# DEVELOPMENT OF A NONLINEAR FINITE ELEMENT PROGRAM FOR SEISMIC ANALYSIS OF SIMPLE STRUCTURES

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## ABSTRACT

Advanced, nonlinear modelling is becoming an accepted tool to predict the plastic response of structures. This modelling is typically carried out using software purchased from a third party, that are powerful and proprietary and cannot be significantly modified by those performing the analysis. A framework for nonlinear, dynamic finite element analysis has been developed in the C++ programming language. This framework, called Enoch, was designed to be easily extended, enabling the modeling of new types of structures and complex structural behaviour. A shear wall was modelled with a plastic hinge at the base to explore the potential of the Enoch framework. The model was analysed with and without the nonlinear hinge enabled. The results of the two analyses are compared. Enoch was shown to properly predict the response of both linear and non-linear systems. The plastic hinge element behaved as expected during validation. The analysis of a shear wall demonstrated the difference between linear and non-linear structures. Changing the plastic hinge model and adding new elements is simple and straightforward. The accessibility of the code allows engineers to develop their own models and check every aspect of the computation for themselves.

## 1 INTRODUCTION

Designing buildings to behave plastically during earthquakes is a currently accepted method to reduce the likelihood of collapse. Formation of plastic hinges limits the forces in members, which will prevent unpredictable failure in these members. Plastic behaviour is highly complex and not easily predicted by simplified methods. As a result advanced, nonlinear modelling is becoming an accepted tool to predict the plastic response of structures.

Nonlinear modeling is typically carried out using software purchased from a third party, such as SAP2000 or Raumoko. While these programs are powerful, they are proprietary and cannot be significantly modified by those performing the analysis. In many instances an engineer may wish to change the behaviour of an element to more closely reflect results obtained from experimentation or field observation. With commercial software this can be a complicated process requiring interaction with the software vendor, which increases the cost and time to completion of the project.

An open framework that can be easily extended to model new materials and execute new methods of analysis would give engineers the flexibility they need to model complex, nonlinear behaviour. In order to achieve this goal a three-stage process was implemented. First a finite element method (FEM) program, called Enoch, was developed and validated in the dynamic linear elastic case against theoretical response. Secondly a non-linear plastic hinge element was developed. The hinge was validated against NonLin, a program for non-linear analysis of simple structures developed by Finley A. Charney, for the Federal Emergency Management Agency.

Finally a multi story, multi degree of freedom structure was modeled, and subjected to a push pull analysis.

## 2 PROGRAM DESCRIPTION

Enoch is an object-oriented framework written in C++. It provides an application programming interface (API) that can be used within other programs to represent and analyze finite element models. There is no user interface built into Enoch—it is up to the application programmer to gather data from users and pass it on to the Enoch code.

A structural model consists of an array of Node objects and an array of Element objects. The Node objects store position, displacement, velocity, acceleration and mass values for an arbitrary number of degrees of freedom. The Element class is associated with a set of Node classes and defines an interface for accessing an element's stiffness matrix.

A structural model is passed to an Enoch object that performs a Newmark-Beta Time History Analysis, as described by Filiatrault (1998). Loads are specified for each degree of freedom at each time step, allowing arbitrary, complex load conditions. The damping matrix is generated using Rayleigh's model of damping.

If all the elements are linear, the stiffness and damping matrices are generated at the beginning of the analysis and remain constant throughout. Structural models incorporating non-linear elements are included in the analysis by updating the global stiffness matrix at each time step. The linear elements are assembled into a global linear stiffness matrix, which remains constant throughout the time-history analysis. At each time step, the tangent stiffness matrix of each nonlinear element is acquired through the interface defined in the Element class and added to the global stiffness matrix.

Non-linear elements are added by creating a new class derived from the Element class. When an Enoch object requests the element's stiffness matrix, any computation to generate the tangent stiffness matrix may be performed.

The abstraction of elements allows many different elements, both linear and non-linear, to be included in the same analysis. Since the time-history analysis interacts with each element through the same interface, new elements can be developed without changing the analysis code. Using the same interface, it is also possible to develop a new analysis module that uses the existing elements.

## 3 ELEMENT TYPES

Currently Enoch contains only two element types—a linear elastic beam and a non-linear plastic hinge. These elements are of primary importance in the modeling of a shear wall and were therefore the first elements to be created. Enoch is not limited to these types, and elements programmed to interact with Enoch in the way described above can be added easily

### 3.1 Linear Beam

A two-dimensional beam element was implemented. The beam spans two nodes, each of which has three degrees of freedom and only accounts for bending deformations. Each element is describe by its area, Young's modulus, moment of inertia and length. The length is calculated from the position of the endpoints.

