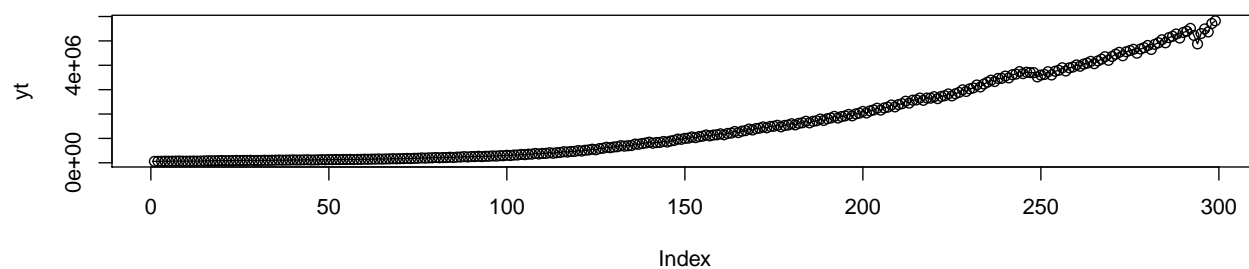# FIN6103 Final Project

Haoyue Heather Tan
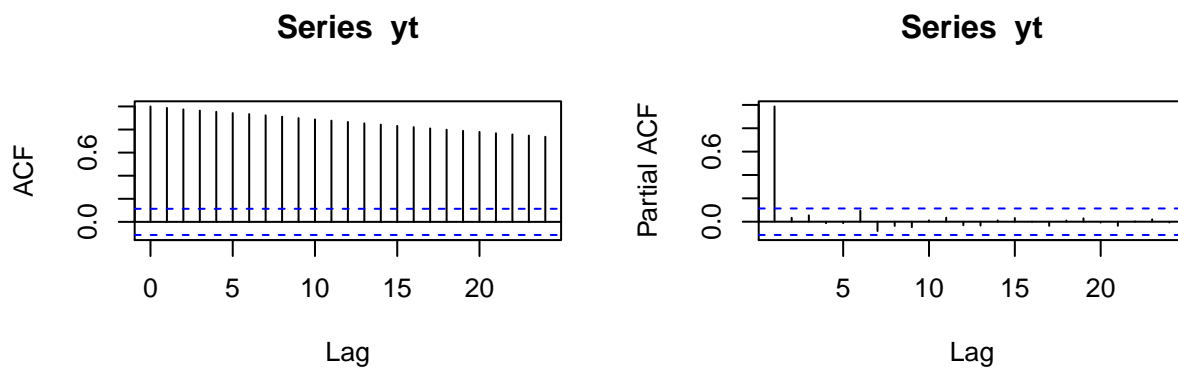
12/8/2021

```r
df <- read.csv("GDP_US_Quarterly.csv",header=TRUE, row.names="X")
#df
```

```r
yt <- df$GDP
fig1 <- plot(yt, type = "o")
```
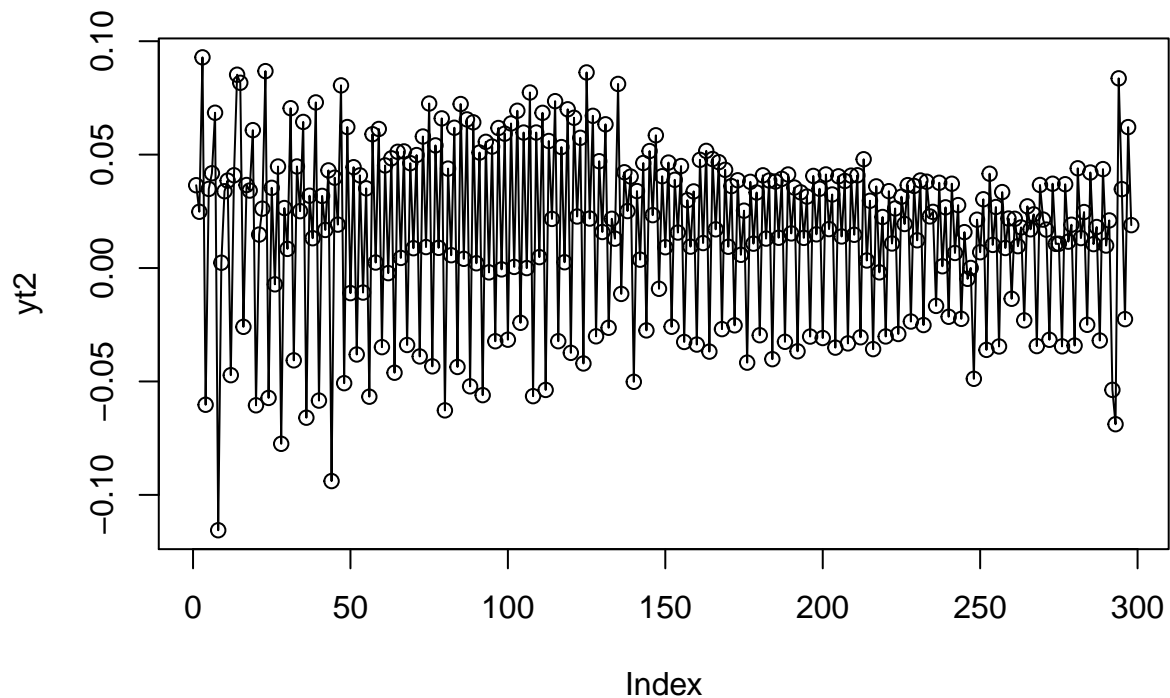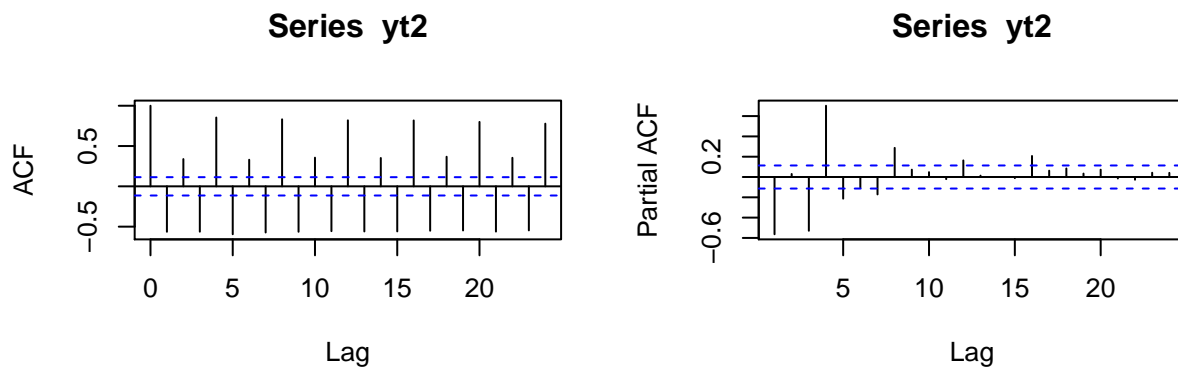


```r
par(mfrow=c(2,2))
acf(yt)
pacf(yt)
```



```r
yt1 <- log(yt)
yt2 <- diff(yt1)
fig2 <- plot(yt2, type = "o")
```

```r
par(mfrow=c(2,2))
acf(yt2)
pacf(yt2)
```

