§3 有理函数的不定积分

一、有理函数的积分

定义 两个多项式的商表示的函数称之为有理函数.

$$\frac{P(x)}{Q(x)}=\frac{a_0x^n+a_1x^{n-1}+\cdots+a_{n-1}x+a_n}{b_0x^m+b_1x^{m-1}+\cdots+b_{m-1}x+b_m}$$

其中m、n都是非负整数；$a_0,a_1,\cdots,a_n$及$b_0,b_1,\cdots,b_m$都是实数，并且$a_0\neq 0$，$b_0\neq 0$.

假定分子与分母之间没有公因式

(1) $n < m$，这有理函数是**真分式**；

(2) $n \geq m$，这有理函数是**假分式**；

利用多项式除法，假分式可以化成一个多项式和一个真分式之和.

例如

$$\frac{x^5}{1-x^2} = \frac{x^5 - x^3 + x^3}{1-x^2} = -x^3 + \frac{x^3 - x}{1-x^2} + \frac{x}{1-x^2} = -x^3 - x + \frac{x}{1-x^2}.$$

真分式有理函数化为部分分式之和的一般规律：

（1）分母中若有因式 $(x-a)^k$，则分解后有

$$\frac{A_1}{x-a}+\frac{A_2}{(x-a)^2}+\cdots+\frac{A_k}{(x-a)^k}$$

其中$A_1, A_2, \cdots, A_k$都是常数.

特殊地：$k=1$, 分解后为$\dfrac{A}{x-a}$;

（2）分母中若有因式 $(x^2+px+q)^k$，其中 $p^2-4q<0$ 则分解后有

$$\frac{M_1x+N_1}{x^2+px+q}+\frac{M_2x+N_2}{(x^2+px+q)^2}+\cdots++\frac{M_kx+N_k}{(x^2+px+q)^k}$$

其中M_i, N_i都是常数$(i=1,2,\cdots,k)$.

特殊地：$k=1$，分解后为 $\dfrac{Mx+N}{x^2+px+q}$；

定理3.1（有理函数的分解定理）

若有理真分式 $\dfrac{P(x)}{Q(x)}$ 的分母$Q(x)$有分解式

$$Q(x)=(x-a)^{\alpha}\cdots(x-b)^{\beta}(x^2+px+q)^{\mu}\cdots(x^2+rx+s)^{\nu},$$

$a,\cdots b,p,q,\cdots,r,s,\mu,\cdots,\nu$为实数，且$p^2-4q<0,\cdots,$
$r^2-4s<0,\alpha,\cdots,\beta,\mu,\cdots,\nu$为正整数，则

$$\frac{P(x)}{Q(x)}=\frac{A_1}{x-a}+\frac{A_2}{(x-a)^2}+\cdots+\frac{A_\alpha}{(x-a)^\alpha}+\cdots+\frac{B_1}{x-b}+\frac{B_2}{(x-b)^2}$$

$$+\cdots+\frac{B_\beta}{(x-b)^\beta}+\frac{C_1x+D_1}{x^2+px+q}+\frac{C_2x+D_2}{(x^2+px+q)^2}+\cdots$$

$$+\frac{C_\mu x+D_\mu}{(x^2+px+q)^\mu}+\cdots+\frac{E_1x+F_1}{x^2+rx+s}+\frac{E_2x+E_2}{(x^2+rx+s)^2}+\cdots+\frac{E_\nu x+F_\nu}{(x^2+rx+s)^\nu},$$

其中$A_1,\cdots,A_\alpha,B_1,\cdots,B_\beta,C_1,\cdots,C_\mu,D_1,\cdots,D_\mu,E_1,\cdots,E_\nu,$
$F_1,\cdots,F_\nu$都为实数.

