



Unit 3: Algorithms

...

Recursion

Roadmap

- **What is recursion?**
- **Why use recursion?**
- **Types of Recursion**

What is Recursion?

A function is recursive if...

It calls itself within its own definition!

It calls itself within its own definition!

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It calls itself within its own definition!

Consider the following function...

```
function sumNaturalsBelow(n){  
    return n + sumNaturalsBelow(n - 1);  
}
```

Recursion, like a for/while loop, requires terminating conditions

```
// Terminating function
function sumNaturalsBelow(n){
  if (n <= 0) return 0;
  return n + sumNaturalsBelow(n - 1);
}
```

Common Traits - Iterative/Recursive

- Initialized Value
- Changing Initial Value
- Terminating Condition

```
// iterative
function sumNaturalsBelow(n) {
  let sum = 0;
  for (let i = 0; i <= n; i++) {
    sum = sum + i;
  }
  return sum;
}

// recursive
function sumNaturalsBelow(n) {
  if (n <= 0) return 0;
  return n + sumNaturalsBelow(n - 1);
}
```

**Why use
recursion?**

Recursive functions are frequently more readable, maintainable, and easier to understand

Without Recursion

```
function contains(LLHead, val) {  
  let current = LLHead;  
  while (current) {  
    if (current.value === val) return true;  
    current = current.next;  
  }  
  return false;  
}
```

Recursive functions are frequently more readable, maintainable, and easier to understand

With Recursion

```
function contains(LLHead, val) {  
    if (!LLHead) return false;  
    else if (LLHead.value === val) return true;  
    else return contains(LLHead.next, val);  
}
```

Why are recursive algorithms easier to understand?

- Non-recursive functions tend to describe **how** to get to a solution.
- Recursive solutions describe **what** the solution is.
- Writing recursive functions often forces you to write **declarative** code instead of **imperative** code

Types of Recursion

Let's Compare Three ways to write a factorial algorithm

$\text{factorial}(6) ==> 6 * 5 * 4 * 3 * 2 * 1$

Factorial: Option #1

```
function factorial(n) {  
  let product = 1;  
  while (n) {  
    product *= n;  
    n -= 1;  
  }  
  return product;  
}
```

Factorial: Option #2

```
function factorial(n) {  
  if (n === 0) return 1;  
  else return n * factorial(n - 1);  
}
```

Factorial: Option #3

```
function factorial(n, product = 1) {  
  if (n === 0) return product;  
  else return factorial(n - 1, product * n);  
}
```


Let's Compare!

- *How is option #1 different from option #2 and #3?*

```
const factorial = n => {  
  let product = 1;  
  while (n) {  
    product *= n;  
    n -= 1;  
  }  
  return product;  
}
```

```
const factorial = n => {  
  if (n === 0) return 1;  
  return n * factorial(n - 1);  
}
```

```
const factorial = (n, product = 1) => {  
  if (n === 0) return product;  
  return factorial(n - 1, product * n);  
}
```

Let's Compare!

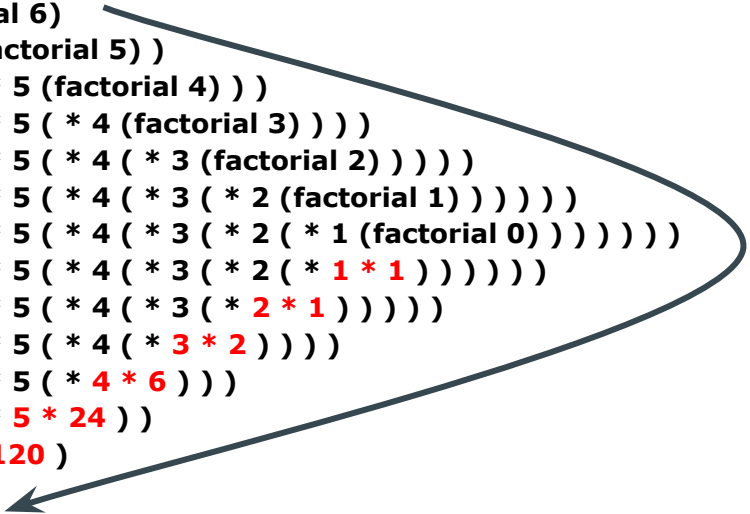
- *How are option #2 and #3 similar? Different?*

```
const factorial = n => {  
  if (n === 0) return 1;  
  return n * factorial(n - 1);  
}
```

```
const factorial = (n, product = 1) => {  
  if (n === 0) return product;  
  return factorial(n - 1, product * n);  
}
```