Series yt2

Series yt2


```r
library(forecast)
```

```
## Warning: package 'forecast' was built under R version 3.6.2
```

```
## Registered S3 methods overwritten by 'ggplot2':
##   method         from
##   [.quosures     rlang
##   c.quosures     rlang
##   print.quosures rlang
```

```
## Registered S3 method overwritten by 'quantmod':
##   method           from
##   as.zoo.data.frame zoo
```

```r
auto.arima(yt2, seasonal = T)
```

```
## Series: yt2
## ARIMA(1,1,2)
```

```
## 
## Coefficients:
##           ar1      ma1     ma2
##       -0.2750  -1.4079  0.4373
## s.e.   0.0914   0.0887  0.0875
## 
## sigma^2 estimated as 0.0009611:  log likelihood=609.57
## AIC=-1211.14   AICc=-1211   BIC=-1196.36
```

```
library(astsa)
```

```
## Warning: package 'astsa' was built under R version 3.6.2
```

```
## 
## Attaching package: 'astsa'
```

```
## The following object is masked from 'package:forecast':
## 
##     gas
```
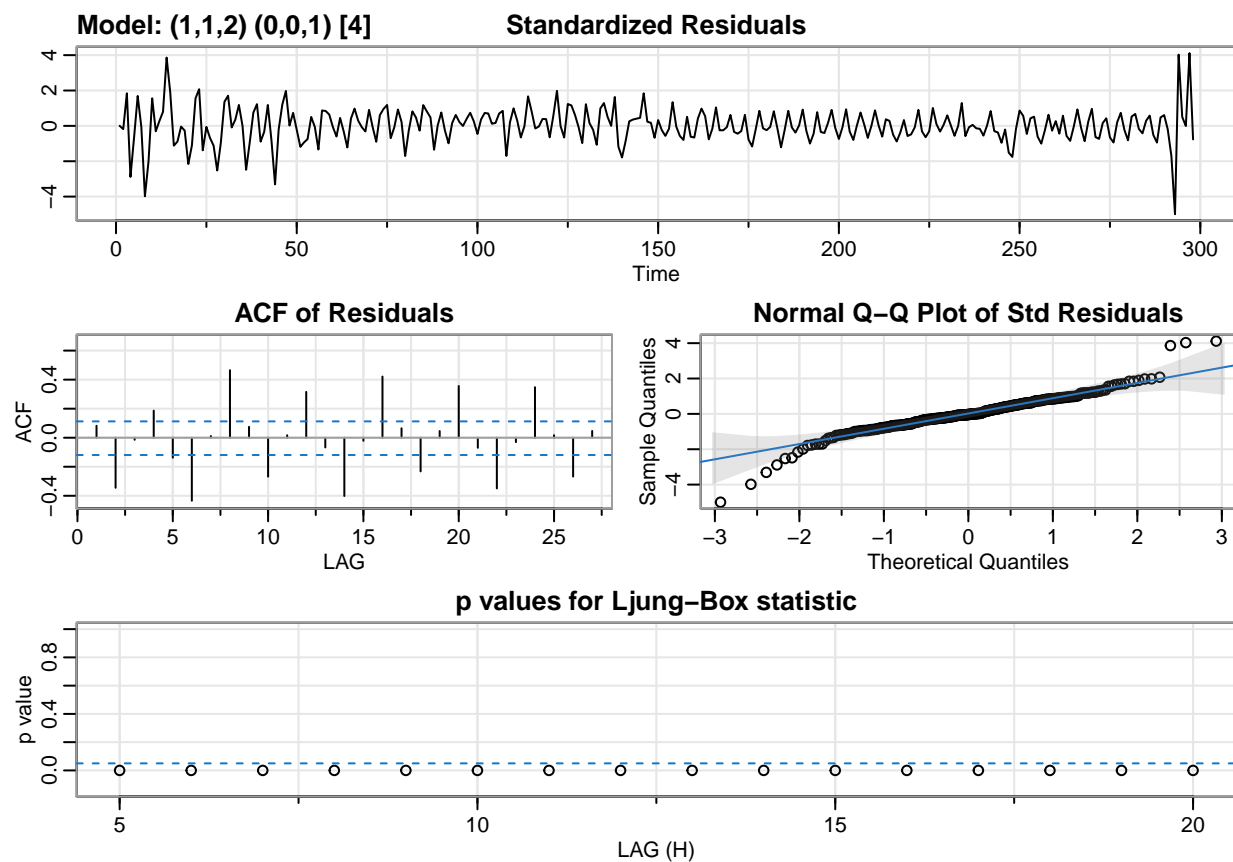
```
sarima (yt2, 1,1,2,0,0,1,4)
```

```
## initial  value -2.694385
## iter   2 value -2.705946
## iter   3 value -3.105776
## iter   4 value -3.421911
## iter   5 value -3.528776
## iter   6 value -3.563423
## iter   7 value -3.587628
## iter   8 value -3.632510
## iter   9 value -3.640338
## iter  10 value -3.646493
## iter  11 value -3.655514
## iter  12 value -3.657134
## iter  13 value -3.658912
## iter  14 value -3.673898
## iter  15 value -3.694596
## iter  16 value -3.715332
## iter  17 value -3.715828
## iter  18 value -3.736361
## iter  19 value -3.774905
## iter  20 value -3.783633
## iter  21 value -3.796686
## iter  22 value -3.798764
## iter  23 value -3.811677
## iter  24 value -3.819902
## iter  25 value -3.823760
## iter  26 value -3.826388
## iter  27 value -3.828585
## iter  28 value -3.828614
## iter  29 value -3.829006
## iter  30 value -3.829374
## iter  31 value -3.830091
## iter  32 value -3.830161
## iter  33 value -3.830291
## iter  34 value -3.830295
```

```
## iter  35 value -3.830295
## iter  35 value -3.830295
## iter  35 value -3.830295
## final  value -3.830295
## converged
## initial  value -3.842651
## iter   2 value -3.844046
## iter   3 value -3.847317
## iter   4 value -3.847771
## iter   5 value -3.848658
## iter   6 value -3.848780
## iter   7 value -3.848848
## iter   8 value -3.848864
## iter   9 value -3.848919
## iter  10 value -3.849096
## iter  11 value -3.849314
## iter  12 value -3.849541
## iter  13 value -3.849564
## iter  14 value -3.849586
## iter  15 value -3.849591
## iter  16 value -3.849598
## iter  17 value -3.849599
## iter  18 value -3.849600
## iter  19 value -3.849602
## iter  20 value -3.849608
## iter  21 value -3.849617
## iter  22 value -3.849626
## iter  23 value -3.849630
## iter  24 value -3.849630
## iter  25 value -3.849630
## iter  25 value -3.849630
## iter  25 value -3.849630
## final  value -3.849630
## converged
```

