Circular Trajectory Approach (CTA) for Sine Wave Distortion Monitoring and Visualization

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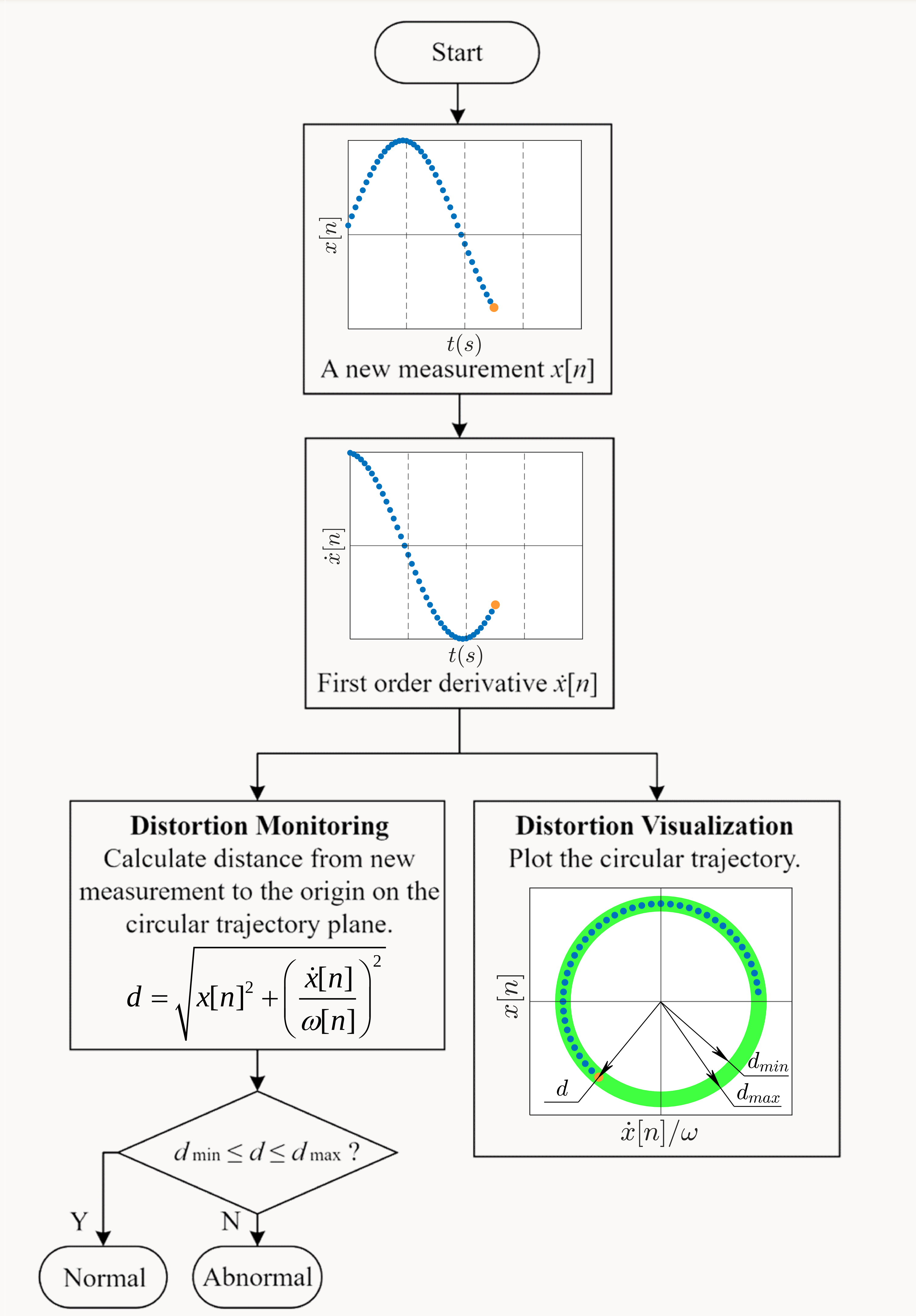
Main Idea

To transpose a fluctuating <u>Sine Wave</u> into a <u>Static Shape</u> so that its distortions are easier to monitor.

