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Strategy of Cycling Time Trial

Summary

At the Tokyo Olympics in 2021, Dr. Anna Kissenhoff, a mathematician, won the women's road cycling race on her own. It was strength and sound mathematical planning that contributed to her success. This also provides a new way of thinking about road cycling: using mathematical modeling to optimize the athletes' physical strength allocation. In this paper, we use mathematical and physical methods, combining theory and practice, to develop an **"Optimal Power Distribution"** model. We aim to help athletes find the most suitable power allocation solution to shorten the race time.

Model 1: Power - Velocity Model, we get the relation between the rider's power output and speed through energy conservation. Considering the rider's upward movement on a slope, the rider's energy output will be used to overcome air resistance (which is the main resistance), converted into gravitational potential energy, consumed by rolling friction, and lost as heat through the friction of the bicycle's mechanical components. Finally, we get the expression of output power and speed after reasonable approximation: $\mathrm{P} = mg\alpha \cdot v + \frac{1}{2}\rho \cdot AC_D \cdot (v - v_{windparallel})^2 v$. Meanwhile, we give the detailed model of the Sprinter, Time Specialist and Female bicyclist .

Model 2: Optimal Power Distribution Model. Firstly, we chose a three-parameter model to get the **Power Profile** and fit the power curve for three kinds of chosen riders. Then, we give the models of situations of curve, climbing, downhill, departure and Sprint. When climbing, output power will increase in order to the velocity doesn't drop too much. When descending, output power will decrease to keep a safe velocity. And we give the most effective cornering strategy for turning a corner. All of that will give the riders an indication to help them win the race. Finally, we give the optimal power allocation strategy for three types of riders based on specific sites.You can see the optimal power distribution for the three types of riders on the Tokyo Olympic track and the 2021 UCI World Championship time trial track in Figures 14 and 16.

Model 3: Longitudinal model for team time trials, we give the Basic Longitudinal Formation. And the following distance is given, which is about 0.7m. Each member of the team realizes tactics and saves energy by constantly changing his position in the group. Meanwhile, the optimal lead rider output power and lead riding time is given. According to this way of distribution can realize the timely supplement and effective use of energy, has a great help to the improvement of grades. Finally we discuss the influence of the wind on Longitudinal Model and give other useful suggestions.

Keywords: Individual Time Trial, Power Profile, Power Distribution, Team Time Trial

Contents

1 Introduction

1.1 Problem Background

Road cycling is a sport that challenges speed and endurance and is held on roads with various terrain variations. It is divided into standard races, team time trials and individual time trials. Different races have different rules, but the core rule is to finish the race in the shortest time possible. Different types of riders have their own unique power curves, influenced by physical fitness. Riders need to adjust their power on different stages to finish the race as fast as possible by distributing their energy wisely.

Figure 1: bicycle road races

1.2 Restatement of the Problem

In order to help riders understand more clearly how to allocate their physical strength, we need to build a mathematical model. This model needs to be able to combine conditions such as rider quality, stage terrain, weather conditions, and race type to give riders the best physical strength allocation plan. According to the problem context and the requirements of the problem, we can divide this problem into the following six steps(the last two of which can be considered as sensitivity analysis):
Task 1. Build a power profiles modal, i.e., a model used to solve for the relationship between track position and the power the rider applies. Make it possible to apply it to any situation.
Task 2. Create a power curve model of a rider. By evaluating the various qualities of different types of riders, the corresponding power curves are created. Among them, a time trial specialist and another type of rider are selected for a more detailed analysis to solve their power profiles.
Task 3. Apply the power profiles model to the following three tracks to solve for the corresponding power profiles.

- Olympic Time Trial course in Tokyo, Japan
- 2021 UCI World Championship time trial course in Flanders, Belgium
- Customized tracks

Task 4. Expanding the model to a team competition.
Task 5. Adjust the weather indicator to see how the weather affects the results.
Task 6. Tests the effect on race results when the actual power configuration deviates slightly from the power profile.

1.3 Our work

The following figure is a brief summary of our work:

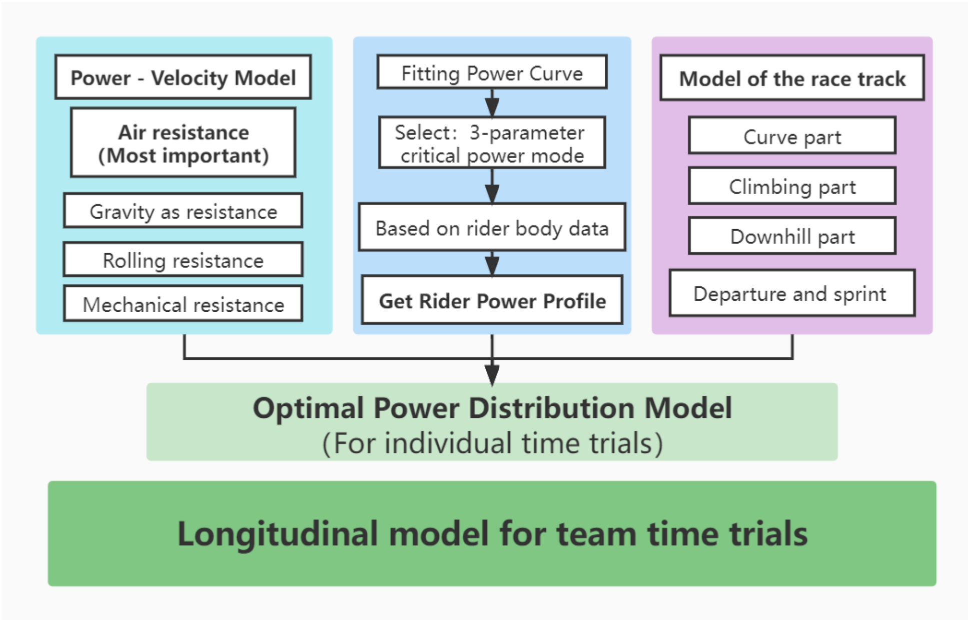

Figure 2: The frame of this paper

2 Assumptions and Justifications

To simplify our model, we make the following basic assumptions, each of which is properly justified.

- **The friction of a bicycle tire against the ground is sufficient**
  At normal race speeds of 30-60 KMH, the friction between the wheels and the ground is large enough to ensure that the wheels only roll with the ground without relative sliding
- **The heat produced by bicycle wheels and chains can be negligible**
  At normal race speeds of 30-60 KMH, the friction between the wheels and the ground is large enough to ensure that the wheels only roll with the ground without relative sliding
- **The slope is not too steep**
  That means, The slope conforms to the general time trial rules, we do not consider specific race types such as mountain time trials or hill-climbing time trials, and generally consider the slope to be less than 5%.
- **No extreme weather or road conditions**
  That means there are no strong winds during the race. Moreover, the road conditions meet the requirements of regular competitions, and the influence of mechanical failures and other objective conditions on power output is not considered

3 Glossary and Notations

3.1 Glossary

- **Criterium**: A bicycle race that takes place on a closed course. The length can be specified by a fixed number of laps or the most laps in a predetermined time period.
- **Directeur Sportif**: A team's director who is responsible for managing the riders and staff, making race decisions, and deciding the team composition for a given race.
- **Individual Time Trial**: An event in which riders traverse a predetermined course one at a time.The riders are not allowed to work together or ride near one another. The time required to traverse the course is recorded for each rider. The lower the time the better the rider's final placement.
- **Power Curve**: A visual representation of the maximum power a rider can maintain for a particular length of time.
- **Climber**: A rider that specializes in races that have multiple long climbs.
- **Puncheur**: A rider that specializes in races that include many short, steep climbs or many sharp accelerations.
- **Rouleur**: A rider that is a generalist and can do well in races with a wide variety of terrains.
- **Sprinter**: A rider that specializes in producing extremely high power for short periods of time. These riders generally focus on winning at the end of a race or during the intermediate sprints (if a race has intermediate sprints).
- **Time Trial Specialist**: A rider that specializes in the individual time trial events.
- **MMP**: Mean Maximal Power output.
- **FTP**: Functional Threshold Power.
- **TTF**: Time to task failure.
- **CP**: Critical power the athlete can provide.

