**Name:**

**Instructions**

1. Write your name at the top of the *first* page and your initials at the bottom of *every* page.
2. Do *not* staple the exam.
3. Return the exam with *all* the pages, arranged in *ascending* order.
4. This is a closed-book exam.
5. No electronic devices are permitted.
6. You may use the blank spaces for any scratch work.
7. Discussing the exam before the solutions have been posted is a violation of the Honor Code.
8. There are 13 problems on this exam and you have 180 minutes to answer them.
9. Problems 1 – 11 involve 19 multiple-choice questions, each worth 4 points. Each question must have *exactly one* response clearly marked in the circle provided — or else your answer will be marked incorrect.
10. Problems 12 (worth 16 points) and 13 (worth 8 points), must be answered clearly in the boxed space provided for those problems.

**Problem 1.** Consider the following recursive method:

```
public static int mystery(int a, int b) {
    if (b == 0) {
        return 1;
    }
    return a * mystery(a, b - 1);
}
```

a. What does `mystery(3, 5)` return?

- ○ 125
- ○ 5
- ○ 15
- ○ 3
- ● 243

b. What does `mystery(a, b)` return in general about $a$ and $b$?

- ○ $a$
- ● $a^b$
- ○ $b$
- ○ $b^a$
- ○ $ab$

**Problem 2.** Consider the following table, which gives the running time $T(N)$ in seconds for a program for various values of the input size $N$:

| $N$ | $T(N)$ |
|---|---|
| 1000 | 3 |
| 2000 | 24 |
| 4000 | 192 |
| 8000 | 1536 |

What is the order of growth of $T(N)$?

- ○ Linearithmic
- ○ Linear
- ○ Quadratic
- ● Cubic
- ○ Logarithmic

**Problem 3.** What does the following code fragment write when $n$ is 113?

```
Stack<Integer> s = new Stack<Integer>();
while (n > 0) {
    s.push(n % 2);
    n = n / 2;
}
while (!s.isEmpty()) {
    StdOut.print(s.pop());
}
```

- ○ `1110101`
- ○ `1101001`
- ○ `1010001`
- ○ `1110011`
- ● `1110001`

**Problem 4.** Suppose we use the `QuickFindUF` data structure to solve the dynamic connectivity problem with 10 sites and input pairs $(8,1)$, $(7,8)$, $(6,0)$, $(7,6)$, $(4,6)$, $(9,2)$, and $(4,1)$, arriving in that order; the code for the `union()` method in `QuickFindUF` is shown below.

```
public void union(int p, int q) {
   int pID = id[p];
   int qID = id[q];
   if (pID == qID) {
       return;
   }
   for (int i = 0; i < id.length; i++) {
        if (id[i] == pID) {
            id[i] = qID;
        }
   }
   count--;
}
```

a. What are the values in the `id` array after all the pairs are processed?

- (●) `id = {0, 0, 2, 3, 0, 5, 0, 0, 0, 2}`
- ( ) `id = {0, 1, 0, 2, 0, 0, 0, 2, 8, 0}`
- ( ) `id = {2, 0, 2, 0, 0, 0, 6, 0, 8, 0}`
- ( ) `id = {0, 0, 2, 3, 1, 5, 1, 6, 1, 2}`
- ( ) `id = {0, 0, 2, 0, 0, 2, 6, 0, 8, 0}`

b. What is the identifier of the largest component?

- ( ) 4
- ( ) 2
- ( ) 1
- ( ) 3
- (●) 0

**Problem 5.** Consider sorting an array `a[]` containing the following strings, using quick sort (shown below):

```
                    P   T   H   C   N   R   Q   X   Z   V
```

```
public static void sort(Comparable[] a) {
    sort(a, 0, a.length - 1);
}

private static void sort(Comparable[] a, int lo, int hi) {
    if (hi <= lo) return;
    int j = partition(a, lo, hi);
    sort(a, lo, j - 1);
    sort(a, j + 1, hi);
}
```

```
private static int partition(Comparable[] a, int lo, int hi) {
    int i = lo;
    int j = hi + 1;
    Comparable v = a[lo];
    while (true) {
        while (less(a[++i], v)) { if (i == hi) { break; } }
        while (less(v, a[--j])) { if (j == lo) { break; } }
        if (i >= j) { break; }
        exch(a, i, j);
    }
    exch(a, lo, j);
    return j;
}
```

a. What is the final (destination) index of the pivot element `P` after the first call to `partition()`?