Workflow

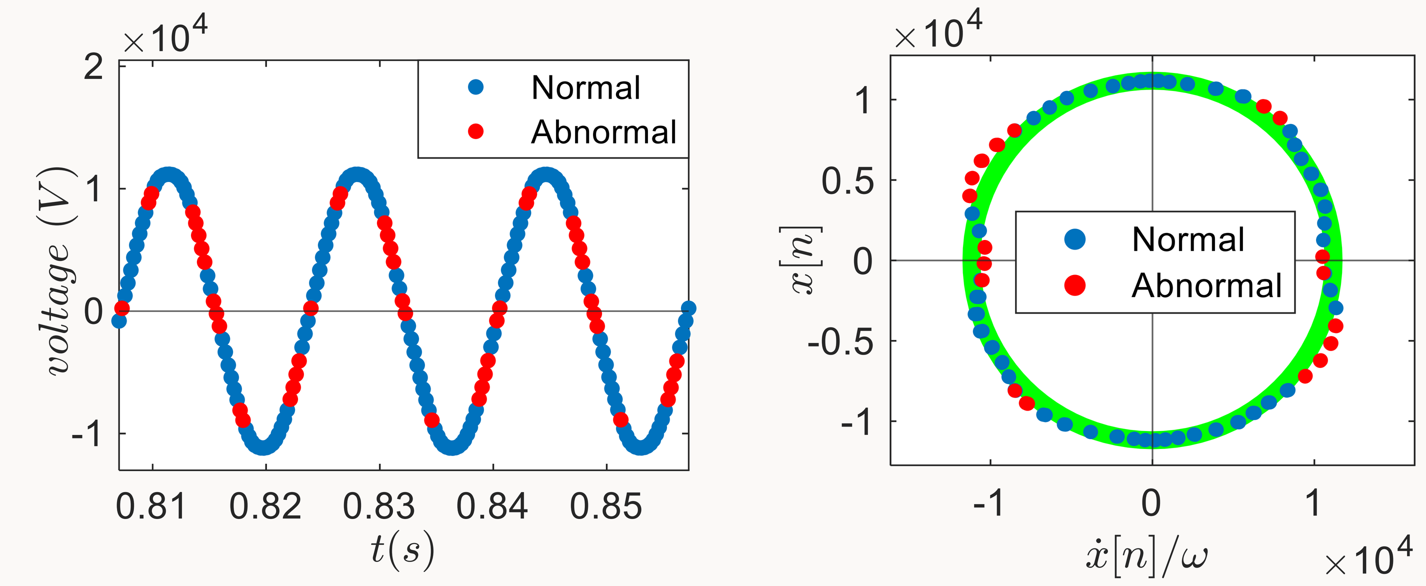


Applications

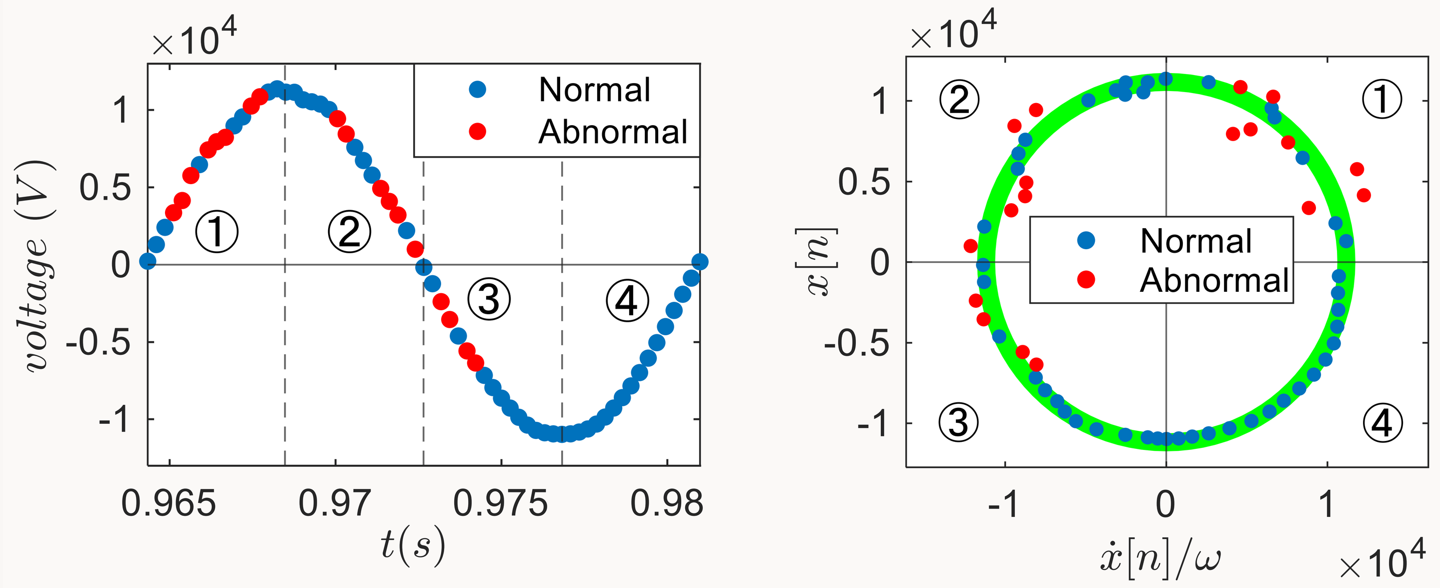
Advantages

- High-resolution
- Adjustable sensitivity
- Easy computation
- Immediate detection
- Apply to all distortions
- Suitable for online use