3.2 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations of our paper

Symbol	Definition
ρ	Air density.
AC_D	Effective Frontal Area
$v_{windparallet}$	Component of wind velocity in the same direction as the bicycle velocity
FA	The Frontal Area
P_{mech}	The power of mechanical resistance to do work
W'	The work above critical power
P_{max}	Maximum momentary power the athlete can provide
α	Slope gradient
β	The inclination angle of the bike

4 Model 1: Power - Velocity Model

Let the total power output of the cyclist is P . The energy will be expended in the following four ways, Air resistance, climbing, Rolling resistance and Mechanical resistance.

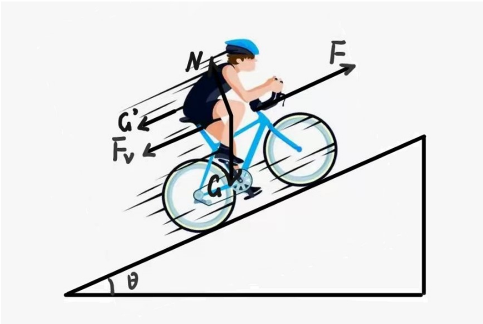

Figure 3: Force Analysis

4.1 Air resistance work

Air resistance is the most important drag factor in bicycle racing, so it should be considered.The power of air resistance doing work is

$$P_{air} = -\frac{1}{2}\rho \cdot AC_D \cdot (v - v_{windparallet})^2 v$$

- ρ is the air density.
- AC_D is Effective Frontal Area.
- v is the velocity of the bicycle in m/s.
- $v_{windparallet}$ is the component of wind velocity in the same direction as the bicycle velocity in m/s.

4.1.1 Solve for each data

1. The density of air (ρ) is considered as $1.2224 kg/m^3$ in dry air and $1.1955 kg/m^3$ in wet air. The specific data are as follows:

type of air	dry	wet
temperature / degree centigrade	15	20
air pressure / Pa	1.013×10^5	1.013×10^5
relative humidity	30%	80 %
air density(ρ) / kg/m^3	1.2224	1.1955

Table 2: Air Density Data

2. Effective Frontal Area (AC_D) is generally less than Frontal Area(FA). Based on individual differences, it is too difficult to find Effective Frontal Area for any athletes. In order to simplify the module, we just consider AC_D= FA.

For FA , SA. Bassett et al.[1] persent an equation, utilising the cyclist's height and weight that will estimate the cyclist's total FA in terms of height and mass while in the aero-racing position utilising aero bars:

$$FA = 0.0293H^{0.725}M^{0.425} + 0.0604$$

- FA=Frontal Area in m^2;
- H is height in m;
- M is mass in kg

3. The **Sprinter** is generally 180-185cm tall, weighing 70-75kg, with a BMI of approximately $22kg/m^2$, which allows for the best Absolute Power Output (APO).

Time Specialist and Climber, on the other hand, require the maximum Relative Power Output, RPO. Therefore they are relatively short in stature, typically 170-175cm tall, 60-66kg, and 19-20kg/m^2 BMI.

On average, cyclists have **a Body Area between 1.54-2.08** m^2 and **a Frontal Area between 0.28-0.38** m^2.[2]

For convenience of out calculation, we consider:

- The weight of **male Sprinter** is 75kg. FA is 0.35 m^2.
- The weight of **male Time Specialist or Climber** is 65kg. FA is 0.28 m^2.
- The weight of **female bicyclist** is 55kg. FA is 0.25 m^2.

4.2 Gravity as resistance to do work

The power of gravity as resistance to do work is

$$\mathrm{P}_G = -mg\sin(\theta)\cdot v$$

It is worth noting that because the start and finish in the time trial are in the same place, the total gravitational potential energy of the race changes to 0, i.e.

$$W_G = \oint_{course} P_G dt = 0$$

In the course of the race, because each part of the track needs to calculate the power separately, so we still need to consider the power of gravity do the work.

4.3 Other resistance does work

4.3.1 Rolling resistance

Bicycle wheels are not rigid bodies, their surfaces are irregular and the ground is not absolutely smooth. Therefore, rolling resistance is generated.

The power of rolling resistance doing work is

$$\mathrm{P}_{roll} = -\mu_r mg\cos(\theta)\cdot v$$

Of course, in bicycle racing, rolling resistance is a very small amount compared to air resistance.

4.3.2 Mechanical resistance

The power of mechanical resistance to do work is P_{mech} .However, that power is difficult to calculate simply quantitatively. For a given track, a cyclist rides a bike of similar quality. The energy consumed by their mechanical drag work should be approximately equal. Therefore, we only need to add a time correction term t_{mech} due to mechanical drag work to the final timing result.Since mechanical drag is a very small quantity compared to air drag in cycling, we believe that the effect of mechanical drag on the total time is also very small.

4.4 The Power-Velocity Model

The air resistance component while cycling is proportional to the cube of speed;consequently, it is the primary energy cost factor at high speeds. This aerodynamic resistance represents >90% of the total resistance the cyclist encounters at >30 kmh. At speeds >50 kmh, aerodynamic resistance is the determining variable.[3]

As shown in Figure 3, the curves of air resistance and other resistance about speed are shown below when tested on flat ground and other conditions are the same. It can be considered that the range of bicycle racing is 30-60kph, the effect of air resistance is much higher than other resistance.