The beam element is linear. It calculates a stiffness matrix when it is generated and does not make any subsequent adjustments.

### 3.2 Non-Linear Hinge

To capture non-linear behaviour a plastic hinge that exhibits simple bi-linear response was developed. A more complex plastic hinge was also developed. Thought the more advance hinge would better model the response of a yielding shear wall, the simpler element was used, since it corresponds to the model used by NonLin.

The hinge is fully defined by an initial stiffness (k1), a yield moment (My) and a plastic stiffness (k2), as shown in Figure 1. It is important to note that the element itself converts the yield moment into an equivalent yield rotation, so that it only needs to know the rotation of the node to which it is attached, and not the loads being applied. Up until the yield rotation is reached, the hinge behaves elastically with a stiffness of k1. Once the yield rotation is exceeded, the hinge behaves with a stiffness of k2. When the loading is reversed, the hinge immediately responds with a stiffness of k1, which leads to an unrecovered plastic deformation.

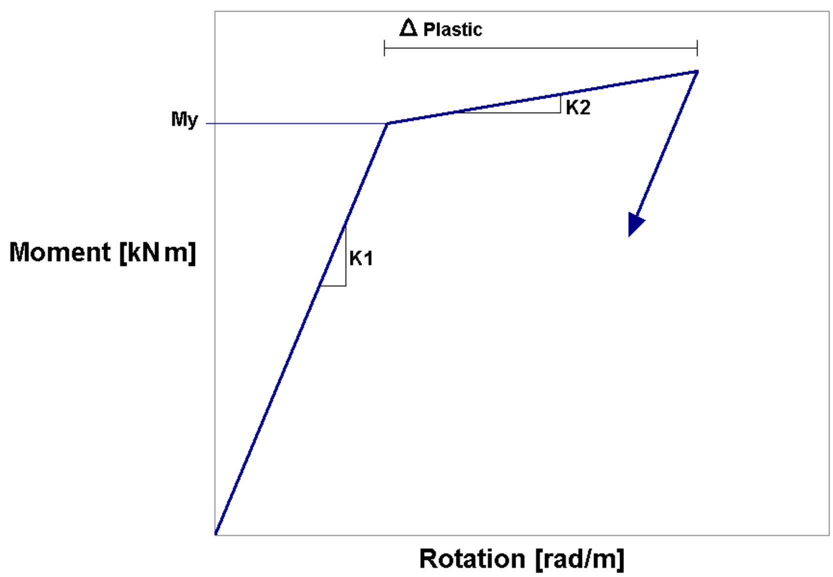


**Figure 1 - Plastic Hinge Behaviour**

As mentioned above, validation of the element response in Enoch was critical. To mimic the nonlinear behaviour of Nonlin's plasticity, the hinge was given a constant range of elastic response. This means that as the element rotates plastically, both the positive and the negative yield points increase by an amount equal to the plastic deformation.

## 4 VALIDATION

Validation was carried out in simple scenarios where the response could be easily compared to theoretical or accepted responses. The response of linear and nonlinear single degree of freedom systems was tested.

### 4.1 Linear, Dynamic Behaviour – Linear Beam

The response of a system composed of a mass at the end of a beam was checked against the theoretical response of a single degree of freedom. A constant force was instantaneously applied to the free end of the beam, which showed both the dynamic and static response of the system. Properties such as the stiffness, length, and damping of the member, its associated mass and the applied force were altered to validate the program for a variety of conditions. Figure 2 shows the response calculated by Enoch, which was identical to the theoretical response, which is described by Equation 1. Table 1 shows the system properties used for this validation.

