Kellogg School of Management
Northwestern University
Finance 520, Viktor Todorov, Winter 2018

Problem Set 1

This problem set is due in class on Wednesday January 15th, 2018

Problem 1 Suppose that the sequences $\{X_t\}$ and $\{Y_t\}$ are independent of each other i.i.d. with $\mathbb{P}(X_t = 0) = \mathbb{P}(X_t = 1) = 1/2$ and $\mathbb{E}(Y_t) = 0$ and $\mathbb{E}|Y_t|^p < \infty$ for any $p > 0$. Is the process

$$Z_t = X_t(1 - X_{t-1})Y_t$$

white noise? Is it i.i.d.?

Problem 2 Consider the ARMA(2,5) model:

$$y_t = 0.8y_{t-1} - 0.1y_{t-2} + \epsilon_t + 0.9\epsilon_{t-1} - 0.8\epsilon_{t-4} - 0.1\epsilon_{t-5}$$

where ϵ_t is an i.i.d. white noise process with mean zero and variance equal to one.

(a) Is this ARMA process for y_t covariance stationary?

(b) Is this ARMA process for y_t invertible?

(c) Calculate the first 50 autocovariances for y_t.

(d) Calculate and plot the first 50 autocorrelations for y_t.

(e) Suppose that you have data on y_t for t = n, n-1, n-2, n-3. Specifically,

$$y_n = 2.0, \quad y_{n-1} = 1.7, \quad y_{n-2} = 0.5, \quad y_{n-3} = -0.2.$$

Calculate the minimum MSE forecast for y_{n+1}.

(f) What is the variance of the forecast error that you calculated in part (e)?

(g) Suppose now that you have data for y_t for t = n, n-1, ..., n-N, where N is a large positive integer. Briefly discuss how you would forecast y_{n+1} based on this expanded information set. What would be the variance of the forecast error associated with this forecast for y_{n+1}?