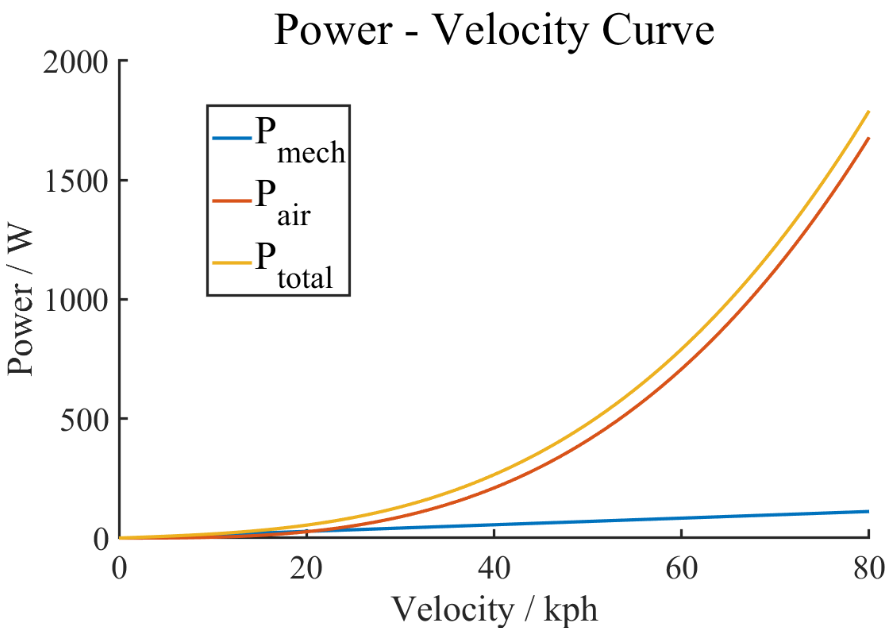

Figure 4: Power-Velocity Curve

From the law of conservation of energy, we know that

$$P + P_G + P_{air} = 0$$

i.e.

$$P = mg\sin(\theta)\cdot v + \frac{1}{2}\rho \cdot AC_D \cdot (v - v_{windparallel})^2 v$$

When the bike is moving on flat ground, only air resistance does the work.

$$P = \frac{1}{2}\rho \cdot AC_D \cdot (v - v_{windparallel})^2 v$$

When the bike is going uphill and the slope is relatively small,it can be approximated by the slope α.That is,$\sin(\theta)=\alpha$

$$\mathrm{P}=mg\alpha\cdot v+\frac{1}{2}\rho\cdot AC_D\cdot(v-v_{windparallel})^2v$$

4.5 The Application of The Power-Velocity Model

The data of Weight and FA for Sprinter, Time Specialist and Female Bicyclist in our work are

	Weight / kg	Fount Area / m^2
Male Sprinter	75	0.35
Male Time Specialist	65	0.28
Female bicyclist	55	0.25

Considering the effect of weather on power, out model for Sprinter, Time Specialist and Female Bicyclist are:

- In dry air:
  For Male Sprinter, $\mathrm{P}=204.17\alpha v+0.00458(v-v_{wind})^2v$
  For Male Time Specialist, $\mathrm{P}=176.94\alpha v+0.00367(v-v_{wind})^2v$
  For Female bicyclist, $\mathrm{P}=149.72\alpha v+0.00328(v-v_{wind})^2v$
- In wet air:
  For Male Sprinter, $\mathrm{P}=204.17\alpha v+0.00448(v-v_{wind})^2v$
  For Male Time Specialist, $\mathrm{P}=176.94\alpha v+0.00359(v-v_{wind})^2v$
  For Female bicyclist, $\mathrm{P}=149.72\alpha v+0.00320(v-v_{wind})^2v$

Attention, the Unit of Velocity is KM/H here.

5 Model 2: Optimal Power Distribution Model

5.1 Power Curve

5.1.1 measuring method during cycling

muscles created a force which is applied perpendicular to the bicycle crank arms. Kinesiologists use physical expressions to describe the properties of force generation such as mean power output or mean torque. Kinesiologists measure the product of many impulses over a given period of time and expressed that as **Power Output**. Although some kinesiologists argred it a more proper expression to use 'time avaerage power output', we still just use 'Power Output'in this articles. In addition, that power output does not include the forces applied in the different direction of the bicycle moving.

To describe power outputs for individual efforts, kinesiglogists use the **mean maximal power output**(MMP). The value of MMP represents the highest average power recorded for a arbitrary given duration such as 1 min MMP.

Besides, we use **Functional Threshold Power**(FTP) to describe the maximal average power output during a period of an hour. It means the aerobic capacity of the bicyclist.

5.1.2 Obtain the power curve

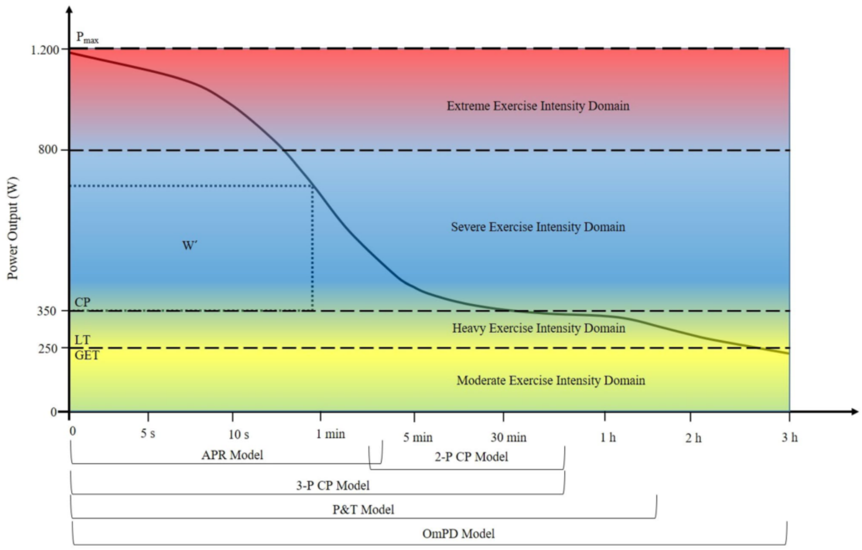

Figure 5: Power Curve

When power output is plotted against **time to task failure**(TTF), we can give the Power Curve.

Monod and Scherrer(1965) firstly analysed muscle fatigue during static and dynamic work of knnee extension exercise and give a mathematics module describing the relationship of Power and TTF. After decades of development of exercise physiology, Power Curve is considered as an effective integrative approach to the physical condition of a cyclist during a race.

According to Burnley and Jones(2007)[4], the Power - TTF relationship consists of four distinct exercise intensity domains: Moderate, Heavy, Severe and Extreme.(figure 5) And the character of four states can be distinct whole-body pyhsiological responses. (Jamnick et al. 2020[5]; Vanhatalo et al. 2016[6]; Whipp 1996[7])

5.1.3 3-parameter critical power model

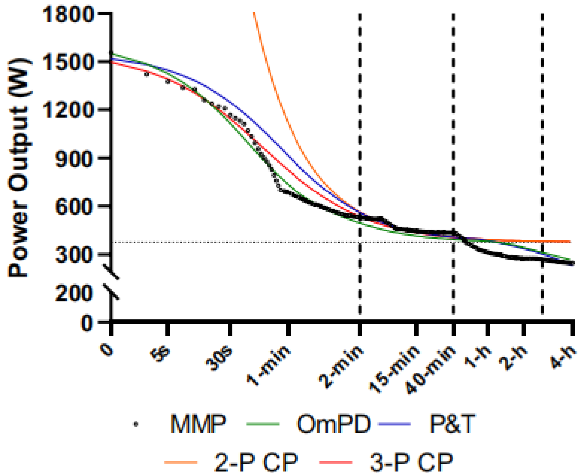

Figure 6: Various power duration modelling approaches applied to the same MMP data.

Fitting to the actual data shows that there are respective best-fit models for different power intervals.We should choose the model that best predicts the power-duration relationship within the intensity range of the athlete's training and competition.In individual time trial of bicycle road races, a large portion of power output falls into the realm of heavy and severe intensity.Therefore, we selected the 3-parameter critical power model(3-P CP), which fits better in the heavy and severe intensity domain, for the following analysis.

