

Mechanical Behavior of Materials

Midterm Review

Haoxuan Zeng

UM-SJTU Joint Institute
zenghaoxuan@sjtu.edu.cn

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JOINT INSTITUTE
交大密西根学院

This range of the midterm exam will cover lecture 2~12, including the following topics:

- Bonding in Solids & Crystalline Structures (Lec.2)
- Crystallographic Directions/Planes and Defects (Lec.3)
- Physics of Elastic and Inelastic Deformation (Lec.4)
- Phase Diagrams (Lec.5&6)
- Phase Transformation & Kinetics (Lec.7)
- Survey of Engineering Materials (Lec.8&9)
- Basic Mechanical Testing Methods (Lec.10)
- Elastic Stress-Strain Behavior (Lec.11)
- Plane Stresses, Mohr's Circle & 3D Stresses (Lec.12)

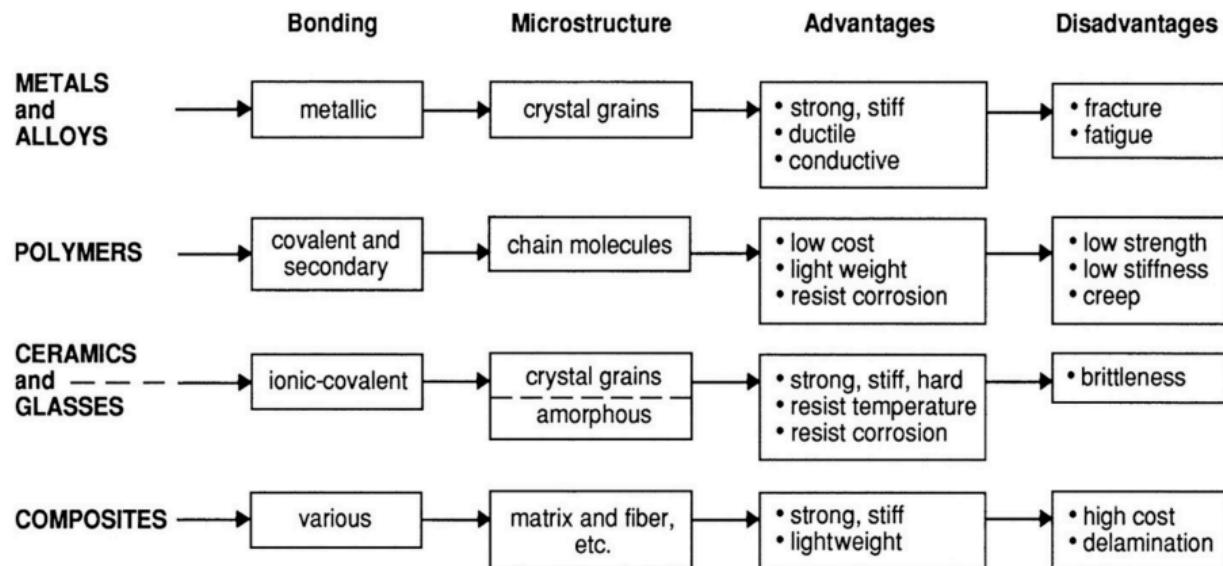
In today's review session, we will go through some key concepts and problems in HW.

Types of Interatomic Bonds

Type	Bond Energy	Comment
Ionic	Large	Non-directional, high T_m and E
Covalent	Large	Directional, high T_m and E
Metallic	Variable	Non-directional, moderate T_m and E
Van der Waals	Small	Non-directional, low T_m and E
Hydrogen	Small	Directional, low T_m and E

Types of Materials

Materials can be classified by the type of dominant bonding present between the atoms.



Exercise 1

HW1 ques1

Table 2.2(b) in Dowling book 4th edition gives a value of $E = 160$ GPa for a fiber of linear polyethylene, in which the polymer chains are aligned with the fiber axis. Why is this value so much higher than the typical $E = 3$ Gpa mentioned for polymers, in fact almost as high as the value for iron and steel?

Exercise 1

HW1 ques1

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Solution:

- For a fiber of linear polyethylene, the polymer chains are highly aligned along the fiber axis, for which the dominant bonding is covalent bond.
- However, typical polymers exhibit an amorphous structure, where the chains are disordered and not aligned. The loading direction may be perpendicular to the chain, on which the weak secondary bond is dominant.

Crystal Structures



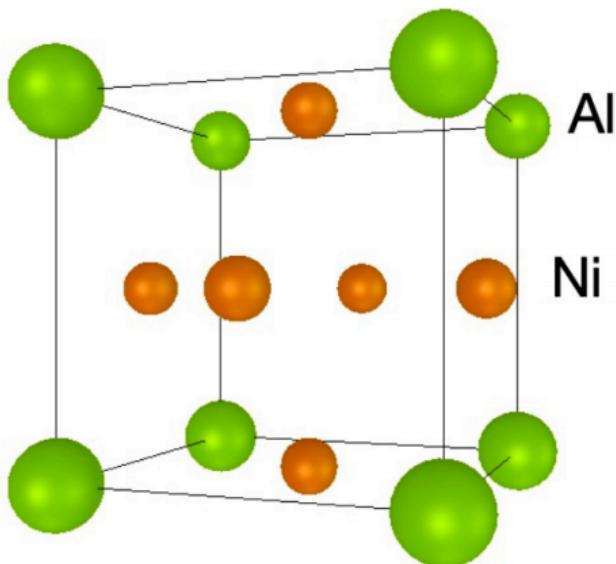
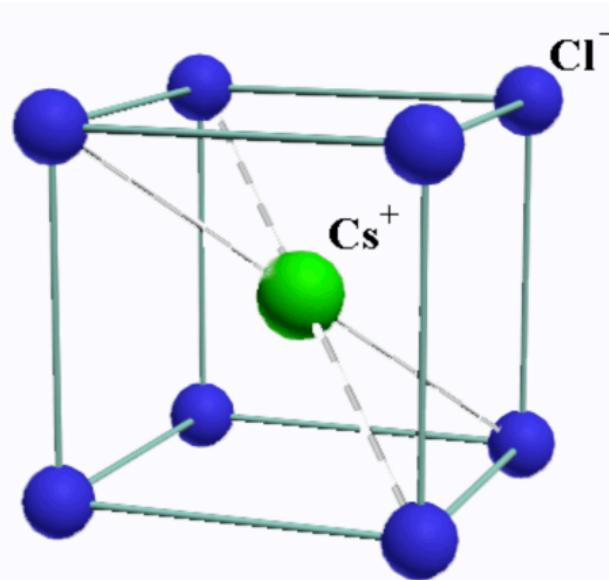
Crystal structure = Lattice structure + Basis