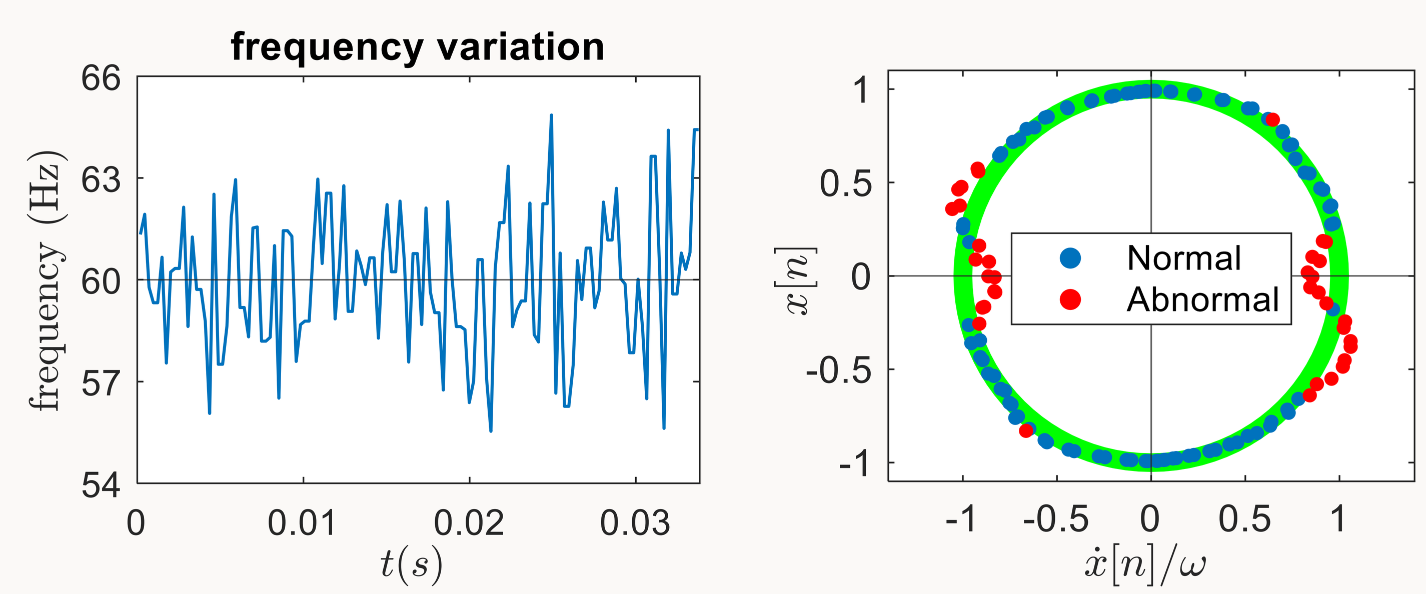
Revelation of Small Distortions



Revelation of Distorted Sections



Resistance to Frequency Variations



Theory

Origin of This Idea

$$\sin^2(\omega t) + \cos^2(\omega t) = 1$$

Ideal Case

Perfect Continuous Sine Wave

$$x(t) = A\sin(\omega t + \varphi)$$

$$\dot{x}(t) = A\omega\cos(\omega t + \varphi)$$

$$x(t)^2 + \left(\frac{\dot{x}(t)}{\omega}\right)^2 = A^2 \qquad ☆$$

Real Case

Discrete Signal + Frequency Fluctuation

$$x[n] = A\sin\left(\sum_0^n \omega[n]\Delta t + \varphi\right)$$

$$\dot{x}[n] = A\omega[n]\cdot\cos\left(\sum_0^n \omega[n]\Delta t + \varphi\right)$$

$$x[n]^2 + \left(\frac{\dot{x}[n]}{\omega[n]}\right)^2 = A^2 \qquad \dot{x}[n] = \frac{x[n+1] - x[n-1]}{2\Delta t}$$

Theoretical Basis

Proposition: Equation ☆ holds ⟺ $x(t)$ is a Sine Wave.

Proof :

$$\frac{dx(t)}{dt} = \pm\omega\sqrt{A^2 - x(t)^2} \qquad \arcsin\left(\frac{x(t)}{A}\right) = \pm\omega t + C_1$$

$$\frac{dx(t)}{\sqrt{A^2 - x(t)^2}} = \pm\omega dt \qquad x(t) = \pm A\sin(\omega t + \varphi). \blacksquare$$

Conclusion

- This work provides a general solution for sine wave distortion monitoring and visualization — the Circular Trajectory Approach (CTA).
- <u>Future works</u> include how CTA may help with <u>anomaly classification</u>.

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Interested? Check out our full paper.

Haoyuan Sun *et al.*, "Circular Trajectory Approach for Online Sinusoidal Signal Distortion Monitoring and Visualization," IEEE Trans. on Smart Grid (PES letter), July 2022.

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