The equation of 3-P CP model is

$$t = \frac{W^{'}}{P - CP} + \frac{W^{'}}{CP - P_{\max}}$$

- t is time in minutes;
- $W^{'}$ is work above critical power in $1/60$ J;
- P is power output in W;
- CP is critical power in W;
- $P_{\max}$ is peak power in W.

5.1.4 Power Profile of Chosen Bicyclists

We choose Three-parameter critical power model: $t = \frac{W^{'}}{P-CP} + \frac{W^{'}}{CP-P_{\max}}$

We selected time trial specialists (male) and sprinter (male) and cyclists (female) for our study. They are all capable of competing in world class races. For each bicyclist choosed, we use the 1 sec MMP, 5 sec MMP, 1 min MMP, 5 min MMP and FTP to fit the power curve(Power - TTF).The table 3 and table 4 are the example data for each kind of rider.

Table 3: Maximal Power Output per Weight(in W/kg)

	1sec	5sec	1min	5min	FTP
Male Sprinter	22.94	21.03	9.33	4.81	3.77
Male Time Specialist	18.42	16.89	8.74	5.99	5.44
Female bicyclist	18.06	16.55	8.11	5.74	4.09

Table 4: Maximal Power Output (in W)

	weight	1sec	5sec	1min	5min	FTP
Male Sprinter	75	1720.64	1577.25	699.75	360.75	282.75
Male Time Specialist	65	1197.65	1097.85	568.1	389.35	353.6
Female bicyclist	55	993.42	910.25	446.05	300.85	224.95

The Power Profile are:

Table 5: Power Profile

	$W^{'}$	CP	P_{max}
Male Sprinter	574.75	263.25	1798.64
Male Time Specialist	328.35	353.70	1250.65
Female bicyclist	302.45	224.95	1031.42

Attention:the horizontal coordinate in the graph is not uniform, its taken log10 of the original time.

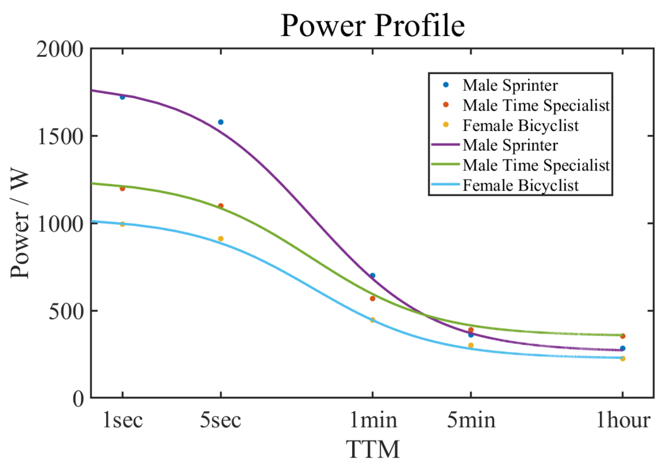

Figure 7: Power Profile

5.2 Solving for the optimal solution

In road cycling, except for a few special time trial courses around the world, the individual time trial and team time trial courses are generally less undulating, flatter and shorter, ranging from 5 to 40km.For the majority of the race, the competitors rode at near-uniform speeds on the flat at approximately CP power.Its **power burst** is mostly at **the beginning of the race**, **the exit phase of the curve**, **the uphill phase**, and **the finish sprint phase**.And in the **downhill phase**, the output power usually decreases while the speed is faster compared to the flat ground.

The following will focus on the physical processes in the **curve part**, **uphill phase**, **downhill phase**,and**departure and sprint phase**, and give the corresponding power output strategies.

5.2.1 Curve part

When cornering, cyclists often use a strategy of leaning the bike body.The rider will enter from the outside of the track, pass the inside boundary of the track in the corner, and finally exit the corner from the outside of the track in a tangential direction. To simplify the model, we only consider cyclists using a general turning course strategy. It means taking a corner along a segment of a circle. As shown in Figure 8, the pink curve depicts the bicycle cornering route.

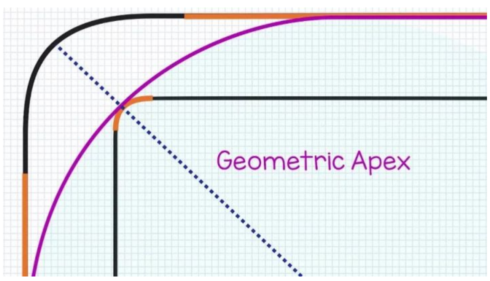

Figure 8: bicycle cornering route

We only consider the sharp corners. There is no need to consider turns that require no speed reduction at all and where the bike leans low enough that you can comfortably pedal the bike for power output.

For general curves,

$$\frac{m{v_r}^2}{R} \le \mu mg$$

$$\frac{m{v_r}^2}{R} = mg\tan\beta$$

Therefore,

$$v_r \leq \sqrt{\mu g R} = v_{r\max}$$

$$\tan\beta = \frac{{v_r}^2}{gR} \leq \mu$$

However, during cornering, the rider needs to avoid touching the pedals to the ground because the angle between the bike and the ground is relatively small (i.e., the inclination angle of the bike β is relatively large). At the same time, in order to gain better control of the bike during cornering, the pedals cannot be pedaled to output power and maintain a constant speed v_r. **So that the exit speed v_{rf} will be slightly less than the entry speed v_{ri}.**

In the straight before entering the corner, we consider that the cyclist is riding at a cruising speed v, when the power output is PC.When approaching a corner, the rider should decide whether to brake according to the relevant parameters of the corner.

- When $v \leq v_{r\max}$, no brakes are needed, at which point $v_{ri} = v$.
- When $v > v_{r\max}$, the rider needs to brake so that $v_{ri} = v_{r\max}$.

It is worth noting that for most of the corners in the time trial stage, only very light braking is required to achieve the turn-in speed v_{ri} to meet the turn conditions.

After exiting the corner, the rider should quickly correct this speed difference with a larger output in a few seconds. The straight afterwards will resume cruising speed v.

The cyclist needs to provide energy $E = P\Delta t$ after exiting a corner. The magnitude of E_{loss} is the difference between the cruising kinetic energy and the kinetic energy out of the corner.