- Lattice describes the repetition pattern or periodicity, or intrinsically, the translational symmetry of the crystal.
- Basis describes the motif, i.e. the detailed atom arrangement, at each lattice point

Crystal Structures

Example

What is the lattice and basis of the following crystal structure?



Exercise 2

HW1 ques3

Sketch three-dimensional views of the unit cell of b.c.c lattice, draw and label the closed packed planes, and determine the linear atomic density along the slip direction^a.

^aThis question is modified from HW.

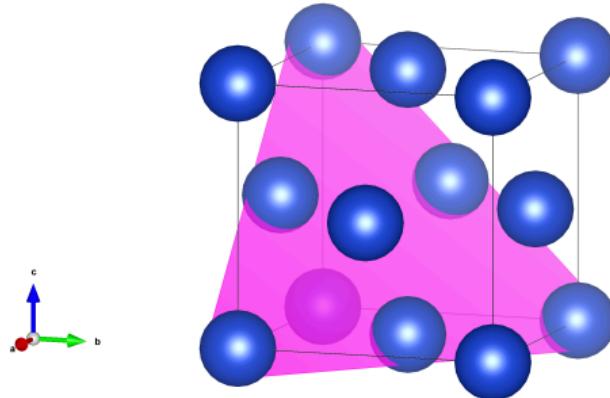
Exercise 2

HW1 ques3

Sketch three-dimensional views of the unit cell of b.c.c lattice, draw and label the closed packed planes, and determine the linear atomic density along the slip direction^a.

^aThis question is modified from HW.

Solution:



The slip directions for FCC copper are $\langle 110 \rangle$.
The linear density along these directions is

$$LD_{110} = \frac{2}{\sqrt{2}a} = \frac{\sqrt{2}}{a}$$

Interatomic Force

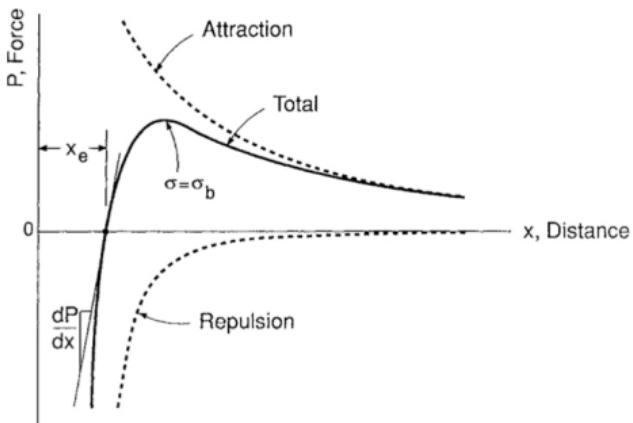


Figure 2.16 Variation with distance of the attractive, repulsive, and total forces between atoms. The slope dP/dx at the equilibrium spacing x_e is proportional to the elastic modulus E ; the stress σ_b , corresponding to the peak in total force, is the theoretical cohesive strength.

- Total force is attractive at large distance, repulsive at short distance, and zero at equilibrium atomic spacing x_e .
- Elastic deformation represent a small perturbation about x_e .

Exercise 3

HW1 ques2

For the force-distance curve between atoms, could you describe how the curve will change when the bonding type change from primary bond to secondary bond. Please comment on how these changes are related to the melting temperature and the young' s modulus of the material.

Exercise 3

HW1 ques2

For the force-distance curve between atoms, could you describe how the curve will change when the bonding type change from primary bond to secondary bond. Please comment on how these changes are related to the melting temperature and the young' s modulus of the material.

Solution:

- When changing from primary bond to secondary bond, x_e increases , σ_b decreases, and $\left.\frac{dP}{dx}\right|_{x=x_e}$ decreases.
- The smaller σ_b indicates that the secondary bond is weaker than the primary bond. As T rises, the secondary bond is more likely to break, thus the melting temperature will become lower.
- As $E \propto \left.\frac{dP}{dx}\right|_{x=x_e}$, the young's modulus will also become lower.

Elastic Deformation

Definition

Elastic deformation is macroscopic view of accumulation of inter-atomic distance changes through out the material, which recovers immediately when external stress removed.

The difficulty of elastic deformation is characterized by the elastic modulus:

$$E = \frac{d\sigma}{d\epsilon} \Big|_{x=x_e} = \frac{x_e}{A} \frac{dP}{dx} \Big|_{x=x_e}$$

- E is the intrinsic property of materials dominated by the bonding of the materials. Microstructure and defects do not affect E .
- Strong primary bonds are resistant to stretching and so result in high value of E (Typically, metal \approx ceramic $>$ polymer).
- As T rises, bonds are easier to separate from equilibrium spacing, so E decreases.
- E is isotropic for polycrystalline materials, but anisotropic for single crystals.