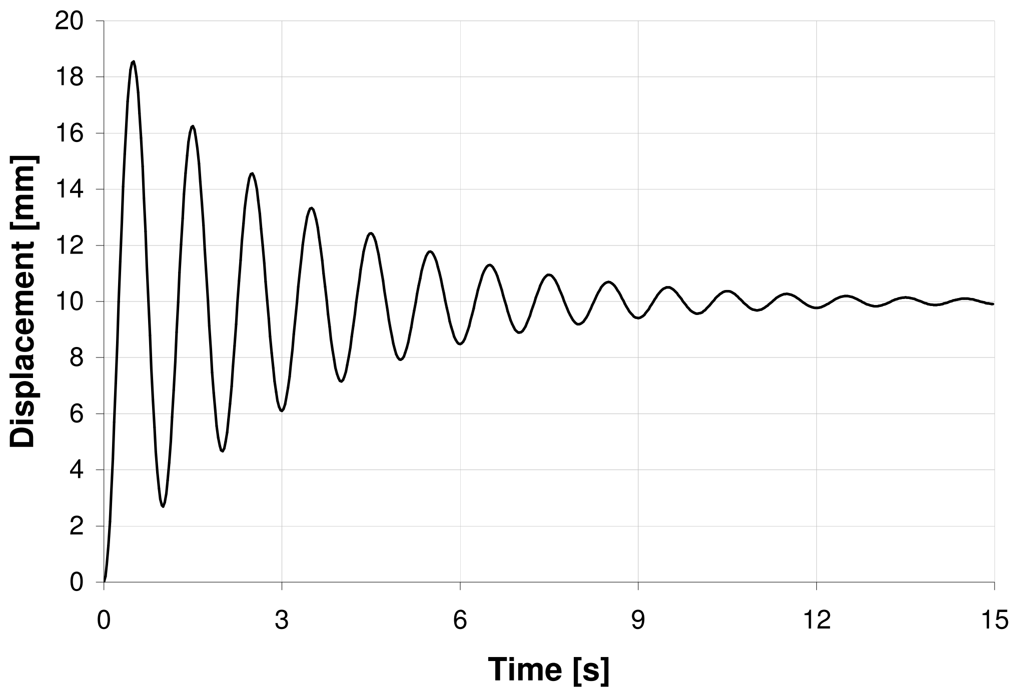


**Figure 2 - Linear, Dynamic Response**

| Mass [N s^2 / mm] | Length [mm] | E [kN/m^2] | I [10^9 mm^4] | K [N/mm] | ξ [% crit.] | ω [rad/s] | T [s] |
|---|---|---|---|---|---|---|---|
| 1.00 | 4000 | 200 | 4.213 | 39.5 | 5.0 | 6.28 | 1 |

**Table 1**

$$x(t) = \frac{F}{k}\left(1 - e^{-\xi\omega t}\cos(\omega t)\right)$$

**Equation 1**

Initially the system did not respond as desired, so the time step had to be lowered. For the range of stiffnesses and masses that were used we achieved convergence with a time step of approximately 0.05s. The ranges that were used were those that represented periods and damping in the range considered important for buildings. For systems with larger periods convergence could likely be achieved with a larger time step, while a smaller time step might be needed for systems with smaller periods. For most subsequent testing, it was decided to use a time step of 0.02s or smaller in order to assure adequate response of the system.

### 4.2 Nonlinear, Dynamic Behaviour – Non-Linear Hinge

The non-linear hinge element was validated in two ways. It was first loaded monotonically to ensure that it returned proper stiffness values depending on its current rotation. It behaved as anticipated.

Subsequently the hinge was subjected to a cyclic load, composed of four complete cycles of a sin wave, where the maximum moment was twice the initial yield moment of the hinge. An equivalent system was input into NonLin. The time history response is shown in Figure 3.

The response from Enoch and NonLin are very similar in overall behaviour. The time at which the system begins to yield is identical in both cases. There is some difference in peak deflections, but this could also be due to differences in the method used to deal with damping.

Figure 4 presents the hysteresis loop of the system. The energy dissipation predicted by Enoch is the same as that predicted by Nonlin.

