EL2700 - Model Predictive Control
Assignment 3 : Linear Quadratic Regulator

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1 PART I : LQR Implementation

Q1: A key question in LQR design is how to choose the weight matrices Q, and R to ensure desired performance specification. You are encouraged to try different weight matrices and comment on the resulting performance without disturbance.

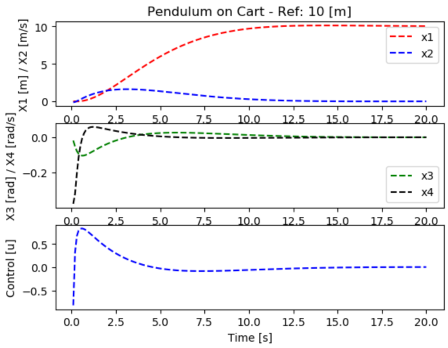
A1: According to the Bryson's rule, we normalize the weight matrix Q and R. There are different methods to determine z_{max} and u_{max}. One is to use the results from *Assignment 1.* We can get the max values of all states and control input from the output figure conveniently. Another method is to combine the design criteria and the experience of *Assignment 1.*

$$[\bar{Q}]_{ii} = \frac{1}{\left(z_{\max}^{(i)}\right)^2}, \quad [\bar{R}]_{ii} = \frac{1}{\left(u_{\max}^{(i)}\right)^2} \tag{1}$$

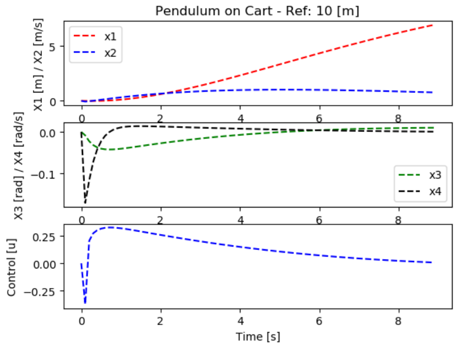
In order to ensure desired performance specification, we set Q and R matrices as follows:

$$Q = \begin{bmatrix} \frac{1}{100} & 0 & 0 & 0 \\ 0 & \frac{1}{25} & 0 & 0 \\ 0 & 0 & \frac{1}{0.03} & 0 \\ 0 & 0 & 0 & \frac{1}{0.5} \end{bmatrix}, R = \rho \times \frac{1}{4} \tag{2}$$

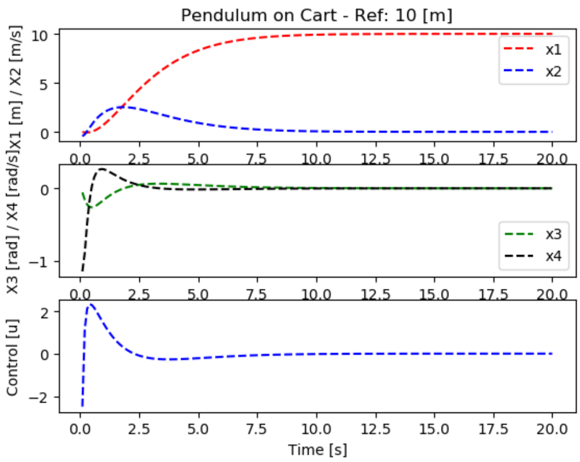
The results are shown in Figure.1. From the Figure.1a, we can see that it finally reaches 10 meters position. In 10 seconds, it has reached 9.72m. The pendulum has been kept within 5.67°around the vertical position. Under this parameters setting, three criteria are achieved.



(a) Q: diag[0.01, 0.04, 33.33, 2], R: 0.25



(b) Q: diag[0.01, 0.04, 333.33, 20], R: 0.25



(c) Q: diag[0.1, 0.4, 33.33, 2], R: 0.25

Figure 1: Simulation with different Q values

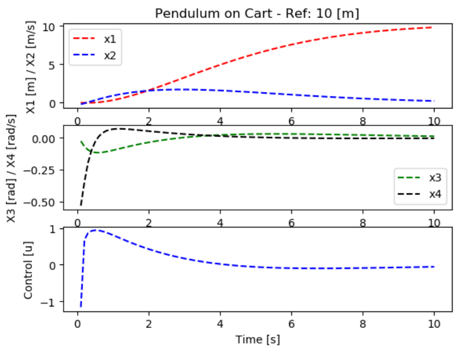
Then we modify the weights for x_3 and x_4 which are related with degrees. The result are shown in Figure.1b. Apparently, the degree range is narrower. The maximum degree becomes 2.36°. This meets our expectation because larger weights means that we try to guarantee the degree is in range first.