Plastic Deformation

Definition

Deformation that does not disappear when the stress is removed is called plastic deformation.

- Plastic deformation occurs mainly by shear.
- Theoretical shearing through breaking all the atomic bonds between two layers at the same time.

$$\tau_b = \frac{Gb}{2\pi h} \approx \frac{G}{10} \quad \text{where } G = \frac{d\tau}{d\gamma}$$

- Actual shearing through slip. Discrepancy exists between theoretical and actual strength:
 - Actual ultimate tensile strength smaller than theoretical cohesive tensile strength $\sigma_b \approx \frac{E}{10}$ by factor of 10 to 100
 - Actual shear strength smaller than theoretical shear strength $\tau_b \approx \frac{G}{10}$ by factor of 300 to 10000

Exercise 4

HW1 ques4

Using Table 2.2 in Dowling, compare the strengths of Al_2O_3 whiskers versus Al_2O_3 fibers, can you explain the large difference observed?

Exercise 4

HW1 ques4

Using Table 2.2 in Dowling, compare the strengths of Al_2O_3 whiskers versus Al_2O_3 fibers, can you explain the large difference observed?

Solution:

- According to Table 2.2 in Dowling, the tensile strength of Al_2O_3 whisker is 23.3 GPa, while the tensile strength of Al_2O_3 fiber is 2.1 GPa. Al_2O_3 whiskers are much stronger than Al_2O_3 fibers.
- The reason is that the whiskers are nearly perfect single crystals with low dislocation density, while the fibers are polycrystalline with higher dislocation density.
- As the incremental atom shift is much more easier than simultaneously breaking all atomic bonds, the tensile strength of whiskers is much higher than that of fibers.

Dislocation and Hardening Mechanisms

If there are **obstacles** that impede dislocation motion, the strength of the material increases. Hardening mechanisms include:

- Strain hardening: **dislocation** density increases with plastic deformation, which increases the resistance to further deformation.
- Solid solution hardening: **impurity atoms** distort the lattice, which increases the resistance to dislocation motion.
- Precipitation hardening: **precipitates** impede dislocation motion.
- Grain size reduction: **grain boundaries** impede dislocation motion.

Exercise 5

HW1 ques5

Cold working a metal by rolling it to a lesser thickness or hammering it introduces a large number of dislocations into the crystal structure. Would you expect the yield strength (beyond which material starts to have plastic deformation) and ductility to be affected by this; and if so, should it increase or decrease. And why? How about the elastic modulus and shear modulus?

Exercise 5

HW1 ques5

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Solution:

- Yield strength will increase. The reason is that cold working like rolling and hammering can increase the dislocation density, hence decrease the average separation between dislocations. As on average, the interactions between dislocations are repulsive, the motion of dislocations will be hindered, thus plasticity deformation will become harder and the yield strength will increase.
- The elastic modulus and shear modulus will not change. The reason is that the elastic modulus and shear modulus are intrinsic properties of the material, which are determined by the atomic structure and bonding. The cold working process only does not change the atomic structure, so the elastic modulus and shear modulus will not change.

Binary Phase Diagrams Calculation

Given a binary phase diagram, the composition of an alloy, its temperature, and assuming the system is at equilibrium, we can determine:

- **What phase(s) is (are) present**

- Locate the $T\text{-}C$ point on the phase diagram and notes the phases with which the corresponding phase field is labeled.

- **The composition(s) of the phase(s)**

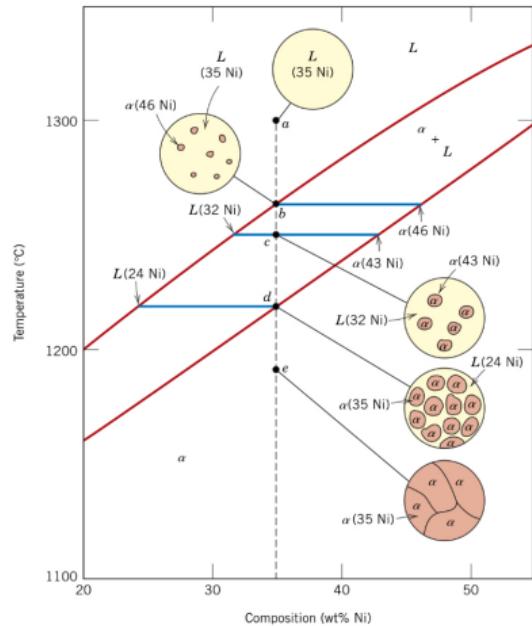
- For single-phase regions, the composition of the phase is the same as the overall composition of the alloy.
- For two-phase regions, construct a tie line through the $T\text{-}C$ point and intersect the phase boundary.

- **The mass fraction(s) of the phase(s)**

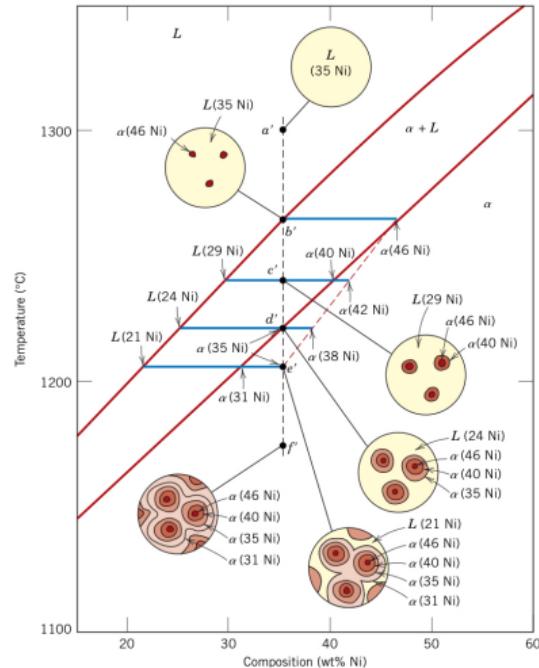
- For single-phase regions, the mass fraction of the phase is 1.
- For two-phase regions, the mass fraction is determined by the lever rule.

