Correct Overfitting in Two Stage OLS Regression: an Immigration Example

Methods and Initial Results

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In this research, I want to correct the overfitting in a two stage OLS regression and find more accurate and powerful parameters. I will use an immigration model to do the two stage OLS regression and explore the impact of foreign student on high skilled labor market. In specific, the immigration shock is measured by the percentage of foreign student and the high skilled labor market outcomes are measured by the annual earnings of doctoral recipients. I will use full sample to do the estimation first and then use bootstrap method to the estimation. By comparing the mean squared error of these two estimations, I want to find whether the parameters got from bootstrap method are much more accurate.

1 data

1.1 Data Source

The data used in this research are from two surveys. One is the 'Survey of Earned Doctorates (SED)'. The other is the 'Survey of Doctoral Recipients (SDR)'.

The Survey of Earned Doctorates (SED) is an annual census conducted since 1957 of all individuals receiving a research doctorate from an accredited U.S. institution in a given academic year. The SED collects information on the doctoral recipient's educational history, demographic characteristics, and postgraduation plans. Results are used to assess characteristics of the doctoral population and trends in doctoral education and degrees. SED data is available until 2017, but I will just use SED data from 1966 to 2000, because the major categories in this dataset changed since 2000 and there are many missing data after 2000.

The Survey of Doctorate Recipients (SDR) provides demographic, education, and career history information from individuals with a U.S. research doctoral degree in a science, engineering, or health (SEH) field. Conducted since 1973, the SDR is a unique source of information about the educational and occupational achievements and career movement of U.S.-trained doctoral scientists and engineers in the United States and abroad. The SDR uses a fixed panel design with a sample of new doctoral graduates added to the panel in each biennial survey cycle. For example, all doctorates who were included in the 2015 SDR sample and who are less than 76 years old in 2017, will be retained in 2017 survey, and a sample of new graduates who had earned their degrees were added in 2017 survey, so one person can be observed in several years' SDR sample. I will use SDR data from 2003 to 2013.

1.2 Data Processing and Cleaning

The WebCASPAR database constructed by National Science Foundation provide statistical data resources for the SED. I will use three variables in this survey: the variables of cohort, citizenship and field of study. The cohort is the graduation year of

doctorate student, which is the survey conducted year of the SED. The citizenship status contains four categories: US citizens, permanent residents, temporary residents and unknown citizenship. Since unknown citizenship just contains few samples, I drop samples in this category. I also combine permanent residents and temporary residents into non-us citizens category. The field of study contains 47 majors, I make similar majors into one category and drop the majors which are not contained in the SDR, since the SDR just have the majors in science, engineering, and health (SEH) field. The final dataset contains 19 categories of majors, which are same with the majors in the SDR.

We can see the trend of doctorate recipients use the citizenship variable in the SED. The following figure shows the trend of the number of native doctoral student and foreign student from 1966 to 2000.

From figure1, we can see that native doctoral students increased rapidly in 1966 to 1970. Then it fluctuated from 23000 to 28000. The number of non-native doctoral students increased rapidly before 1996. In 1966, the number of non-native doctoral students was just 2500, while the number of native students is around 15000. In 1996, the number of non-native doctoral students is 13500, while the number of native students is around 28000. Nearly half of the doctoral students are non- native in 1996. The non-native students' ratio increased a lot through 1966 to 2000, which shows the increasing immigrant supply shock in labor market.

I used samples in non-us citizenship category as 'immigrant'. Then the immigrant share in each field and cohort is calculated by the following equation:

