Parametric Representation and Basics in Plane

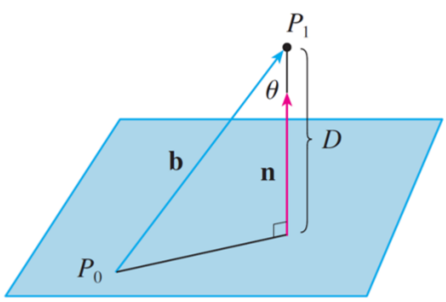
1. $x = p + tv$,p and v is a three dimensional vector(平面的情况)
2. $x = p + c_1 \cdot v_1 + c_2 \cdot v_2$
3. if $x_1 + x_2 + x_3 = 1$, find parametric representation

   Answer :$(x_1 - 1) + x_2 + x_3 = 0$,the start point, p= $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $v_{vertical} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$,all we have to do is to find two orthogonal vectors. （只要找到是与 $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ 垂直的两个向量）

   $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ 和 $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$,确保是正交
4. **Projection** : to find the projection of vector a on vector b\

   The $Proj_v = \frac{b \cdot a}{a \cdot a} \cdot a$
5. **The distance** :

   The distance of a dot from a plane can be described as



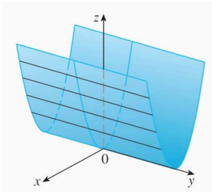
   The$D = |\frac{\vec{n} \cdot \overrightarrow{P_0P_1}}{\vec{n}}| = |\frac{\vec{n} \cdot \overrightarrow{OP_0} - \vec{n} \cdot \overrightarrow{OP_1}}{\vec{n}}| = \frac{|(ax_1 + by_1 + cz_1) - (ax_0 + by_0 + cz_0)|}{\sqrt{a^2 + b^2 + c^2}}$

6. 若 $ax_0 + by_0 + cz_0 + d = 0$. $ax_0 + by_0 + cz_0 + d = 0$, $D = |\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}|$

Cylindrical surfaces(圆柱面)

1. **The definition** : cylinder is a surface that consists of all lines (called rulings) that are parallel to a given line and pass through a given plane curve. 数条平行的直线围绕着一个曲线进行移动的曲面，称为cylinder
2. **Expression**: $f(x, y, z) = Ax^2 + By^2 + Cz^2 + Dx + Ey + Gz + H = 0,$

3. **parabolic cylinder(抛物柱)**:

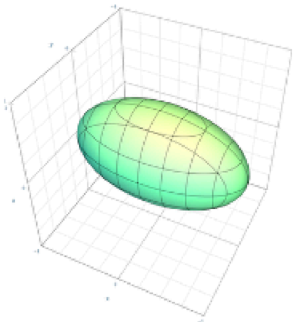
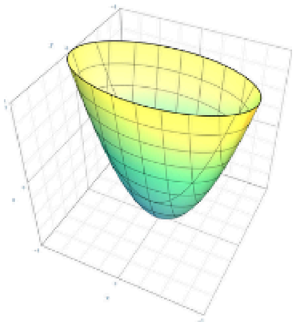
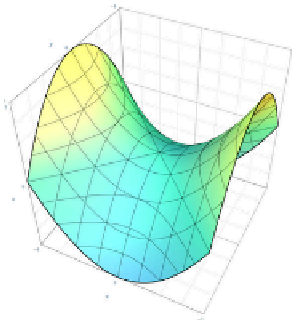
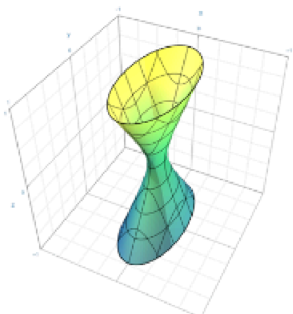
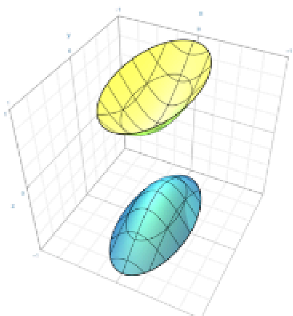


The surface $z = x^2$ is a parabolic cylinder

Figure 1

Quadric surfaces(二次曲面)

1. **Definition and Expression** : A quadric surface is the graph of a second-degree equation in three variables x, y, and z. The most general such equation is $Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+HY+Iz+J=0$
2. By rotation and transition,it can be sth. like $Ax^2+By^2+Cz^2+I=0$ or $Ax^2+By^2+I=0$
3. **Classical Quadrics**:

Non-degenerate real quadric surfaces		
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	
Elliptic paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$	
Hyperbolic paraboloid	$\frac{x^2}{a^2}-\frac{y^2}{b^2}-z=0$	
Hyperboloid of one sheet or Hyperbolic hyperboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	
Hyperboloid of two sheets or Elliptic hyperboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$	

Supplementary

Lines:

$$t = \frac{x - x_0}{a} \quad t = \frac{y - y_0}{b} \quad t = \frac{z - z_0}{c}$$

Planes:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

The point that is passed by the planes or the lines is (x_0, y_0, z_0)

For the plane , the $\overrightarrow{n} = (a, b, c)$

Distance between skew lines:

$D = \frac{|(\mathbf{b}-\mathbf{a})\cdot(\mathbf{u_1}\times\mathbf{u_2})|}{|\mathbf{u_1}\times\mathbf{u_2}|}$。

First line： $\mathbf{r_1} = \mathbf{a} + \lambda_1 \mathbf{u_1}$

Second line： $\mathbf{r_2} = \mathbf{b} + \lambda_2 \mathbf{u_2}$

其中：

- $\mathbf{a}$ and $\mathbf{b}$ is a position that is known；
- $\mathbf{u_1}$ and $\mathbf{u_2}$ are two direction vectors ；
- λ_1 and λ_2 are parameters
- $\mathbf{u_1} \times \mathbf{u_2}$ the cross product can be understood as the $\overrightarrow{n}$ that is perpendicular to the given plane
- $(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{u_1} \times \mathbf{u_2})$ is the projection