Development of Microstructure

For binary isomorphous system, the microstructure development is as follows:



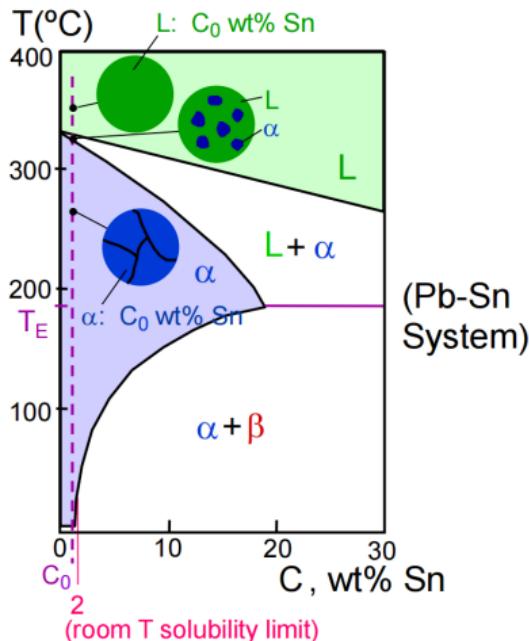
Equilibrium cooling



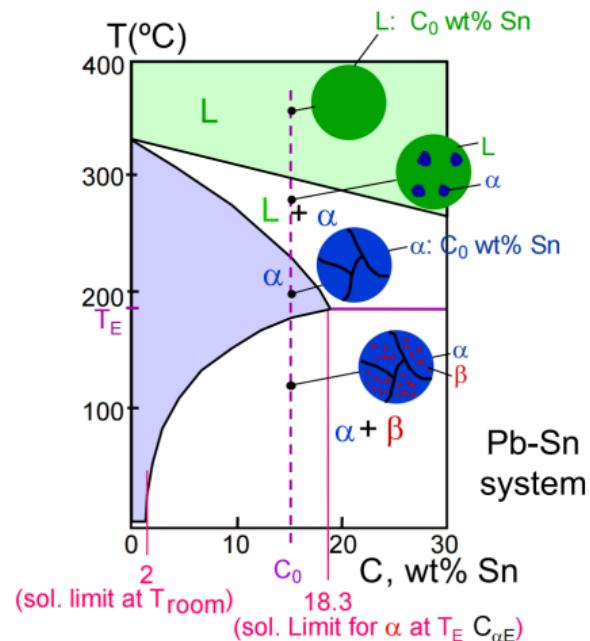
Nonequilibrium cooling

Development of Microstructure

For binary eutectic system, the microstructure development is as follows:



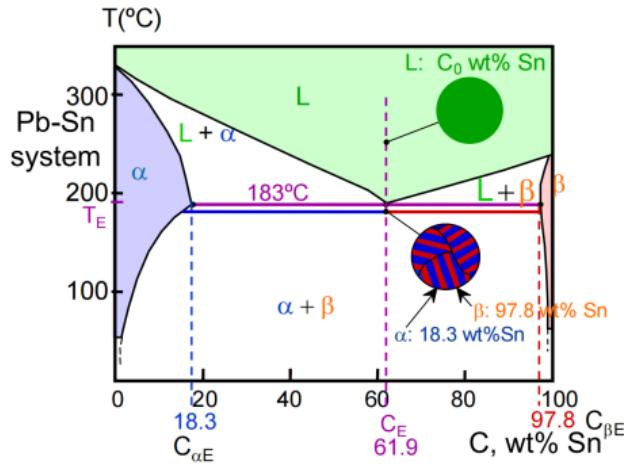
1. Below RT solubility limit



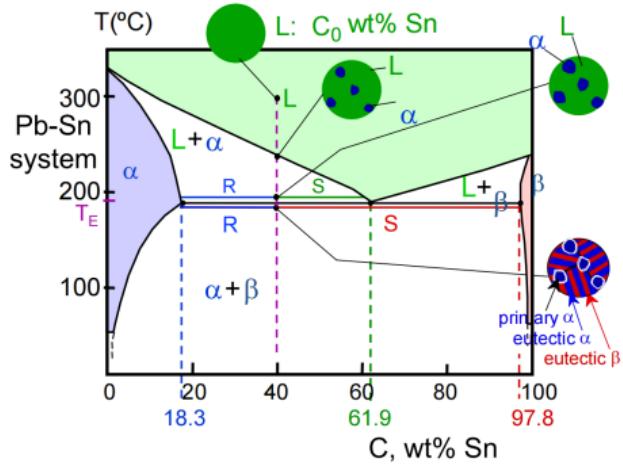
2. Between solubility limit at RT and T_E

Development of Microstructure

Continued:



