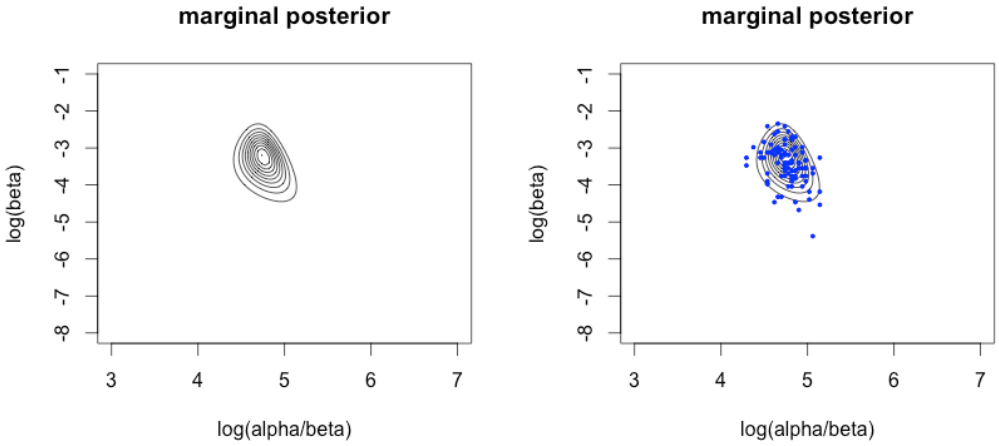
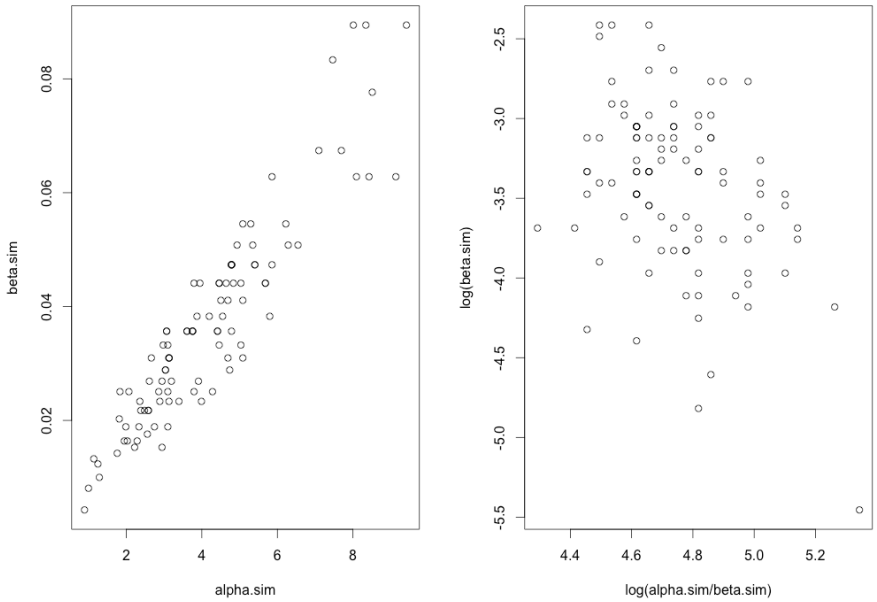
5.14

(a)

(b)

