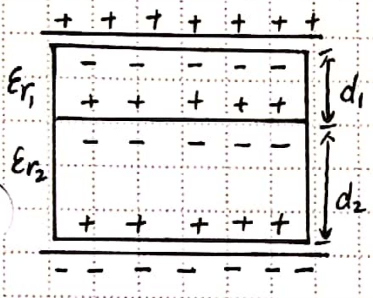
电位移矢量 $\vec{D}=\varepsilon_0\varepsilon_r\vec{E}=\varepsilon\vec{E}$ (均匀各向同性介质)

有介质时的高斯定理 $\oint_S \vec{D}\cdot d\vec{s}=\sum_i Q_{0i}$

$$\vec{D}=\begin{cases}\vec{P}+\varepsilon_0\vec{E} & \text{(任何介质)}\\ \varepsilon\vec{E} & \text{(均匀介质)}\end{cases}$$

~~$\oint_S D=\sigma$~~ 在上一例中.
$\oint_S \vec{D}\cdot d\vec{s}=Q_0$
$DS=\sigma_0 S$
$D=\sigma_0$

例: 求 σ_1', σ_2'

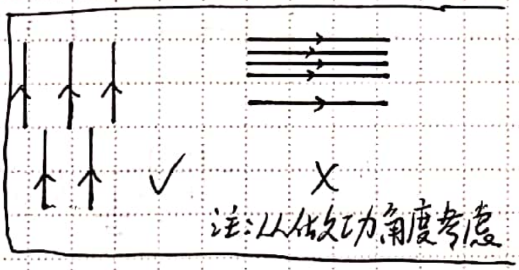


$$\sigma_1'=\frac{\varepsilon_{r_1}-1}{\varepsilon_{r_1}}\sigma_0$$

$$\sigma_2'=\frac{\varepsilon_{r_2}-1}{\varepsilon_{r_2}}\sigma_0$$

$$D=\varepsilon\vec{E}=\varepsilon_0\varepsilon_r\cdot\frac{E_0}{\varepsilon_r}=\varepsilon_0E_0$$

上下E不连续, D是一样的.



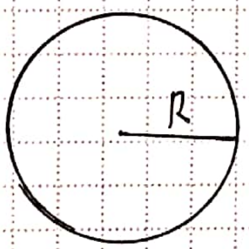
2.2 电容和电容器

一、电容

单位 $1F=1C/V$
$1\mu F=10^{-6}F$
$1pF=10^{-12}F$

$C=\frac{Q}{\varphi}$, 选 $\varphi_\infty=0$

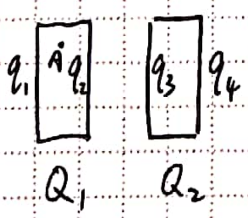
(φ也可写成V)



$$\varphi=\begin{cases}k\frac{Q}{R} & (r\le R)\\ k\frac{Q}{r} & (r>R)\end{cases}$$

孤立导体球电容 $C=\frac{Q}{V}=\frac{Q}{k\frac{Q}{R}}=\frac{R}{k}=4\pi\varepsilon_0R$

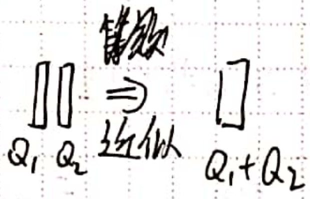
电容器电容 $C=\frac{Q}{\varphi_A-\varphi_B}=\frac{Q}{U}$ ← 指的是$|q_2|$ or $|q_3|$



法一:
$$\begin{cases}q_1+q_2=Q_1\\ q_3+q_4=Q_2\\ q_2+q_3=0 \rightarrow \text{发射电场线数目=接收电场线数}\\ q_1=q_4 \rightarrow \text{A点}E=0\end{cases}$$

$$\Rightarrow\begin{cases}q_1=q_4=\frac{Q_1+Q_2}{2}\\ q_2=-q_3=\frac{Q_1-Q_2}{}\end{cases}$$

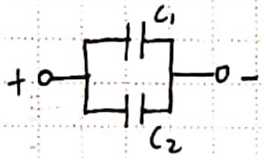
法二：



故 $q_1=q_4=\frac{Q_1+Q_2}{2}$

最外的面的电荷量一定是总电荷的一半。

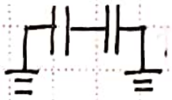
电容器的并联



$C=C_1+C_2$

证明：$C_{总}U=Q_{总}=q_1+q_2=C_1U+C_2U$
$=(C_1+C_2)U$

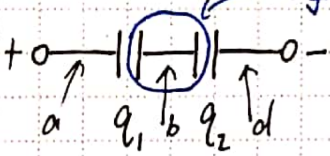
注：



也是并联

电容器的串联

净电荷为0，才可看作电容器的串联



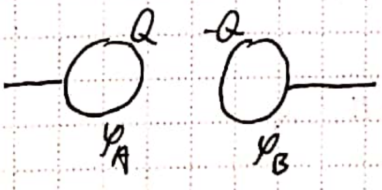
仅 $q_1=q_2$ 时，才能看作一个电容

证：若 $q_1=q_2=q$

$U_{ad}=U_{ab}+U_{bd}$

$\frac{q}{C_{总}}=\frac{q_1}{C_1}+\frac{q_2}{C_2}$

$\therefore C_{总}=\frac{C_1C_2}{C_1+C_2}$



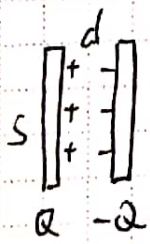
$C_1=\frac{Q}{\varphi_A}$

$C_2=-\frac{Q}{\varphi_B}$

$C=\frac{Q}{\varphi_A-\varphi_B}$

※ C与电荷量无关

一、平行板电容器



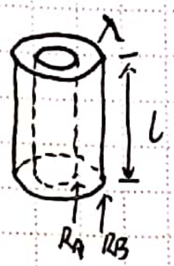
$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 S}$