Another trial is enlarging weights on x_1 and x_2. As shown in Figure.1c, the convergence speed is faster. The convergence time decreases from 11s in Figure.1a to 9.25s in Figure.1c. Of course, the control input range is wider.

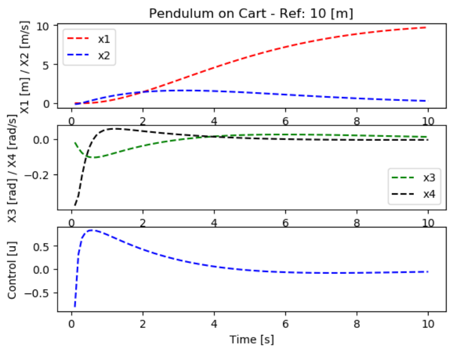
In the end, we can let some elements in Q matrix equal zero. It means we do not apply requirements on these states.

Q2: Add a small constant disturbance to mimic the presence of horizontal wind force and comment on the tracking ability of the LQR controller.

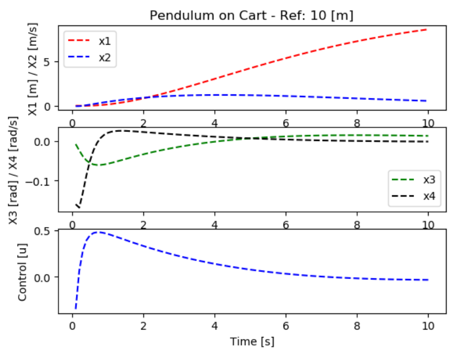
A2: The comparison result is shown in Figure.2. From Figure.2, we notice that disturbance causes a stationary error which is about 0.5m. Larger R value implies narrower range for the control in put.



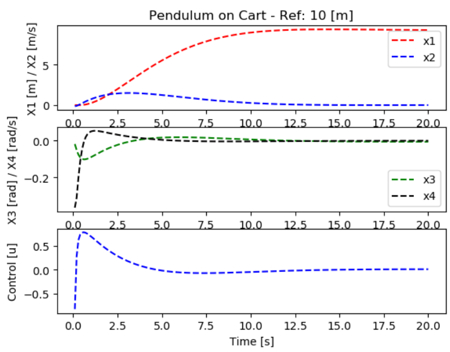
(a) $\rho = 0.1$, no disturbance



(b) $\rho = 1$, no disturbance



(c) $\rho = 10$, no disturbance



(d) $\rho = 1$, disturbance $w = 0.01$

Figure 2: Simulation with disturbance and different R values

2 PART II : Adding integral action

Q3: Form the augmented system matrices from (7) by creating the system matrices in `set_augmented_discrete_system` and creating its dy-

namics in `pendulum_augmented_dynamics`. Adjust the weight matrices Q (with q_i) and R, and design the controller (8) in `lqr_ff_fb_integrator` that meets the desired performance specification. Motivate your choice of the weight matrices. Comment if the controller is eliminating the step disturbance.

A3: After completing the codes for part2, we ran the simulation under parameter setting the same as Figure.1a. The fifth element in Q matrix is set as 1. And we set the disturbance w as 1 rather than 0.01 in order to check if the controller is able to eliminate the disturbance. Result picture is shown in Figure.3.

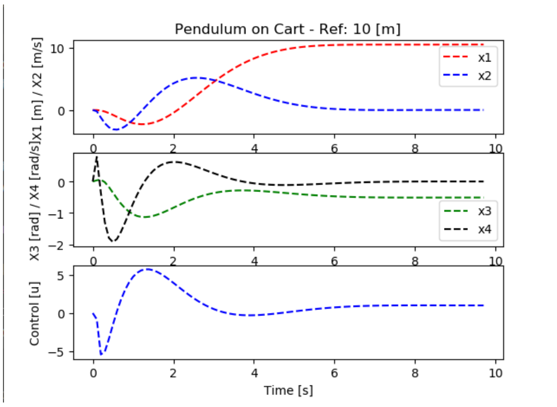
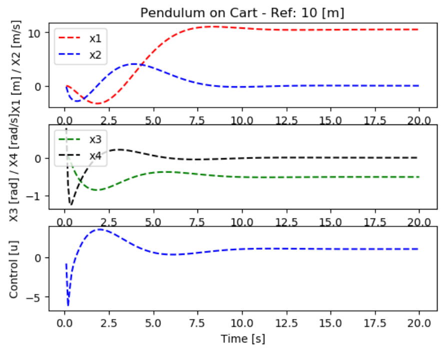