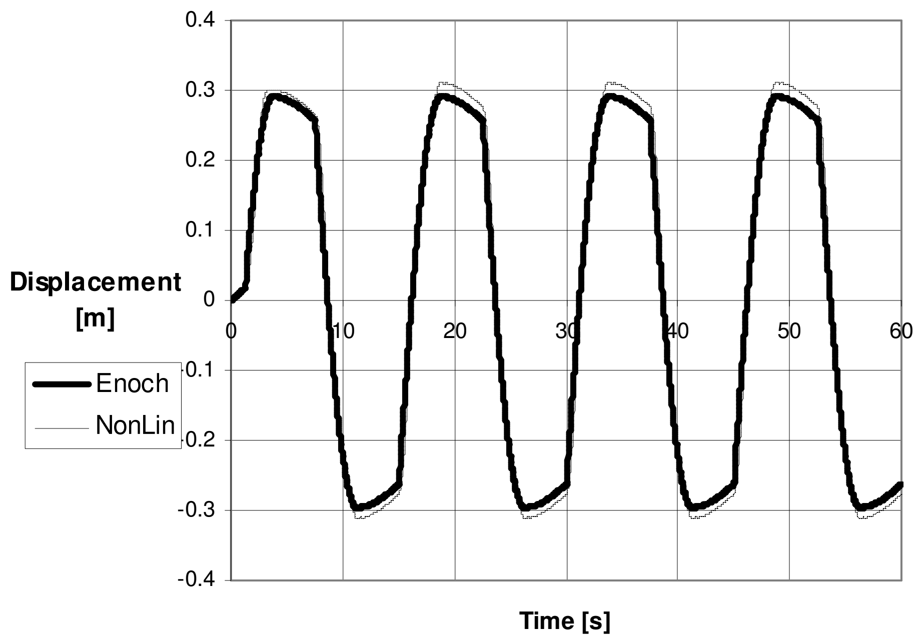


**Figure 3 - Time history response of Enoch and NonLin**

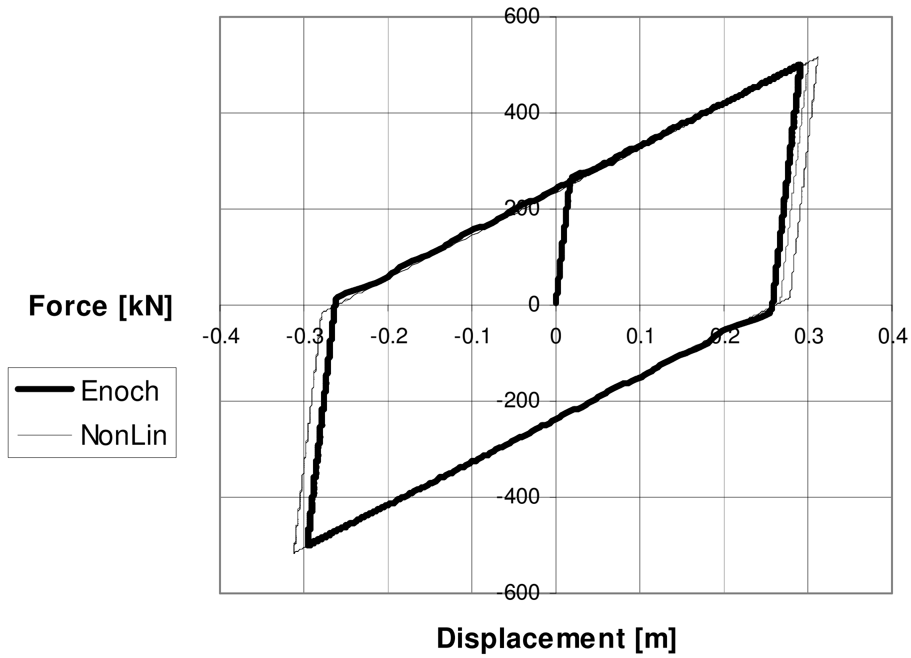


**Figure 4 - Force deflection response of Enoch and NonLin**

## 5 SHEAR WALL

To investigate the behaviour of multi-degree of freedom structure, a shear wall was modeled. The wall was idealized as three beams stacked vertically, as shown in Figure 5. The vertical and horizontal displacements of the base of the wall were fixed, which the rotational degree of freedom was connected to a fixed node through a plastic hinge.

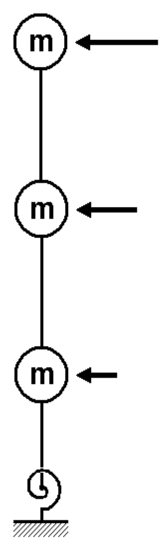


**Figure 5 - Shear Wall Model**

The wall was a reinforced concrete shear wall. To determine section properties, a representative wall was modeled in Response 2000 (Bentz). The wall was 300mm thick, and 6000mm wide, with longitudinal reinforcement all along the wall but concentrated at either end. Stirrups enclosed the longitudinal reinforcement in six equally sized hoops. Properties determined by response 2000 were as follows:

| | |
|---|---|
| A [mm^2] | 1800000 |
| I [10^9 mm^4] | 1.67 |
| E [MPa] | 30 000 |

The wall was modeled with beam elements that only account from bending deformation of the wall. Though this is not the best way to model a shear wall, is adequate to demonstrate the difference between the plastic response of the hinge and the elastic response of the wall. The nodes were placed 5 m apart, which gave the wall an overall height of 15 m.

The plastic hinge was used to only account for the plastic deformations. It had a high initial stiffness of 5000000 kN*m/rad to ensure that the elastic deformations were primarily those of the beams. A yield moment of 25000 kN*m and a post-yield stiffness of 30000 kN*m/rad were also used.

The model was subjected to a push-pull analysis. Load was applied to each node in an inverted triangular distribution, with the loads on the first and second story 1/3 and 2/3 of the third story load respectively. The load at the top story rose from 0 to 200 kN, decreased to –200 kN and rose again to zero.

