

Probabilistic Machine Learning Session 1

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Introductions

University of Oxford







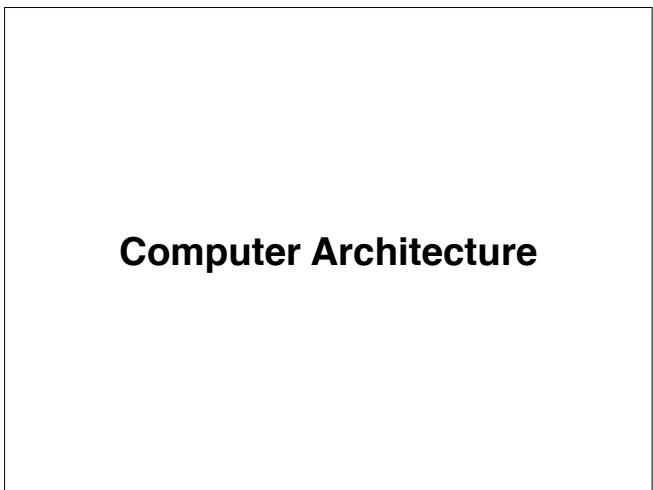




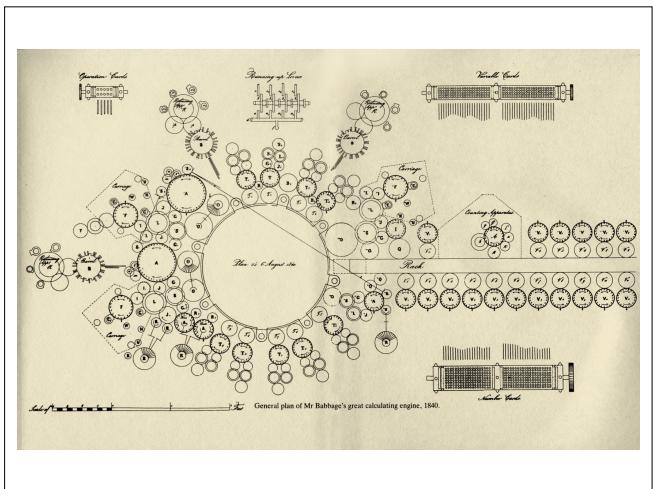
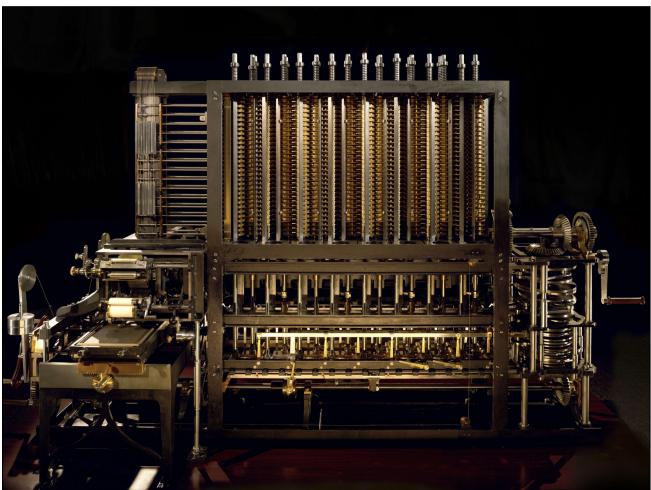


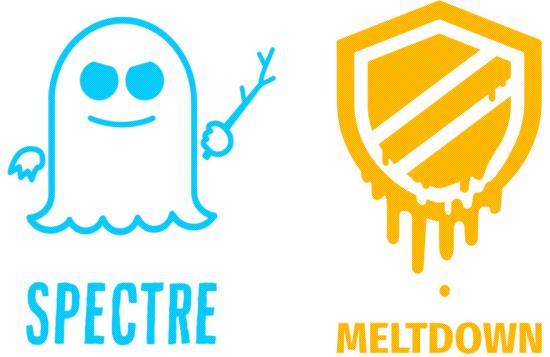
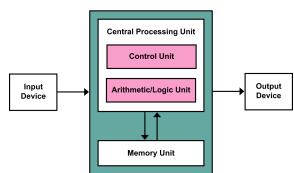
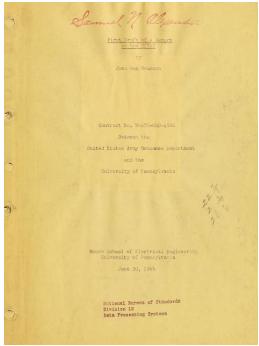