$$P_f c = N_f c / (N_f c + U_f c)$$

where $N_f c$ is the number of non-US citizens, which are also the immigrants in field f and cohort c, and $U_f c$ is the corresponding number of US citizens.
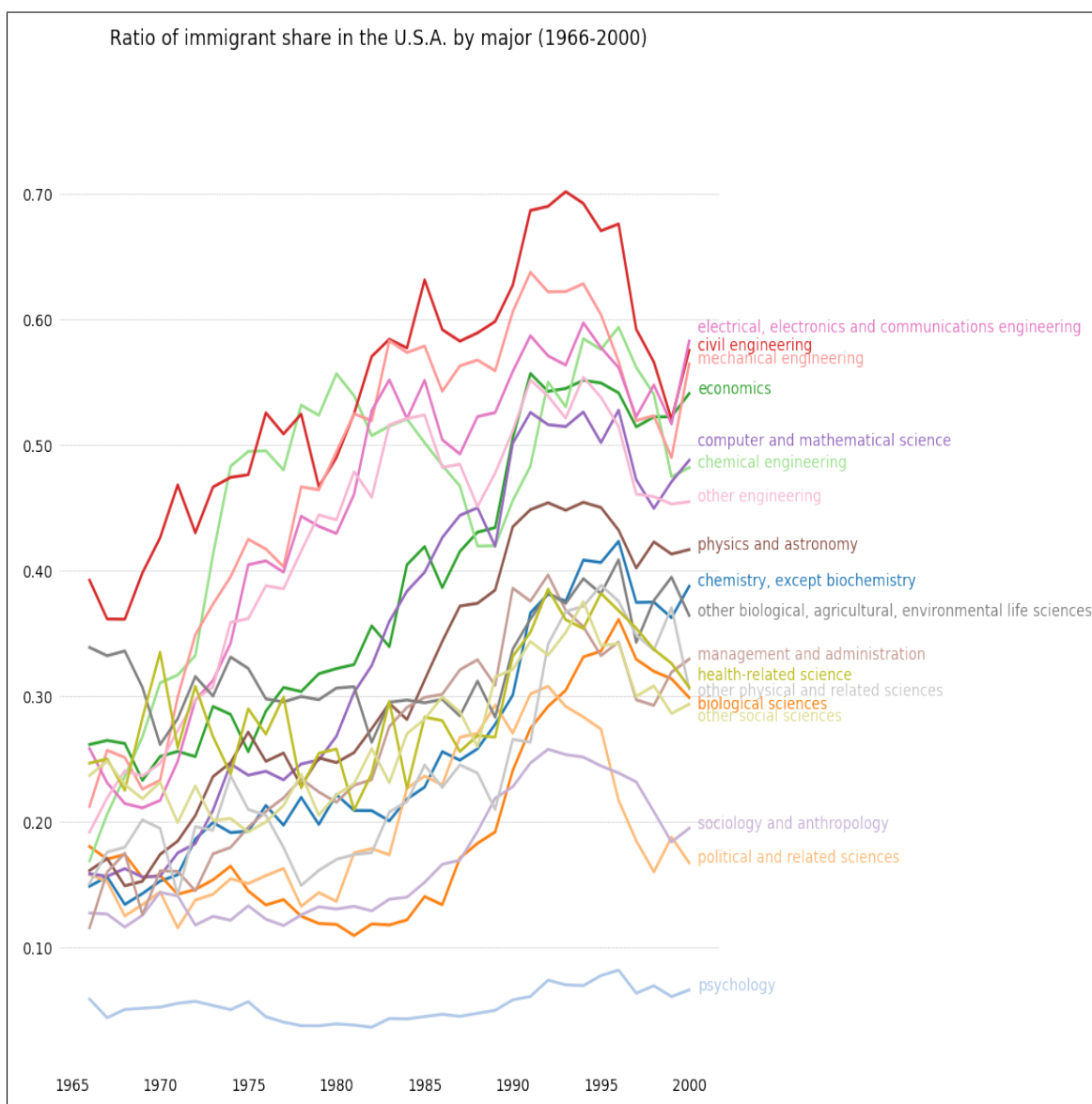
By defining the immigrant share, we can know the trend of immigrant supply shock in each field of study and each cohort through 1996 to 2000 from figure2.