Model: (1,1,2) (0,0,1) [4]   Standardized Residuals

ACF of Residuals

Normal Q-Q Plot of Std Residuals
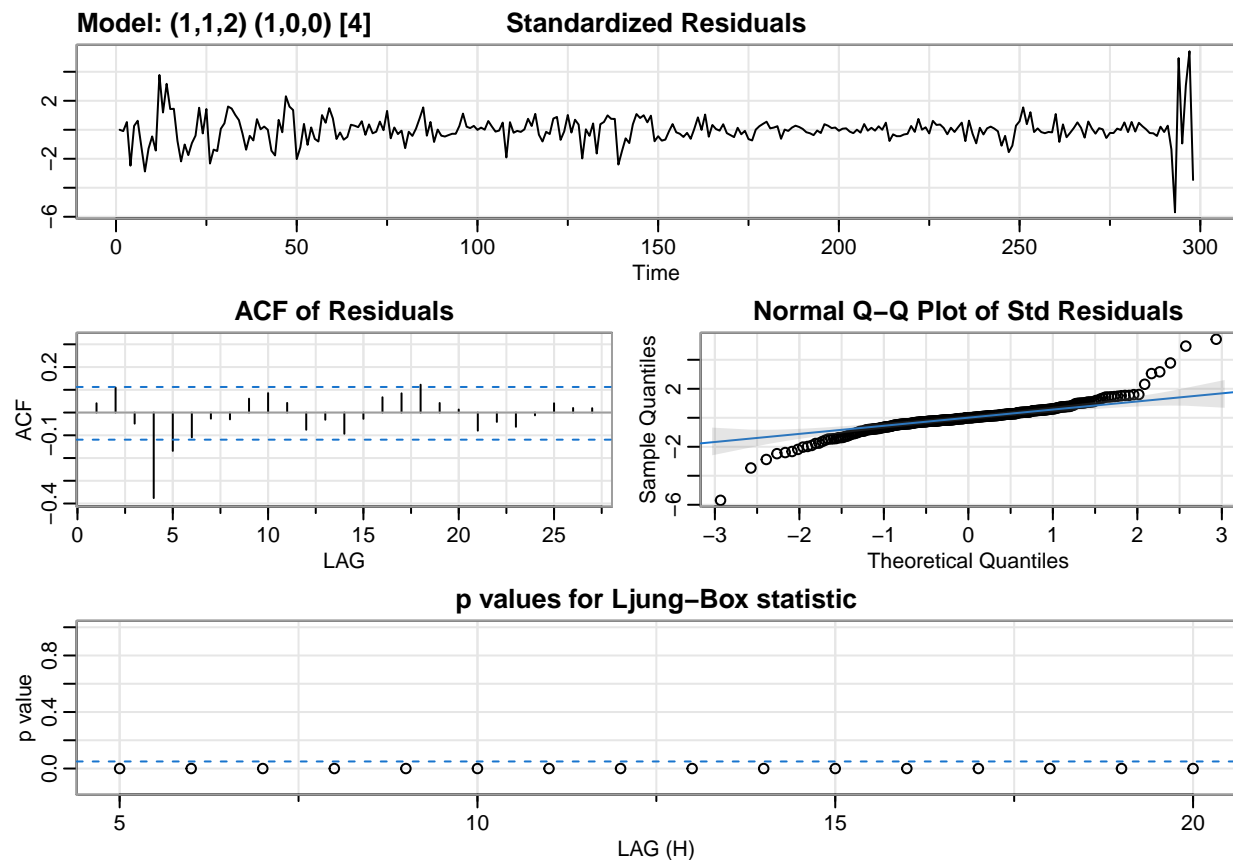

p values for Ljung-Box statistic


```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = constant, transform.pars = trans, fixed = fixed, optim.control = list(trace = trc,
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1      ma2     sma1  constant
##      -0.9996  -0.0259  -0.9740   0.4870      0e+00
## s.e.   0.0007   0.0155   0.0153   0.0436      1e-04
##
## sigma^2 estimated as 0.0004399:  log likelihood = 721.92,  aic = -1431.83
##
## $degrees_of_freedom
## [1] 292
##
## $ttable
##          Estimate      SE   t.value p.value
## ar1       -0.9996  0.0007 -1406.9261  0.0000
## ma1       -0.0259  0.0155    -1.6767  0.0947
## ma2       -0.9740  0.0153   -63.8391  0.0000
## sma1       0.4870  0.0436    11.1815  0.0000
## constant   0.0000  0.0001    -0.4985  0.6185
##
## $AIC
```

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```
## [1] -4.820979
##
## $AICc
## [1] -4.820285
##
## $BIC
## [1] -4.746358
```

