Instructions:

- Write your name in the space provided above.
- Please consult the Syllabus for HW grading and late-submission policies.
- Please first solve the problem in draft paper and present your **complete** final answer in *clean* form on the space provided. **Answer all of the questions in the spaces provided. Answers are expected to be succinct but complete.** Please only use extra space (attach to the end of your submission) if absolutely necessary.
- The HW is **due Friday October 11 9:00am sharp**.
- The problem statements might be long. This does not seem that the problems are hard to solve! The questions are designed to guide you through the problem step by step. Give them a shot. Read the lecture notes. Talk to your classmates. Ask for help in the lab and in the office hours if you need!
- The HW set includes **Programming Assignments** marked as **(P.A.)**. Create a new Jupyter notebook and write code for each one of them. Execute the code to obtain the desired results (e.g., plot of the signals). Upload the notebook **including the code and its output** on GauchoSpace.
- For this HW, you are expected to upload **two** notebooks. The first one contains your answers to problems 3b, 3d, 3e. The second one contains your work in Problem 4.
- **Return a paper copy of your HW to the homework box in HF. Upload your P.A.s to Gauchospace.** **.**

1. **Problem 1 [Jargon].**

(a) (3 points) Give a definition for the term "*signal*" as used in Signal Processing (SP). Give two examples of signals. Are they continuous or discrete? Also, briefly describe why you might want to process these signals.

Signal is a measurable change that carries information.
Such as WiFi & radio wave, they are typically continuous, digital signals can be considered as discrete depending on how you think of them.

Raw signal is hard to be used directly, some translation would be necessary for the CPU to perform an action, we need to process the signal to be something that can be understand by human or machines (Such as serial-communication, without processing it would only be a series of highs & lows & oscilloscopes.)

(b) (2 points) What is "*sampling*" and why is it important in SP?

Sampling is recording a signal at a certain time. The more frequent the signal is sampled, the better the original data can be recreated.

(c) (1 point) Explain in your own words what is the "*spectrum*" of a signal.

Spectrum of a signal describes the frequencies that is used to makeup the signal. Any signal can be described as the linear combination of sin and cos waves at discrete frequencies, hence a spectrum of frequency

(d) (3 points) Give two examples where it is beneficial to think of the signal in the "*frequency domain*".

When we need to clearly see the frequency of something. Such as in music we might want to express the sound signal in frequency domain to identify which exact notes are being played.
Or, we might want to tell what color makes up the colored light.

2. **Problem 2 [Building blocks].**

(a) (3 points) The phase of a sinusoid can be related to *time shift* as follows:

$\frac{2\pi \times t_1}{T_0}$

$$s(t) = A\cos(2\pi f_0 t + \phi) = A\cos(2\pi f_0(t - t_1)).$$

$T_0 = 8$

$f = 0.125$ Hz

Assume the period of the signals is $T_0 = 8$s. *Explain* whether the following are True or False.

1. "When $t_1 = -2$s, the value of the phase is $\phi = 3\pi/4$."

$2\pi f_0 t + \phi = 2\pi f_0 t - 2\pi f_0 t_1$ $\phi = \frac{\pi}{2}$

False $\phi = 4\pi/8$

2. "When $t_1 = 3$s, the value of the phase is $\phi = 3\pi/4$."

False, $-\frac{3}{4}\pi$

3. "When $t_1 = 7$s, the value of the phase is $\phi = \pi/4$."

True, $\phi = -\frac{7}{4}\pi$, $2\pi - \frac{7}{4}\pi = \frac{\pi}{4}$

(b) (2 points) Let $A, B, f_0 \in \mathbb{R}$ be given. Find C and ϕ (in terms of the given variables) such that

$$A\cos(2\pi f_0 t) + B\sin(2\pi f_0 t) = C\cos(2\pi f_0 t + \phi), \qquad \text{for all } \; t \in \mathbb{R}.$$

RHS $\Rightarrow C\cos(2\pi f_0 t)\cos(\phi) - C\sin(2\pi f_0 t)\sin\phi$

In which $C\cos(\phi)$ & $C\sin\phi$ coefficients.

hence $A\cos(2\pi f_0 t) + B\sin(2\pi f_0 t) =$ RHS

(c) (3 points) Let $f_0, \{A_k\}_{k=1}^N, \{\phi_k\}_{k=1}^N \in \mathbb{R}$ and $N \in \mathbb{N}$ be given. Consider the signal

$$s(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_0 t + \phi_k)$$

that is a linear additive combination of sinusoidal signals of the same frequency, but different amplitudes and phases. Show that there exist amplitude A and phase ϕ such that $s(t)$ can be equivalently expressed as: $s(t) = A\cos(2\pi f_0 t + \phi)$.