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说明 将有理函数化为部分分式之和后，只出现三类情况：

(1) 多项式；(2) $\dfrac{A}{(x-a)^n}$；(3) $\dfrac{Mx+N}{(x^2+px+q)^n}$；

讨论积分 $\displaystyle\int\frac{Mx+N}{(x^2+px+q)^n}dx$，

$$\because\quad x^2+px+q=\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4},$$

令 $x+\dfrac{p}{2}=t$， $q-\dfrac{p^2}{4}=a^2$， $N-\dfrac{Mp}{2}=b$，

则 $x^2+px+q=t^2+a^2$，$Mx+N=Mt+b$

$$\therefore \int \frac{Mx+N}{(x^2+px+q)^n}dx = \int \frac{Mt}{(t^2+a^2)^n}dt + \int \frac{b}{(t^2+a^2)^n}dt$$

(1) $n=1$, $\displaystyle\int \frac{Mx+N}{x^2+px+q}dx$

$$= \frac{M}{2}\ln(x^2+px+q) + \frac{b}{a}\arctan\frac{x+\frac{p}{2}}{a} + C;$$

(2) $n>1$, $\displaystyle\int \frac{Mx+N}{(x^2+px+q)^n}dx$

$$= -\frac{M}{2(n-1)(t^2+a^2)^{n-1}} + \boxed{b\int \frac{1}{(t^2+a^2)^n}dt}.$$

记 $I_n=\int\frac{dt}{(t^2+a^2)^n}$ 分部积分

$$I_n=\frac{t}{(t^2+a^2)^n}+2n\int\frac{t^2}{(t^2+a^2)^{n+1}}dt$$

$$=\frac{t}{(t^2+a^2)^n}+2n\int\frac{t^2+a^2-a^2}{(t^2+a^2)^{n+1}}dt$$

$$=\frac{t}{(t^2+a^2)^n}+2nI_n-2na^2I_{n+1}$$

$$I_{n+1}=\frac{1}{2na^2}\left\{\frac{t}{(t^2+a^2)^n}+(2n-1)I_n\right\}$$

$$I_1=\int\frac{dt}{a^2+t^2}=\frac{1}{a}\arctan\frac{t}{a}+c$$

递推即可

真分式化为部分分式之和的**待定系数法**

$$\frac{x+3}{x^2-5x+6}=\frac{x+3}{(x-2)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3},$$

$$\because \quad x+3=A(x-3)+B(x-2),$$

$$\therefore \quad x+3=(A+B)x-(3A+2B),$$

$$\Rightarrow\begin{cases}A+B=1,\\-(3A+2B)=3,\end{cases}\quad\Rightarrow\begin{cases}A=-5\\B=6\end{cases},$$

$$\therefore \quad \frac{x+3}{x^2-5x+6}=\frac{-5}{x-2}+\frac{6}{x-3}.$$

$$\frac{1}{x(x-1)^2}=\frac{A}{x}+\frac{B}{(x-1)^2}+\frac{C}{x-1},$$

$$1=A(x-1)^2+Bx+Cx(x-1) \qquad (1)$$

代入特殊值来确定系数 A,B,C

取 $x=0,\ \Rightarrow A=1$ 取 $x=1,\Rightarrow B=1$

取 $x=2,$ 并将 A,B 值代入(1) $\Rightarrow C=-1$

$$\therefore \frac{1}{x(x-1)^2}=\frac{1}{x}+\frac{1}{(x-1)^2}-\frac{1}{x-1}.$$

$$\frac{1}{(1+2x)(1+x^2)}=\frac{A}{1+2x}+\frac{Bx+C}{1+x^2},$$

$$1=A(1+x^2)+(Bx+C)(1+2x),$$

整理得 $1=(A+2B)x^2+(B+2C)x+C+A,$

$$\begin{cases} A+2B=0, \\ B+2C=0, \\ A+C=1, \end{cases} \Rightarrow A=\frac{4}{5}, B=-\frac{2}{5}, C=\frac{1}{5},$$

$$\therefore \frac{1}{(1+2x)(1+x^2)}=\frac{\frac{4}{5}}{1+2x}+\frac{-\frac{2}{5}x+\frac{1}{5}}{1+x^2}.$$

思考：可否通过取x特殊值，确定A,B,C

$$x=-\frac{1}{2}\Rightarrow 1=\frac{5}{4}A,\ x=\mathrm{i},\Rightarrow 1=(B\mathrm{i}+C)(1+2\mathrm{i})$$

