Mechanical Behavior of Materials
Lec.07-12

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This recitation class will cover lecture 7 and lecture 10~12, the topics include:

- Phase Transformation & Kinetics (Lec.7)
- Basic Mechanical Testing Methods (Lec.10)
- Elastic Stress-Strain Behavior (Lec.11)
- Plane Stresses, Mohr's Circle & 3D Stresses (Lec.12)

The contents of Survey of Engineering Materials (Lec.8&9) will be covered in the RC for midterm review.

Agenda

Process of Phase Transformation

The process of a phase Transformation is broken down into two distinct stages:

- **Nucleation:** the appearance of nuclei of the new phase, which are capable of growing.
  - Homogeneous: nuclei of the new phase form uniformly throughout the parent phase.
  - Heterogeneous: nuclei form preferentially at structural inhomogeneities such as grain boundaries, dislocations, and interfaces.
- **Growth:** the nuclei increase in size, which results in the disappearance of the parent phase.
- The Transformation reaches completion when equilibrium fraction of the new phase is attained.

Homogeneous Nucleation

For solidification, the driving force for the phase transformation is as follows:

$$\Delta G = \frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma$$

- ΔG_v: the volume free energy difference between solid and liquid; $\Delta G_v < 0$ when $T < T_m$.
- γ: the solid-liquid surface energy; $\gamma > 0$.
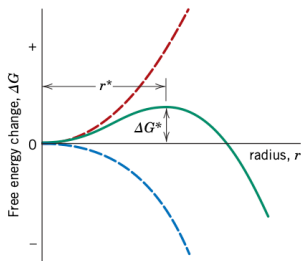
- Growth will continue when $r > r^*$.
- Otherwise the nuclei will shrink and redissolve.
- Activation free energy (energy barrier) ΔG^* is required for the formation of a stable nucleus.

Homogeneous Nucleation

It can be shown that ΔG_v is a function of temperature as:

$$\Delta G_v = \frac{\Delta H_f(T_m - T)}{T_m}$$

where ΔH_f is the latent heat of fusion. Then the critical radius is given by:

$$r^* = -\frac{2\gamma}{\Delta G_v} = \left(-\frac{2\gamma T_m}{\Delta H_f}\right)\left(\frac{1}{T_m - T}\right)$$

The activation free energy is given by:

$$\Delta G^* = \frac{16\pi\gamma^3}{3\Delta G_v^2} = \left(\frac{16\pi\gamma^3 T_m^2}{3\Delta H_f^2}\right)\frac{1}{(T_m - T)^2}$$

Heterogeneous Nucleation

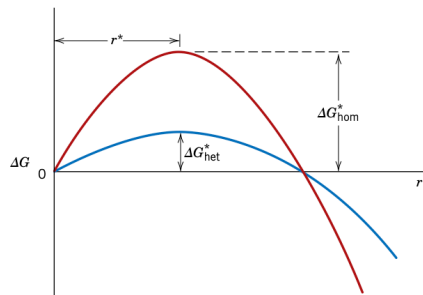The critical radius for heterogeneous nucleation is given by:

$$r^*_{\text{het}} = -\frac{2\gamma_{\text{SL}}}{\Delta G_v} = r^*_{\text{hom}}$$

The activation free energy is given by:

$$\Delta G^*_{\text{het}} = \left(\frac{16\pi\gamma^3_{\text{SL}}}{3\Delta G_v^2}\right) S(\theta) = \Delta G^*_{\text{hom}} S(\theta)$$

where

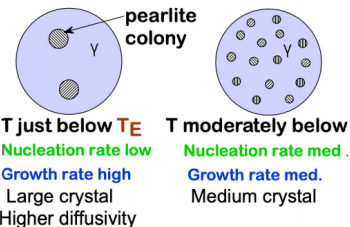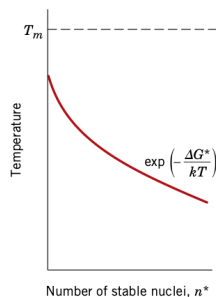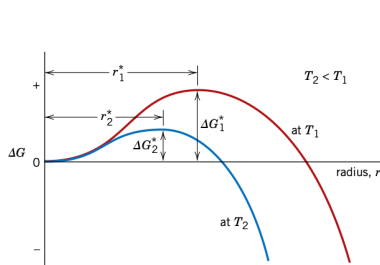$$S(\theta) = \frac{2 - 3\cos\theta + \cos^3\theta}{4}$$

- The critical radius is the same as that for homogeneous nucleation.
- The activation energy is lowered by factor $S(\theta)$ because the surface energy is reduced.