That is

$$P\Delta t = E_{loss}$$

In the following parts, E_{loss} is calculated :

We only consider air resistance as the resistance, in the previous model we considered air resistance as:

$$F_{air} = -\frac{1}{2}\rho \cdot AC_D \cdot (v - v_{windparallel})^2$$

Make $\gamma = \frac{1}{2}\rho \cdot AC_D$. Then the above equation simplifies to

$$F_{air} = -\gamma(v - v_{windparallel})^2$$

we consider the case where there is no wind. it means,

$$F_{air} = -\gamma v^2$$

Then,

$$m\frac{dv}{dt} = -\gamma v^2$$

$$\int_{v_{ri}}^{v(t)} \frac{dv}{v^2} = \int_0^t (-\frac{\gamma}{m})dt$$

we get

$$\frac{1}{v_{ri}} - \frac{1}{v(t)} = \frac{\gamma}{m}t$$

which is,

$$v(t) = \frac{1}{1/v_{ri} - \gamma t/m}$$

Let the difference in travel distance during the turn be ds and its total travel distance through the turn be $s = R\theta$.In this, R is the turning radius (note that it is not the radius of curvature of the curve) and θ is the steering angle,i.e. the angle of speed change during the whole turn.Then we can get;

$$\int_0^{R\theta} ds = \int_0^T \frac{1}{1/v_{ri} - \gamma t/m} dt$$

Then,we can get T,

$$T = \frac{m}{\gamma v_{ri}} \cdot \left[1 - \exp(-R\theta\gamma/m)\right]$$

Therefore,

$$v_{rf} = v_{ri} \cdot \exp[R\theta\gamma/m]$$

We can derive loss Energy:

$$E_{loss} = \frac{1}{2}m(v^2 - {v_{ri}}^2 \cdot \exp[2R\theta\gamma/m])$$

$$t_{accelerate} = \begin{cases} E_{loss} = \frac{1}{2}mv^2(1 - \exp[2R\theta\gamma/m]) & ,v \le v_{r\max} \\ E_{loss} = \frac{1}{2}m(v^2 - \mu gR \cdot \exp[2R\theta\gamma/m]) & ,v > v_{r\max} \end{cases}$$

Because γ/m has a magnitude of 10^{-5}, so that

$$\exp[2R\theta\gamma/m] \approx 1 + 2R\theta\gamma/m$$

Therefore, the model simplifies to:

$$t_{accelerate} = \begin{cases} E_{loss} = R\theta\gamma v^2 & ,v \le v_{r\max} \\ E_{loss} = \frac{1}{2}m(v^2 - \mu gR \cdot [1 - 2R\theta\gamma/m]) & ,v > v_{r\max} \end{cases}$$

Based on the data, we note that the vast majority of road cycling courses can support a cruising speed of v, or slightly less than v for cornering. Therefore, in the following specific calculations, we only use the formula

$$E_{loss} = R\theta\gamma v^2$$

to perform calculations.

In the acceleration phase, we choose 10sec power for acceleration according to the Power Profile.An athlete accelerating at that power is usually enough to replace the kinetic energy lost in cornering and not to fatigue too much.The acceleration time is:

$$t_{accelerate} = E_{loss}/P_{10\,\mathrm{sec}} = R\theta\gamma v^2/P_{10\,\mathrm{sec}}$$

Table 6: The acceleration time (in s)

	$R*\theta$	γ	v	CP	P_{10sec}	$t_{accelerate}$
Male Sprinter	20	0.21392	37.225	263.25	1400	4.235
Male Time Specialist	20	0.17114	45.848	353.70	1000	7.194
Female bicyclist	20	0.15280	40.933	224.95	840	6.096

5.2.2 Climbing

In cycling, the uphill part of the race is often considered a tougher stage.In this stage, the cyclists will increase their power output, but their speed will be reduced to some extent.In the following we will analyze how to choose the right power according to the slope and slope length to maintain a good speed for the race.

In uphill, the output power and the power consumed by the resistance are given by the previous:

$$\mathrm{P} = mg\alpha \cdot v + \frac{1}{2}\rho \cdot AC_D \cdot (v - v_{wind})^2 v$$

Similarly, we disregard the effect of wind direction and wind speed on our model for the moment, and introduce

$$\gamma = \frac{1}{2}\rho \cdot AC_D$$

Then the power equation is

$$\mathrm{P(v)} = mg\alpha \cdot v + \gamma \cdot v^3$$

For arbitrary power, the relationship between duration and output power is given by the three-parameter model above.

$$T(P) = \frac{W'}{P - CP} + \frac{W'}{CP - P_{\max}}$$

It is easy to know that our theoretical solution must be on the power curve when seeking the minimum value of time spent uphill.

Assuming that we know the terrain gain in the uphill phase is h and the slope is α, we can find the slope length $L = h/\alpha$

We know,

$$vt = h/\alpha$$

	$\alpha(\%)$	$P(W)$	$v_{climb}(km/h)$
Male Sprinter	0.5	343.81	35.89
	1	403.35	32.82
	2	475.77	26.54
	3	509.85	21.16
	5	533.83	14.17
Time Specialist	0.5	392.58	41.41
	1	419.49	36.90
	2	449.85	28.58
	3	463.11	22.15
	5	472.70	14.49
Female bicyclist	0.5	262.90	36.82
	1	289.02	32.58
	2	317.62	24.81
	3	329.37	18.98
	5	337.29	12.29

Figure 9: Climbing power

Bringing in the equations of P(v) and T(P) above, we can get the quadratic equation in one element:

$$\frac{\gamma W'}{CP - P_{\max}} \cdot v^4 - \frac{h\gamma}{\alpha} \cdot v^3 + \frac{mg\alpha \cdot W'}{CP - P_{\max}} \cdot v^2 + (W' - mgh) \cdot v + (\frac{CP \cdot h}{\alpha} - \frac{CP \cdot W'}{CP - P_{\max}}) = 0$$

The equation does not have an analytical solution, but we can solve it using numerical values. After solving for the speed, the theoretical output power when going uphill can be obtained by bringing it into the P(v) equation.

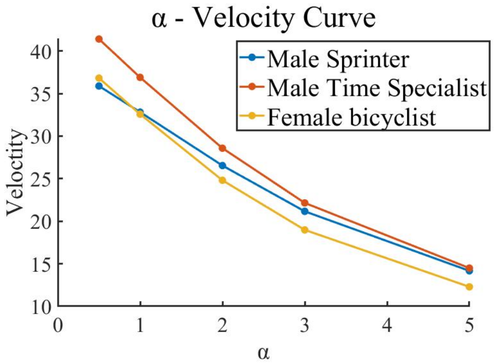

Figure 10: Climbing α-Velocity curve

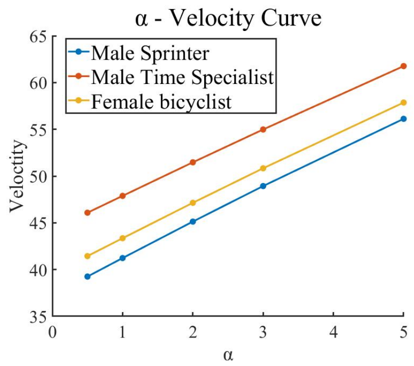

Figure 11: Downhill α-Velocity curve

5.2.3 Downhill

In the downhill stage, the rider can output less power and gain more speed to finish the downhill, which is a relatively easy stage in the race. There is no need to build a rigorous mathematical model to solve for this stage. It will depend more on the rider's control of the road conditions on the downhill section. We will give reference power output ranges for different courses.

According to Brianna, "Be calm going down the ramp, avoid pushing on the pedals too hard or you might slip out! I fishtailed slightly on the ramp at 2014 Worlds.You won't gain time on the ramp,so better to be conservative and just chill out."

We generally recommend that cyclists race at a balanced, consistent and relatively conservative speed on downhill sections. The recommended downhill speeds v_{down} are shown in the table below. We can conclusion, 0.90CP is an appropriate choice.