例1 求积分 $\int \frac{1}{x(x-1)^2}dx.$

解 $$\int \frac{1}{x(x-1)^2}dx = \int \left[\frac{1}{x} + \frac{1}{(x-1)^2} - \frac{1}{x-1}\right]dx$$

$$= \int \frac{1}{x}dx + \int \frac{1}{(x-1)^2}dx - \int \frac{1}{x-1}dx$$

$$= \ln|x| - \frac{1}{x-1} - \ln|x-1| + C.$$

例2 求积分 $\int \frac{1}{1+x^4}dx.$

解 $$\frac{1}{1+x^4}=\frac{1}{1+x^4+2x^2-2x^2}=\frac{1}{(1+x^2)^2-(\sqrt{2}x)^2}$$

$$=\frac{1}{(1+x^2-\sqrt{2}x)(1+x^2+\sqrt{2}x)}$$

$$=\frac{Ax+B}{1+x^2-\sqrt{2}x}+\frac{Cx+D}{1+x^2+\sqrt{2}x}$$

$$1=(A+C)x^3+(B+D+\sqrt{2}A-\sqrt{2}C)x^2$$
$$+(A+C+\sqrt{2}B-\sqrt{2}D)x+B+D$$

$$A=-\frac{\sqrt{2}}{4},B=D=\frac{1}{2},C=\frac{\sqrt{2}}{4}.$$

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$$\int \frac{1}{1+x^4}dx = \int \left(\frac{-\frac{\sqrt{2}}{4}x+\frac{1}{2}}{1+x^2-\sqrt{2}x} + \frac{\frac{\sqrt{2}}{4}x+\frac{1}{2}}{1+x^2+\sqrt{2}x} \right) dx$$

$$\int \frac{-\frac{\sqrt{2}}{4}x+\frac{1}{2}}{1+x^2-\sqrt{2}x}dx = \int \frac{-\frac{\sqrt{2}}{4}x+\frac{1}{2}}{\frac{1}{2}+(x-\frac{\sqrt{2}}{2})^2}dx$$

令$t = x-\frac{\sqrt{2}}{2}$ $$= \int \frac{-\frac{\sqrt{2}}{4}t}{\frac{1}{2}+t^2}dx + \int \frac{\frac{1}{4}}{\frac{1}{2}+t^2}dx$$

$$= -\frac{\sqrt{2}}{8}\ln|t^2+\frac{1}{2}| + \frac{\sqrt{2}}{4}\arctan\sqrt{2}t + C$$

$$= -\frac{\sqrt{2}}{8}\ln|x^2-\sqrt{2}x+1| + \frac{\sqrt{2}}{4}\arctan(\sqrt{2}x-1) + C_1$$

类似地， $\int \frac{\frac{\sqrt{2}}{4}x+\frac{1}{2}}{1+x^2+\sqrt{2}x}dx$

$$= \frac{\sqrt{2}}{8}\ln|x^2+\sqrt{2}x+1| + \frac{\sqrt{2}}{4}\arctan(\sqrt{2}x+1) + C_2$$

$$\int \frac{1}{1+x^4}dx = \frac{\sqrt{2}}{8}\ln\left|\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right| + \frac{\sqrt{2}}{4}\arctan(\sqrt{2}x+1)$$

$$+\frac{\sqrt{2}}{4}\arctan(\sqrt{2}x-1) + C$$

例3 $\displaystyle\int \frac{5x+3}{(x^2-2x+5)^2}dx$

解 $\displaystyle\int \frac{5x+3}{(x^2-2x+5)^2}dx = \int \frac{\frac{5}{2}(2x-2)+8}{(x^2-2x+5)^2}dx$

$$= \frac{5}{2}\int \frac{1}{(x^2-2x+5)^2}d(x^2-2x+5) + \int \frac{8}{(x^2-2x+5)^2}dx$$

$$\boxed{I_2 = \int \frac{1}{(t^2+a^2)^2}dt = \frac{1}{2a^2}\left(\frac{t}{t^2+a^2} + I_1\right)}$$

$\boxed{u = x - 1}$

$$\int \frac{dx}{\left(x^2-2x+5\right)^2} = \int \frac{dx}{\left(\left(x-1\right)^2+4\right)^2} = \int \frac{d\left(x-1\right)}{\left(\left(x-1\right)^2+4\right)^2} = \int \frac{du}{\left(u^2+4\right)^2},$$