Effect of Temperature

Both the critical radius r^* and the activation free energy ΔG^* decrease as temperature T decreases. The number of stable nuclei n^* is a function of temperature:

$$n^* = K_1 \exp\left(-\frac{\Delta G^*}{kT}\right)$$

where K_1 is a constant related to total number of nuclei.

T just below T_E
Nucleation rate low
Growth rate high
Large crystal
Higher diffusivity

T moderately below
Nucleation rate med .
Growth rate med.
Medium crystal

Effect of Time

For a fixed temperature, the fraction of transformation y is a function of time given by the **Avrami equation**:

$$y = 1 - \exp(-kt^n)$$

where k and n are time-independent constants. By convention, the rate of a transformation is defined as

$$\text{rate} = \frac{1}{t_{0.5}}$$

where $t_{0.5}$ is the time required for $y = 0.5$. The rate of transformation is a function of temperature and the Avrami equation can be used to determine the rate of transformation.

Logarithm of heating time, t
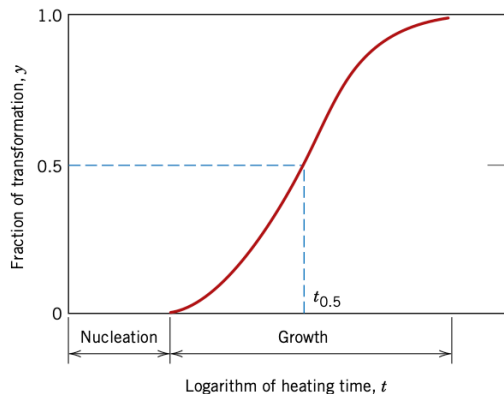
TTT Diagram

- TTT stands for **Time**-**Temperature**-**Transformation**.
- TTT diagrams provide the following information:
  - Nature and type of transformation.
  - Stability of phases under isothermal transformation conditions.
  - Temperature or time required to start or finish transformation and rate of transformation.
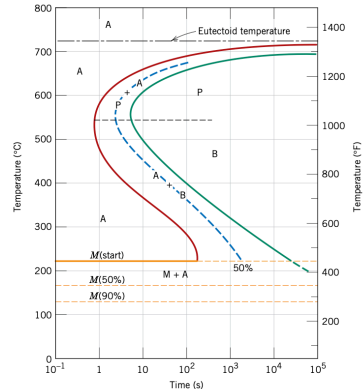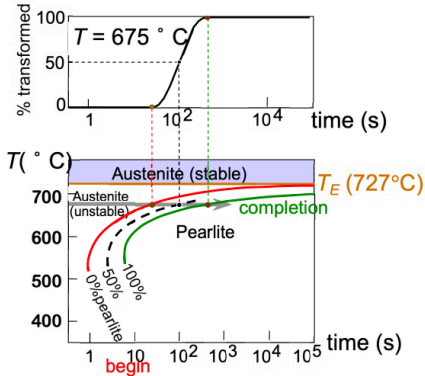
Microstructures in Fe-Fe$_3$C System

- Pearlite: alternating ferrite+cementite layers.
  - High T, both C and Fe diffuse.
  - Coarse pearlite: high diffusion rate and low nucleation rate, smaller ΔT
  - Fine pearlite: low diffusion rate and high nucleation rate, larger ΔT
- Bainite: non layered ferrite+cementite.
  - Moderate T, C diffuses.
  - Note that bainite forms directly from austenite and not from pearlite.
- Martensite: ferrite with supersaturated carbon.
 - Low T, C and Fe do not diffuse but just slight atomic displacement.

Example

Specify the nature of the final microstructure of a small specimen of eutectoid composition that has been subjected to the following time-temperature treatments. In each case assume that the specimen begins at $760°C$ and that it has been held at this temperature long enough to have achieved a complete and homogeneous austenitic structure.

(a) Rapidly cool to $350°C$, hold for 10^4 s, and quench to room temperature.

(b) Rapidly cool to $250°C$, hold for 10^2 s, and quench to room temperature.

