Anhui University Semester 2, 2020-2021 Final Examination
Numerical Analysis (Paper A)

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I Single-choice Questions (3 marks for each question, 15 marks in total)

1.Suppose that $f(x) \in C[a,b]$. If the range of $y = f(x)$ satisfies $y \in [a,b]$ for all $x \in [a,b]$, then $f(x)$ has ______in $[a,b]$.
(A) a fixed point (B) no fixed point (C) a unique fixed point (D) a simple root

2.The matrix $\begin{pmatrix} 4 & 1 & -1 \\ 1 & -5 & -1 \\ 2 & -1 & -6 \end{pmatrix}$ is ______.
(A) a strictly diagonally dominant matrix; (B) not a strictly diagonally dominant matrix;
(C) a singular matrix; (D) a matrix whose determinant is equal to 0.

3.If $T_n(x)$ represents Чебышев(Chebyshev) Polynomials, then $T_n(x)$ is ______for $n = 2, 3, \cdots$.
(A) $T_{n-1}(x) - 2T_{n-2}(x)$ (B) $2xT_{n-1}(x) - T_{n-2}(x)$ (C) $4xT_{n-1}(x) - 2T_{n-2}(x)$ (D) $4xT_{n-1}(x) - T_{n-2}(x)$

4.Suppose that $\{(x_k, y_k)\}_{k=0}^{n}$ are $n+1$ points, where $a = x_0 < x_1 < \cdots < x_n = b$. If the endpoints constraints of a cubic spline are $S''(a) = S''(b) = 0$, then the cubic spline is ______.
(A) clamped cubic spline (B) parabolically terminated spline
(C) natural cubic spline (D) curvature-adjusted cubic spline

5.Assume that $[a,b]$ is subdivided into M subintervals with width $h = \dfrac{b-a}{M}$. If the composite trapezoidal rule is used to approximate the integral $\displaystyle\int_a^b f(x)\mathrm{d}x$, then the error $E_T(f,h)$ is ______.
(A) $O(1)$ (B) $O(h)$ (C) $O(h^2)$ (D) $O(h^3)$

II Fill-in-the-blanks Questions (3 marks for each question, 15 marks in total)

6.According to Gaussian elimination, the triangular factorization of the matrix $\begin{pmatrix} 1 & 1 & 6 \\ -1 & 2 & 9 \\ 1 & -2 & 3 \end{pmatrix}$ is ______________________.

7.Assume that $L_{N,k}(x)$ is the Lagrange coefficient polynomial of degree N, and $x_0, x_1, \cdots, x_N$ is $N+1$ nodes, then for all $j = 0, \cdots, N$, $\displaystyle\sum_{k=0}^{N} L_{N,k}(x_j) =$ ______.

8.If $f(x) = x^2 + 1$, then the divided difference $f[1,2,3,4]$ is ___________.

9.The recurrence relation of Бернштейн(Bernstein) polynomial $B_{i,N}(t)$ is ___________.

10. The degree of precision for Simpson's rule is ___________.

III Computation Problems (10 marks for each problem; keep the forth decimal place with truncation error)

11. Consider the function $f(x) = x\mathrm{e}^{-x}$.
(a)Find the Newton-Raphson formula $p_k = g(p_{k-1})$; (b)If $p_0 = 0.4$, then find p_1, p_2, p_3, p_4 and $\lim\limits_{k \to \inf} p_k$.

12. In the following linear equation system:

$$4x - y + z = 7 \qquad 4x - 8y + z = -21 \qquad -2x + y - 5z = 15$$

(a)Starting with $P_0 = (1, 2, 2)$ and use Gauss-Seidel iteration to find P_1, P_2;
(b)Prove this Gauss-Seidel iteration is convergent.

13. Let $f(x) = \log_2(x)$,use quadratic Newton interpolation polynomial based on the nodes $x_0 = 1, x_1 = 2, x_2 = 4$ to approximate $f(3)$.

14. Find the least-squares polynomial approximation of degree 2 to the following data:

x	0	1	2	4	6
y	3	1	0	1	4

15. Use the three-point Gauss-Legendre rule to approximate $\displaystyle\int_1^5 \frac{\mathrm{d}t}{t}$ and compare the result with Simpson's rule $S(f,h)$ with h=2.

IV Proof Problems (10 marks for each question, 20 marks in total)

16. Use Heun's method to solve the initial value problem $y' = \frac{t-y}{2}, t \in [0,3]$ with $y(0) = 1$, for the step size $h = 1$.

17. Suppose that $[a,b]$ is subdivided into M subintervals $[x_k, x_{k+1}]$ of width $h = \frac{b-a}{M}$. The composite trapezoidal rule $T(f,h)$ is an approximation to the integral

$$\int_a^b f(x)\mathrm{d}x = T(f,h) + E(f,h).$$

If $f \in C^2[a,b]$, prove there exists a value $c \in (a,b)$ such that the error $E(f,h)$ has the form

$$E(f,h) = -\frac{b-a}{12} f''(c)h^2 = O(h^2).$$