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$$\int \frac{du}{\left(u^2+4\right)} = \frac{u}{\left(u^2+4\right)} + 2\int \frac{u^2du}{\left(u^2+4\right)^2} = \frac{u}{\left(u^2+4\right)} + 2\int \frac{du}{\left(u^2+4\right)} - 8\int \frac{du}{\left(u^2+4\right)^2}.$$

$$\text{因此}\int \frac{du}{\left(u^2+4\right)^2} = \frac{1}{8}\left(\frac{u}{\left(u^2+4\right)} + \int \frac{du}{\left(u^2+4\right)}\right) = \frac{1}{8}\left(\frac{u}{\left(u^2+4\right)} + \frac{1}{2}\arctan\frac{u}{2}\right) + C$$

$$\text{所以}\int \frac{5x+3}{\left(x^2-2x+5\right)^2}dx = -\frac{5}{2}\left(\frac{1}{x^2-2x+5}\right) + \frac{x-1}{x^2-2x+5} + \frac{1}{2}\arctan\frac{x-1}{2} + C.$$

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二、三角函数有理式的积分

由三角函数和常数经过有限次四则运算构成的函数称之．一般记为 $R(\sin x, \cos x)$

令 $x = 2\arctan u$，（万能代换公式）

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2\tan\frac{x}{2}}{\sec^2\frac{x}{2}} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2u}{1+u^2},$$

$$\cos x = 2\cos^2\frac{x}{2} - 1 = \frac{2}{1+\tan^2\frac{x}{2}} - 1 = \frac{1-u^2}{1+u^2},$$

$$dx = \frac{2}{1+u^2}du,$$

$$\int R(\sin x, \cos x)dx = \int R\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right)\frac{2}{1+u^2}du.$$

例4 求积分 $\int \frac{\sin x}{1+\sin x+\cos x}dx.$

解 由万能置换公式 $\sin x = \frac{2u}{1+u^2}$,

$$\cos x = \frac{1-u^2}{1+u^2} \qquad dx = \frac{2}{1+u^2}du,$$

$$\int \frac{\sin x}{1+\sin x+\cos x}dx = \int \frac{2u}{(1+u)(1+u^2)}du$$

$$= \int \frac{2u}{(1+u)(1+u^2)} du$$

$$= \int \frac{1+u}{1+u^2} du - \int \frac{1}{1+u} du$$

$$= \arctan u + \frac{1}{2} \ln(1+u^2) - \ln|1+u| + C$$

$$\because \quad u = \tan\frac{x}{2}$$

$$= \frac{x}{2} + \ln\left|\sec\frac{x}{2}\right| - \ln\left|1 + \tan\frac{x}{2}\right| + C.$$

例5 求积分 $\int \frac{1}{\sin^4 x} dx.$

解（一） $u = \tan\frac{x}{2}$, $\sin x = \frac{2u}{1+u^2}$, $dx = \frac{2}{1+u^2} du$,

$$\int \frac{1}{\sin^4 x} dx = \int \frac{1+3u^2+3u^4+u^6}{8u^4} du$$

$$= \frac{1}{8}[-\frac{1}{3u^3} - \frac{3}{u} + 3u + \frac{u^3}{3}] + C$$

$$= -\frac{1}{24\left(\tan\frac{x}{2}\right)^3} - \frac{3}{8\tan\frac{x}{2}} + \frac{3}{8}\tan\frac{x}{2} + \frac{1}{24}\left(\tan\frac{x}{2}\right)^3 + C.$$

解（二） 修改万能置换公式, $\int R(\tan x)dx$

令 $u = \tan x$ $\quad x = \arctan u \quad dx = \dfrac{1}{1+u^2}du$

$$\int R(\tan x)dx = \int R(u)\frac{1}{1+u^2}du$$

$$\int \frac{1}{\sin^4 x}dx = \int \frac{dx}{\tan^4 x\cos^4 x} = \int \frac{(1+\tan^2 x)^2}{\tan^4 x}dx$$

$$= \int \frac{(1+u^2)^2}{u^4}\frac{1}{1+u^2}du = \int \frac{1+u^2}{u^4}du$$

$$= -\frac{1}{3u^3} - \frac{1}{u} + C = -\frac{1}{3}\cot^3 x - \cot x + C.$$

解（三） 可以不用万能置换公式.

$$\int \frac{1}{\sin^4 x} dx = \int \csc^2 x (1 + \cot^2 x) dx$$

$$= \int \csc^2 x dx + \int \cot^2 x \boxed{\csc^2 x dx} \quad = d(\cot x)$$

$$= -\cot x - \frac{1}{3}\cot^3 x + C.$$

结论 比较以上三种解法, 便知万能置换不一定是最佳方法.