(Hint: You don't need to explicitly compute A and ϕ. Just find a way to argue that such values exist. Part (a) of this problem together with its reverse statement that we saw in class might be useful.)

$$s(t) \equiv \sum_{k=1}^{N} \left(A_k \cos(2\pi f_0 t) \cos\phi_k - A_k \sin(2\pi f_0 t) \sin\phi_k \right)$$

let $A_k \cos\phi_k$ be B_k, $A_k \sin\phi_k$ be B_{1k}

$$\sum_{k=1}^{N} \left(B_k \cos(2\pi f_0 t) + B_{1k} \sin(2\pi f_0 t) \right) \Longleftrightarrow \sum_{k=1}^{N} (B_k) \cos(2\pi f_0 t) + \sum_{k=1}^{N} (B_{1k}) \sin(2\pi f_0 t)$$

$$\Downarrow$$

$$C_1 = \sum_{k=1}^{N} (B_k) \quad C_2 = \sum_{k=1}^{N} (B_{1k})$$

$$\therefore \; C_1 \cos(2\pi f_0 t) + C_2 \sin(2\pi f_0 t)$$

hence can be expressed as

$$A\cos(2\pi f_0 t + \phi) \text{ where } C_1 = A\cos\phi \quad C_2 = -A\sin\phi$$

$$\therefore s(t) = A\cos(2\pi f_0 t + \phi)$$

3. **Problem 3 [Modulation].** Let f_c, f_Δ two frequencies such that $f_c \gg f_\Delta$. Consider the signal resulting by multiplying two pure cos-sinusoidal signals with these frequencies.

$$s(t) = \cos(2\pi f_\Delta t)\cos(2\pi f_c t).$$

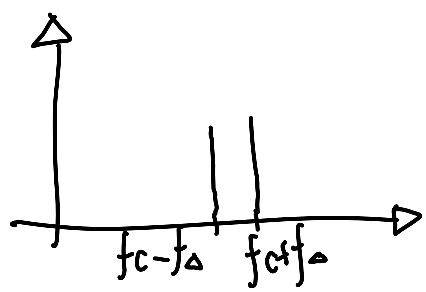
(a) (3 points) Compute the frequency spectrum of the signal $s(t)$ and make a sketch of it.
(Hint: Recalling the trigonometric identities from high-school might be helpful!)

$$\cos P + \cos Q \equiv 2\cos\left(\tfrac{1}{2}(P+Q)\right)\cos\left(\tfrac{1}{2}(P-Q)\right)$$

hence $s(t) \Longleftrightarrow \frac{1}{2}\left(\cos(2\pi f_\Delta t - 2\pi f_c t) + \cos(2\pi f_\Delta t + 2\pi f_c t)\right)$

since $f_c \gg f_\Delta$

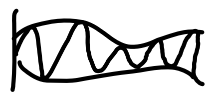
$$s(t) \simeq \frac{1}{2}\left(\cos(2\pi t(f_\Delta - f_c)) + \cos(2\pi t(f_\Delta + f_c))\right)$$



(b) (3 points) **(P.A.)** Take $f_c = 200$Hz and $f_\Delta = 20$Hz. Use Python to plot the following signals on the same plot: $s(t)$, $\cos(40\pi t)$ and $-\cos(40\pi t)$ for $0 \le t \le 100$.

(c) (1 point) The plots of $\cos(40\pi t)$ and $-\cos(40\pi t)$ are called the *envelope* of the signal $s(t)$. Give an explanation for this term.

The fitting function of the extreme points



(d) (1 point) **(P.A.)** Can you play the signals $s(t)$ and $\cos(2\pi 200t)$ on your computer? What do you observe?

Same frequency of one of the wave
that makes up the signal
So they sounded almost the same

For the remaining part of the exercise, consider the signal $s(t) = (5 + 4\cos(40\pi t))cos(400\pi t)$.