Figure 1: Number of Doctorate recipients from 1966 to 2000 in US
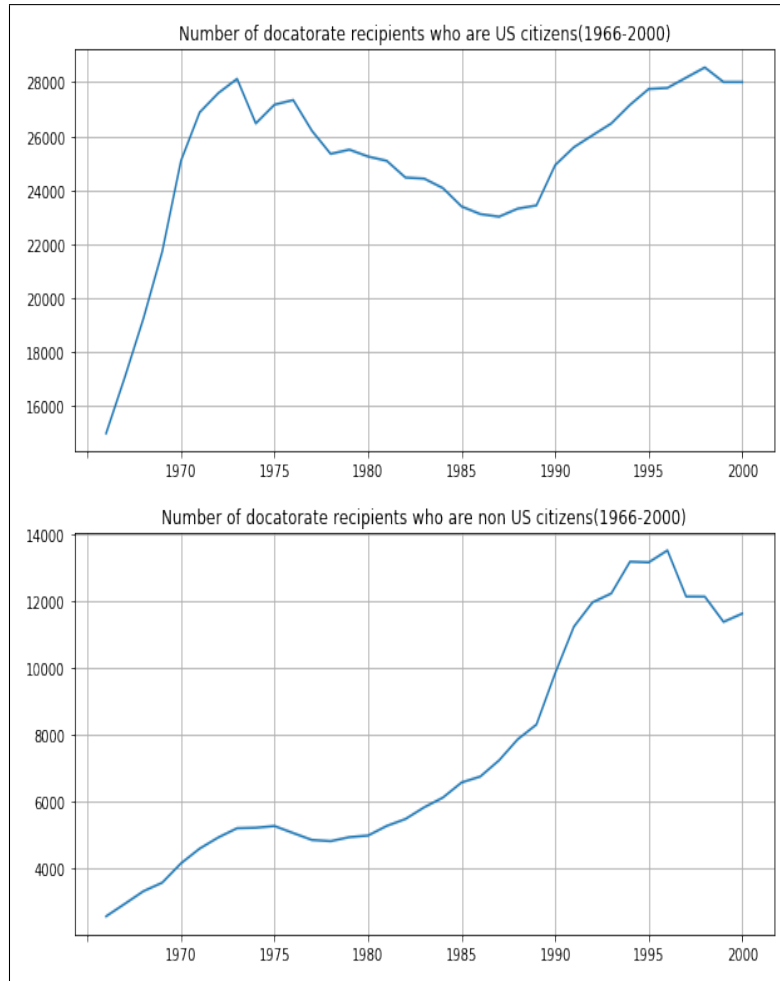
From figure2, we can see that although it shows some fluctuations in these years, the trend of immigrant supply showed increase trend in each field and cohort.

Figure 2: The ratio of immgrant share in US

Ratio of immigrant share in the U.S.A. by major (1966-2000)

electrical, electronics and communications engineering
civil engineering
mechanical engineering
economics
computer and mathematical science
chemical engineering
other engineering
physics and astronomy
chemistry, except biochemistry
other biological, agricultural, environmental life sciences
management and administration
health-related science
other physical and related sciences
biological sciences
other social sciences
sociology and anthropology
political and related sciences
psychology

The IPUMS Higher Ed is intended to facilitate access to data on the STEM labor force. I got the data from the Survey of Doctorate Recipients (SDR) from 1993 to 2003 in this database. I will use five variables in this survey: the variables of person id, annual earning, field of study, graduation year(cohort) and survey conducted year. Since the sample size in the field of 'management and administration' and 'other non-science and engineering' are too small, I dropped this two fields from dataset.

Finally, I linked the SED data and the SER data by field of study and graduation year (cohort). Since the graduation year is a large span, which is from 1966 to 2000, I divided the data into six five-year cohorts.

The summary statistics for interested variable earnings and immigrant share shows in table1.

Table 1: Summary Statistics of Key Variables

variables	Annual Earnings	Immigrant Share
Count	92078	92078
Mean	94150.731	0.288
Std	40833.232	0.164
Min	0.000	0.041
25%	65000	0.154
50%	95000	0.293
75%	130000	0.396
Max	150000	0.702

