Q1

Initialize two subsets A and B from the vertex set V with $A = \emptyset$ and $B = \emptyset$

Proceed through a series of iterations as follows:

1. At each iteration k, select the vertices indexed $2k-1$ and $2k$.
2. Decide the optimal placement of these vertices between A and B to increase the number of connecting edges across the partition.

For any given iteration where considering the placement of vertices x and y:

- Define D_{Ax} and D_{Bx} as the counts of neighbours of vertex x within subsets A and B
- Similarily, let D_{Ay} and D_{By} be the neighbour counts for vertex y within A and B.

Then, assign x to A and y to B contributes $D_{Ax} + D_{By}$ new edges to the partition interface. Conversely, positioning x in B and y in A adds $D_{By} + D_{Ax}$ new edges.

In each decision point, choose the vertex arrangement that maximizes the increase in the partition edge count, ensuring the larger of $D_{Bx} + D_{Ay}$ or $D_{By} + D_{Ax}$ is selected.

By consistently opting for the superior outcome, the algorithm ensures that each pair of vertices contributes optimally to the cross-partition edge count. As each choice adds at least half of the possible new edges for that pair, the method secures at least half of the total possible edges $|E|$ in the final edge count $|E(A,B)|$ of the partition.

Given that the maximum achievable edge count across any optimal partition could not surpass $|E|$, the approach described here delievers a performance that is at least a half-approximation of the ideal solution. aligning with the constraints of the Max Equal Cut problem.

Q2.

a) Define s_i as the number of students placed in classroom i, where i ranges from 1 to 7.

b) Objective function:

Maximize $\sum_{i=1}^{7} \alpha_i (C_i - S_i)$

where α_i are known coefficients, and C_i are the capacities of the classrooms.

c). Subject to $S_i \le \frac{1}{2} C_i \quad \forall i = 1, 2, \ldots, 7.$

$S_i \ge 0$ and $S_i \in \mathbb{Z} \quad \forall i = 1, 2, \ldots, 7$

$\sum_{i=1}^{7} S_i = 650$

Q3

1). x_u: where $x_u = 1$ if vertex u is on the side of the source s in the cut, and $x_u = 0$ otherwise.

$x_{(u,v)}$: where $x_{(u,v)} = 1$ if the edge (u,v) cross the cut from the side of s to the side of t, and $x_{(u,v)} = 0$ otherwise.

$C(u,v)$: the capacity of edge (u,v).

b) objective function:

$$\text{Minimize} \sum_{(u,v)\in E} C(u,v)\, x_{(u,v)}$$

c) subject to:

Ensure the source s and sink t are on different sides of cut

$$\begin{cases} x_s = 1 \\ x_t = 0 \end{cases}$$

$$x_u \in \{0,1\} \quad \forall u \in V: u \neq s,t$$

binary variables

$$x_{(u,v)} \in \{0,1\} \quad \forall (u,v) \in E.$$

$$x_v - x_u + x_{(u,v)} \geq 0 \quad \forall (u,v) \in E.$$

If u is with s ($x_u = 1$) and v is with t ($x_v = 0$), then for the inequality to hold, $x_{(u,v)}$ must be set to 1, indicating the edge (u,v) across the cut.

If u and v are on the same side of the cut, then $x_u = x_v$, and $x_{(u,v)}$ can be 0, as the edge does not cross the cut.