Figure 3: Q: diag[0.01, 0.04, 33.33, 2, 1], R: 0.25, w = 1

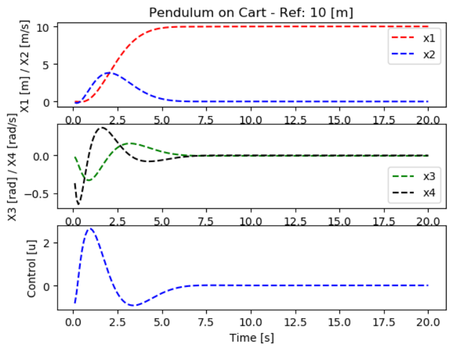
We can see that the controller with integral action can eliminate the disturbance. But it needs more control input. And the degree criteria is not met. The largest degree reaches 64.55°. And a stationary error of x_3 can been seen due to the strong wind.

Therefore, we set q_3 as $20 \times \frac{1}{0.03}$ instead to improve the system response. Modified model's simulation result is shown in Figure.4a. The degree becomes smaller but still cannot fulfill the requirement for the degree. Intuitively, the strong disturbance(wind) has changed the equilibrium point far away from original [10,0,0,0].

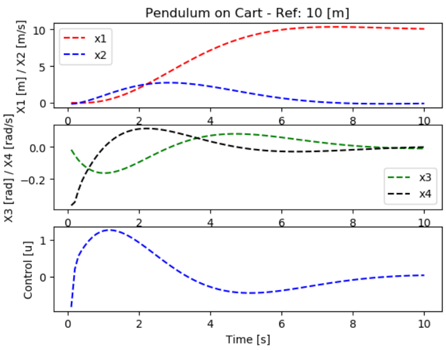
Therefore, we change the disturbance back to 0.01. The initial Q matrix is firstly used, which is shown in Figure.4b. The degree is still large and the control is too much. We apply two methods to improve the performance. The first method is to increase q_3. The second method is to increase R 10 times. They are shown in Figure.4c and Figure.4d. By increasing q_3, we can get really good result. And increasing R alone can not give us content result.



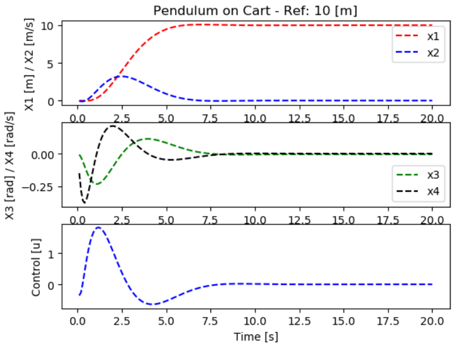
(a) Q: diag[0.01, 0.04, 666.6, 2, 1], R: 0.25, w = 1