2 Model and Methods

2.1 The immigration model

The general regression model for immigration impact for labor market is as follows:

$$\log W_{ifc}(t) = \theta\, p_{fc} + x_{ifc}(t) + d_f + y_c + \pi_t + \varepsilon_{ifc}(t)$$

where $W_{ifc}(t)$ is the annual earnings for individual i who graduated in cohort c, majored in field f and participated SER in year t. $x_{ifc}(t)$ is the fixed effects of experience, which is defined as the year in the labor market for individual i. It is calculated by the year between individual i's graduation year and observed year in SER. I treat experience variable as fixed effect rather than continuous variable. Because that the intervals of graduation year is from 1966 to 2000 and the interval of SER conducted year is from 2003 to 2013, the largest experience is 47 years and the smallest experience year is 3 years. Since the interval of experience is just from 3 to 47 while the sample size is 92000, experience is better to be treated as fixed effects rather than continuous variable. d_f, y_c and π_t are fixed effects for field, cohort and observed year. This model can be added in interaction terms between field fixed effects and SER year fixed effects. This model can not be added in other variables because other variables are not significantly influence the dependent variable or perfectly colinear with the independent variables.

θ is the parameter of immigrant shock, but it is hard to interpret directly. Borjas (2009) converted it to elasticity to interpret the impact of foreign student on high skilled labor market by the following way.

$$\frac{\partial \log W_{fc}}{\partial P_{fc}} = \theta$$

$$n_{fc} = \frac{N_{fc}}{U_{fc}} = \frac{P_{fc}}{1 - P_{fc}}$$

where n_{fc} can be interpreted as the percentage increase of labor supply con-

tributed by immigrant. Then we can get the elasticity of wage and labor supply.

$$\frac{\partial \, log \, W_{fc}}{\partial \, n_{fc}} = \theta(1 - P_{fc})^2$$

However, since one individual can appear several times in the database in different SER conducted year and p_{fc} does not change for individuals who graduated in same year and studied in same field, so the error terms for OLS regression will be incorrect. Borjas(2009) handled this model into a two stage OLS regression to make the error terms more correct:

$$\log W_{ifc}(t) = V_{ifc} + x_{ifc}(t) + \pi_t + \varepsilon_{ifc}(t)$$

$$\widehat{V_{fc}} = \theta \, p_{fc} + d_f + y_c + w_{fc}$$

where V_{ifc} is the individual effect in field f and cohort c and $\widehat{V_{fc}}$ is the average individual fixed effect for all individuals in field f and cohort c.

This two stage OLS regression can adjust the standard errors for the individual's correlation and the clustering of immigrant share in same cohort and field. I will use this two stage OLS regression model as base model for my research.

2.2 The bootstrap model

Full sample estimation may have overfitting problems. When maximizing the accuracy, the model may pick incorporate noise from the data. Bootstrap method is a good way to detect the overfitting problems in this two stage OLS regression and correct the overfitting problems by randomly splitting the whole dataset into training set and testing set. The randomness and repetition can help leverage the risk of picking noise from the dataset and thus correct overfitting problems.

I will measure the accuracy of the model by Mean Squared Error (MSE):

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \hat{Y_i})^2$$

Firstly, I will use Borjas' model and method to do this two stage OLS regression. He used the full sample to do this regression. We can get the estimation parameter $\hat{\beta}^{FS}$ and mean squared error MSE^{FS} from this full sample regression.

$$MSE^{FS} = \frac{1}{n} \sum_{i=1}^{n} (Y_i^{FS} - \hat{Y}_i^{FS})^2$$

Then I will use bootstrap method to do this regression. I will randomly split the whole data set into training set and testing set. Then use the training set to get all the estimation parameters: $\hat{V}_{fc}$, $\hat{x}_{ifc}(t)$, $\hat{\pi}_t, \hat{\theta}$, $\hat{d}_f$ and $\hat{y}_c$, and use these estimation parameters into the testing set to get the prediction of log wage. Then use the estimated log wage and the real log wage to calculate the MSE^{BS}. Then I repeat this procedure in testing set but using the estimation parameters got from full sample estimation. Particularly, in the fitting procedure in testing set, I will use the estimated $\hat{V}_{fc}$ instead of $\hat{V}_{ifc}$ in the first stage.

$$MSE^{BS} = \frac{1}{n} \sum_{i=1}^{n} (Y_i^{BS} - \hat{Y}_i^{BS})^2$$

Then repeat this procedure N times and calculate the average mean squared error of regression using full sample estimation parameters and using bootstrap estimation parameters.

$$\bar{MSE} = \frac{1}{N} \sum_{i=1}^{M} MSE$$

Finally, compare $\bar{MSE}^{FS}$ and $\bar{MSE}^{BS}$ to find whether bootstrap method can make the model better fit. If we find that:

$$\bar{MSE}^{BS} < \bar{MSE}^{FS}$$

we can have a conclusion that bootstrapping can detect and correct overfitting problems in this two stage OLS model.