- (●) 3
- ( ) 2
- ( ) 1
- ( ) 4
- ( ) 5

b. What is pivot element in the next call to `partition()`?

- ( ) T
- ( ) P
- ( ) H
- ( ) N
- (●) C

**Problem 6.** Insert the following keys in that order into a minimum-oriented heap-ordered binary tree:

C Y V J P G Q F H

If we perform a `delMin()` operation on the tree, what is the key that will replace the current minimum (the key `C`) before it is sunk down?

- ( ) P
- ( ) V
- (●) J
- ( ) Y
- ( ) Q

**Problem 7.** Consider inserting the following keys (assume values to be non `null` and arbitrary) into a binary search tree (BST) symbol table `st`, an object of type `BST`.

I K C W U R F L E T N A Q Z

a. What is the height of the BST (assume root to be at height 0)?

- (●) 7
- ( ) 5
- ( ) 8
- ( ) 4
- ( ) 6

b. What is the order in which the keys are visited if we traverse the BST in pre-order?

- ( ) I C A F E K W U R T L Q Z N
- ( ) I C A F E K W U R L Z T N Q
- (●) I C A F E K W U R L N Q T Z
- ( ) I C K A F W E U Z R L T N Q
- ( ) A C E F I K L N Q R T U W Z

**Problem 8.** Consider inserting the following keys into an initially empty 2-3 search tree:

J I B Y F W V U A P L

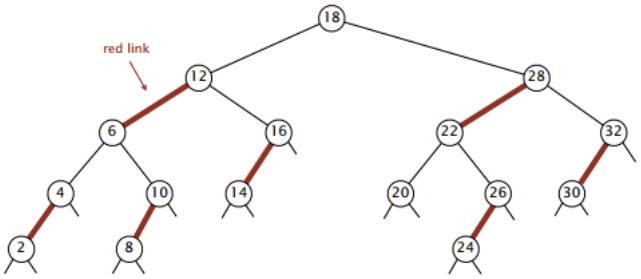
a. How many 3-nodes does the tree contain?

- ( ) 4
- ( ) 2
- ( ) 0
- (●) 1
- ( ) 3

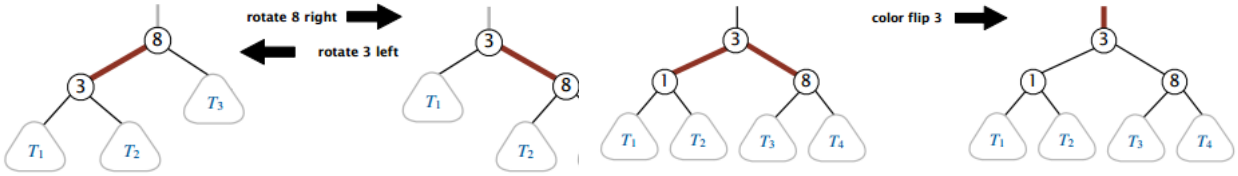
b. Which of the following keys is part of the root node?

- (●) U
- ( ) A
- ( ) F
- ( ) L
- ( ) P

**Problem 9.** Suppose you insert the key 11 into the following left-leaning red-black BST:




Allowed operations (rotations and color flip):




a. What is the *first* operation that results?

- ○ `Rotate 12 left`
- ○ `Rotate 6 left`
- ● `Color flip 10`
- ○ `Rotate 6 right`
- ○ `Rotate 12 right`

b. What is the *third* operation that results?

- ○ `Rotate 12 left`
- ● `Rotate 12 right`
- ○ `Color flip 10`
- ○ `Rotate 6 right`
- ○ `Rotate 6 left`

**Problem 10.** Consider inserting the following keys (assume values to be non `null` and arbitrary) into an initially empty hash table of $M = 5$ lists, using separate chaining. Use the hash function $h(k) = k \bmod M$ to transform the $k$th letter of the alphabet into a table index, where $1 \leq k \leq 26$.

