table 2: Comparison of $\hat{\beta}_{OLS}$ using gradient descent, L-BFGS algorithm and Nelder-Mead algorithm

| **Coefficient** | **Gradient Descent** | **L-BFGS algorithm** | **Nelder-Mead algorithm** |
|---|---|---|---|
| $\beta_1$ | 1.5010518 | 1.5010518 | 1.1770684 |
| $\beta_2$ | -1.0008296 | -1.0008296 | -0.9164661 |
| $\beta_3$ | -0.2516480 | -0.2516480 | -0.1601914 |
| $\beta_4$ | 0.7490406 | 0.7490406 | 0.9990248 |
| $\beta_5$ | 3.5005531 | 3.5005531 | 3.0740315 |
| $\beta_6$ | -2.0008185 | -2.0008185 | -2.2658981 |
| $\beta_7$ | 0.4987148 | 0.4987148 | 0.5961485 |
| $\beta_8$ | 1.0028269 | 1.0028269 | 0.8454130 |
| $\beta_9$ | 1.2465102 | 1.2465102 | 1.4415925 |
| $\beta_{10}$ | 2.0010012 | 2.0010012 | 2.0331941 |

The results from gradient descent and L-BFGS algorithm are nearly identical to the true $\beta$, confirming their reliability. Nelder-Mead algorithm, while still converging to a reasonable result, performs worse in this context and is less accurate.

## 3 Answer for question 9

Table 3: Comparison of True $\beta$ and Estimated $\hat{\beta}_{OLS}$ Using `lm()`

| **Coefficient** | **True** $\beta$ | `lm()` Estimate |
|---|---|---|
| $\beta_1$ | 1.50 | 1.5010518 |
| $\beta_2$ | -1.00 | -1.0008296 |
| $\beta_3$ | -0.25 | -0.2516480 |
| $\beta_4$ | 0.75 | 0.7490406 |
| $\beta_5$ | 3.50 | 3.5005531 |
| $\beta_6$ | -2.00 | -2.0008185 |
| $\beta_7$ | 0.50 | 0.4987148 |
| $\beta_8$ | 1.00 | 1.0028269 |
| $\beta_9$ | 1.25 | 1.2465102 |
| $\beta_{10}$ | 2.00 | 2.0010012 |

Overall, these estimates are identical to those obtained from the closed-form solution, gradient descent, and the L-BFGS algorithm. This confirms that the `lm()` estimates in R are very close to the ground truth and are a fast and convenient method.