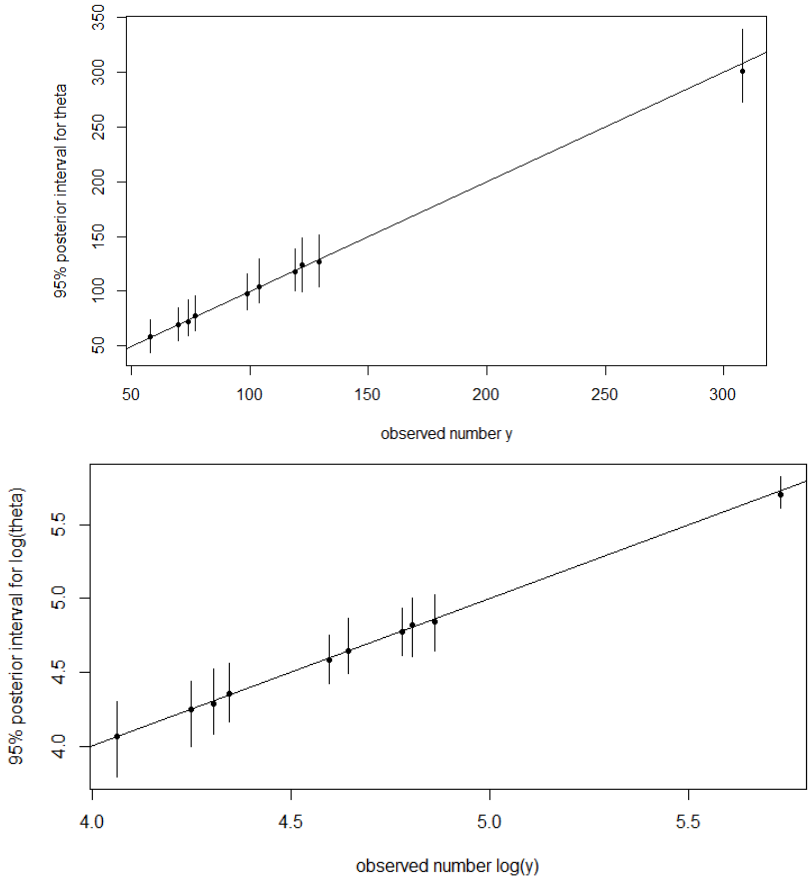

marginal posterior
log(beta)
log(alpha/beta)
marginal posterior
log(beta)
log(alpha/beta)


beta.sim
alpha.sim
log(beta.sim)
log(alpha.sim/beta.sim)

```
y <- c(16+58, 99, 58, 70, 19+103, 77, 18+86, 17+112, 35+273,
55+64)
n <- 100
log_alpha_over_beta <- seq(3,7, length.out=n)
log_beta <- seq(-8,-1,length.out=n)
log.m.post <- matrix(0,n,n)
phi<-0.001

     for (i in 1:n){
           for (j in 1:n){
                 beta <- exp(log_beta[j])
                 alpha <- exp(log_alpha_over_beta[i]) * b

                     su <- 0
                     for (s in 1:10){
                           su <- su + alpha * log(beta) - (alpha
+ y[s]) * log(beta+1) + lgamma(alpha + y[s]) - lgamma(alpha)
                          }

                          log.m.post[i,j] <- phi * (log(alpha *
beta) - alpha - beta) + su
                       }
         }
 log.m.post <- log.m.post - max(log.m.post)
 m.post <- exp(log.m.post)
 contour(log_alpha_over_beta, log_beta, m.post,drawlabels=F,
           xlab="log(alpha/beta)", ylab="log(beta)")
  m <- 100
  alpha.sim<-rep(0, m)
  beta.sim<-rep(0, m)
  log_alpha_over_beta.sim <- rep(0, m)
  log_beta.sim <- rep(0, m)
  dev.m.post <- rowSums(m.post)

      for (s in 1:m){
            i<-sample(m,1,prob=dev.m.post)
            j<-sample(m,1,prob=m.post[i,])
            log_alpha_over_beta.sim[s]<-log_alpha_over_beta[i]
            log_beta.sim[s] <- log_beta[j]
            beta.sim[s] <- exp(log_beta[j])
            alpha.sim[s] <- exp(log_alpha_over_beta.sim[i]) *
beta.sim[s]

              }
```

```
points(log_alpha_over_beta.sim, log_beta.sim, pch=20, cex=0.6,
col=4)

par(mfrow=c(1,2))
plot(beta.sim ~ alpha.sim)
plot(log_beta.sim ~ log_alpha_over_beta.sim)
```