`C J F Y K H P S Q R G U`

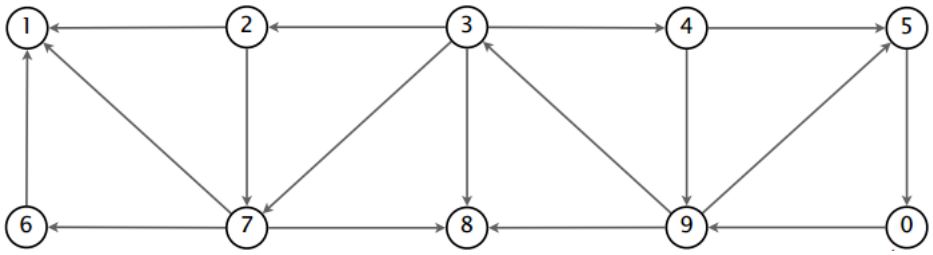
a. What is the index of the longest chain?

- ○ 3
- ○ 4
- ● 1
- ○ 2
- ○ 0

b. Which of the following keys is *not* in the longest chain?

- ○ K
- ● Q
- ○ U
- ○ P
- ○ F

**Problem 11.** Assume that the adjacency lists for the following digraph are in sorted order: for example, when iterating over the edges pointing from 3, process the edge $3 \to 2$ before either $3 \to 7$ or $3 \to 8$.




a. Do a breadth-first search with 0 as the source vertex, and list the order in which vertices are processed by the algorithm?

- ○ 0 9 3 6 1 4 5 2 7 8
- ○ 0 9 3 7 4 1 2 6 8 5
- ● 0 9 3 5 8 2 4 7 1 6
- ○ 0 9 3 2 7 8 6 5 1 4
- ○ 0 9 3 2 7 8 1 6 5 4

b. Do a depth-first search with 0 as the source vertex, and list all vertices in pre-order.

- ○ 0 9 3 2 4 6 1 5 8 7
- ● 0 9 3 2 1 7 6 8 4 5
- ○ 0 9 3 6 1 5 4 7 8 2
- ○ 0 9 3 8 2 6 4 5 1 7
- ○ 0 9 3 7 8 4 1 2 6 5

**Problem 12.** (16 points) Implement a comparable data type `Country` that represents a country, and supports the following API:

| | Method/Class | Description |
|---|---|---|
| a. (2 points) | `Country(String name, String code, long population)` | construct a country given its name, code, and population |
| b. (2 points) | `boolean equals(Country that)` | is this country's code the same as that country's code? |
| c. (2 points) | `String toString()` | a string representation in the format `"name, code, population"` |
| d. (2 points) | `int compareTo(Country that)` | comparison of this and that country by name |
| e. (2 points) | `static class ReverseCodeOrder` | a comparator for comparing two counries in reverse order of their codes |
| f. (2 points) | `static class PopulationOrder` | a comparator for comparing two counries by population |

```
import java.util.Comparator;

public class Country implements Comparable<Country> {
    private final String name;
    private final String code;
    private long population;

    public Country(String name, String code, long population) {
        this.name = name;
        this.code = code;
        this.population = population;
    }

    public boolean equals(Country that) {
        return this.code.equals(that.code);
    }

    public String toString() {
        return name + ", " + code + ", " + population;
    }

    public int compareTo(Country that) {
        return this.name.compareTo(that.name);
    }

    public static class ReverseCodeOrder implements Comparator<Country> {
        public int compare(Country x, Country y) {
            return -x.code.compareTo(y.code);
        }
    }

    public static class PopulationOrder implements Comparator<Country> {
        public int compare(Country x, Country y) {
            if (x.population < y.population) {
                return -1;
```

```
            }
            else if (x.population == y.population) {
                return 0;
            }
            else {
                return 1;
            }
        }
    }
}
```

g. (2 points) Suppose `countries` is an array of `Country` objects. Write down a statement that uses `Arrays.sort()` to sort `countries` by name.

```
Arrays.sort(countries);
```

h. (2 points) Write down a statement that uses `Arrays.sort()` to sort `countries` by population.

```
Arrays.sort(countries, new Country.PopulationOrder());
```

**Problem 13.** a. (6 points) Given an array `a` containing $N$ integers, provide a crisp and concise English description of an algorithm for finding the *farthest* pair of integers. For example, `(-1, 9)` is the farthest pair in the array `a = {4, 9, 3, -1, 6}`.

The farthest pair is $(m, M)$, where $m$ and $M$ are the minimum and maximum values in the array `a` and which can be discovered by scanning through the array.

b. (2 points) What is the order of growth of the worst case running time of your algorithm?

$N$ (linear)