(c) Rapidly cool to $650°C$, hold for 20 s, rapidly cool to $400°C$, hold for 10^3 s, and quench to room temperature.
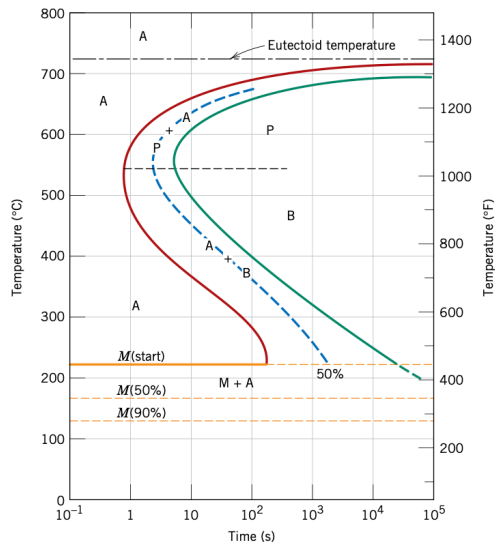
Example

(a) At $350°C$, reaction takes about 10s to start and reach final state (bainite) after about 500s $\Rightarrow$ 100% bainite.

(b) At $250°C$, reaction takes more than 100s to reach final state, but the process only takes 100s $\Rightarrow$ 100% austenite. Austenite then cools down into martensite $\Rightarrow$ 100% martensite.

(c) At $650°C$, after 20 s, the reaction stops on the blue line $\Rightarrow$ 50% austenite & 50% pearlite. At $400°C$, all remaining austenite changes into bainite after 100s $\Rightarrow$ 50% bainite & 50% pearlite.
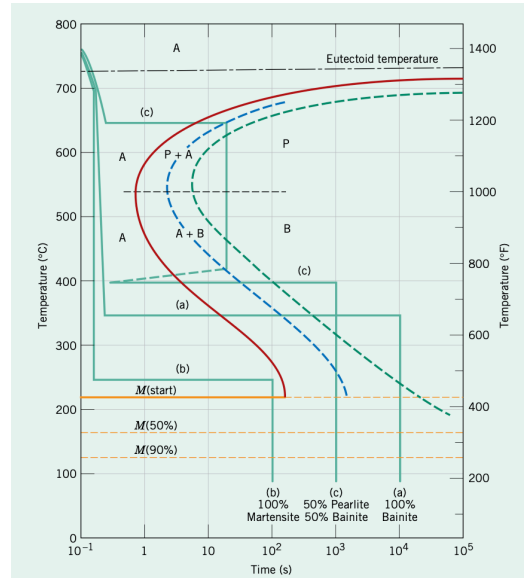
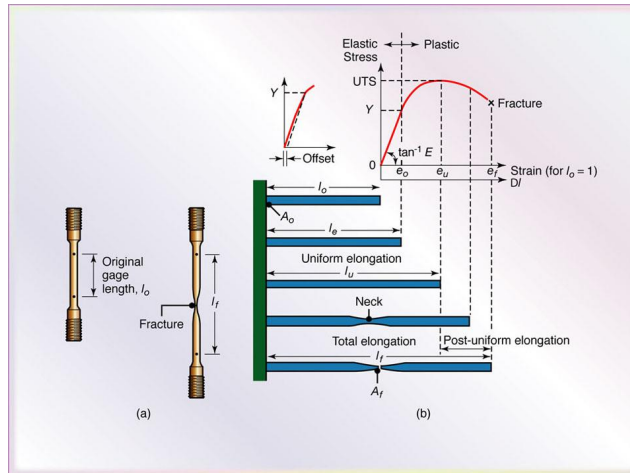# Agenda

1. Phase Transformation & Kinetics

2. **Basic Mechanical Testing Methods**

3. Elastic Stress-Strain Behavior

4. Plane Stresses, Mohr's Circle & 3D Stresses

Stress-Strain Curve

Stress-strain curve can be used to determine the following mechanical properties: (1) ultimate tensile strength; (2) yield strength; (3) ductility; (4) toughness; (5) modulus of elasticity.

Strength

Ultimate Tensile Strength

The ultimate tensile strength σ_u is the highest engineering stress on the stress-strain curve.

$$\sigma_u = \frac{P_{\max}}{A_i}$$

For brittle materials, $\sigma_u = \sigma_f$, for ductile materials, $\sigma_u > \sigma_f$, where σ_f is the stress at fracture.

Yield Strength

The yield strength σ_y is maximum stress that can be developed in a material without causing plastic deformation.

Ductility

Ductility

Ductility is the ability of materials to accommodate inelastic deformation without breaking.