```r
sarima (yt2, 1,1,2,1,0,0,4)
```

```
## initial  value -2.701530
## iter   2 value -3.327635
## iter   3 value -3.754964
## iter   4 value -3.826833
## iter   5 value -3.852803
## iter   6 value -3.865783
## iter   7 value -3.883374
## iter   8 value -3.915000
## iter   9 value -3.917028
## iter  10 value -3.918660
## iter  11 value -3.921024
## iter  12 value -3.924306
## iter  13 value -3.933728
## iter  14 value -3.951328
## iter  15 value -3.955760
## iter  16 value -3.959153
## iter  17 value -3.961923
## iter  18 value -3.965793
## iter  19 value -3.967152
## iter  20 value -3.971977
## iter  21 value -3.973331
## iter  22 value -3.974727
## iter  23 value -3.977995
## iter  24 value -3.979685
## iter  25 value -3.980675
## iter  26 value -3.981078
## iter  27 value -3.981373
## iter  28 value -3.981600
## iter  29 value -3.981751
## iter  30 value -3.981845
## iter  31 value -3.981918
## iter  32 value -3.982007
## iter  33 value -3.982068
## iter  34 value -3.982075
## iter  35 value -3.982076
## iter  35 value -3.982076
## iter  35 value -3.982076
## final  value -3.982076
## converged
## initial  value -3.959899
## iter   2 value -3.970525
## iter   3 value -3.972459
## iter   4 value -3.972801
## iter   5 value -3.976029
## iter   6 value -3.976295
```

```
## iter   7 value -3.976545
## iter   8 value -3.976561
## iter   9 value -3.976565
## iter  10 value -3.976568
## iter  11 value -3.976569
## iter  11 value -3.976570
## iter  11 value -3.976570
## final  value -3.976570
## converged
```

Model: (1,1,2) (1,0,0) [4]   **Standardized Residuals**

ACF of Residuals   **Normal Q–Q Plot of Std Residuals**

p values for Ljung–Box statistic

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = constant, transform.pars = trans, fixed = fixed, optim.control = list(trace = trc,
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##           ar1      ma1      ma2    sar1  constant
##       -0.7049  -0.0832  -0.9168  0.9861      0e+00
## s.e.   0.0604   0.0394   0.0394  0.0127      2e-04
##
## sigma^2 estimated as 0.0003388:  log likelihood = 759.62,  aic = -1507.23
##
## $degrees_of_freedom
## [1] 292
##
```
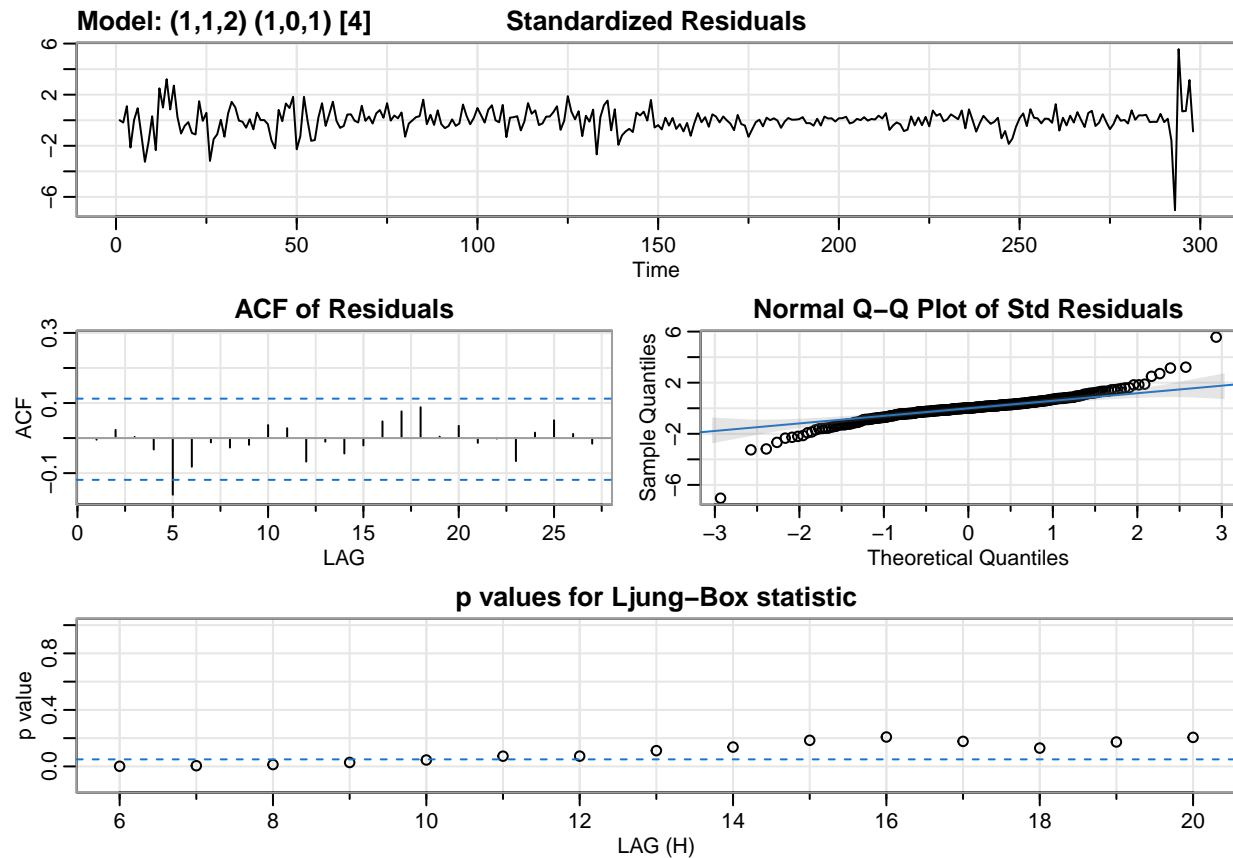
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```
## $ttable
##           Estimate     SE  t.value p.value
## ar1        -0.7049 0.0604 -11.6708  0.0000
## ma1        -0.0832 0.0394  -2.1093  0.0358
## ma2        -0.9168 0.0394 -23.2600  0.0000
## sar1        0.9861 0.0127  77.5923  0.0000
## constant   0.0000 0.0002  -0.0547  0.9565
##
## $AIC
## [1] -5.074858
##
## $AICc
## [1] -5.074164
##
## $BIC
## [1] -5.000237
```

```r
sarima (yt2, 1,1,2,1,0,1,4)
```

```
## initial  value -2.701530
## iter   2 value -3.263302
## iter   3 value -3.676538
## iter   4 value -3.794141
## iter   5 value -3.908196
## iter   6 value -4.007556
## iter   7 value -4.062079
## iter   8 value -4.082248
## iter   9 value -4.105972
## iter  10 value -4.123244
## iter  11 value -4.129590
## iter  12 value -4.133126
## iter  13 value -4.135032
## iter  14 value -4.138029
## iter  15 value -4.138122
## iter  16 value -4.139572
## iter  17 value -4.141067
## iter  18 value -4.142793
## iter  19 value -4.144416
## iter  20 value -4.147954
## iter  21 value -4.149618
## iter  22 value -4.150599
## iter  23 value -4.152451
## iter  24 value -4.157258
## iter  25 value -4.158139
## iter  26 value -4.158286
## iter  27 value -4.158341
## iter  28 value -4.158348
## iter  29 value -4.158374
## iter  30 value -4.158376
## iter  31 value -4.158378
## iter  32 value -4.158380
## iter  33 value -4.158381
## iter  34 value -4.158382
## iter  34 value -4.158382
## iter  34 value -4.158382
```