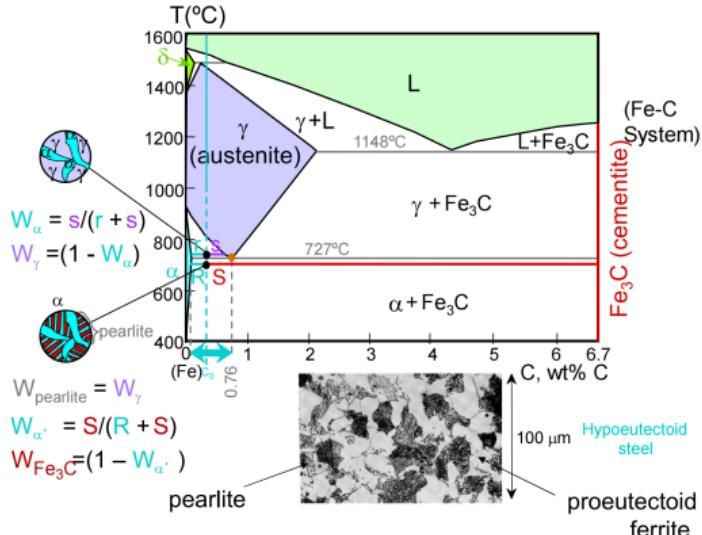
3. At C_E



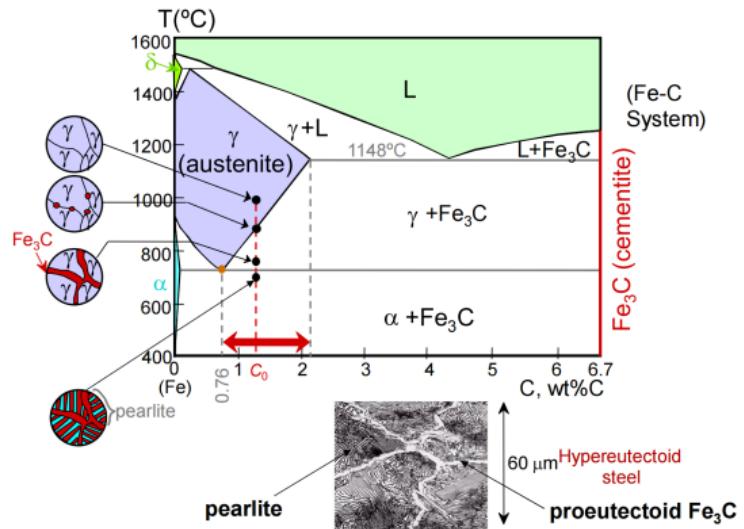
4. Other points on eutectic line except C_E

Development of Microstructure

For Iron-Carbon (Fe-C) system, the microstructure development is as follows:



Hypoeutectoid Steel



Hypereutectoid Steel

Exercise 6

HW2 ques1

Consider 8 kg of austenite containing 0.5 wt% C, cooled from 1000 °C to below 727 °C (1341 °F).

- a What is the proeutectoid phase?
- b How many kilograms each of total ferrite and cementite form?
- c How many kilograms each of pearlite and the proeutectoid phase form?
- d Schematically sketch and label the microstructure development during the cooling process.
(Assume that equilibrium phases can be reached and only sketch the microstructures at critical points which illustrate the changes.)

Exercise 6

Solution:

(a) Since

$$0.022 < C_0 = 0.5 < 0.76$$

According to the phase diagram, the proeutectoid phase is α – Fe or ferrite.

(b) The fractions of ferrite and cementite are calculated using the lever rule as follows

$$W_\alpha = \frac{C_{\text{Fe}_3\text{C}} - C_0}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{6.7 - 0.5}{6.7 - 0.022} = 0.9284$$

$$W_{\text{Fe}_3\text{C}} = \frac{C_0 - C_\alpha}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{0.5 - 0.022}{6.7 - 0.022} = 0.0716$$

Thus

$$m_\alpha = m_0 W_\alpha = 8 \text{ kg} \times 0.9284 = 7.427 \text{ kg}$$

$$m_{\text{Fe}_3\text{C}} = m_0 W_{\text{Fe}_3\text{C}} = 8 \text{ kg} \times 0.0716 = 0.573 \text{ kg}$$

Exercise 6

- (c) The fractions of pearlite and the proeutectoid α phase are calculated using the lever rule as follows.

$$W_{\alpha} = \frac{0.76 - 0.5}{0.76 - 0.022} = 0.3523$$

$$W_{\gamma} = \frac{0.5 - 0.022}{0.76 - 0.022} = 0.6477$$

As the pearlite is formed by austenite, the mass of the pearlite is the same as the mass of the austenite, i.e.

$$W_p = W_{\gamma} = 0.6477$$

Thus

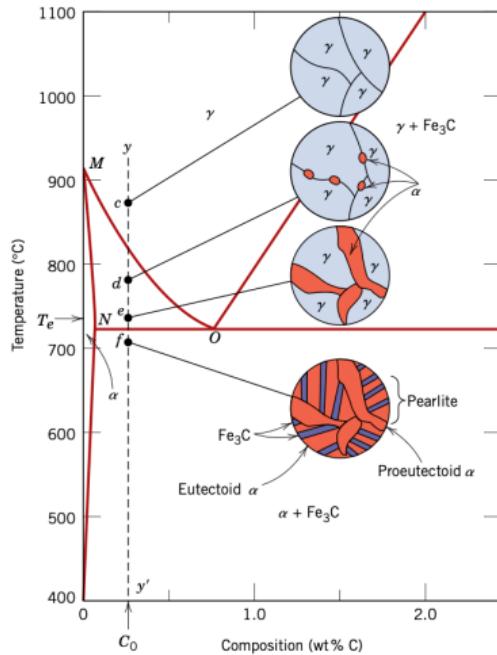
$$m_{\alpha} = m_0 W_{\alpha} = 8 \text{ kg} \times 0.3523 = 2.8184 \text{ kg}$$