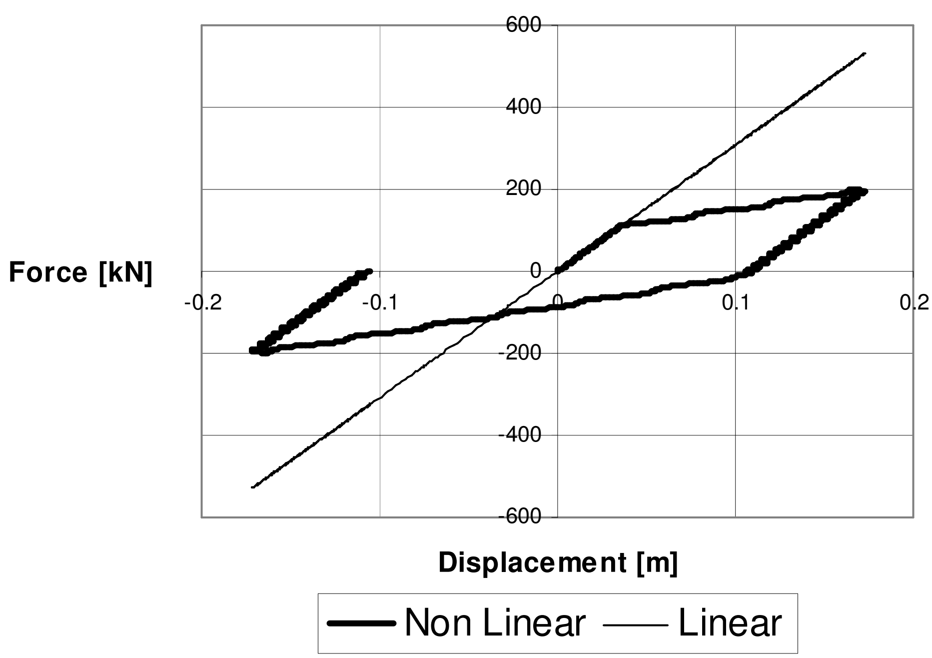


**Figure 6 - Force Deflection Response of Top of Wall**

Figure 6 shows the force-displacement response of at the top of the wall. Though the beam element connected to the top of the wall responds linearly throughout the loading, the effect of the hinge's plastic response can be seen clearly. This shows that isolating non-linear behaviour to one part of a structure reduces the load imposed on the rest of the structure.

## 5 LIMITATIONS

The Enoch framework is far from complete. Currently, though damping must be specified in terms of Rayleigh's alpha and beta coefficients, the solutions diverge unless beta equals zero. Only lumped mass is considered, making Enoch unsuitable for modeling some torsional problems.

In general, writing custom finite element analysis code is time-consuming. The program must be both written and validated. The payoff from a flexible, extensible modeling tool is huge and a decision must be made about whether the payoff is worth the trouble. A standardized framework reduces the overhead cost of writing custom code, while maintaining the flexibility of an open model.

## 6 FUTURE WORK

There still remains much work to be done on this program. The data structure used to store the loading information needs to be changed. Currently the load is stored as matrix with (number of degrees of freedom) x (number of time steps) elements. This is very inefficient, especially for the case of a constant load.

A better solution would be to develop a Load class and associate an instance of that class with each degree of freedom. The Load class would be inherited for different types of loads, for instance ConstantLoad, EarthquakeLoad, SinusoidalLoad, etc... This would simplify the task of using many different loads.

The data output behaviour currently leaves much to be desired. The ultimate solution would be for Enoch to make the data available to another class that would deal with the task of logging the data.

Finally, more validation is needed. Once the above changes have been made, many structures should be tested in many load cases to ensure that the program gives accurate results and to gain an understanding of how small the time step needs to be to guarantee convergence.

# 7 CONCLUSION

Enoch, a flexible framework for nonlinear, dynamic finite element analysis was developed. Written in C++ and taking advantage of the language's object-oriented features, it is easily extended to include new types of elements and new methods of analysis.

The Enoch framework has been validated by comparing its linear and non-linear dynamic response to theoretical response and the response generated by an existing program, Nonlin. This validation has demonstrated that Enoch correctly models the dynamic behaviour of linear and nonlinear systems.

A system composed of both linear and nonlinear elements was analysed. The effect of the nonlinear elements was observed throughout the structure.

The object-oriented nature of the framework allows both the analysis method and the elements that make up the structural system to be extended or replaced without affecting other parts of the framework.

## REFERENCES

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