Linear Recursion:

(factorial 6)
(* 6 (factorial 5))
(* 6 (* 5 (factorial 4)))
(* 6 (* 5 (* 4 (factorial 3))))
(* 6 (* 5 (* 4 (* 3 (factorial 2)))))
(* 6 (* 5 (* 4 (* 3 (* 2 (factorial 1))))))
(* 6 (* 5 (* 4 (* 3 (* 2 (* 1 (factorial 0)))))))
(* 6 (* 5 (* 4 (* 3 (* 2 (* 1 * 1))))))
(* 6 (* 5 (* 4 (* 3 (* 2 * 1)))))
(* 6 (* 5 (* 4 (* 3 * 2))))
(* 6 (* 5 (* 4 * 3 * 2)))
(* 6 (* 5 (* 4 * 6)))
(* 6 (* 5 * 24))
(* 6 * 120)
= 720



```
function factorial(n) {  
  if (n === 0) return 1;  
  return n * factorial(n - 1);  
}
```

Time complexity: $O(n)$
Space complexity: $O(n)$

Linear Recursion:

```
1  function factorial(n) {  
2      if (n === 0) return 1;  
3      return n * factorial(n - 1);  
4  }  
5  factorial(6);
```

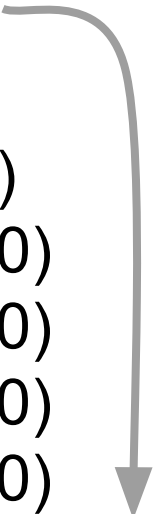
Time complexity: $O(n)$

Space complexity: $O(n)$

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Iterative Recursion (a.k.a. tail call recursion):

(factorial 6)
⇒ (factorial 5, 6)
⇒ (factorial 4, 30)
⇒ (factorial 3, 120)
⇒ (factorial 2, 360)
⇒ (factorial 1, 720)
⇒ (factorial 0, 720)
⇒ 720



```
1 function factorial(n, product = 1) {  
2   if (n === 0) return product;  
3   return factorial(n - 1, product * n);  
4 }  
5 factorial(6);
```

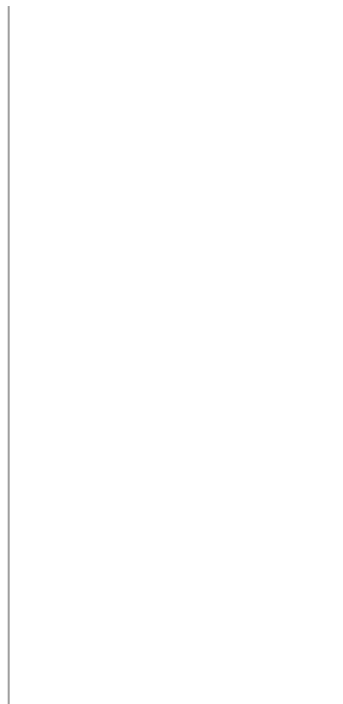
Time complexity: $O(n)$
Space complexity: $O(1)$

Iterative Recursion (a.k.a. tail call recursion):

```
1  function factorial(n, product = 1) {  
2    if (n === 0) return product;  
3    return factorial(n - 1, product * n);  
4  }  
5  factorial(6);
```

