Matrix Analysis Homework 8

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For A symmetric $n \times n$ matrix, we assume the following ordering on its eigenvalues:

$$\lambda_1(A) \geq \lambda_2(A) \geq \cdots \geq \lambda_n(A)$$

Problem 1 : *(Finish the proof of the second part of the Weyl-II theorem)*
Let A,B be $n \times n$ symmetric matrices. Prove that $\lambda_j(A) + \lambda_k(B) \leq \lambda_{j+k-n}(A+B), \forall i, j = 1, \cdots, n$.

Proof.
Done. ☐

Problem 2 :*(Finish the proof of the second part of the Interlacing-II theorem)*
Let A be $n \times n$ symmetric matrix. Let B be an $r \times r$ principal submatrix of A, obtained by deleting rows/columns $i_1, \cdots, i_{n-r}$. Show that $\lambda_k(B) \leq \lambda_k(A), \forall k = 1, \cdots, r$.

Proof. Done. ☐

Problem 3 :*(Finish the proof of the second part of the variational characterization of sums of eigenvalues)*
Let A be $n \times n$ symmetric matrix. Prove that $\sum_{i=n-k+1}^{n} \lambda_i(A) = \min_{U \in \mathbb{R}^{n \times k}, U^T U = I_k} trace(U^T AU)$.

Proof. Done. ☐

Problem 4 :
Let A be an $n \times n$ symmetric matrix. Prove that $\lambda_n(A) \leq a_{ii} \leq \lambda_1(A), \forall i = 1, \cdots, n$.

Proof. Done. ☐

Problem 5 :
Let A, B be $n \times n$ symmetric matrices. Prove that $\sum_{i=1}^{k}(\lambda_i(A) + \lambda_i(B)) \geq \sum_{i=1}^{k} \lambda_i(A+B)$, $\forall k = 1, \cdots, n$.
Hint: Use the variational characterization of the sum of eigenvalues.

Proof.
Done. ☐