	$\alpha(\%)$	0.95CP	$v_{down}(0.95CP)$	0.90CP	$v_{down}(0.90CP)$
Male Sprinter	-0.5	250.09	39.89	236.93	39.25
	-1		41.84		41.23
	-2		45.68		45.13
	-3		49.41		48.93
	-5		56.52		56.12
Male Time Specialist	-0.5	336.02	46.85	318.33	46.08
	-1		48.63		47.88
	-2		52.15		51.47
	-3		55.61		54.98
	-5		62.29		61.76
Female bicyclist	-0.5	213.70	42.12	202.46	41.44
	-1		44.00		43.35
	-2		47.72		47.14
	-3		51.36		50.83
	-5		58.30		57.86

Figure 12: The downhill speed v_{down}

5.2.4 Departure and Sprint Phase

At the start of the ride, the rider quickly reaches cruising speed with a brief burst. Peter, a professional cyclist, says that the first six pedals are the key to power generation, and within 10 seconds of the start of the run, the bike should be at cruising speed. By analyzing the starting data of world-class professional road cycling riders, we recommend using 95% of 10sec MMP to accelerate until the bike reaches cruising speed.

In the last 150m-200m sprint, the rider should charge to the finish line as fast as possible. Different riders have different charging strategies, and we think 200 meters is a more appropriate distance to charge. Sprint takes about 13-18s. Since the power output of the sprint process is greatly affected by the physical state and excitement of the athletes during the competition, 10sec MMP is given as the reference power.

5.3 Solution of the Optimal Power Distribution Model

In the condition of dry air and no wind, the cruising velocity of three riders are,

	Cruising Velocity / $m \cdot s^{-1}$
Male Sprinter	38.59
Male Time Specialist	45.85
Female bicyclist	40.93

5.3.1 2021 Olympic Time Trial course in Tokyo, Japan

It is worth noting that the model does not take into account the power output stop in the process of turning and the short burst of power after turning. This part requires the rider to select the appropriate rhythm according to the track. See above for reference values.

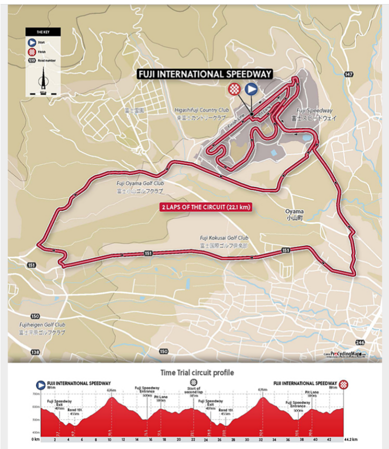

Figure 13: 2021 Olympic Time Trial course

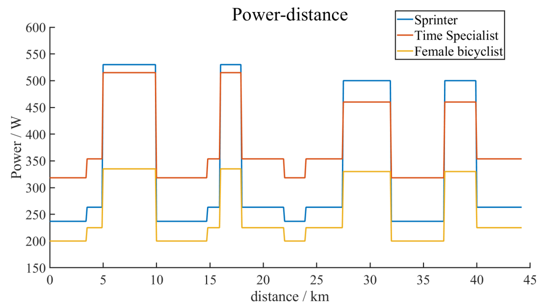

Figure 14: Results for 2021 Olympic

5.3.2 2021 UCI World Championship time trial course in Flanders, Belgium

The track is mainly flat with few big ups and downs, and we can think that in general, riders only need to ride with CP as the output power. It is worth noting that in this simplified model, we also did not include the extra power output when turning in the model, which requires players to choose the appropriate rhythm according to the track. For reference, please refer to the above.

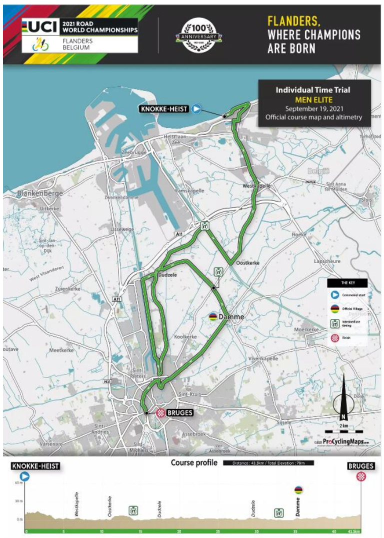

Figure 15: 2021 UCI World Championship time trial course

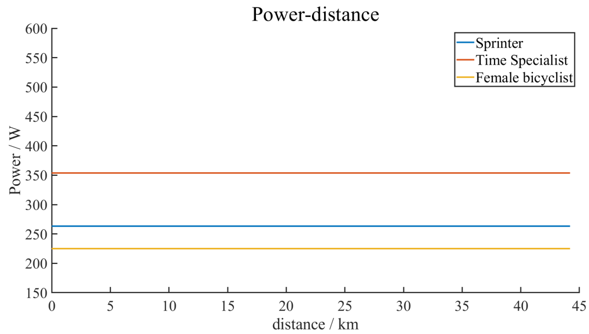

Figure 16: Results for 2021 UCI World Championship

6 Model 3: Longitudinal model for team time trials

Team time trial, refers to the team form to participate in the time trial, each team interval more than 30s to start, so there is no need to consider the mutual influence between teams. According to the requirements of the question, the team members are considered to be six in this paper, and the fourth team member crossing the line time is used as the final timing result.

6.1 Basic Formation

In a Team Time Trial (TTT), the team usually advances in a column, with the riders spaced a relatively small distance apart. In the process of moving forward, the leader will take on more air resistance, while the following riders will take on relatively less air resistance. After a period of time (usually 10s-30s, depending on the speed, environment and rider's ability), the leader will give way to the side and gradually recede to the end of the group, while the rider in second place will become the leader. This process is repeated until the race is completed. In the following, we elaborate on the optimal distance between the front and rear riders and the power and duration of the leader's output in the longitudinal model. Finally, we will discuss the effect of the bias wind on the whole team.

6.2 Following distance

The following distance refers to the distance between any two adjacent team members in front and behind in the column model. Specifically, it is the shortest distance between the rear wheel of the former and the front wheel of the latter.

6.2.1 Reasons for resistance reduction

In the follow ride, the air resistance of both the former and the latter is significantly reduced. The reasons for this are as follows:

A single bicycle in a fast ride will have a large number of air molecules hitting the bicycle and the front of the rider, and the area of impact is called the effective windward area (EFA). In addition, at the rear of the bike, the airflow flows at high speed and merges quickly, creating a small area of low air pressure, which can further increase air resistance. When an additional rider is following, the aerodynamic properties of the rear bike cause it to generate a lot of turbulence, which disrupts the low air pressure area behind the front body and reduces the front bike's air resistance. At the same time, because the wake of the front bike does not merge, the rear bike can use the wake of the front bike to reduce its own air resistance, an effect called the wake effect.

6.2.2 Experiments on this content

According to JIN Li-ying(2005)[],we were able to obtain the effect of different following distances on the air resistance of the front and rear bikes.Experiments require the use of a wind tunnel for wind tunnel air head-on resistance testing.The wind tunnel adopts Beijing Aerodynamic Research Institute FD-09 single wild flow closed-port bottom speed wind tunnel.

- The cross-sectional shape of the experimental section is 3m*3m, with rounded corners.
- The length of the experimental section is 12m.
- The experimental wind speeds are 8m/s, 11m/s, 14m/s, 16m/s, 18m/s, 20m/s.