$$m_p = m_0 W_p = 8 \text{ kg} \times 0.6477 = 5.1816 \text{ kg}$$

Exercise 6

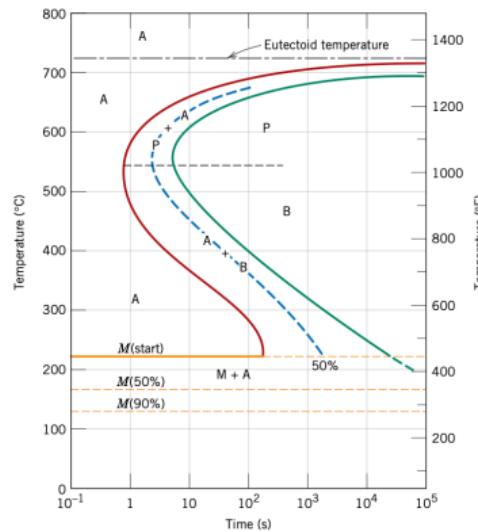
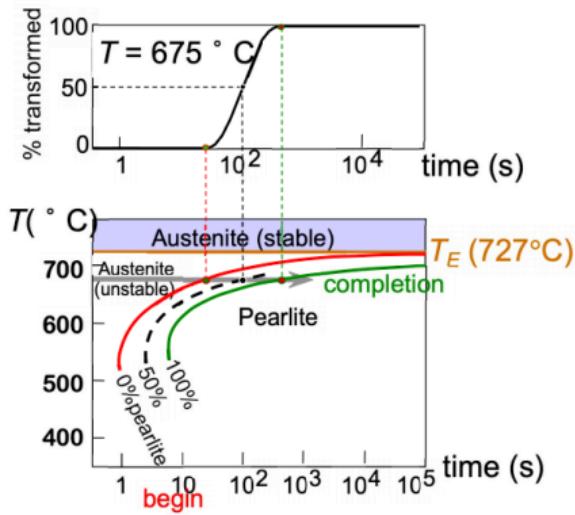
- (d) The schematically sketch of the microstructure development during the cooling process is shown below.

Figure 9.29 Schematic representations of the microstructures for an iron–carbon alloy of hypoeutectoid composition C_0 (containing less than 0.76 wt% C) as it is cooled from within the austenite phase region to below the eutectoid temperature.



TTT Diagram

- TTT stands for **Time-Temperature-Transformation**.
- TTT diagrams provide the following information:
 - Nature and type of transformation.
 - Stability of phases under isothermal transformation conditions.
 - Temperature or time required to start or finish transformation and rate of transformation.



Exercise 7

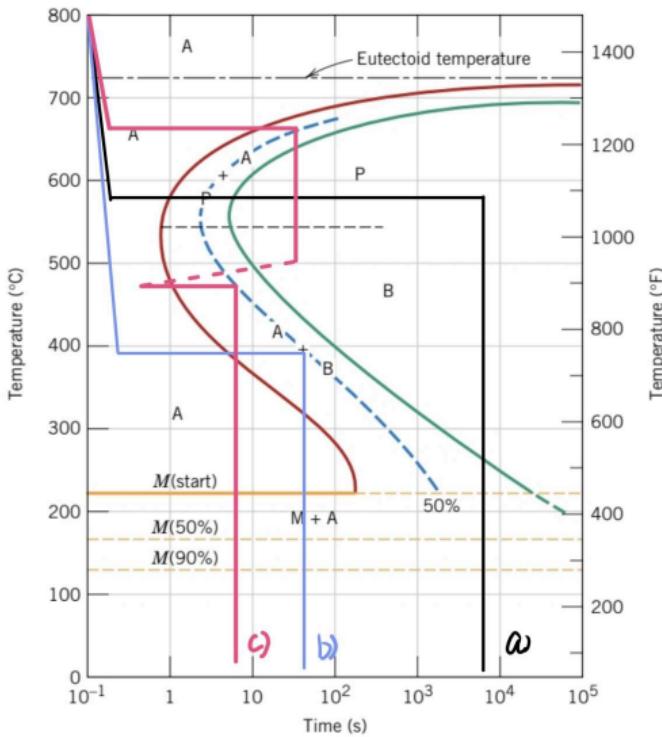
HW2 ques3

Make a copy of the isothermal transformation diagram for an iron-carbon alloy of eutectoid composition (Figure 10.22 in the Callister book), and then sketch and label time-temperature paths on this diagram to produce the following microstructures:

- a 100% fine pearlite
- b 50% tempered martensite, 50% bainite
- c 50% coarse pearlite, 25% bainite, and 25% martensite

Exercise 7

Solution:



