CSCI-570 Spring 2023

Practice Midterm 2

INSTRUCTIONS

- The duration of the exam is 140 minutes, closed book and notes.
- No space other than the pages on the exam booklet will be scanned for grading! Do not write your solutions on the back of the pages.
- If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.

1. True/False Questions

a) (T/F) There is a feasible circulation with demands d_v if $\sum_v d_v = 0$.

b) (T/F) If all capacities in a flow network are integers, then every maximum flow in the network is such that flow value on each edge is an integer.

c) (T/F) If you are given a maximum $s-t$ flow in a graph $G = (V, E)$ then you can find a minimum $s - t$ cut in time $O(E)$ where E is the number of the edges in the graph.

d) (T/F) The optimization version of the Linear Programming is not in NP.

e) (T/F) Maximize

$$\sum_{1 \leq i \leq n} x_i(1 - x_i)$$

subject to

$$x_i + x_j < 1$$

and

$$x_i \in \{0, 1\}$$

where $1 \leq i \leq n, 1 \leq j \leq m$ and n and m are constants, is an integer linear program

f) (T/F) Every optimization problem has an equivalent decision problem.

g) (T/F) If problem A can be reduced to problem B and B can be solved in polynomial time, then so can be A.

h) (T/F) All the NP-hard problems are in NP.

i) (T/F) If SAT $\leq_P$ A, then A is NP-hard.

j) (T/F) Every problem in NP can be solved in polynomial time by a nondeterministic Turing machine.

2. Multiple Choice Questions

a) What is the role of the objective function in a linear programming problem?

a) To define the feasible region
b) To set the constraints
c) To measure the profit or cost of a decision
d) To set the decision variables

b) What is the relationship between NP-hard and NP-complete problems?

a) All NP-complete problems are also NP-hard
b) All NP-hard problems are also NP-complete
c) NP-hard problems are a subset of NP-complete problems
d) NP-hard problems are not related to NP-complete problems

c) Assuming $P \neq NP$, which of the following is true?

a) NP-complete = NP
b) NP-complete $\cap$ P $\neq \emptyset$
c) NP-hard = NP
d) P = NP-complete

d) Let X be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to X and X is polynomial-time reducible to R. Which one of the following statements is true?

a) Q is NP-complete
b) R is NP-complete
c) R is NP-hard
d) Q is NP-hard

e) Which of the following statements is correct ?

a) Every LP problem has at least one optimal solution.
b) Every LP problem has a unique solution.
c) If an LP problem has two optimal solutions, then it has infinitely many solutions.
d) If a feasible region is unbounded then LP problem has no solution

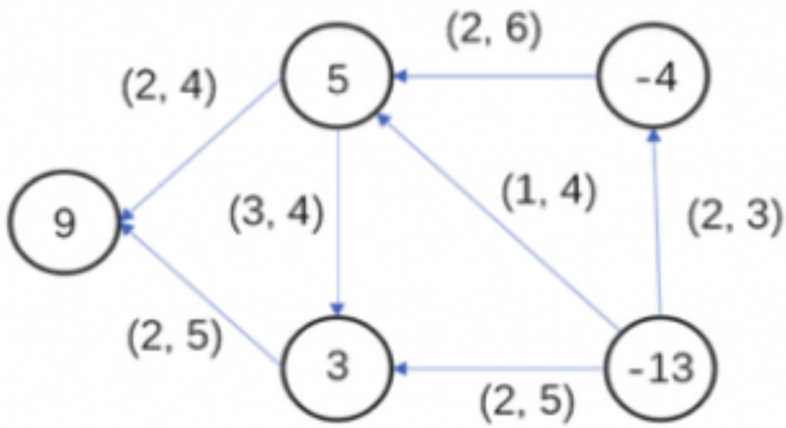

Figure 0.1: Question 3

3. Network Flow

Given the network below with the demand values on vertices and lower bounds on edge capacities, determine if there is a feasible circulation in this graph.

a) Turn the circulation with lower bounds problem into a circulation problem without lower bounds

b) Turn the circulation with demands problem into the max-flow problem

c) Does a feasible circulation exist? Explain your answer.

4. Duality

Consider the following linear programming problem:

$$\max(3x_1 + 8x_2)$$

subject to:

$$\begin{aligned} x_1 + 4x_2 &\leq 20 \\ x_1 + x_2 &\geq 7 \\ x_1 &\geq -1 \\ x_2 &\leq 5 \end{aligned}$$

Write the dual associated to the above problem.

5. Linear Programming

A paper company is building warehouses to supply its $m \geq 1$ customers. The company is considering whether to build a warehouse at each of $n \geq 1$ potential construction sites. The cost of building a warehouse at site i is denoted by $b_i \geq 0$. After the warehouses are built, the company assigns each customer to a warehouse that will supply it. Each warehouse that is built may supply paper to any number of customers, but each customer is supplied by exactly one warehouse. The cost of supplying customer j from a warehouse built at site i is denoted by $s_{ij} \geq 0$. Find a subset of construction sites and an assignment of customers to warehouses such that the total cost of building a warehouse at each selected site and supplying each customer from their assigned warehouse is minimized. Formulate this problem as a LP problem

a) Describe what your LP variables represent.

b) Show your objective function.

c) Show your constraints.

6. NP Completeness

Let there be a set ground set G and $S_i \subset G$, $i = 1...n$ be subsets of G. The hitting set problem is defined as finding a set $H \subset G$ of size at-most k such that it intersects all the sets S_i. Prove that this problem is NP complete by reducing from 3SAT.

7. Approximation Algorithm

There are k restaurants and each restaurant receives various numbers of customer ratings, ranging from 1 star to 5 stars. Specifically, the i-th restaurant get n_{ij} ratings of j star, $1 \le j \le 5$. The average rating of the i-th restaurant is

$$\frac{\sum_{j=1}^{5} j \cdot n_{ij}}{\sum_{j=1}^{5} n_{ij}}$$

and it is guaranteed that $\sum_{j=1}^{5} n_{ij} > 0$. To find the highest rating among all the restaurants, we consider the following approximation algorithms

a) Prove that randomly choosing 1 restaurant is a 1/5-approximation algorithm.

b) Prove that choosing $\arg\max_{1 \le i \le k} \frac{n_{i5}}{\sum_{j=1}^{5} n_{ij}}$, a restaurant with the greatest portion of 5-star ratings is a 1/4-approximation algorithm.

c) Similarly, we can choose $\arg\min_{1 \le i \le k} \frac{n_{i1}}{\sum_{j=1}^{5} n_{ij}}$, a restaurant with the *least* portion of 1-star ratings. Does this method produce a better approximation? If so, prove that this is a ρ-approximation for some $\rho > 1/4$. Otherwise, find an example showing that the output of this method is a β-approximation for some $\beta \le 1/4$.