1. **Two - ride experiment**

   The two - ride experiment uses different distance as a variable to measure the percentage reduction of air resistance compared with the single person riding.The optimal following distance can be observed by this experiment.

   The experiments were conducted by selecting 0.10m, 1.60m and 3.60m as the heel-riding distance measurement points. The two bicycles were placed in the wind tunnel at the above heel-riding distances, front and rear, and the air resistance of the front and rear bicycles were measured under different wind speed conditions during the experiments.

   In the two ride experiments, we found that compared with a bicycle riding, two bicycle longitudinal queue for the rear bicycle air resistance value is proportional to the following distance, that is, the smaller the following distance, the smaller the air resistance, and vice versa. The specific data are as follows.

Table 7: Comparison of double rider resistance and single rider resistance

	Wind speed (m/s)							
	8	11	14	16	18	20	Average	Errors
single person	0.95	1.72	2.79	3.53	4.61	5.82		
0.10m	0.47	0.82	1.39	1.73	2.43	3.02		
$P_{air}^{0.1m}/P_{air} \times 100\%$	49.15	47.49	49.65	48.17	52.7	51.93	49.98	2.03
1.60m	0.63	1.03	1.62	2.18	2.9	3.52		
$P_{air}^{1.6m}/P_{air} \times 100\%$	66.73	59.72	58.10	60.59	62.76	60.56	60.34	1.50
3.60m	0.67	1.24	2.08	2.53	3.37	4.22		
$P_{air}^{3.6m}/P_{air} \times 100\%$	70.00	72.40	74.29	70.50	73.12	72.54	72.57	1.23

As for the former bike, when the following distance is 0.10-0.20m, the air resistance is reduced by 3%-4%, and the interference effect basically disappears outside 0.30m (speed is 16m/s-20m/s).

In order to ensure the maneuverability of the bike, to ensure the reaction time required by the rider to change the speed and to obtain a large wake effect, in the actual race, riders usually choose a distance of 0.3-1m as the following distance, and the choice of the specific distance may also vary according to the level of the rider.When entering and exiting curves or climbing or descending slopes, the following distance will be increased or decreased appropriately, and **the following experiment uses 0.70m as the following distance to continue the experiment**.

2. **Four - ride experiment**

This part of the riding experiment are using 0.7m as the riding distance, which is also the more practical riding distance in the competition.

We need to ensure that each rider has the same body size and riding posture (i.e., the same effective wind area) and that the following distance between each neighboring bike in front and behind is the same in the three-person and four-person following experiments.

The experimental data are shown in the table

Table 8: Comparison of four−rider resistance and single−rider resistance

	Wind speed (m/s)						
	11	14	16	18	20	average	error
single person	1.67	2.81	3.78	5	6.21		
Second person	0.95	1.57	2.02	2.65	3.40		
$P^2_{air}/P^1_{air} \times 100\%$	57.71	56.04	53.36	53.04	54.83	54.79	1.41
Third person	0.89	1.43	1.78	2.41	3.04		
$P^3_{air}/P^1_{air} \times 100\%$	53.41	50.80	47.21	48.23	48.90	49.71	2.18
Fourth person	0.77	1.31	1.74	2.24	2.84		
$P^4_{air}/P^1_{air} \times 100\%$	46.22	46.77	46.28	44.77	45.82	45.95	0.66

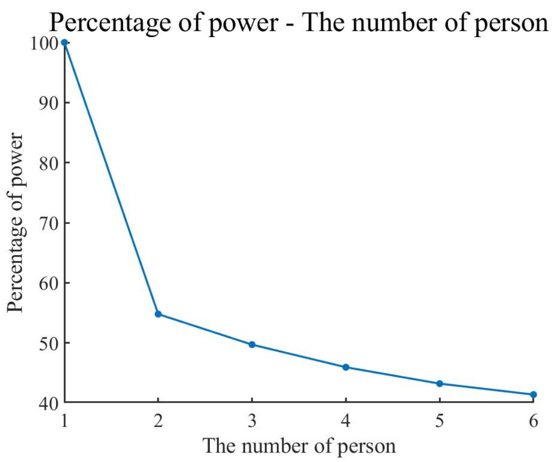

Figure 17: Percentage of power - The number of person

Another study showed that after the fourth rider, the air resistance to each rider was roughly the same and could all be considered the same as the fourth rider.

3. **Optimal longitudinal queue model**

In our six-rider longitudinal formation model, 0.7m was chosen as the following distance. The resistance to each rider and the corresponding power at cruising speed v=50km/h are shown in Table and Figure.

6.3 Lead Rider Output Power

Generally speaking, the lead time is 10s-30s, which depends on the overall team strategy and the individual rider's ability. We will use the Power Profile of two different riders as a reference, assuming three riders for each of the two, and give their lead times.

From the above, it is known that at a following distance of 0.7m, the air resistance of the leader is barely attenuated compared to its solo riding. We selected the rider with the lowest 10s power among the six team members and used 95% of his 10s MMP as the actual output power P_{out}. Let him go forward for 10s at the speed corresponding to that power.The second rider then leads the race, still using P_{out} as his actual power output, but adjusting the lead time accordingly according to his own Power Profile. This process is repeated until the finish line is reached.

6.4 The influence of the wind

If a rider encounters a headwind in a race that is nearly the same direction as the forward direction, the same solution as before should be used to maintain power forward at the expense of losing speed.

If riders encounter crosswinds (or winds at a greater angle to the forward direction), they should change the way the group moves forward to some extent. The longitudinal queue needs to be changed to the mode shown in the following figure.

6.5 Other suggestions

Pacing on Climbs: When the team approach the climbs from the flat road, the first rider should ease into the climb so that other riders can have a time to adjust their output power. steadily climbing is the most efficient way to finish the rising segment. The first rider should check frequently to see that the group is together. And the output power on a climb is given by our previous model.

Pacing on Descents: The riders should take shorter pulls on descents and generally a quick rotation depending on the grade. It's ultimately up to the second rider to determine pulling through or not, so if the second rider sense an increase in momentum, they should give the leader an indication to control the team in a proper rate.

7 Sensitivity analysis

Wet air and wind in the opposite direction of the rider's movement will provide some resistance to the rider's progress, and wind in the same direction of the rider's movement will help the rider forward.Assume that a male Sprinter is riding normally on a flat road with constant power CP and the wind speed is 2m/s 4m/s.

According to $\mathrm{P} = 204.17\alpha v + 0.00448(v - v_{wind})^2 v$ (We have proved it above.). We get the curve for $v_{wind} - v$.

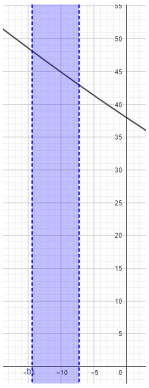

Figure 18: curve for $v_{wind} - v$

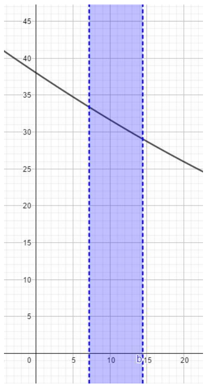

Figure 19: curve for $v_{wind} - v$

From this, we can know that, under the condition that the power remains unchanged, the greater adverse wind speed will have a greater impact on the rider's progress; Therefore, when the wind is coming from the opposite direction, the rider should choose to increase power appropriately to maintain speed.Similarly, with the same power, the wind in the same direction will help the rider advance and achieve better results. So when the wind is in the same direction, the rider can choose to reduce power and save energy. The +3m/s wind gives the rider about 21% speed, which is a very good thing.

