Logic

1. Propositional logic（命题逻辑）

1. $p \to q = (\neg p) \vee q$ （useful law）
2. $p \to q$ is read in a variety of equivalent ways:
    1. q unless ¬p
    2. p only if q
3. $w \wedge (u \vee v) = (w \wedge u) \vee (w \wedge v)$
   $w \vee (u \wedge v) = (w \vee u) \wedge (w \vee v)$
4. The statement is false: This is not a proposition
5. conditional statement:
   The convers of $p \to q$ is $q \to p$
   The contrapositive of $p \to q$ is $\neg q \to \neg p$
   The inverse of $p \to q$ is $\neg p \to \neg q$
6.

2. Predicates and Quantifiers(谓词和量词)

1. predicate(谓语)

Predicate 是用来表示$P(x_1, x_2, \ldots\ldots, x_n)$在取什么的时候值，使得$P(x_1, x_2, \ldots\ldots, x_n)$ =T or F

简单来说谓词就是人是可以行走的中的“可以行走的”这个元素

domain 是域内，表示x_i的取值范围

truth set是所有的使得$P(x_1, x_2, \ldots\ldots, x_n) = True$的集合$set(x_i)$

An Example :

Let $Q(x, y)$ be the predicate “$x = y + 3$” with domain of the real numbers.

1. What are the truth values of $Q(1, 2)$ and $Q(3, 0)$?
   $Q(1, 2) = \mathrm{F}$, $Q(3, 0) = \mathrm{T}$
2. What is the truth set of $Q(x, y)$?
   $(a, a - 3)$ for all real numbers a

2. Compound Statements in Predicate Logic(谓词逻辑中的复合陈述句)

An Example:函数和逻辑运算符的组合模式↓

Compound statements are obtained via logical connectives.

$P(x)$: x is a prime
$Q(x)$: x is an integer

- $P(2) \wedge P(3)$: Both 2 and 3 are primes. (T)
- $P(2) \wedge Q(2)$: 2 is a prime or an integer. (T)
- $Q(x) \rightarrow P(x)$: If x is an integer, then x is a prime.

4. Quantifiers (量词)

$$\text{Propositional function } P(x) \xrightarrow{\text{for all/some } x \text{ in domain}} \text{Proposition}$$

相较于之前，限定数量，∀, ∃等等

Two types of quantified statements:

- Universal quantifier $\forall x P(x)$

- Existential quantifier $\exists x P(x)$

5. Negation for Quantifiers（量词的否定）

1. $\neg(\forall x P(x)) \equiv \exists x(\neg P(x))$

   所有反证法的逻辑化表达

2.

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x\ P(x)$	$\forall x\ \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg \forall x\ P(x)$	$\exists x\ \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

6. Order of Quantifiers(量词的顺序)

1. 当符号是相同的时候，顺序可以交换，否则不行 $\exists x \exists y P(x,y) \equiv \exists y \exists x P(x,y)$: $\forall x \forall y P(x,y) \equiv \forall y \forall x P(x,y)$:

7. Negating Nested Quantifiers(否定嵌套量词)

1. Note: $\neg(\forall x P(x)) \equiv \exists x(\neg P(x))$, $\neg(\exists x P(x)) \equiv \forall x(\neg P(x))$
2. $$\begin{aligned} & \neg\forall x \exists y\ (xy = 1) \\ \equiv\ & \exists x \neg\exists y\ (xy = 1) \\ \equiv\ & \exists x \forall y\ \neg(xy = 1) \\ \equiv\ & \exists x \forall y\ (xy \neq 1) \end{aligned}$$

守则就是顺着not过去，一直到not（$'\neg'$）的符号消失

8. Universal Quantifiers (全称量词)

The universal quantification of $P(x)$ is the statement

$P(x)$ for all values of x in the domain.

The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. We read $\forall x P(x)$ as "for all $xP(x)$" or "for every $xP(x)$."

（注意任何的判断都必须有domain,否则没法去判断T,F）

1. '∀' is the key to show it's universal quantifiers
2. domain is important

9.Existential Quantifiers (全称量词)

1. '∃' is the key to show it's universal quantifiers
2. domain is important

10. Translation

1. The sum of two positive integers is always positive.
   - $P(x, y)$: $(x > 0) \wedge (y > 0)$
   - $Q(x, y)$: $x + y > 0$
   - Domain of x and y: all integers
   - $\forall x \forall y (P(x, y) \rightarrow Q(x, y))$
   - Or, we can write it as $\forall x \forall y ((x > 0) \wedge (y > 0) \rightarrow x + y > 0)$
2. Every real number except zero has a multiplicative inverse.
   - Domain of x: all real numbers
   - $\forall x ((x \neq 0) \rightarrow \exists y (xy = 1))$

11. Argument

An argument form in propositional logic is a sequence of compound propositions involving propositional variables.

- p: "You have a current password"
- q: "You can log onto the network" or "You can change your grade"

$$\begin{array}{rl} & p \rightarrow q \\ & p \\ \hline \therefore & q \end{array}$$

- **modus tollens** 否定后件式

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

corresponding tautology:
$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

- **hypothetical syllogism** 假言三段论

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

corresponding tautology:
$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

- **disjunctive syllogism** 选言三段论

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

corresponding tautology:
$(\neg p \wedge (p \vee q)) \rightarrow q$

- **Addition**

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$$

corresponding tautology:
$p \rightarrow (p \vee q)$

- **Simplication**

$$\begin{array}{c} p \wedge q \\ \hline \therefore q \end{array}$$

corresponding tautology:
$(p \wedge q) \rightarrow p$

- **Conjunction**

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

corresponding tautology:
$((p) \wedge (q)) \rightarrow (p \wedge q)$

- **Resolution**

$$\begin{array}{c} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

corresponding tautology:
$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

- **Universal Instantiation** (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

- **Universal Generalization** (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

- **Existential Instantiation** (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

- **Existential Generalization** (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$