```
## final   value -4.158382
## converged
## initial  value -4.132813
## iter    2 value -4.146165
## iter    3 value -4.152422
## iter    4 value -4.153691
## iter    5 value -4.159096
## iter    6 value -4.162018
## iter    7 value -4.164549
## iter    8 value -4.166629
## iter    9 value -4.167175
## iter   10 value -4.168418
## iter   11 value -4.171541
## iter   12 value -4.173498
## iter   13 value -4.176718
## iter   14 value -4.179183
## iter   15 value -4.181672
## iter   16 value -4.182969
## iter   17 value -4.183503
## iter   18 value -4.183956
## iter   19 value -4.184070
## iter   20 value -4.184070
## iter   21 value -4.184101
## iter   22 value -4.184120
## iter   23 value -4.184134
## iter   24 value -4.184206
## iter   25 value -4.184270
## iter   26 value -4.184310
## iter   27 value -4.184316
## iter   28 value -4.184317
## iter   29 value -4.184320
## iter   30 value -4.184325
## iter   31 value -4.184342
## iter   32 value -4.184352
## iter   33 value -4.184352
## iter   34 value -4.184355
## iter   35 value -4.184357
## iter   36 value -4.184359
## iter   37 value -4.184372
## iter   38 value -4.184436
## iter   39 value -4.184449
## iter   40 value -4.184450
## iter   41 value -4.184466
## iter   42 value -4.184470
## iter   43 value -4.184473
## iter   44 value -4.184495
## iter   45 value -4.184545
## iter   46 value -4.184612
## iter   47 value -4.184745
## iter   48 value -4.184992
## iter   49 value -4.185197
## iter   50 value -4.185719
## iter   51 value -4.186333
## iter   52 value -4.186387
```

```
## iter  53 value -4.186388
## iter  54 value -4.186429
## iter  55 value -4.186459
## iter  56 value -4.186465
## iter  57 value -4.186490
## iter  58 value -4.186593
## iter  59 value -4.186814
## iter  60 value -4.187244
## iter  61 value -4.187689
## iter  62 value -4.188017
## iter  63 value -4.188034
## iter  64 value -4.188060
## iter  65 value -4.188060
## iter  66 value -4.188072
## iter  67 value -4.188073
## iter  68 value -4.188111
## iter  69 value -4.188244
## iter  70 value -4.188509
## iter  71 value -4.188543
## iter  72 value -4.188550
## iter  73 value -4.188552
## iter  74 value -4.188585
## iter  75 value -4.188619
## iter  76 value -4.188651
## iter  77 value -4.188657
## iter  77 value -4.188657
## final  value -4.188657
## converged
```

Model: (1,1,2) (1,0,1) [4] Standardized Residuals

ACF of Residuals

Normal Q–Q Plot of Std Residuals
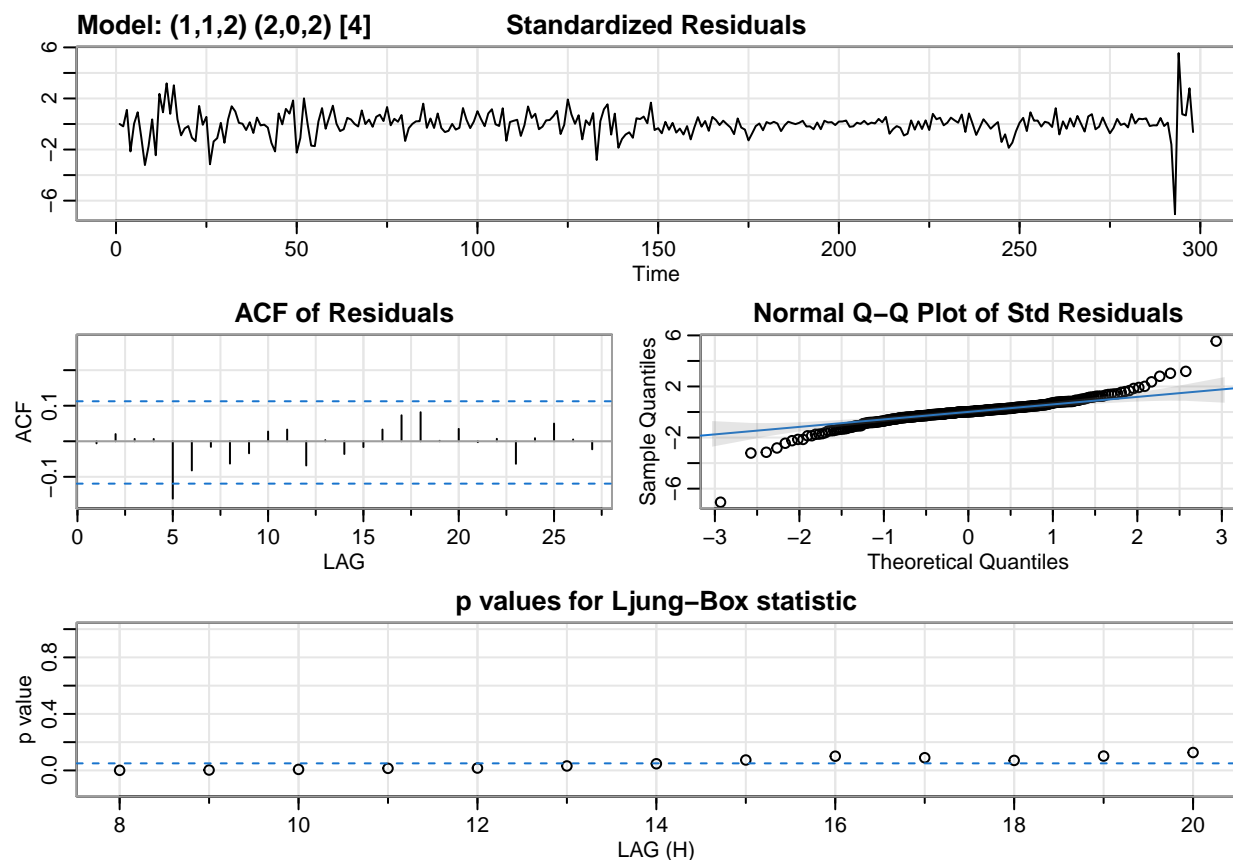
p values for Ljung–Box statistic

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = constant, transform.pars = trans, fixed = fixed, optim.control = list(trace = trc,
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1     ma2    sar1     sma1  constant
##       0.2669  -1.1754  0.1754  0.9973  -0.7725    0e+00
## s.e.  0.3428   0.3441  0.3441  0.0022   0.0352    1e-04
##
## sigma^2 estimated as 0.0002211:  log likelihood = 822.61,  aic = -1631.21
##
## $degrees_of_freedom
## [1] 291
##
## $ttable
##          Estimate      SE  t.value p.value
## ar1        0.2669  0.3428   0.7784  0.4370
## ma1       -1.1754  0.3441  -3.4154  0.0007
## ma2        0.1754  0.3441   0.5097  0.6106
## sar1       0.9973  0.0022 449.4292  0.0000
## sma1      -0.7725  0.0352 -21.9534  0.0000
## constant   0.0000  0.0001  -0.3155  0.7526
##
```