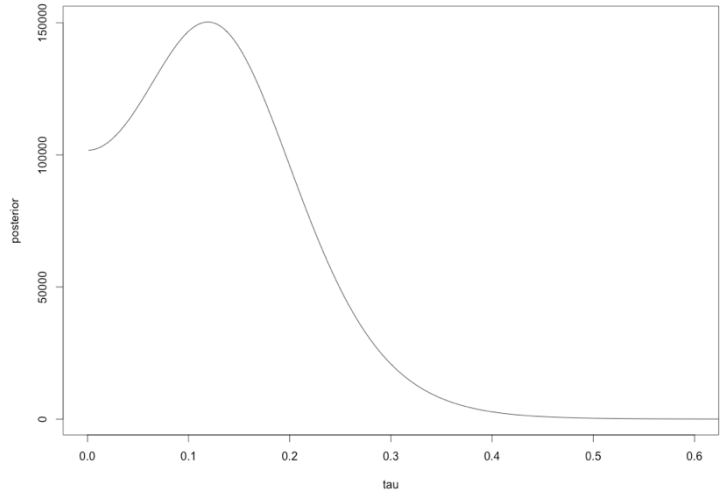
(e)




```
theta<-matrix(0,100,10)
for (i in 1:100){
      for (j in 1:10){
            theta[i,j]<-rgamma(1,a.sim[i]+y[j],b.sim[i]+1)
          }
    }

med<-apply(theta,2,median)
conf<-function(a){
      quantile(a,c(0.025,0.975))
    }
```

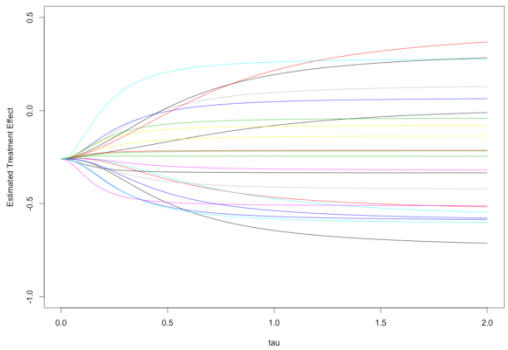
```
ci<-apply(theta,2,FUN=conf)

    plot(med~y,type='p',pch=20,ylim=c(min(ci),max(ci)),
             xlab='observed rate',ylab='95% posterior interval')
for (i in 1:10){
      lines(x=c(y[i],y[i]),y=c(ci[1,i],ci[2,i]))
    }
abline(a=0,b=1)
plot(log(med)~log(y),type='p',pch=20,ylim=c(log(min(ci)),log(max
(ci))), xlab='observed number log(y)',ylab='95% posterior
interval for log(theta)')
for (i in 1:10){

lines(x=c(log(y[i]),log(y[i])),y=c(log(ci[1,i]),log(ci[2,i])))
    }
abline(a=0,b=1)
```

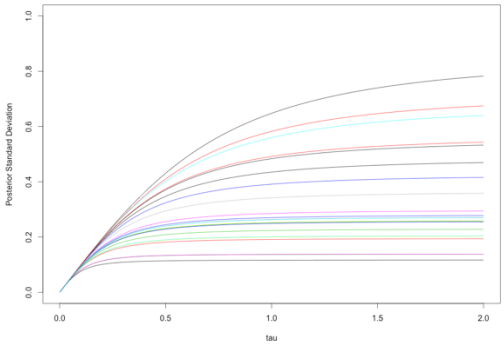
5.15
(a)




```
x <- read.table("meta.asc.txt", head = T)
x <- x[,-1]
y <- -log(x[,1]/(x[,2]-x[,1]))+log(x[,3]/(x[,4]-x[,3]))
sigma <- sqrt(1/x[,1]+1/(x[,2]-x[,1])+1/x[,3]+1/(x[,4]-x[,3]))

tau <- seq(0.001,2.0,0.001)
posterior <- vector()
mu <- vector()
for(i in 1:length(tau))
{
  mu.est <- sum(y / (sigma^2 + tau[i]^2)) / sum(1 / (sigma^2 +
tau[i]^2))
  v.mu <- 1 / sum(1 / (sigma^2 + tau[i]^2))
  prior <- 1
  mu[i] <- rnorm(1, mean = mu.est, sd = sqrt(v.mu))
  posterior[i] <- prior * sqrt(v.mu) * prod((sigma^2 + tau[i]^2)
^ (-0.5) * exp(-(y - mu.est) ^ 2 / (2 * (sigma^2 + tau[i]^2))))
}

plot(tau, posterior, type='l', xlim=c(0, 0.6))
```

