Session 1 - Probabilistic Machine Learning - Exercises

1. Download the notebook files from the github repository (`https://github.com/AlexRogersCS/probabilistic_machine_learning`). Either install the Python package from Anaconda (`https://www.anaconda.com/products/individual`) and PyMC3 (`https://docs.pymc.io`), or open them in Google Colab (`https://colab.research.google.com`).

2. Calculate by hand, and verifying in Python (by editing the `probability.ipynb` script as necessary) the following probabilities when two dice are rolled:

   (a) $P(D_1 + D_2 = 6)$

   (b) $P(D_1 + D_2 > 10)$

   (c) $P(D_1 = 1\ ,\ D_2 < 4)$

   (d) $P(D_1 = 1 \mid D_1 + D_2 = 4)$

   (e) $P(D_1 < 5 \mid D_1 + D_2 = 10)$

3. Use Bayes rule to answer each of these three settings:

   (a) You are planning to go on a picnic but the morning is cloudy. This is bad as 50% of all rainy days start of cloudy. However, cloudy morning are common (about 40% of days start cloudy) and this is usually a dry month (only 3 out of 30 days, or 10% of days, tend to be rainy). What is the chance of rain during the day?

   (b) Two production lines produce the same part. Line 1 produces 1,000 parts per week of which 100 are defective. Line 2 produces 2,000 parts per week of which 150 are defective. If you choose a part randomly from the stock what is the probability it is defective? If it is defective what is the probability it was produced by line 1?

   (c) A robot tasked with patrolling an office building has a Lidar sensor to determine if any doors are open or closed. The sensor will successfully detect an open door 60% of the time. However, it will also mistake a closed door for open door 30% of the time. If half the doors in the office building are typically left open, and the robot's sensor detects that a particular door is open, what is the robot's belief state about this door?

Session 1 - Probabilistic Machine Learning - Solutions

1. No solution needed.
2. (a) 5 / 36 : `probability=np.sum(total==6)/N_REPEATS`
   (b) 3 / 36 : `probability=np.sum(total>10)/N_REPEATS`
   (c) 3 / 36 : `probability=np.sum(np.logical_and(dice_1==1,dice_2<4))/N_REPEATS`
   (d) 1 / 3 : `probability=np.sum(np.logical_and(dice_1==1,total==4))/np.sum(total==4)`
   (e) 1 / 3 : `probability=np.sum(np.logical_and(dice_1<5,total==10))/np.sum(total==10)`
3. (a) The information we know is $P(\text{Rain}) = 0.1$, $P(\text{Cloud}|\text{Rain}) = 0.5$, and $P(\text{Cloud}) = 0.4$. Applying Bayes rules gives $P(\text{Rain}|\text{Cloud}) = 0.125$.
   (b) The information we know is $P(D|L_1) = 1/10$, $P(D|L_2) = 3/40$, $P(L_1) = 1/3$ and $P(D) = 5/60$. Applying Bayes rules gives $P(L_1|D) = 2/5$.
   (c) The information we know is $P(\text{Open}) = 0.5$, $P(\text{Sensor Reading}|\text{Open}) = 0.6$, and $P(\text{Sensor Reading}|\text{Closed}) = 0.3$. Applying Bayes rules gives $P(\text{Open}|\text{Sensor Reading}) = 2/3 = 0.67$.