Name:

Instructions

1. Write your name at the top of the *first* page and your initials at the bottom of *every* page.
2. Do *not* staple the exam.
3. Return the exam with *all* the pages, arranged in *ascending* order.
4. This is a closed-book exam.
5. No electronic devices are permitted.
6. You may use the blank spaces for any scratch work.
7. Discussing the exam before the solutions have been posted is a violation of the Honor Code.
8. There are 9 problems on this exam and you have 75 minutes to answer them.
9. Problems 1 – 7 involve 19 multiple-choice questions, each worth 4 points. Each question must have *exactly one* response clearly marked in the circle provided — or else your answer will be marked incorrect.
10. Problems 8 (worth 16 points) and 9 (worth 8 points), must be answered clearly in the boxed space provided for those problems.

Problem 1. Insert the following keys in that order into a maximum-oriented heap-ordered binary tree:

`B Z Q K V F S N I`

a. What is the key with index 1?

○ `B`

○ `I`

○ `Z`

○ `N`

○ `F`

b. What is the key with index 6?

○ Q

○ F

○ S

○ K

○ N

c. If we perform a `delMax()` operation on the tree, what is the key that will replace the current maximum before it is sunk down?

○ F

○ B

○ I

○ Q

○ K

Problem 2. Consider inserting the following key-value pairs in that order into a symbol table `st`.

key:	R	Q	J	G	L	R	M	I	Q	H	R	V
value:	1	2	3	4	5	6	7	8	9	10	11	12

a. What is the value returned by `st.size()`?

○ 12

○ 11

○ 8

○ 9

○ 10

b. What is the value returned by `st.get("R")`?

○ 6

○ 11

○ 3

○ 18

○ 1

Problem 3. Consider inserting the following keys (assume values to be non `null` and arbitrary) into a binary search tree (BST) symbol table `st`, an object of type `BST`.

```
G   T   J   Q   H   Z   K   A   O   C   M   B
```

a. What is the height of the BST (assume root to be at height 0)?

○ 6

○ 5

○ 7

○ 4

○ 8

b. What is the value returned by `st.rank("M")`?

○ 5

○ 8

○ 7

○ 6

○ 4

c. What is the order in which the keys are visited if we traverse the BST in pre-order?

○ A B C G H J K M O Q T Z

○ G A C B T J H Q O K Z M

○ B C A H M O K Q J Z T G

○ G A C B T J H M K Z O Q

○ G A C B T J H Q K O M Z

d. What is the order in which the keys are visited if we traverse the BST in in-order?

○ A B C G H J K M O Q T Z

○ A B C G H J K M Z Q O T

○ A B C G H J K Q Z T O M

○ B C A H M O K Q J Z T G

○ G A C B T J H Q K O M Z

e. What is the order in which the keys are visited if we traverse the BST in post-order?

○ B C A H M O K Z Q J G T

○ B C A H M O K J G Q Z T

○ A B C G H J K M O Q T Z

○ B C A H M O K Q J Z T G

○ G A C B T J H Q K O M Z

Problem 4. Consider inserting the following keys into an initially empty 2-3 search tree.

```
B   Q   P   F   N   W   G   J   L   H   U   X
```

a. What is the height of the tree that results (assume root to be at height zero)?

○ 3

○ 5

○ 2

○ 4

○ 1

b. How many 2-nodes does the tree contain?

○ 6

○ 5

○ 3

○ 7

○ 4

c. How many 3-nodes does the tree contain?

○ 4

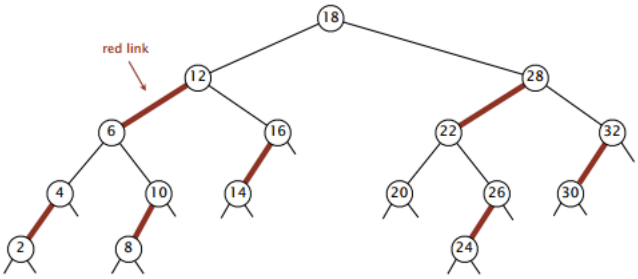
○ 6

○ 5

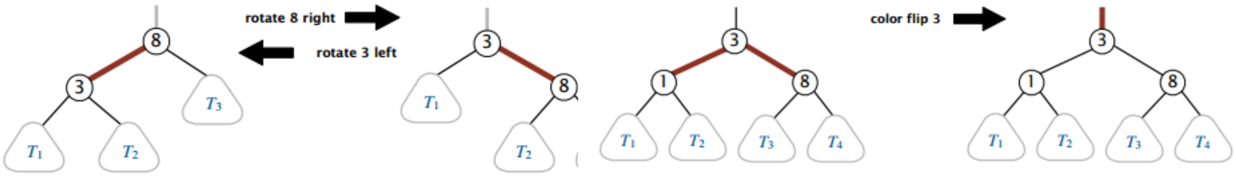
○ 3

○ 7

Problem 5. Suppose you insert the key 9 into the following left-leaning red-black BST:



Allowed operations (rotations and color flip):



a. What is the *third* operation that results?