3 Analysis and Results

3.1 Full Sample Estimation

Firstly, I use full sample to do the estimation and get each parameter. The result showed in table2.

Table 2: Full sample estimation results

	Annual earnings	
	(1)	(2)
Immigrant share	0.2296	0.2267
	(0.0397)	(0.0397)
controls		
(field * period) interactions	No	Yes

Full sample estimation shows that the estimated parameter is approximately 0.23. While P_{fc} is approximately 0.3 at 2000, we can calculate the elasticity by equation that when the number of immigrants increase by 10%, the wage of native student will decrease approximately by 11.3%.

The result shows that the influx of foreign students has a significant negative impact on native students' earnings. But this estimation may have overfitting problems and we need to use bootstrap method to further study the impact of foreign student on high skilled labor market.

3.2 Bootstrap Estimation

I then randomly split the whole dataset into training set and testing set. Get estimation parameters using training set and then calculate MSE using testing set. And repeated this procedure by 10 times, 20 times and 50 times. The parameters are calculated by the following way. I don't include the interaction term of field and period at this time, since it doesn't have a significant influence on the results.

$$\widehat{\beta} = \frac{1}{N} \sum_{i=1}^{M} \widehat{\beta}^{trainnig}$$

The results showed in table3.

Table 3: Bootstrap estimation results

	Full Sample	10 times bootstrap	20 times bootstrap	50 times bootstrap
Immigrant share	0.2297	0.2286	0.2271	0.2243
controls				
(field * period) interactions	No	No	No	No

We can see that θ is a little bit different by using different times of bootstrap. In 50 times bootstrap, θ is 0.2243, which is 2% smaller than full sample estimation parameter. Then 10% increase of immigrant will decrease native's earnings by approximately 10.99%. Is 10.99% much better and accurate than 11.3%? We need to see the mean squared error of each method.

3.3 Compare the Mean Square Error

Using bootstrap estimation parameters and full sample estimation parameters in each testing set and calculate the mean square error of these two kinds of estimations. The results showed in figure3, figure4, figure5 and table4.

Table 4: MSE using different estimation methods

	Full Sample	10 times bootstrap	20 times bootstrap	50 times bootstrap
$\bar{MSE}^{FS}$	0.4309	0.4187	0.4190	0.4186
$\bar{MSE}^{BS}$		0.3780	0.3775	0.3772

