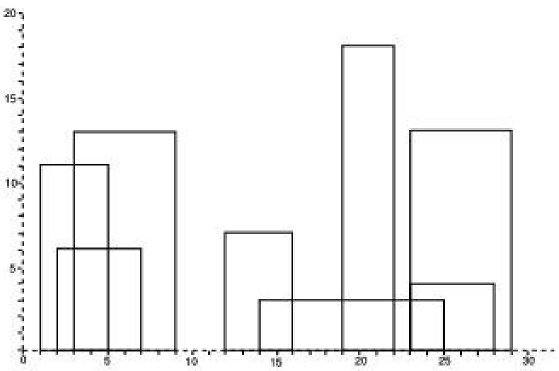
Homework 5

Due: Februaru 11, 11.59 PM PST

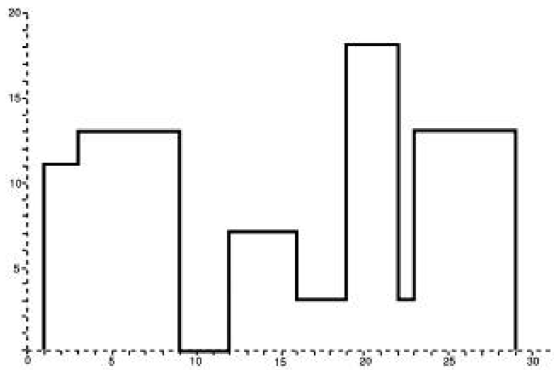
Graded Questions

1. [20 points] Solve the following recurrences by giving tight Θ-notation bounds in terms of n for sufficiently large n. Assume that T(n) represents the running time of an algorithm, i.e., T(n) is a positive and non-decreasing function of n. For each part below, briefly describe the steps along with the final answer:
   a. $T(n) = 9T(n/3) + n^2 logn$
   b. $T(n) = (4.01)T(n/2) + n^2 logn$
   c. $T(n) = \sqrt{6000}T(n/2) + n^{\sqrt{6000}}$
   d. $T(n) = 10T(n/2) + 2^n$
   e. $T(n) = 2T(\sqrt{n}) + log_2 n$

2. [25 points] Solve Kleinberg and Tardos, Chapter 5, Exercise 5.
3. [20 points] Assume that you have a blackbox that can multiply two integers. Describe an algorithm that when given an n-bit positive integer a and an integer x, computes x^a with at most $O(n)$ calls to the blackbox.
4. [25 points] A city's skyline is the outer contour of the silhouette formed by all the buildings in that city when viewed from a distance. A building B_i is represented as a triplet (L_i, H_i, R_i) where L_i and R_i denote the left and right x coordinates of the building, and H_i denotes the height of the building. Describe an O(n log n) algorithm for finding the skyline of n buildings.
   For example, the skyline of the buildings {(3, 13, 9),(1, 11, 5),(12, 7, 16),(14, 3, 25),(19, 18, 22),(2, 6, 7),(23, 13, 29),(23, 4, 28)} is {(1, 11),(3, 13), (9, 0), (12, 7), (16, 3), (19, 18), (22, 3), (23, 13), (29, 0)}.
   (Note that the x coordinates in a skyline are sorted)



buildings



skyline

Ungraded Questions

5. Emily has received a set of marbles as her birthday gift. She is trying to create a staircase shape with her marbles. A staircase shape contains k marbles in the kth row. Given n as the number of marbles help her to figure out the number of rows of the largest staircase she can make.(Time complexity <O(n))
   For example a staircase of size 4 looks like:

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Table 0.1: Staircase of size 4

6. Solve Kleinberg and Tardos, Chapter 5, Exercise 3.