Time complexity: $O(n)$

Space complexity: $O(1)$



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ES6 support for TCO

Tail call optimization in ECMAScript 6

[2015-06-30] esnext, dev, javascript

**Recursion optimization
- where is it? PTC, FUD**

Proper Tail Calls (PTC) in JavaScript

#javascript #webdev #node #ecmascript

Published at
Jun 18 2017

Updated at
2 years ago

Reading time
2min

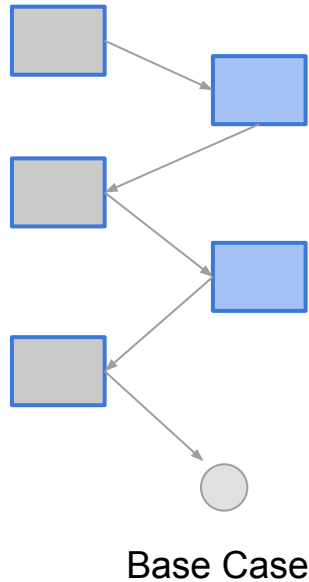
Views
264



Chris David

ECMAScript 6 Proper Tail Calls in WebKit

Other types of recursion: Mutual Recursion



```
function isEven(n) {  
    if (n === 0) return true;  
    else return isOdd(n - 1);  
}  
  
function isOdd(n) {  
    if (n === 0) return false;  
    else return isEven(n - 1);  
}
```


Other types of recursion: Mutual Recursion

JSON Parser!

Other types of recursion: Tree recursion (a.k.a multiple recursion)

Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34...

The Rule

The Fibonacci Sequence can be written as a "Rule" (see [Sequences and Series](#)).

First, the terms are numbered from 0 onwards like this:

$n =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
$x_n =$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	...

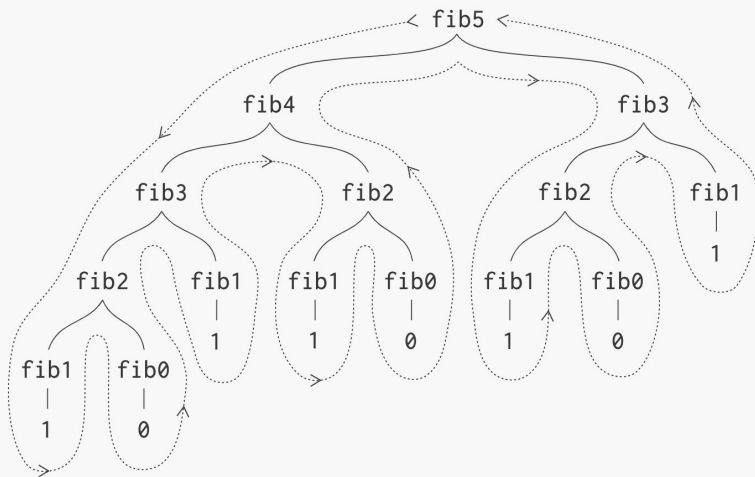
So term number 6 is called x_6 (which equals 8).

Other types of recursion: Tree recursion (a.k.a multiple recursion)

Finding the corresponding Fibonacci number at the nth term

```
function fib(n) {  
  if (n === 0) return 0;  
  else if (n === 1) return 1;  
  else return fib(n - 1) + fib(n - 2);  
}
```

Other types of recursion: Tree recursion (a.k.a multiple recursion)



```
function fib(n) {
  if (n === 0) return 0;
  else if (n === 1) return 1;
  else return fib(n - 1) + fib(n - 2);
}
```

Finding the corresponding Fibonacci number at the nth term

Summary

- A function is recursive if it calls itself in its own definition.
- Recursion allows us to write **declarative** code, rather than **imperative** code.
- Not all browsers implement Tail Call Optimization (TCO)
- Types of recursion:
 - Linear recursion
 - Tail recursion
 - Mutual recursion
 - Tree recursion

Further Reading

- [Recursion in functional JavaScript](#) (SitePoint)
- [Recursion \(JavaScript\)](#) (Microsoft)