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```
## $AIC
## [1] -5.492299
##
## $AICc
## [1] -5.491323
##
## $BIC
## [1] -5.405241
```

```r
sarima (yt2, 1,1,2,2,0,2,4)
```

```
## initial  value -2.713818
## iter   2 value -3.585435
## iter   3 value -3.819654
## iter   4 value -3.963592
## iter   5 value -4.015298
## iter   6 value -4.057941
## iter   7 value -4.061939
## iter   8 value -4.062097
## iter   9 value -4.090600
## iter  10 value -4.096392
## iter  11 value -4.138966
## iter  12 value -4.163349
## iter  13 value -4.175447
## iter  14 value -4.187409
## iter  15 value -4.194618
## iter  16 value -4.200955
## iter  17 value -4.203849
## iter  18 value -4.205684
## iter  19 value -4.207479
## iter  20 value -4.209346
## iter  21 value -4.210538
## iter  22 value -4.211243
## iter  23 value -4.213228
## iter  24 value -4.213626
## iter  25 value -4.214941
## iter  26 value -4.215852
## iter  27 value -4.217235
## iter  28 value -4.217555
## iter  29 value -4.218817
## iter  30 value -4.219933
## iter  31 value -4.219962
## iter  31 value -4.219962
## iter  32 value -4.220003
## iter  33 value -4.220169
## iter  33 value -4.220169
## iter  34 value -4.220174
## iter  34 value -4.220174
## iter  34 value -4.220174
## final  value -4.220174
## converged
## initial  value -4.157511
## iter   2 value -4.159142
## iter   3 value -4.163404
## iter   4 value -4.166509
```

```
## iter    5 value -4.175868
## iter    6 value -4.179379
## iter    7 value -4.183188
## iter    8 value -4.185653
## iter    9 value -4.187008
## iter   10 value -4.188013
## iter   11 value -4.188957
## iter   12 value -4.189078
## iter   13 value -4.189136
## iter   14 value -4.189171
## iter   15 value -4.189195
## iter   16 value -4.189228
## iter   17 value -4.189311
## iter   18 value -4.189476
## iter   19 value -4.189482
## iter   20 value -4.189482
## iter   21 value -4.189526
## iter   22 value -4.189547
## iter   23 value -4.189587
## iter   24 value -4.189693
## iter   25 value -4.189813
## iter   26 value -4.189891
## iter   27 value -4.189926
## iter   28 value -4.189943
## iter   29 value -4.190011
## iter   30 value -4.190146
## iter   31 value -4.190394
## iter   32 value -4.190573
## iter   33 value -4.190671
## iter   34 value -4.190680
## iter   34 value -4.190680
## iter   34 value -4.190680
## final  value -4.190680
## converged
```

Model: (1,1,2) (2,0,2) [4]   **Standardized Residuals**

ACF of Residuals   **Normal Q–Q Plot of Std Residuals**

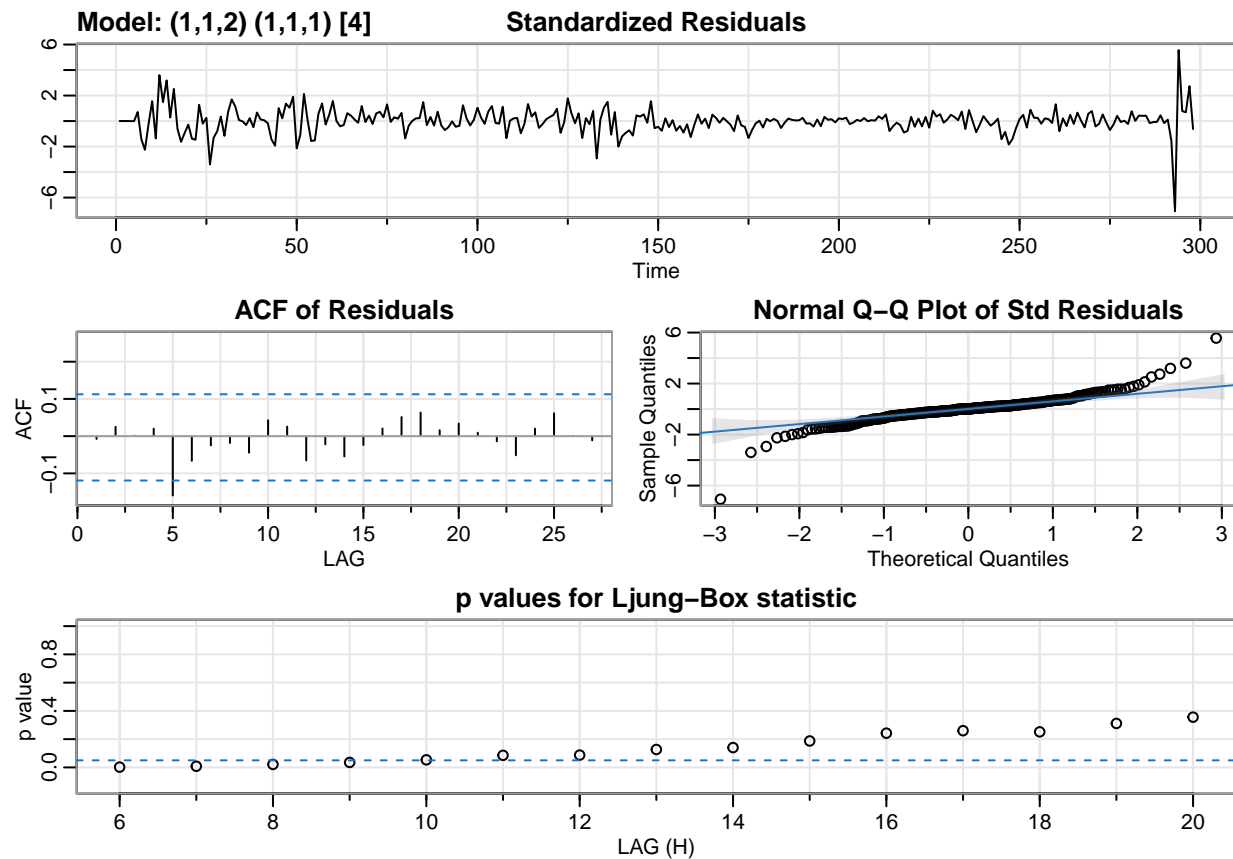**p values for Ljung–Box statistic**