The figure below shows the influence of wind and no wind state at +3m/s on the rider's speed at the same power. From the figure, we can clearly know that the wind speed has a certain influence on the rider's performance. The black line represents the effect of +3m/s wind.

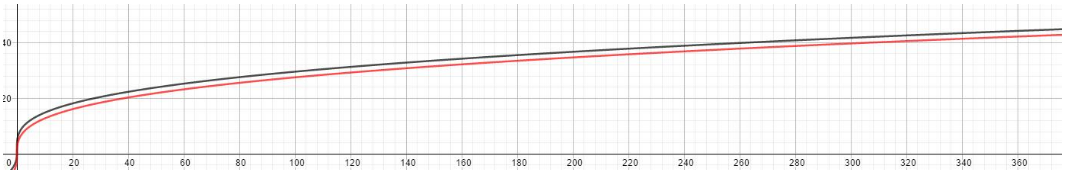

Figure 20: The influence of downwind on speed

8 Strengths and Weaknesses

8.1 Strengths

- We study extensively the riding strategies in realistic cycling races to make our model as close to reality as possible.
- Our model is built on the basis of physics with a high degree of confidence.
- Our power allocation optimal solution model takes into account various factors, such as: weather, terrain, etc.

8.2 Weaknesses

- We used some approximations in the model solving process, which makes the results subject to small errors.
- The modeling of the track is imperfect due to the lack of detailed track data

Rider's race guidance

This cycling guide will explain the power allocation optimal solution model by analyzing the performance of a competitor on the Olympic time trial course in Tokyo, Japan in 2021 after applying the power allocation optimal solution model.

By reading this cycling guide, you will learn about the following

★ How to allocate physical strength can achieve the fastest performance.
★ What strategy should be used in the starting, uphill, downhill, turning, and sprinting phases.

Note: This guide will be for individual time trials.

Rider Profile:

Rider type: Time trial specialist **Gender**: Male

Height: 175cm **Weight**: 65kg

Body Area: $1.6m^2$ **Frontal Area**:0.28 m^2

Power output:

Maximal Power Output (in W)						
	weight	1sec	5sec	1min	5min	FTP
Male Time Specialist	65	1189.37	1088.69	563.1	354.55	321.37

Power curve:

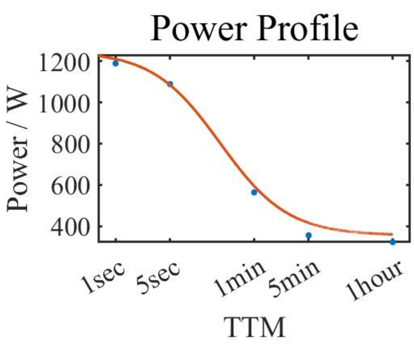

Figure 21: power curve

Figure 22: Competition Photos

Track Overview:

The total length of the time trial course for the 2021 Tokyo Olympics in Japan is 44.2km.The cross-sectional view of the track is as follows:

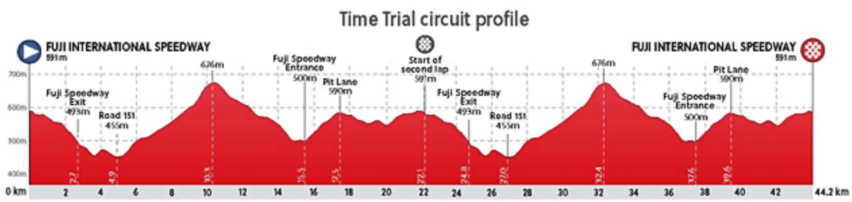

Figure 23: cross-sectional view of the track

Optimal solution:

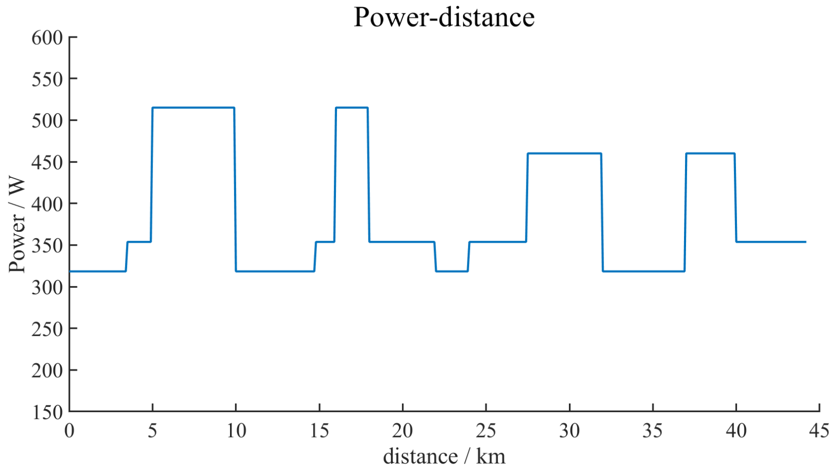

Figure 24: Results

What can be learned:

To achieve the fastest finish time, riders need to:

1. Riding at a constant speed on flat ground at approximately critical power (CP).
2. Accelerate quickly after the corner and use the rider's own power profile to select the power corresponding to 10sec to replace the kinetic energy lost during the corner.
3. The appropriate output power needs to be selected according to the slope and slope length for uphill.The output power can be referred to Figure25.
4. We recommend that cyclists race at a balanced, consistent and relatively conservative speed on downhill sections. The recommended downhill speeds v_{down} are shown in the Figure26.
5. At departure, we recommend that runners use a short burst of power at 95% of 10 second MMP to quickly reach cruising speed.
6. In the final 150m-200m sprint, the rider should charge towards the finish as fast as possible.

	$\alpha(\%)$	$P(W)$	$v_{climb}(km/h)$
Male Sprinter	0.5	343.81	35.89
	1	403.35	32.82
	2	475.77	26.54
	3	509.85	21.16
	5	533.83	14.17
Time Specialist	0.5	392.58	41.41
	1	419.49	36.90
	2	449.85	28.58
	3	463.11	22.15
	5	472.70	14.49
Female bicyclist	0.5	262.90	36.82
	1	289.02	32.58
	2	317.62	24.81
	3	329.37	18.98
	5	337.29	12.29

Figure 25: Climbing power

	$\alpha(\%)$	0.95CP	$v_{down}(0.95CP)$	0.90CP	$v_{down}(0.90CP)$
Male Sprinter	-0.5	250.09	39.89	236.93	39.25
	-1		41.84		41.23
	-2		45.68		45.13
	-3		49.41		48.93
	-5		56.52		56.12
Male Time Specialist	-0.5	336.02	46.85	318.33	46.08
	-1		48.63		47.88
	-2		52.15		51.47
	-3		55.61		54.98
	-5		62.29		61.76
Female bicyclist	-0.5	213.70	42.12	202.46	41.44
	-1		44.00		43.35
	-2		47.72		47.14
	-3		51.36		50.83
	-5		58.30		57.86

Figure 26: The downhill speed v_{down}

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