Figure 3: The MSE of 10 times bootstrap
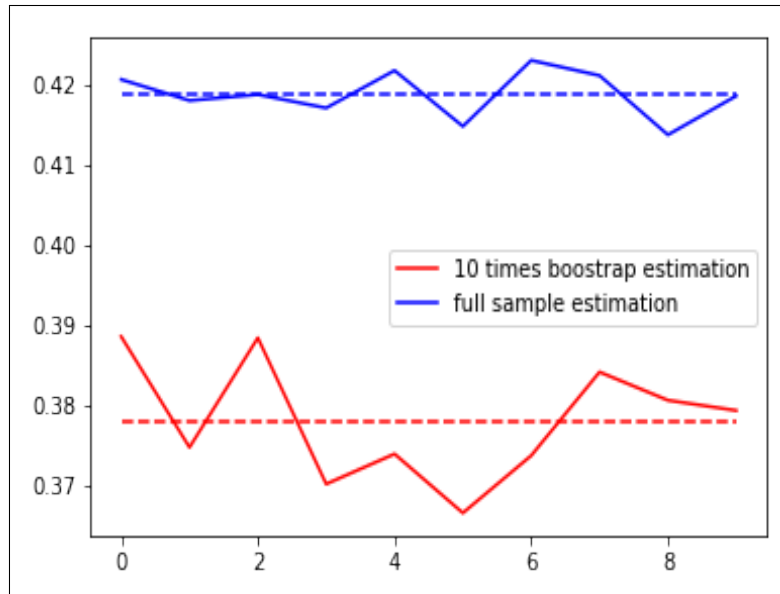

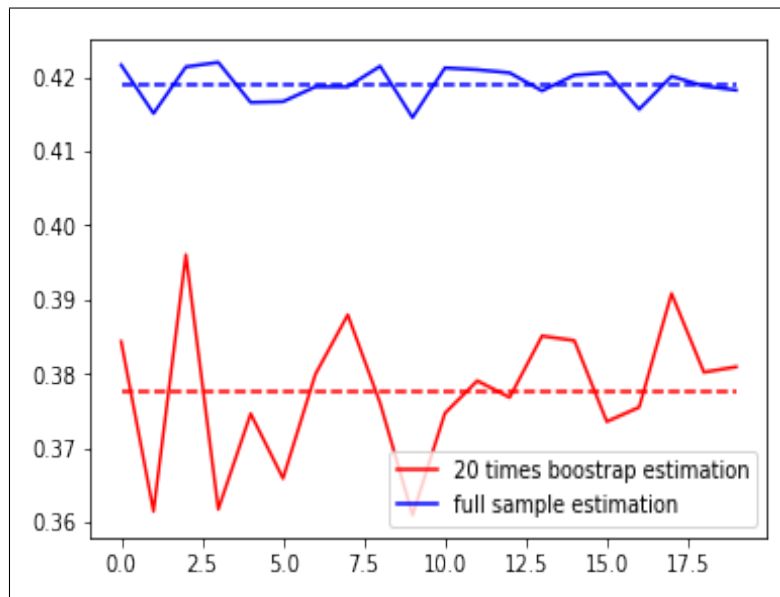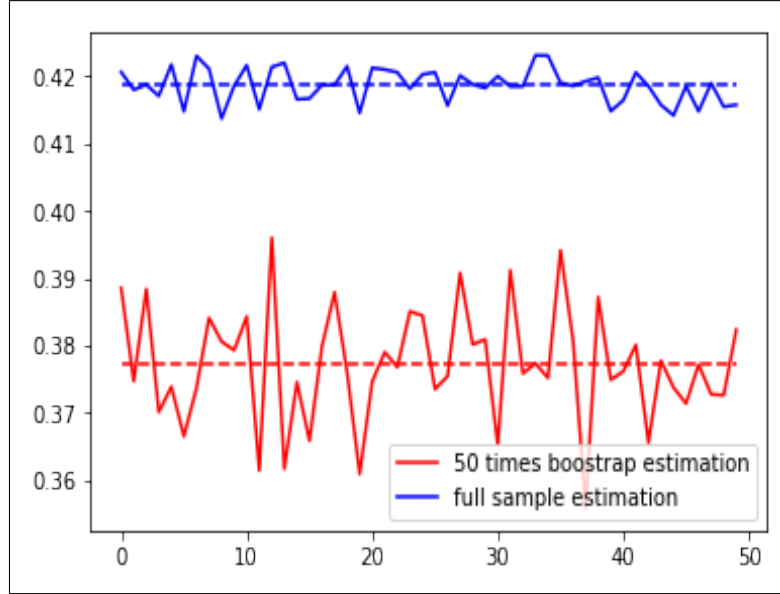**Figure 4: The MSE of 20 times bootstrap**

Figure 5: The MSE of 50 times bootstrap

From the table and figures, we can see that bootstrap estimation has smaller mean squared error than full sample estimation. Increasing bootstrap times doesn't influence mean squared error so much. So we can use the parameters get from 50 times bootstrap estimaton. θ is 0.2243 in 50 times bootstrap. We know that the more accurate impact of foreign students on high skilled labor market is that a 10% increase of immigrant will decrease native's earnings by approximately 10.99%. By using bootstrap method, I correct overfitting problems in this two stage OLS estimation.

Reference

Borjas, George J. "Immigration in high-skill labor markets: The impact of foreign students on the earnings of doctorates." In *Science and engineering careers in the United States: An analysis of markets and employment*, pp. 131-161. University of Chicago Press, 2009.