```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = constant, transform.pars = trans, fixed = fixed, optim.control = list(trace = trc,
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1     ma2    sar1    sar2     sma1     sma2  constant
##       0.3122  -1.2245  0.2245  0.3326  0.6624  -0.1517  -0.4537    0e+00
## s.e.  0.3431   0.3474  0.3474  0.3543  0.3534   0.3689   0.2906    1e-04
##
## sigma^2 estimated as 0.0002202:  log likelihood = 823.21,  aic = -1628.41
##
## $degrees_of_freedom
## [1] 289
##
## $ttable
##        Estimate      SE t.value p.value
## ar1      0.3122  0.3431  0.9098  0.3637
## ma1     -1.2245  0.3474 -3.5250  0.0005
## ma2      0.2245  0.3474  0.6463  0.5186
## sar1     0.3326  0.3543  0.9388  0.3486
## sar2     0.6624  0.3534  1.8745  0.0619
## sma1    -0.1517  0.3689 -0.4112  0.6812
## sma2    -0.4537  0.2906 -1.5612  0.1196
```

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```
## constant    0.0000 0.0001 -0.3180   0.7507
##
## $AIC
## [1] -5.482877
##
## $AICc
## [1] -5.481193
##
## $BIC
## [1] -5.370945
```

```r
sarima (yt2, 1,1,2,1,1,1,4)
```

```
## initial  value -3.703801
## iter   2 value -4.058901
## iter   3 value -4.121359
## iter   4 value -4.132447
## iter   5 value -4.136222
## iter   6 value -4.140773
## iter   7 value -4.141844
## iter   8 value -4.143709
## iter   9 value -4.149091
## iter  10 value -4.150259
## iter  11 value -4.153915
## iter  12 value -4.159803
## iter  13 value -4.159903
## iter  14 value -4.168032
## iter  15 value -4.168791
## iter  16 value -4.171488
## iter  17 value -4.174885
## iter  18 value -4.176185
## iter  19 value -4.177962
## iter  20 value -4.181534
## iter  21 value -4.182430
## iter  22 value -4.182440
## iter  23 value -4.182464
## iter  23 value -4.182464
## final  value -4.182464
## converged
## initial  value -4.173507
## iter   2 value -4.173674
## iter   3 value -4.182252
## iter   4 value -4.186283
## iter   5 value -4.187349
## iter   6 value -4.187406
## iter   7 value -4.187411
## iter   8 value -4.187418
## iter   9 value -4.187436
## iter  10 value -4.187485
## iter  11 value -4.187588
## iter  12 value -4.187725
## iter  13 value -4.187736
## iter  14 value -4.187835
## iter  15 value -4.187854
## iter  16 value -4.187857
```

```
## iter  17 value -4.187859
## iter  18 value -4.187860
## iter  19 value -4.187863
## iter  20 value -4.187869
## iter  21 value -4.187878
## iter  22 value -4.187887
## iter  23 value -4.187891
## iter  23 value -4.187891
## final  value -4.187891
## converged
```


Model: (1,1,2) (1,1,1) [4]   **Standardized Residuals**

ACF of Residuals   **Normal Q–Q Plot of Std Residuals**

p values for Ljung–Box statistic

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     include.mean = !no.constant, transform.pars = trans, fixed = fixed, optim.control = list(trace =
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      ma1     ma2     sar1     sma1
##       0.3424  -1.2537  0.2537  -0.0879  -0.7337
## s.e.  0.3396   0.3474  0.3472   0.0787   0.0504
##
## sigma^2 estimated as 0.0002213:  log likelihood = 811.3,  aic = -1610.61
##
## $degrees_of_freedom
## [1] 288
```

```
##
## $ttable
##       Estimate     SE  t.value p.value
## ar1    0.3424 0.3396   1.0083  0.3142
## ma1   -1.2537 0.3474  -3.6094  0.0004
## ma2    0.2537 0.3472   0.7309  0.4654
## sar1  -0.0879 0.0787  -1.1171  0.2649
## sma1  -0.7337 0.0504 -14.5593  0.0000
##
## $AIC
## [1] -5.496949
##
## $AICc
## [1] -5.496236
##
## $BIC
## [1] -5.421588
```