例6 求积分 $\int \frac{1+\sin x}{\sin 3x+\sin x}dx.$

解1 $\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$$\int \frac{1+\sin x}{\sin 3x+\sin x}dx=\int \frac{1+\sin x}{2\sin 2x\cos x}dx=\int \frac{1+\sin x}{4\sin x\cos^2 x}dx$$

（一）万能公式 $t=\tan\frac{x}{2}$

$$=\int \frac{1+t^2}{4t(1-t)^2}dt=\frac{1}{4}\int\left(\frac{1}{t}+\frac{2}{(1-t)^2}\right)dt$$

$$=\frac{1}{4}\ln|t|+\frac{1}{2}\frac{1}{1-t}+C=\frac{1}{4}\ln\left|\tan\frac{x}{2}\right|+\frac{1}{2(1-\tan\frac{x}{2})}+C$$

解2 $$=\frac{1}{4}\int\frac{1}{\sin x\cos^2 x}dx+\frac{1}{4}\int\frac{1}{\cos^2 x}dx$$

$$=\frac{1}{4}\int\frac{\sin^2 x+\cos^2 x}{\sin x\cos^2 x}dx+\frac{1}{4}\int\frac{1}{\cos^2 x}dx$$

$$=\frac{1}{4}\int\frac{\sin x}{\cos^2 x}dx+\frac{1}{4}\int\frac{1}{\sin x}dx+\frac{1}{4}\int\frac{1}{\cos^2 x}dx$$

$$=-\frac{1}{4}\int\frac{1}{\cos^2 x}d(\cos x)+\frac{1}{4}\int\frac{1}{\sin x}dx+\frac{1}{4}\int\frac{1}{\cos^2 x}dx$$

$$=\frac{1}{4\cos x}+\frac{1}{4}\ln\left|\csc x-\cot x\right|+\frac{1}{4}\tan x+C.$$

三、其它可化为有理函数的积分

例7 求积分 $\int \frac{1}{1+e^{\frac{x}{2}}+e^{\frac{x}{3}}+e^{\frac{x}{6}}}dx.$

解 令 $t=e^{\frac{x}{6}} \Rightarrow x=6\ln t,\qquad dx=\frac{6}{t}dt,$

$$\int \frac{1}{1+e^{\frac{x}{2}}+e^{\frac{x}{3}}+e^{\frac{x}{6}}}dx=\int \frac{1}{1+t^3+t^2+t}\cdot\frac{6}{t}dt$$

$$=6\int \frac{1}{t(1+t)(1+t^2)}dt=\int\left(\frac{6}{t}-\frac{3}{1+t}-\frac{3t+3}{1+t^2}\right)dt$$

$$= \int \left(\frac{6}{t} - \frac{3}{1+t} - \frac{3t+3}{1+t^2} \right) dt$$

$$= 6\ln t - 3\ln(1+t) - \frac{3}{2}\int \frac{d(1+t^2)}{1+t^2} - 3\int \frac{1}{1+t^2} dt$$

$$= 6\ln t - 3\ln(1+t) - \frac{3}{2}\ln(1+t^2) - 3\arctan t + C$$

$$= x - 3\ln(1+e^{\frac{x}{6}}) - \frac{3}{2}\ln(1+e^{\frac{x}{3}}) - 3\arctan(e^{\frac{x}{6}}) + C.$$

简单无理式的不定积分

$R(\sqrt[n]{x}, \sqrt[m]{x})$, 令 $\sqrt[mn]{x} = t$

$R\left(x, \sqrt[n]{\dfrac{ax+b}{cx+d}}\right)$, 令 $\sqrt[n]{\dfrac{ax+b}{cx+d}} = t$