Ductility can be measured by:

- Engineering strain at fracture:

$$\varepsilon_f = \frac{L_f - L_0}{L_0}$$

- Engineering strain after fracture:

$$\varepsilon_{pf} = \frac{L_{pf} - L_i}{L_i} = \varepsilon_f - \frac{\sigma_f}{E}$$

- Percent reduction in area:

$$\%RA = 100\frac{A_i - A_f}{A_i} = 100\frac{d_i^2 - d_f^2}{d_i^2}$$

Tensile Toughness

Tensile Toughness

The area under the entire engineering stress-strain curve up to fracture is called the tensile toughness, u_f, which is a measure of the ability of the material to absorb energy without fracture.

$$u_f = \int_0^{\varepsilon_f} \sigma \mathrm{d}\varepsilon$$

- For brittle materials:

$$u_f \approx \frac{2}{3} \sigma_f \varepsilon_f$$

- For ductile materials:

$$u_f \approx \varepsilon_f \left(\frac{\sigma_y + \sigma_u}{2} \right)$$

True Stress and Strain

True Stress

True stress is the axial force P divided by the current cross-sectional area A instead of A_i:

$$\tilde{\sigma} = \frac{P}{A} = \sigma \frac{A_i}{A}$$
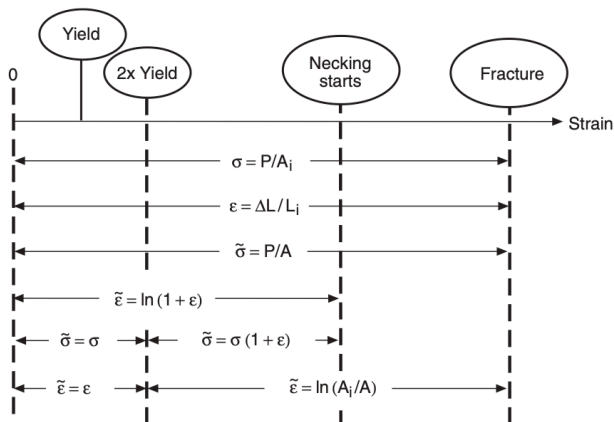
True Strain

The true strain is given as:

$$\tilde{\varepsilon} = \ln\left(\frac{L}{L_i}\right) = \ln\left(1 + \varepsilon\right)$$

Constant Volume Assumption

With constant volume assumption $A_i L_i = AL$ (invalid for small strains), we have

$$\tilde{\sigma} = \sigma(1 + \varepsilon) \quad \text{and} \quad \tilde{\varepsilon} = \ln\frac{A_i}{A}$$

Limitations on True Stress-Strain Calculation

Once necking starts at the engineering ultimate stress point, the engineering strain becomes merely an average over a region of nonuniform deformation. Hence, it does not represent the maximum strain and becomes unsuitable for calculating true stresses and strains.

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Hooke's Law

- For homogeneous and isotropic materials,

$$
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} + \begin{bmatrix} \varepsilon_T \\ \varepsilon_T \\ \varepsilon_T \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

- Volumetric strain: $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{(1-2v)(\sigma_x + \sigma_y + \sigma_z)}{E}$
- Thermal strain: $\varepsilon_T = \alpha \Delta T$
- Three elastic constants, 2 independent: $G = \frac{E}{2(1+v)}$

Example

Strains are measured on the surface of an aluminum alloy part as follows:

$$\varepsilon_x = 400 \times 10^{-6}, \quad \varepsilon_y = 1000 \times 10^{-6}, \quad \gamma_{xy} = 800 \times 10^{-6}$$

Estimate the in-plane stress components σ_x, σ_y and τ_{xy}, and also the strain ε_z normal to the surface. Note that for aluminum alloy,

$$E = 70.3 \times 10^3 \text{ MPa}, \quad \nu = 0.345$$

(Assume that $\sigma_z = \tau_{yz} = \tau_{zx} = 0$.)

Example

According to Hooke's law,

$$\begin{cases} \varepsilon_x = \dfrac{1}{E}\sigma_x - \dfrac{\nu}{E}\sigma_y = 400 \times 10^{-6} \\[2mm] \varepsilon_y = \dfrac{1}{E}\sigma_y - \dfrac{\nu}{E}\sigma_x = 1000 \times 10^{-6} \\[2mm] \gamma_{xy} = \dfrac{1}{G}\tau_{xy} = \dfrac{2(1+\nu)}{E}\tau_{xy} = 800 \times 10^{-6} \end{cases}$$

Therefore,

$$\begin{cases} \sigma_x = 59.4 \text{ MPa} \\ \sigma_y = 90.8 \text{ MPa} \\ \tau_{xy} = 20.9 \text{ MPa} \end{cases}$$

Then

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -7.37 \times 10^{-4}$$

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Ductile vs. Brittle Failure

- Ductile material: if maximum principle shear stress $\tau_{\max}$ is larger than critical shear $\tau_y = \sigma_y/2$, the material yields.
- Brittle material: if maximum principle normal stress $\sigma_{\max}$ is larger than ultimate tensile strength σ_u, the material fractures.