$U=Ed=\frac{Qd}{\varepsilon_0 S}$

$C=\frac{Q}{U}=\varepsilon_0\frac{S}{d}$

有电介质 $C=\frac{\varepsilon_0\varepsilon_r S}{d}=\frac{\varepsilon_r S}{4\pi k d}$

二. 圆柱形电容器



$$E=\frac{\lambda}{2\pi\varepsilon_0 r}\quad (R_A<r<R_B)$$

$$U=\int_{R_A}^{R_B}\frac{\lambda}{2\pi\varepsilon_0}\frac{1}{r}dr=\frac{\lambda}{2\pi\varepsilon_0}\ln r\Big|_{R_A}^{R_B}=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_B}{R_A}$$

$$C=\frac{Q}{U}=\frac{\lambda L}{U}=\frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_B}{R_A}}=\frac{2\pi\varepsilon_0 l}{\ln\frac{R_B}{R_A}}$$

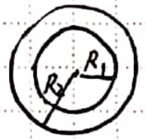
$$\boxed{\lim_{x\to 0}\ln(1+x)\sim x}$$

$$\ln\frac{R_B}{R_A}\ \xlongequal{R_B-R_A\ll R_A}\ \frac{R_B-R_A}{R_A}$$

$$C\approx\frac{2\pi\varepsilon_0 L R_A}{R_B-R_A}=\frac{\varepsilon_0 S}{d}$$

三. 球形电容器

一、



$$\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\cdot\vec{e}_r\quad (R_1<r<R_2)$$

$$U=\int_l \vec{E}\cdot d\vec{l}=\frac{Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2}=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

二、

$$\varphi=\begin{cases}k\frac{Q}{R}, & r\le R\\ k\frac{Q}{r}, & r>R\end{cases}$$

$$U=kQ\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

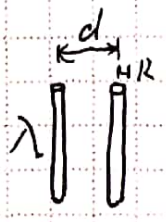
$$\therefore C=4\pi\varepsilon_0\frac{R_1R_2}{R_1-R_2}$$

$R_2-R_1\ll R_1$

$$C=\frac{\varepsilon_0 4\pi R_1R_2}{R_2-R_1}\approx\frac{\varepsilon_0 S}{d}$$

$R_2\to\infty$, $C=4\pi\varepsilon_0 R_1$ ← 孤立导体球电容

孤立导体电容可看作另一极板在无穷远处的电容器



$$E=\frac{\lambda}{2\pi\varepsilon r}+\frac{\lambda}{2\pi\varepsilon(d-r)}$$

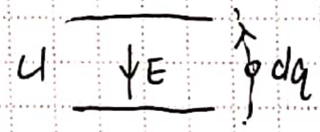
$$U=\int_R^{d-R}\left(\frac{\lambda}{2\pi\varepsilon r}+\frac{\lambda}{2\pi\varepsilon(d-r)}\right)dr=\frac{\lambda}{2\pi\varepsilon}\ln\frac{d-R}{R}-\frac{\lambda}{2\pi\varepsilon}\ln\frac{R}{d-R}\approx\frac{\lambda}{\pi\varepsilon_0}\ln\frac{d}{R}$$

法二：$E_1=\frac{\lambda}{2\pi\varepsilon r}$

$$U=2\int_R^{d-R}\frac{\lambda}{2\pi\varepsilon r}dr=\frac{\lambda}{\pi\varepsilon}\ln\frac{d-R}{R}\approx\frac{\lambda}{\pi\varepsilon_0}\ln\frac{d}{R}$$

$$C=\frac{\pi\varepsilon_0}{\ln\frac{d}{R}}$$

电场能量



$$dW=dq\cdot E\cdot d=dq\cdot U=dq\cdot\frac{q}{C}$$

$$W=\frac{1}{C}\int_0^Q q\,dq=\frac{Q^2}{2C}=\frac{1}{2}QU=\frac{1}{2}CU^2$$

$C=\frac{Q}{U}$

$$W_e=\frac{1}{2}CU^2=\frac{1}{2}\frac{\varepsilon S}{d}(Ed)^2=\boxed{\frac{1}{2}\varepsilon E^2}Sd$$

$$w_e=\underline{\frac{1}{2}\varepsilon\vec{E}^2}=\underline{\frac{1}{2}\vec{E}\cdot\vec{D}}$$

电场能量密度　只适用于各向同性电介质　　适用于任意电介质

物理意义：电场是一种物质，它具有能量

$$W_e=\int_V w_e\,dV=\int_V\frac{1}{2}\varepsilon E^2dV$$

体积分

例：球形电容器的内外半径分别为R_1和R_2，所带电荷为$\pm Q$，若在两球壳间充以电容率为ε的电介质，问此电容器贮存的电场能量为多少

解：$\vec{E}=\frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\vec{e}_r$

$$w_e=\frac{1}{2}\varepsilon E^2=\frac{Q^2}{32\pi^2\varepsilon r^4}$$

$$\cancel{dW_e}\ dW_e=w_e\,dV=\frac{Q^2}{8\pi\varepsilon r^2}dr$$

$$W_e=\int dW_e=\frac{Q^2}{8\pi\varepsilon}\int_{R_1}^{R_2}\frac{dr}{r^2}=\frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

法二：$U=kQ\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$．$W_e=\frac{1}{2}QU=\frac{k}{2}Q^2\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$