CSCI 570 HOMEWORK 2

Spring 2024

Due: Jan 24 , 11:59PM PST

Q1. What is the tight bound on worst-case runtime performance of the procedure below? Give an explanation for your answer. (10 points)

```
int c = 0;
for(int k = 0; k <= log₂n; k++)
        for(int j = 1; j <= 2^k; j++)
                c=c+1
return c
```

Q2. Given an undirected graph G with n nodes and m edges, design an O(m+n) algorithm to detect whether G contains a cycle. Your algorithm should output a cycle if G contains one. (10 points)

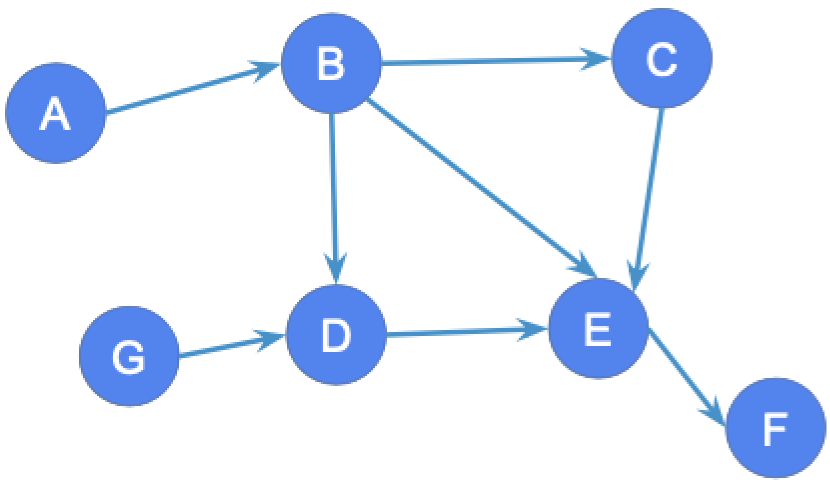
Q3. For each of the following indicate if $f = O(g)$ or $f = \Theta(g)$ or $f = \Omega(g)$ (10 points)

	f(n)	g(n)
1	$n\log(n)$	$n^2\log(n^2)$
2	$\log(n)$	$\log(\log(5^n))$
3	$n^{1/3}$	$(\log(n))^3$
4	2^n	2^{3n}
5	$n^4/\log(n)$	$n(\log(n))^4$

Q4. Indicate for each pair of expressions (A,B) in the table below, whether A is O, Ω, or Θ of B (in other words, whether A=O(B), A= Ω(B), or A= Θ (B)). Assume that k and C are positive constants. You can mark each box with Yes or No. No justification needed. (9 points)
(Note: log is base 2)

A	B	O	Ω	Θ
$n^3+\log(n)+n^2$	$C*n^3$			
n^2	$C*n*2^{\log(n)}$			
$(2^n)*(2^k)$	n^{2k}			

Q5. Find the total number of possible topological orderings in the following graph and list all of them (15 points)



Q6. Given a directed graph with m edges and n nodes where every edge has weight as either 1 or 2, find the shortest path from a given source vertex ‘s’ to a given destination vertex ‘t’. Expected time complexity is O(m+n). (8 points)

Q7. Given functions f_1, f_2, g_1, g_2 such that $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample. (12 points)

(a) $f_1(n) \cdot f_2(n) = O\ (g_1(n) \cdot g_2(n))$

(b) $f_1(n) + f_2(n) = O\ (\max\ (g_1(n), g_2(n)))$

(c) $f_1(n)^2 = O\ g_1(n)^2$

(d) $\log2\ f_1(n) = O\ (\log2\ g_1(n))$

Q8. Design an algorithm which, given a directed graph $G = (V, E)$ and a particular edge $e \in E$, going from node u to node v determines whether G has a cycle containing e. The running time should be bounded by $O(|V| + |E|)$. Explain why your algorithm runs in $O(|V| + |E|)$ time. (8 points)

Q9. Solve Kleinberg and Tardos, **Chapter 3, Exercise 6.** (8 points)

Q10. Solve Kleinberg and Tardos, **Chapter 3, Exercise 9.** (10 points)