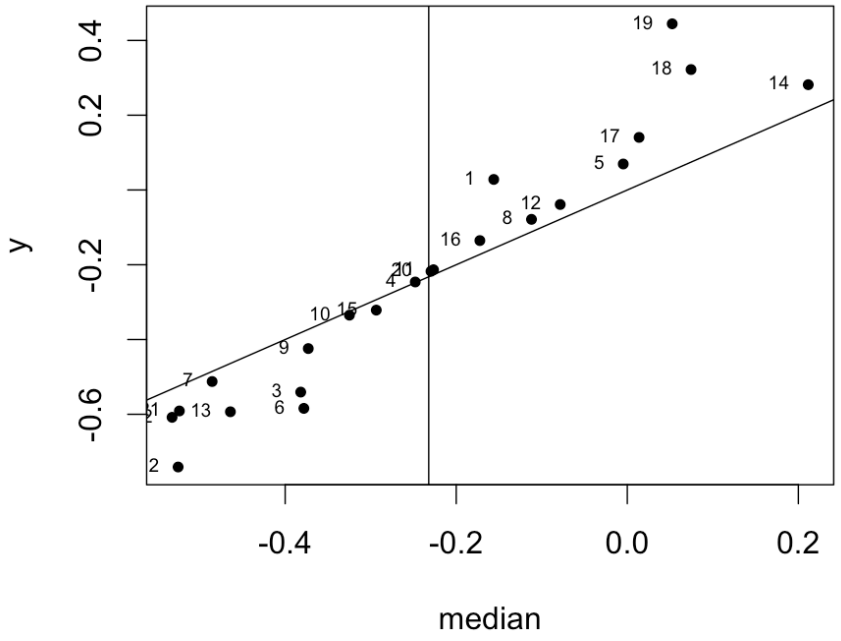
(b)





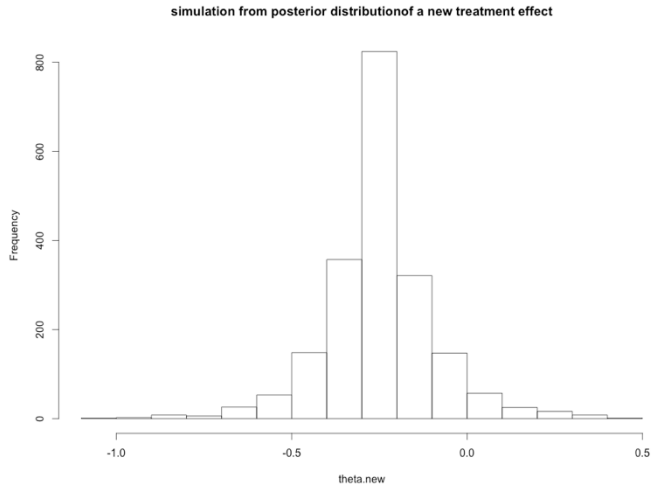

```
theta <- matrix(rep(NA, length(y) * length(tau)), length(y),
length(tau))
theta.mean <- matrix(rep(NA, length(y) * length(tau)),
length(y), length(tau))
theta.sd <- matrix(rep(NA, length(y) * length(tau)), length(y),
length(tau))
for(i in 1:length(y))
  for(j in 1:length(tau))
  {
    {
      mu <- sum(y / (sigma^2 + tau[j]^2)) / sum(1 / (sigma^2 +
tau[j]^2))
      temp_mean <- (y[i] / sigma[i]^2 + mu / tau[j]^2) / (1 /
sigma[i]^2 + 1 / tau[j]^2)
      v<-1/(1/sigma[i]^2+1/tau[j]^2)
      theta[i,j] <- rnorm(1,mean=temp_mean,sd=sqrt(v))
      theta.mean[i,j] <- temp_mean
      theta.sd[i,j] <- sqrt(v)
    }
  }

plot(tau,theta.mean[1,], type='l', ylim=c(-
1,0.5),ylab="Estimated Treatment Effect")
for(i in 1:21)
{
  lines(tau,theta.mean[(i+1),], col=i)
}

plot(tau,theta.sd[1,],type='l', ylim=c(0,1),ylab="Posterior
Standard Deviation")
for(i in 1:21)
{
  lines(tau,theta.sd[(i+1),],col=i)
}
```

(c)




```
median<-apply(theta,1,median)

plot(median,y,pch=20)
text(median,y,1:J,pos=2,cex=0.6)
abline(a=0,b=1)
abline(v=mean(theta))
```