```r
sarima (yt2, 1,1,2,2,2,2,4)
```

```
## initial  value -3.266095
## iter   2 value -3.784253
## iter   3 value -3.902696
## iter   4 value -3.936137
## iter   5 value -3.970843
## iter   6 value -3.981896
## iter   7 value -3.996215
## iter   8 value -4.028918
## iter   9 value -4.036285
## iter  10 value -4.041053
## iter  11 value -4.052421
## iter  12 value -4.059405
## iter  13 value -4.074273
## iter  14 value -4.082583
## iter  15 value -4.087154
## iter  16 value -4.089192
## iter  17 value -4.089844
## iter  18 value -4.090370
## iter  19 value -4.090451
## iter  20 value -4.090455
## iter  21 value -4.090478
## iter  22 value -4.090486
## iter  23 value -4.090491
## iter  24 value -4.090498
## iter  25 value -4.090502
## iter  26 value -4.090504
## iter  26 value -4.090504
## iter  26 value -4.090504
## final  value -4.090504
## converged
## initial  value -4.029334
## iter   2 value -4.047175
## iter   3 value -4.052751
## iter   4 value -4.053846
## iter   5 value -4.054780
```
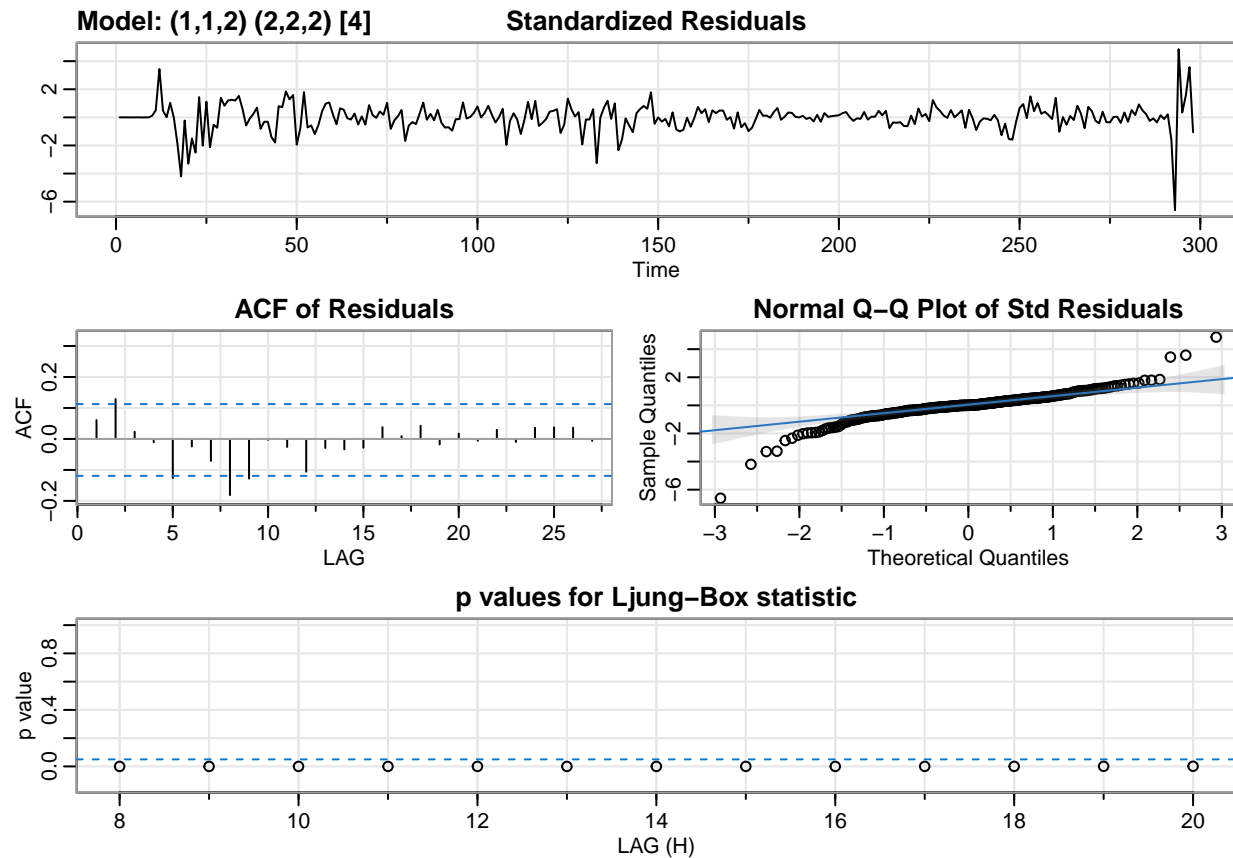
```
## iter    6 value -4.055230
## iter    7 value -4.055997
## iter    8 value -4.056524
## iter    9 value -4.057745
## iter   10 value -4.059020
## iter   11 value -4.059259
## iter   12 value -4.059521
## iter   13 value -4.059531
## iter   14 value -4.059536
## iter   15 value -4.059540
## iter   16 value -4.059549
## iter   17 value -4.059575
## iter   18 value -4.059629
## iter   19 value -4.059639
## iter   20 value -4.059643
## iter   21 value -4.059676
## iter   22 value -4.059694
## iter   23 value -4.059716
## iter   24 value -4.059731
## iter   25 value -4.059761
## iter   26 value -4.059815
## iter   27 value -4.059865
## iter   28 value -4.059897
## iter   29 value -4.059913
## iter   30 value -4.059919
## iter   31 value -4.059926
## iter   32 value -4.059939
## iter   33 value -4.059962
## iter   34 value -4.059983
## iter   35 value -4.059992
## iter   36 value -4.059993
## iter   37 value -4.059993
## iter   38 value -4.059994
## iter   39 value -4.059995
## iter   40 value -4.059995
## iter   40 value -4.059995
## iter   40 value -4.059995
## final  value -4.059995
## converged

## Warning in sqrt(diag(fitit$var.coef)): NaNs produced

## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```

Model: (1,1,2) (2,2,2) [4]   Standardized Residuals

ACF of Residuals

Normal Q–Q Plot of Std Residuals

p values for Ljung–Box statistic


```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     xreg = constant, transform.pars = trans, fixed = fixed, optim.control = list(trace = trc,
##         REPORT = 1, reltol = tol))
##
## Coefficients:

## Warning in sqrt(diag(x$var.coef)): NaNs produced

##           ar1      ma1      ma2     sar1     sar2     sma1     sma2
##       -0.8917  -0.0302  -0.9698  -1.5042  -0.5749  -0.0709  -0.9291
## s.e.   0.0613   0.0434   0.0432   0.0963   0.0658   0.0976   0.0969
##       constant
##          1e-04
## s.e.      NaN
##
## sigma^2 estimated as 0.0002603:  log likelihood = 763.27,  aic = -1508.53
##
## $degrees_of_freedom
## [1] 281
##
## $ttable
##         Estimate     SE  t.value p.value
## ar1      -0.8917 0.0613 -14.5364  0.0000
## ma1      -0.0302 0.0434  -0.6956  0.4873
```

```
## ma2        -0.9698 0.0432 -22.4411  0.0000
## sar1       -1.5042 0.0963 -15.6190  0.0000
## sar2       -0.5749 0.0658  -8.7397  0.0000
## sma1       -0.0709 0.0976  -0.7267  0.4680
## sma2       -0.9291 0.0969  -9.5926  0.0000
## constant   0.0001    NaN      NaN      NaN
##
## $AIC
## [1] -5.219829
##
## $AICc
## [1] -5.218049
##
## $BIC
## [1] -5.105649
```