Comp6211e Homework 4

Assignment date: April 22
Due date: May 6

Theoretical Problems (10 points)

1. (3 points) Let $\hat{\Sigma}$ be a $d \times d$ symmetric positive semidefinite matrix. We want to solve the following problem with $d \times d$ symmetric positive definite matrices X and Z:

$$\min_{X,Z \succ 0} \left[-\ln \det(X) + \mathrm{trace}(\hat{\Sigma} X) + \lambda \|Z\|_1 \right], \qquad X - Z = 0.$$

   Write down the ADMM algorithm, and derive the closed form solutions for the sub-optimization problems.

2. (3 points) Consider the k-class linear structured SVM problem, which has the following loss function:

$$\frac{1}{n} \sum_{i=1}^{n} \max_{y'} u_{i,y'} + \frac{\lambda}{2} \|w\|_2^2,$$

   with the constraints

$$\forall i \in \{1, \ldots, n\}, y' \in \{1, \ldots, k\} : u_{i,y'} = \left[\delta(y', y_i) - w^\top \psi(x_i, y_i) + w^\top \psi(x_i, y') \right].$$

   Consider a decomposition of this problem into two groups of variables $u = \{u_{i,y} : i = 1, \ldots, n; y = 1, \ldots, k\}$ and w. Write down the preconditioned ADMM algorithm for this problem, with closed form solutions for the subproblems.

3. (4 points) Consider the L_1-L_2 regularized loss minimization problem:

$$\min_{w} \left[\frac{1}{n} \sum_{i=1}^{n} \phi_\gamma(w^\top x_i y_i) + \frac{\lambda}{2} \|w\|_2^2 + \mu \|w\|_1 \right],$$

   where $x_i, w \in \mathbb{R}^d$, $y_i \in \{\pm 1\}$, and

$$\phi_\gamma(z) = \begin{cases} 1 - z - \frac{\gamma}{2} & 1 - z > \gamma \\ \frac{1}{2\gamma}(1-z)^2 & 1 - z \in [0, \gamma] \\ 0 & 1 - z < 0 \end{cases}.$$

   Derive the dual formulation, and SDCA algorithm using closed form solution for the dual proximal problem.

Programming Problem (10 points)

- Download data in the mnist/ directory (which contains class 1 (positive) versus 7 (negative) from the MNIST data)
- Use the python template "prog_template.py", and implement functions marked with '# implement'.
- Submit your code and outputs. Compare to the theoretical convergence rates in class, and discuss your experimental results.