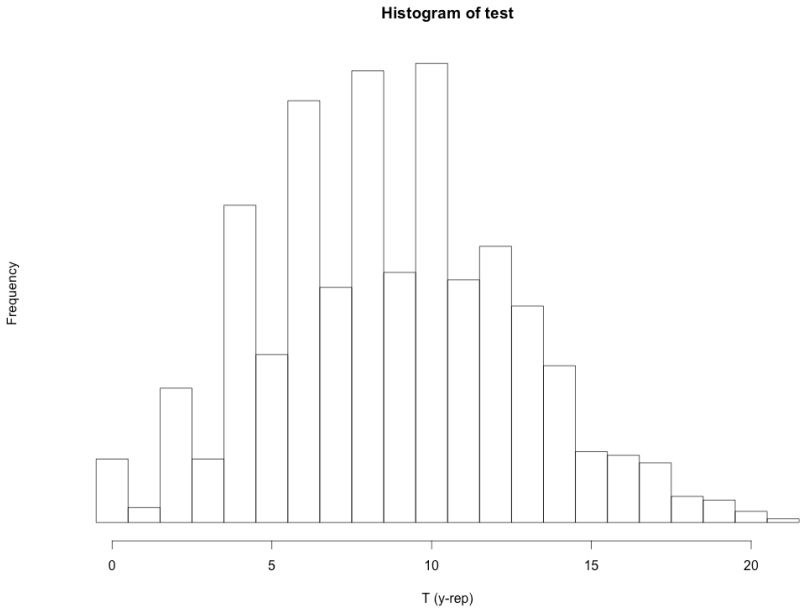
(d)




```
theta.new<-vector()
for(i in 1:length(tau)){
  foo<-sample(length(posterior),1,prob=posterior)
  tau.new<-tau[foo]
  mu.new<-sum(y/(sigma^2+tau.new^2))/sum(1/(sigma^2+tau.new^2))
  theta.new[i]<-rnorm(1,mean=mu.new,sd=tau.new)
}
hist(theta.new,main="simulation from posterior distributionof a
new treatment effect",breaks=20)
```

6.6
(a)

(b)

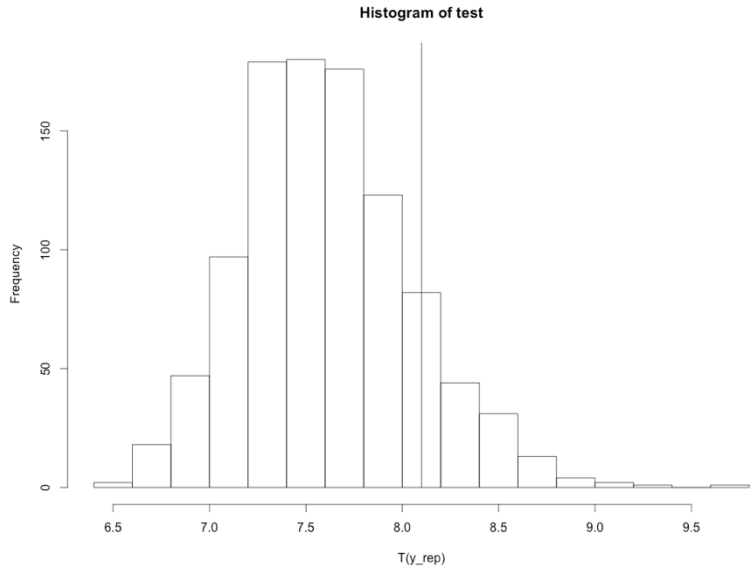


```
test<-vector()
for(i in 1:1000){
  theta <- rbeta (1, 8, 14)
  y.rep <- rbinom (1, 1, theta)
  while(sum(y.rep==0) < 13){
    y.rep <- c(y.rep, rbinom(1, 1, theta))
  }
  n.rep <- length(y.rep)
  test[i] <- sum(y.rep[2:n.rep] != y.rep[1:(n.rep-1)])
}
hist (test, xlab="T (y-rep)", yaxt="n",
      breaks=seq(-.5, max(test) + .5), cex = 2)
```

6.7

(a)

```
> mean(test > 8.1)
[1] 0.138
```




```
test <- vector()
for (i in 1:1000){
  theta <- rnorm (1,5.1,0.1)
  y.rep <- rnorm (100,theta,1)
  test[i] <- max(abs(y.rep))
```

```
}
hist(test, breaks = 20, xlab = "T(y_rep)")
lines(rep(8.1,2), c(0,1000))
mean(test > 8.1)
```

(b)

(c)