○ `Rotate 8 left`

○ `Rotate 12 right`

○ `Rotate 10 right`

○ `Rotate 6 left`

○ `Color flip 9`

b. What is the *fifth* operation that results?

○ `Rotate 12 right`

○ `Rotate 6 left`

○ `Rotate 10 right`

○ `Color flip 9`

○ `Rotate 8 left`

Problem 6. Consider inserting the following keys (assume values to be non `null` and arbitrary) into an initially empty hash table of $M = 5$ lists, using separate chaining. Use the hash function $h(k) = k \bmod M$ to transform the kth letter of the alphabet into a table index, where $1 \leq k \leq 26$.

`J D W E V U L P F K X Y`

a. What is the length of the longest chain?

○ `1`

○ `4`

○ `3`

○ `5`

○ `2`

b. Which of the following keys is in the longest chain?

○ `O`

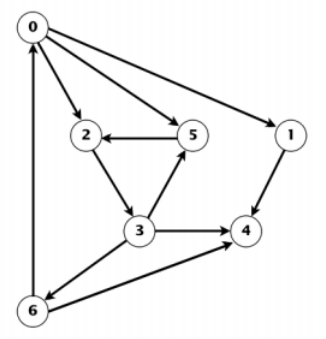
○ `V`

○ `J`

○ `W`

○ `U`

Problem 7. Consider the digraph shown below. Assume that, in the internal representation, all vertices appear in ascending order in each adjacency list.



a. Do a breadth-first search with 0 as the source vertex, and list the order in which vertices are processed by the algorithm?

○ `0 1 4 5 2 3 6`

○ `0 1 2 5 4 3 6`

○ `0 1 3 2 4 5 6`

○ `0 1 2 5 3 4 6`

○ `0 1 2 3 6 4 5`

b. Do a depth-first search with 0 as the source vertex, and list all vertices in pre-order.

○ `0 1 5 6 4 2 3`

○ `0 1 2 3 5 6 4`

○ `0 1 4 2 3 6 5`

○ `0 1 3 5 6 4 2`

○ `0 1 4 2 3 5 6`

Problem 8. (16 points) Design an efficient data structure called `ThreadedSet` to store a *threaded set of strings*, which maintains a set of strings (no duplicates) and the order in which the strings were inserted, according to the following API:

	Constructor/method	**Description**
a. (6 points)	`ThreadedSet()`	create an empty threaded set
b. (3 points)	`void add(String s)`	add the string *s* to the set (if it is not already in the set)
c. (3 points)	`boolean contains(String s)`	is the string *s* in the set?
d. (3 points)	`String previousKey(String s)`	the string added to the set immediately before *s* (`null` if *s* is the first string added; throw `java.util.NoSuchElementException` if *s* is not in the set)

Here is an example:

```
    ThreadedSet set = new ThreadedSet();
    set.add("aardvark");          // { "aardvark" }
    set.add("bear");              // { "aardvark", "bear" }
    set.add("cat");               // { "aardvark", "bear", "cat" }
    set.add("bear");              // { "aardvark", "bear", "cat" }
                                  // (adding a duplicate key has no effect)
    set.contains("bear");         // true
    set.contains("tiger");        // false
    set.previousKey("cat");       // "bear"
    set.previousKey("bear");      // "aardvark"
    set.previousKey("aardvark"); // null
```

Your answer will be graded on correctness, efficiency, and clarity. You may use data types that we have considered in this course.

```
import edu.princeton.cs.algs4.*;
import java.util.NoSuchElementException;

public class ThreadedSet {
    // Instance variables.
```

```
    public ThreadedSet() {
```

```
    }
    public void add(String s) {

    }

    public boolean contains(String s) {

    }

    public String previousKey(String s) {

    }
}
```

e. (1 point) Under reasonable technical assumptions, what is the order of growth of each of the methods as a function of the number of keys N in the data structure? Assume that the length of all strings is bounded by a constant.

	1	$\log N$	$\sqrt{N}$	N	$N \log N$	N^2
`add()`	○	○	○	○	○	○
`contains()`	○	○	○	○	○	○
`previousKey()`	○	○	○	○	○	○

Problem 9. a. (6 points) Given two integer arrays `a[]` and `b[]`, find an integer that appears in both arrays (or report that no such integer exists). Let m and n denote the lengths of `a[]` and `b[]`, respectively, and assume that $m \leq n$.

b. (2 points) What is the order of growth of the worst case running time of your algorithm?