Homework 1

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Problem 1 (4/10, *If G is an abelian group, then $End(G)$ is a ring*):

Let G be an abelian group with group operation $*$. For $\phi, \psi \in End(G)$, consider the definition of $\phi + \psi$ and $\phi\psi$ that we gave in the class. Verify that $End(G)$ is a ring under these two operations, i.e., check that all the axioms of the ring structure are true. For example, verify that for $\phi, \psi, \chi \in End(G)$, we have that $\phi(\psi + \chi) = \phi\psi + \phi\chi$.

Proof.

$$\forall \phi, \psi, \chi \in End(G)$$

We have that,

$$\begin{aligned} [(\phi + \psi) + \chi](g) &= [\phi + \psi](g) * \chi(g) \\ &= [\phi(g) * \psi(g)] * \chi(g) \end{aligned}$$

G is an abelian group.

$$\begin{aligned} [\phi(g) * \psi(g)] * \chi(g) &= \phi(g) * [\psi(g) * \chi(g)] \\ &= [\phi + (\psi + \chi)](g) \end{aligned}$$

So,

$$(\phi + \psi) + \chi = \phi + (\psi + \chi)$$

□

Problem 2 (4/10, *Behavior of identity element and inverses under group homomorphisms*):

Let G, H be groups with group operations $*_G$, $*_H$ and identity elements e_G, e_H respectively. Let $\phi : G \to H$ be a group homomorphism. Show that $\phi(e_G) = e_H$ and that $\forall g \in G$ we have that $\phi(g^{-1}) = (\phi(g))^{-1}$.

Proof. Here goes your anwser. □

Problem 3 (2/10, *Group isomorphism is a transitive relation*):

Let G, H, K be groups such that G is isomorphic to H, and H is isomorphic to K. Show that G is isomorphic to K.

Proof. Here goes your anwser. $\square$