Exercise 8

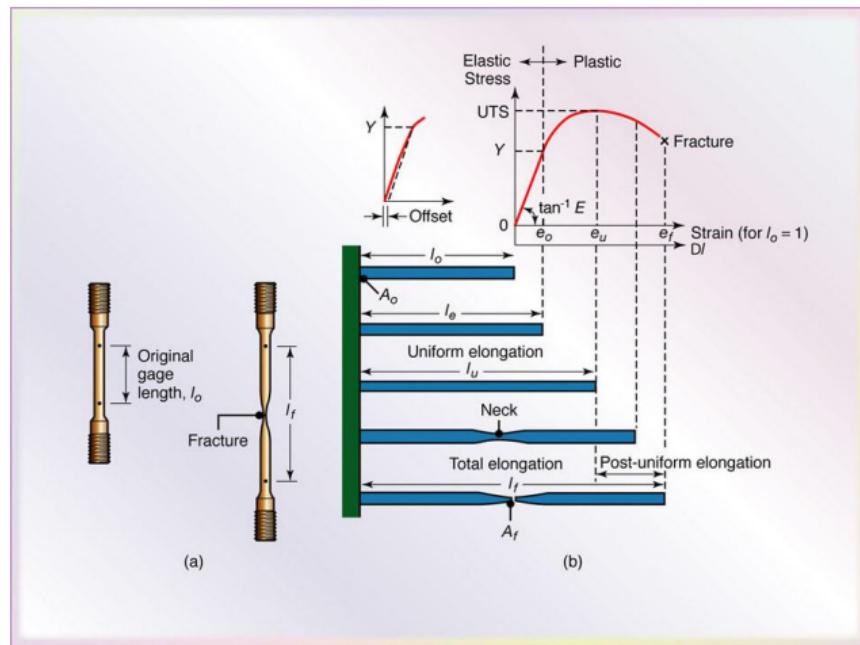
Quiz ques

For eutectoid steel, what of the following statement is correct (multiple choices)?

- Different cooling rate may affect the measured Young's modulus
- Different cooling rate will not affect hardness
- Different cooling rate may not affect the measured Young's modulus
- Different cooling rate will lead to different microstructures

Stress-Strain Curve

Stress-strain curve can be used to determine the following mechanical properties: (1) ultimate tensile strength; (2) yield strength; (3) ductility; (4) toughness; (5) modulus of elasticity.



Strength

Ultimate Tensile Strength

The ultimate tensile strength σ_u is the highest engineering stress on the stress-strain curve.

$$\sigma_u = \frac{P_{\max}}{A_i}$$

For brittle materials, $\sigma_u = \sigma_f$, for ductile materials, $\sigma_u > \sigma_f$, where σ_f is the stress at fracture.

Yield Strength

The yield strength σ_y is maximum stress that can be developed in a material without causing plastic deformation.

Ductility

Ductility

Ductility is the ability of materials to accommodate inelastic deformation without breaking.

Ductility can be measured by:

- Engineering strain at fracture:

$$\varepsilon_f = \frac{L_f - L_0}{L_0}$$

- Engineering strain after fracture:

$$\varepsilon_{pf} = \frac{L_{pf} - L_i}{L_i} = \varepsilon_f - \frac{\sigma_f}{E}$$

- Percent reduction in area:

$$\%RA = 100 \frac{A_i - A_f}{A_i} = 100 \frac{d_i^2 - d_f^2}{d_i^2}$$

Tensile Toughness

Tensile Toughness

The area under the entire engineering stress-strain curve up to fracture is called the tensile toughness, u_f , which is a measure of the ability of the material to absorb energy without fracture.

$$u_f = \int_0^{\varepsilon_f} \sigma d\varepsilon$$

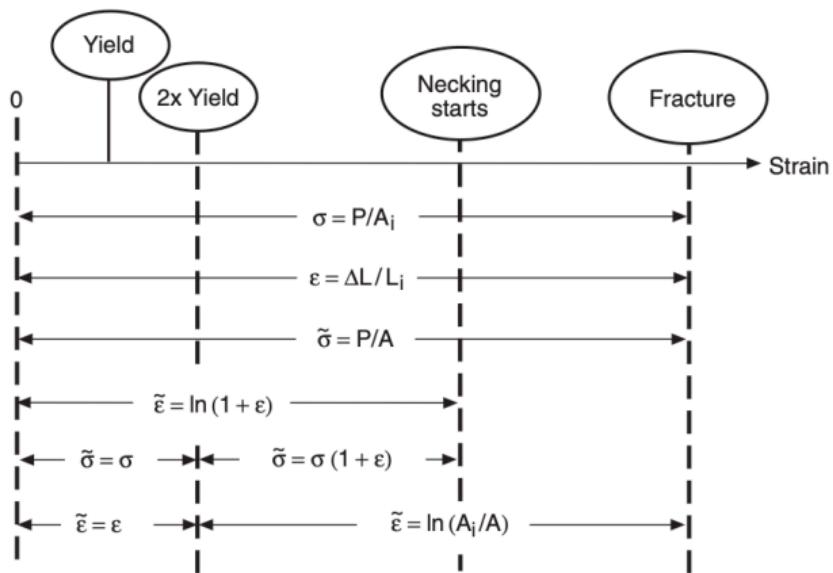
- For brittle materials:

$$u_f \approx \frac{2}{3} \sigma_f \varepsilon_f$$

- For ductile materials:

$$u_f \approx \varepsilon_f \left(\frac{\sigma_y + \sigma_u}{2} \right)$$

Limitations on True Stress-Strain Calculation



Once necking starts at the engineering ultimate stress point, the engineering strain becomes merely an average over a region of nonuniform deformation. Hence, it does not represent the maximum strain and becomes unsuitable for calculating true stresses and strains.

Hooke's Law

- For homogeneous and isotropic materials,

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} + \begin{bmatrix} \varepsilon_T \\ \varepsilon_T \\ \varepsilon_T \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- Volumetric strain: $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{(1-2\nu)(\sigma_x+\sigma_y+\sigma_z)}{E}$
- Thermal strain: $\varepsilon_T = \alpha \Delta T$
- Three elastic constants, 2 independent: $G = \frac{E}{2(1+\nu)}$

Exercise 9

Previous year's HW ques

A block of material is confined by a rigid dies as shown in figure below, so that it can not deform in either x or y directions. A compressive stress is applied in the z direction. Assume that there is no friction along the walls and that no yielding occurs in the metal.

- ① What is the largest value of the compressive stress along z direction that can be applied without the strain in the z -direction exceeding 0.2% or 0.002?
- ② How about if the modulus of the material along x and y equal to each other but are different from z direction?