(e) (1 point) **(P.A.)** Use Python to plot $s(t)$.

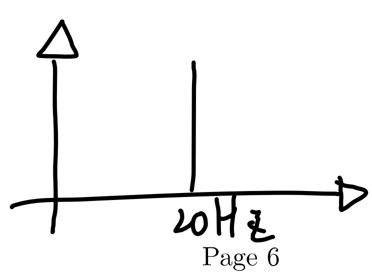
(f) (1 point) In reference to the plot you generated above, *explain* if the following statement is True or False: "The effect of multiplying the higher-frequency sinusoid (called the carrier) by the lower-frequency sinusoid is to change the amplitude of the waveform of the former."

Yes, the amplitude at a certain time
would be varied according to the waveform
of the former

(g) (2 points) Make a sketch of the frequency spectrum of $s(t)$. How is it different from your answer in part (a)?

It is similar, but instead of f something
an actual value is plotted.

(h) (2 points) Make a sketch of the frequency spectrum of the low-frequency signal $5 + 4cos(40\pi t)$. Compare the result with the spectral plot in the previous part (g).



Multiplying sinusoids is commonplace in communication systems. In particular, the process of multiplying a high-frequency sinusoidal signal by a low-frequency signal (such as voice or music signal) is known as *Amplitute Modulation* (AM). It is exactly the technique used in broadcast AM radio! The AM signal is a product of the form

$$s(t) = v(t)\cos(2\pi f_c t),$$

where the frequency f_c (known as the *carrier* frequency) is assumed much higher than the frequencies in $v(t)$ (voice or music signal).

(i) (1 point) The "Santa Barbara News-Press Radio" is on station 1290 kHz. Write an expression for the signals broadcasted by this station and explain the terms involved.

$$s(t) = v(t)\cos(2 \times 1290 \times 10^3 \pi \times t)$$

(j) (3 points) Let $s_1(t) = (A_1 + B_1\cos(2\pi f_1 t))\cos(400\pi t)$ and $s_2(t) = (A_2 + B_2\cos(2\pi f_2 t))\cos(420\pi t)$ be the transmitted signals from two AM radio stations. This (very) simple model allows for the two stations to transmit different information by modifying the corresponding frequencies f_1, f_2 and the amplitudes A_1, A_2, B_1, B_2. Suppose we receive a signal $r(t) = s_1(t) + s_2(t)$. We know the carrier frequencies 400Hz and 420Hz of the two stations, but, at any given instance, we do *not* know the frequencies f_1 and f_2 that the two stations choose to transmit. Argue that it is possible to design a system that is able to reproduce the true content of each station's signal as long as $f_1 + f_2 < 10$Hz.

$$r(t) = S_1(t) + S_2(t) = A_1\cos(400\pi t) + B_1\cos(400\pi t)\cos(2\pi f_1 t)$$
$$A_2\cos(420\pi t) + B_2\cos(420\pi t)\cos(2\pi f_2 t)$$

$\frac{1}{2}(\cos(2\pi f_1 t - 400\pi t) + \cos(2\pi f_1 t + \pi$ [illegible] $t))$

$$r(t) \Leftrightarrow \frac{1}{2}\{\cos[2\pi(f_1 - 200)t] + \cos[2\pi(f_1 + 200)t]\} + A_1\cos(400\pi t)$$
$$+ \frac{1}{2}\{\cos[2\pi(f_2 - 210)t] + \cos[2\pi(f_2 + 210)t]\} + A_2\cos(420\pi t)$$

$S_1(t)$ made of $(200 - f_1)$Hz and $(200 + f_1)$Hz

$S_2(t)$ made of $(210 - f_2)$Hz and $(210 + f_2)$Hz

hence for two signal no interference

$200 + f_1 < 210 - f_2$

$\underline{f_1 + f_2 < 10}$

4. (15 points) **Problem 4 [Find the note]. (P.A.)** Download the files "HW_1_find_the_note.ipynb" and "piano.txt" from GauchoSpace. Open the notebook "HW_1_find_the_note.ipynb" and go through the material. Fill in all the blanks in the code cells (indicated by three dots ...) and write your own lines of code wherever required. When appropriate, answer the questions posed (e.g., "which note was this?"). **Answer everything on the notebook and upload your work on GauchoSpace.**

——— *End of HW #1* ———