(b) Q: diag[0.01, 0.04, 33.33, 2, 1], R: 0.25, w = 0.01



(c) Q: diag[0.01, 0.04, 666.6, 2, 1], R: 0.25, w = 0.01



(d) Q: diag[0.1, 0.4, 33.33, 2, 1], R: 2.5, w = 0.01

Figure 4: Simulation in Part 2

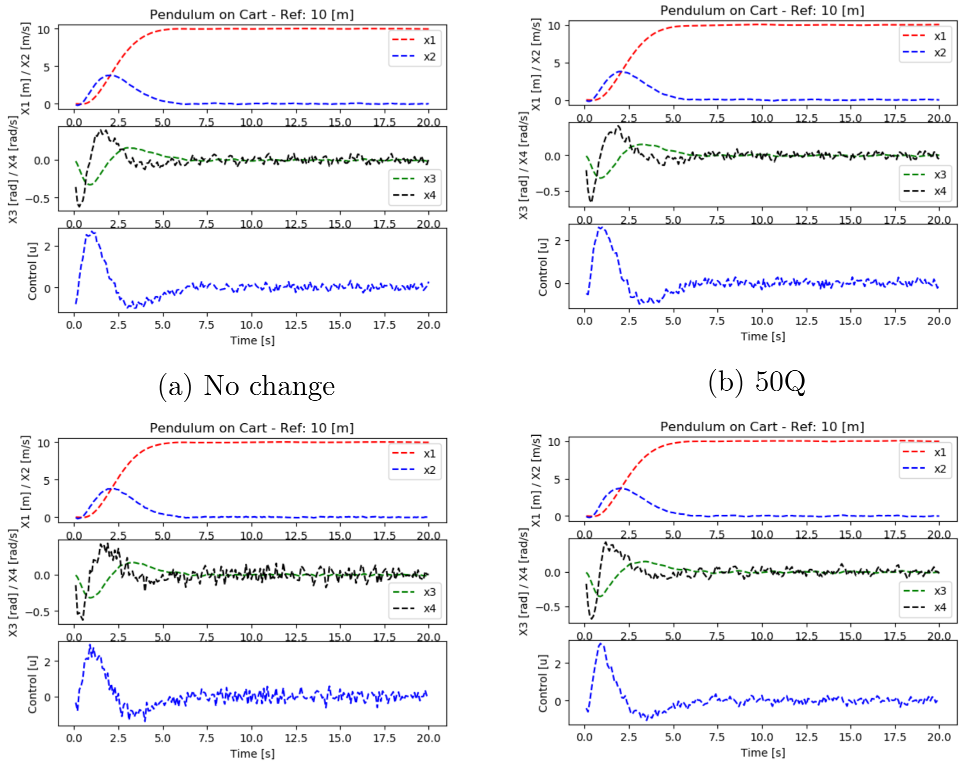
3 Part III : Output feedback controller

Q4: Compare the performance of the output feedback controller to the state feedback one (from Part II). Try different feasible choices of the matrix C_p and values for Q_p and R_n.

A4: We do not change the parameters setting in Part III first. The C_p, Q_p and R_n are

$$C_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, Q_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

Next, we increase Q_p matrix by 50 times. And then we increase some covariance in Q_p. In end, we increase the R_n. We suppose covariance matrix Q_p represents the process noise. And R_n represents measurement noise. Results are shown in Figure.5.



(a) No change

(b) 50Q

(c) Q: diag[10, 1000, 10, 1000]

(d) 1000R

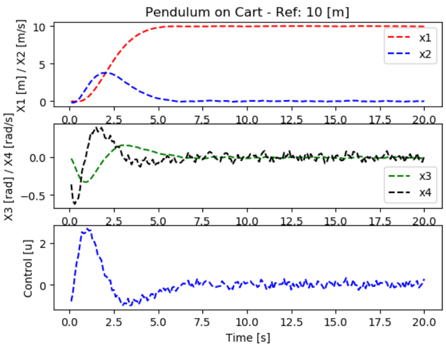
Figure 5: Different Q_p and R_n in Part 3

Compared with Figure.4b, the performance is similar. But after several seconds, the output feedback controller input becomes shaky. It is because the process noise and measurement noise. The shaky performance of control can make the controller design difficult in practice.

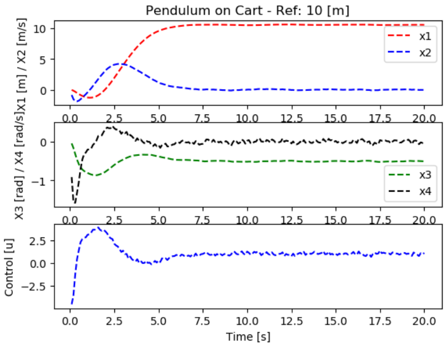
Q5: Comment on the robustness of the output feedback controller.

A5: In order to observe the robustness of the output feedback controller, we increase the disturbance to 1 and 10. Results are shown in Figure.6. We use default C, Q_p, R_n matrices. When considering the distance measurement, we believe the robustness of the output feedback controller is good, especially when disturbance is large. Although the cart needs to go negatively, it can still converge to the correct position.

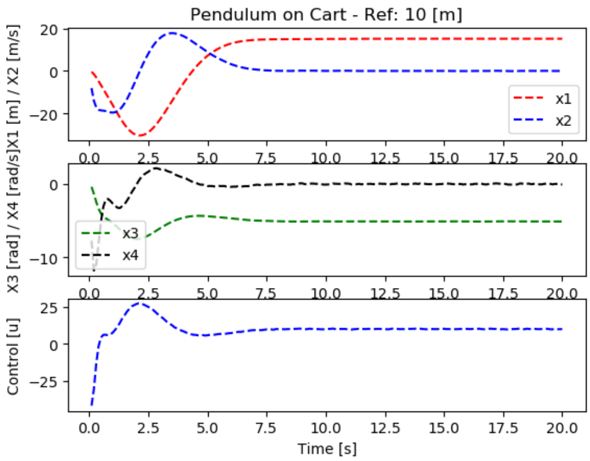
But the angle control is not robust. We believe that we need to adjust the weight for x_3 in order to get satisfactory results with disturbance.



(a) w = 0.01



(b) w = 1



(c) w = 10

Figure 6: Different disturbance in Part 3