Plane Stress Transformation

- Plane stress when:

$$\sigma_z = \tau_{zx} = \tau_{yz} = 0$$

- θ is defined as positive in counterclockwise (CCW) rotation direction for coordinate systems.
- The new stresses are given by:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principle Stresses

- Principle normal stresses:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_n = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

- Principle shear stress:

$$\tau_3 = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{|\sigma_1 - \sigma_2|}{2}$$
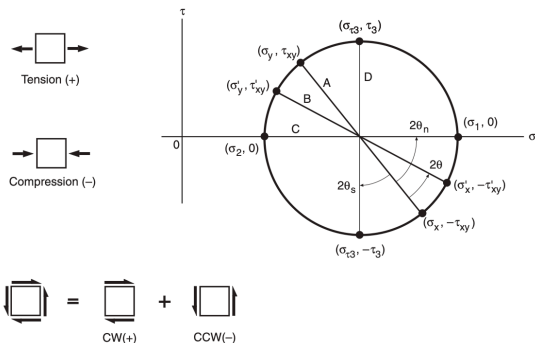
$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}, \quad \sigma_{\tau_3} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

- $|\theta_s - \theta_n| = \frac{\pi}{4}$
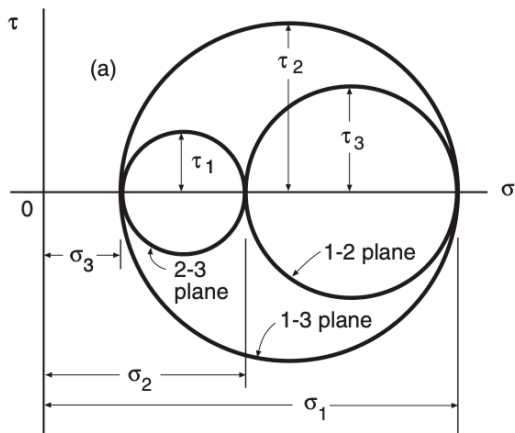
Mohr's Circle

Equation of Mohr's circle:

$$\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

- A rotation of 2θ on Mohr's circle corresponds to a rotation of θ for coordinate system in real space.
- Clockwise (CW) shear stress: positive.
- Counter Clockwise (CCW) shear stresses: negative
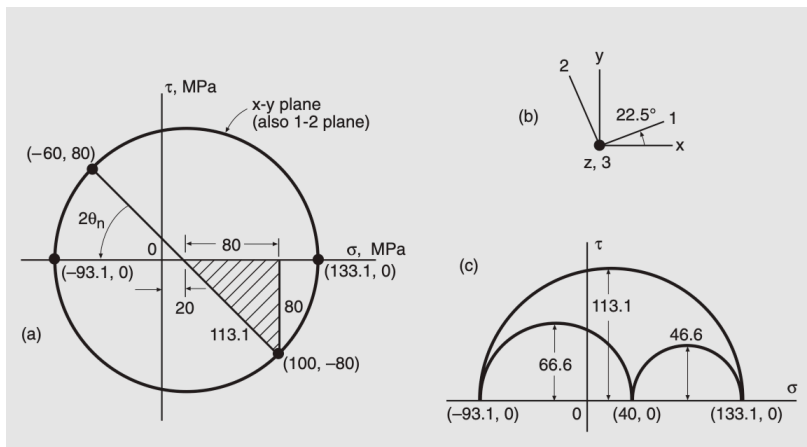
3D State of Stress Transformation

(a)

- Draw three Mohr's circles for *xy*, *yz*, and *zx* planes.
- Usually define $\sigma_1 > \sigma_2 > \sigma_3$
- The maximum shear stress for any plane in the material is the largest of the three principal shear stresses:

$$\tau_{\mathsf{max}} = \max(\tau_1, \tau_2, \tau_3) = \frac{\sigma_1 - \sigma_3}{2}$$

Example 6.3 in *Dowling*

For the following state of stress (in MPa), determine the principal normal stresses, the principal axes, and the principal shear stresses:

$$\sigma_x = 100, \sigma_y = -60, \sigma_z = 40, \tau_{xy} = 80, \tau_{yz} = \tau_{zx} = 0$$

Thank you!