Exercise 9 |

Solution:

- ① As there is no friction along the walls,

$$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

As the material can not deform in x or y directions, according to the Hooke's law, we have

$$\begin{cases} \varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] = 0 \\ \varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] = 0 \end{cases}$$

Then

$$\sigma_x + \sigma_y = \frac{\nu \sigma_z}{1 - \nu}$$

then

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] = \frac{(1 - \nu - 2\nu^2)}{(1 - \nu)E} \sigma_z \leqslant 0.002$$

Exercise 9 II

Therefore, the largest value of σ_z is $\frac{0.002E(1-\nu)}{1-\nu-2\nu^2}$

- ② If ν is still isotropic, the result will be the same as the previous one. If ν is anisotropic, the result will be different. As

$$\nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i} \quad \text{and} \quad E_x = E_y \neq E_z$$

As

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \quad \text{and} \quad \frac{\nu_{xz}}{E_x} = \frac{\nu_{zx}}{E_z} \quad \text{and} \quad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z}$$

we have

$$\nu_{xy} = \nu_{yx} \quad \text{and} \quad \nu_{xz} = \frac{\nu_{zx}E_x}{E_z} \quad \text{and} \quad \nu_{yz} = \frac{\nu_{zy}E_y}{E_z}$$

Then

$$\sigma_x + \sigma_y = \frac{\sigma_z(\nu_{xz} + \nu_{yz})}{1 - \nu_{xy}}$$

Exercise 9 III

$$\sigma_x - \sigma_y = \frac{\sigma_z(\nu_{xz} - \nu_{yz})}{1 + \nu_{xy}}$$

Then

$$\varepsilon_z = \frac{\sigma_z}{E_z} - \frac{\nu_{xz}\sigma_x + \nu_{yz}\sigma_y}{E_x} \leq 0.002$$

The next steps are plugging in σ_x and σ_y and solving the inequality for σ_z .

Plane Stress Transformation

- Plane stress when:

$$\sigma_z = \tau_{zx} = \tau_{yz} = 0$$

- θ is defined as positive in counterclockwise (CCW) rotation direction for coordinate systems.
- The new stresses are given by:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principle Stresses

- Principle normal stresses:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_n = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- Principle shear stress:

$$\tau_3 = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{|\sigma_1 - \sigma_2|}{2}$$

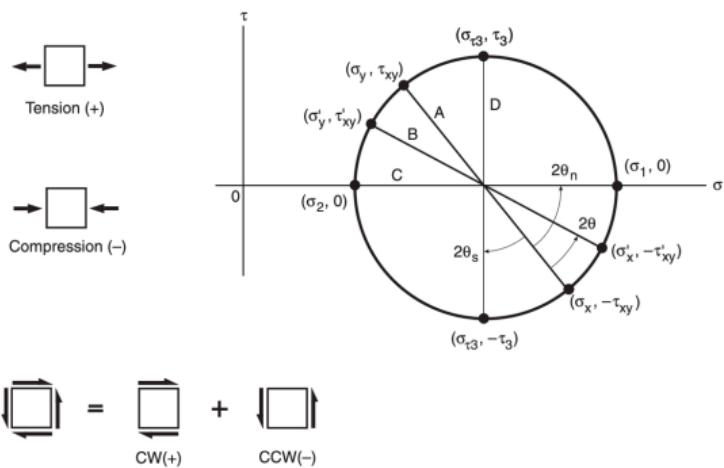
$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}, \quad \sigma_{\tau_3} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

- $|\theta_s - \theta_n| = \frac{\pi}{4}$

Mohr's Circle

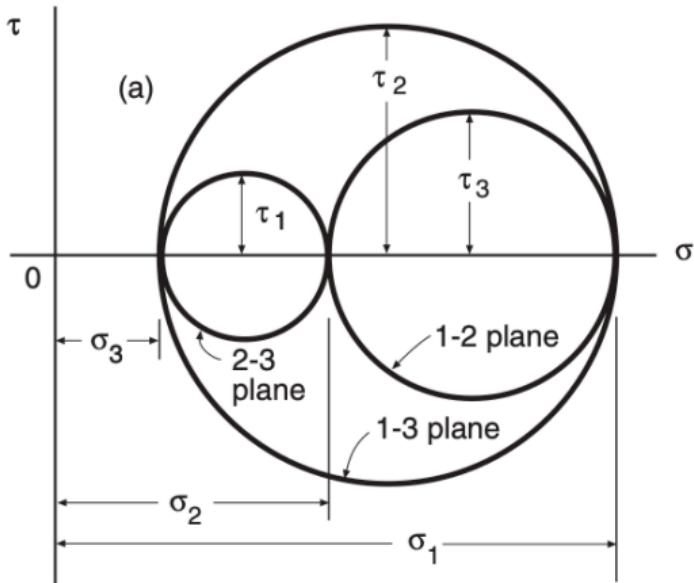
Equation of Mohr's circle:

$$\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$



- A rotation of 2θ on Mohr's circle corresponds to a rotation of θ for coordinate system in real space.
- Clockwise (CW) shear stress: positive.
- Counter Clockwise (CCW) shear stresses: negative

3D State of Stress Transformation



- Draw three Mohr's circles for xy , yz , and zx planes.
- Usually define $\sigma_1 > \sigma_2 > \sigma_3$
- The maximum shear stress for any plane in the material is the largest of the three principal shear stresses:

$$\tau_{\max} = \max(\tau_1, \tau_2, \tau_3) = \frac{\sigma_1 - \sigma_3}{2}$$

Thank you!