$R(x, \sqrt[n]{ax+b})$, 令 $\sqrt[n]{ax+b} = t$

$R(x, \sqrt{ax^2+bx+c}), b^2-4ac<0$, 配方，三角代换

例8 求积分 $\int \frac{1}{x}\sqrt{\frac{1+x}{x}}dx$

解 令 $\sqrt{\frac{1+x}{x}} = t \Rightarrow \frac{1+x}{x} = t^2,\quad x = \frac{1}{t^2-1},$

$$dx = -\frac{2tdt}{\left(t^2-1\right)^2},$$

$$\int \frac{1}{x}\sqrt{\frac{1+x}{x}}dx = -\int \left(t^2-1\right)t\frac{2t}{\left(t^2-1\right)^2}dt = -2\int \frac{t^2dt}{t^2-1}$$

$$= -2\int \left(1+\frac{1}{t^2-1}\right)dt = -2t - \ln\left|\frac{t-1}{t+1}\right| + C$$

$$= -2\sqrt{\frac{1+x}{x}} - \ln\left[x\left(\sqrt{\frac{1+x}{x}}-1\right)^2\right] + C.$$

例9 求 $\int \dfrac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}$.

解 $\because \sqrt[3]{(x+1)^2(x-1)^4} = \sqrt[3]{\left(\dfrac{x-1}{x+1}\right)^4} \cdot (x+1)^2$.

令 $t = \dfrac{x-1}{x+1}$, 则有 $dt = \dfrac{2}{(x+1)^2}dx$,

$$原式 = \int \frac{dx}{\sqrt[3]{\left(\frac{x-1}{x+1}\right)^4} \cdot (x+1)^2} = \frac{1}{2}\int t^{-\frac{4}{3}}dt$$

$$= -\frac{3}{2}t^{-\frac{1}{3}} + C \quad = -\frac{3}{2}\sqrt[3]{\frac{x+1}{x-1}} + C.$$

例10 求积分 $\int \frac{1}{\sqrt{x+1}+\sqrt[3]{x+1}}dx.$

解 令$t=\sqrt[6]{x+1} \Rightarrow 6t^5dt=dx,$

$$\int \frac{1}{\sqrt{x+1}+\sqrt[3]{x+1}}dx=\int \frac{1}{t^3+t^2}\cdot 6t^5dt$$

$$=6\int \frac{t^3}{t+1}dt=2t^3-3t^2+6t-6\ln|t+1|+C$$

$$=2\sqrt{x+1}-3\sqrt[3]{x+1}+3\sqrt[6]{x+1}-6\ln(\sqrt[6]{x+1}+1)+C.$$

说明 无理函数去根号时, 取根指数的最小公倍数.

例11 求积分 $\int \frac{x}{\sqrt{3x+1}+\sqrt{2x+1}}dx.$

解 先对分母进行有理化

原式 $=\int \frac{x(\sqrt{3x+1}-\sqrt{2x+1})}{(\sqrt{3x+1}+\sqrt{2x+1})(\sqrt{3x+1}-\sqrt{2x+1})}dx$

$$=\int(\sqrt{3x+1}-\sqrt{2x+1})dx$$

$$=\frac{1}{3}\int\sqrt{3x+1}d(3x+1)-\frac{1}{2}\int\sqrt{2x+1}d(2x+1)$$

$$=\frac{2}{9}(3x+1)^{\frac{3}{2}}-\frac{1}{3}(2x+1)^{\frac{3}{2}}+C.$$

例12 $\displaystyle\int \frac{x+1}{\sqrt{x^2+x+1}}dx$

解 $$\int \frac{x+1}{\sqrt{x^2+x+1}}dx=\int \frac{x+1}{\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}}dx$$

令$x+\dfrac{1}{2}=\dfrac{\sqrt{3}}{2}\tan t$ $$=\int\left(\frac{\sqrt{3}}{2}\tan t+\frac{1}{2}\right)\sec t dt$$

$$=\frac{\sqrt{3}}{2}\sec t+\frac{1}{2}\ln\left|\sec x+\tan x\right|+C$$

$$=\sqrt{x^2+x+1}+\frac{1}{2}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$

作业:

习题6.3

1(单数)，2(双数)