Computer Architecture





Cyber-Physical Systems



MyJoulo - Home

Order your free logger Upload your data Login to view your usage

Personalised energy advice in three simple steps

- 1 Order your free logger
- 2 Record your data
- 3 View your usage

21°

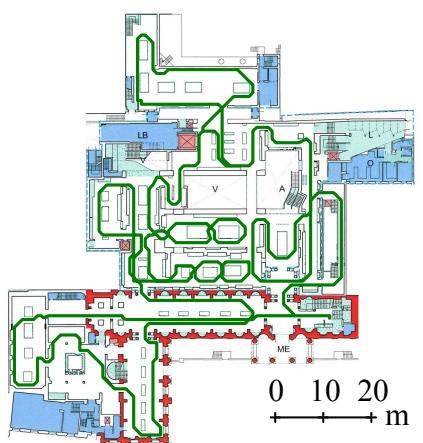
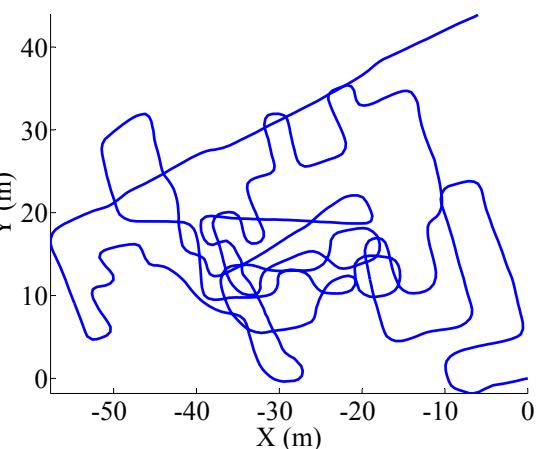
Order your free logger: Register for an account with us and we'll send you a free Joulo logger.

Record your data: Place your Joulo logger on top of the thermostat in your home, and it will log the temperature continuously for one week.

View your usage: Upload the data from the logger here, and you'll receive personalised advice on how you can reduce your heating bill.

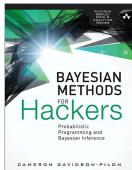
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Probabilistic Machine Learning

- 1.To understand the fundamentals of probability theory, Bayes rule and Bayesian inference.
- 2.To understand how to use PyMC3, a powerful probabilistic programming framework based on Python, to build sophisticated probabilistic models.
- 3.To develop experience applying these models to solve real-world machine learning and data analysis tasks.



Probabilistic Programming and Bayesian Methods for Hackers

Cameron Davidson-Pilon

[https://camdavidsonpilon.github.io/
Probabilistic-Programming-and-Bayesian-
Methods-for-Hackers/](https://camdavidsonpilon.github.io/Probabilistic-Programming-and-Bayesian-Methods-for-Hackers/)



Bayesian Analysis with Python

Osvaldo Martin

GitHub

https://github.com/AlexRogersCS/probabilistic_machine_learning

Python 3.8, Numpy, Matplotlib, Jupyter

<https://www.anaconda.com/products/individual>

PyMC3

<https://docs.pymc.io>

Google Colab Notebooks

<https://colab.research.google.com/>

Motivating Examples

Probability Theory

Single and Joint Probabilities

Probability theory typically describes random variables, $P(X)$, and the probability that these random variables take on actual values.

$$\text{Dice icon}) \quad P(D = 6) = 1/6$$

$$(\text{Dice icon}) \quad P(D_1 = 6, D_2 = 6) = 1/36$$

Independent Probabilities

When events are **independent** their **joint** probability is given by the product of their individual probabilities.

$$(\text{Dice icon}) \quad P(D_1 = 6, D_2 = 6) = 1/36$$

$$P(D_1 = 6, D_2 = 6) = P(D_1 = 6) \times P(D_2 = 6)$$

Conditional Probabilities

Conditional probabilities describe the probability of any event **given** the occurrence of another event.

$$\text{Dice icon}) \quad P(D = 6 | D \text{ is even}) = 1/3$$

$$(\text{Dice icon}) \quad P(D_1 = 6 | D_1 + D_2 > 8) = ?$$

The Rules of Probabilities

There are just two fundamental rules of probability theory.

Sum rule $P(X) = \sum_Y P(X, Y)$

Product rule $P(X, Y) = P(Y|X) \times P(X)$

Plus the observation that probabilities must sum to 1.

Frequentist probability is the interpretation of probability that defines an event's probability as the limit of its relative frequency in a large number of trials.

Bayesian probability is the interpretation of the probability that holds that it can be defined as the degree to which a person (or community) believes that a proposition is true.



Rev. Thomas Bayes (1702 - 1761)

- Framework for plausible reasoning developed over the nineteenth century
- Developed by three physicists:
 - Harold Jeffreys (1891 – 1989)
 - Richard Cox (1898 - 1991)
 - Edwin Jaynes (1922 – 1998)
- Growth of computational power since 2000 has seen this become one of the dominant model for much of machine learning.
- Recent development of probabilistic programming make using Bayesian reasoning ever more accessible.

Cox's axioms describe desirable properties of a framework for reasoning about uncertainty

1. Degrees of plausibility are represented by real numbers.
2. Qualitative correspondence with common sense:
 - Increasing the plausibility of a statement, decreases the plausibility of its negative.
3. Consistency:
 - If a conclusion can be reasoned out in more than one way, then every possible way must lead to the same result.
 - It must be possible to incorporate all the evidence relevant to a question into the reasoning process.
 - All equivalent states of knowledge are represented by equivalent plausibility assignments.

Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes Theorem

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

```

    graph TD
      P[H] --> L[P(D|H)]
      L --> P[D]
      P[H] --> P[P(H)]
      P[D] --> E[Evidence]
      P[P(H)] --> E
  
```

The diagram illustrates the components of Bayes' theorem. At the top, there are three labels: "Posterior" on the left, "Likelihood" in the center, and "Prior" on the right. Below these, the formula $P(H|D) = \frac{P(D|H)P(H)}{P(D)}$ is shown. The term $P(D|H)$ is labeled "Likelihood" with a downward arrow from "Likelihood". The term $P(H)$ is labeled "Prior" with a downward arrow from "Prior". The term $P(D)$ is labeled "Evidence" with a downward arrow from "Evidence".

Example 1



- Suppose that Bob can decide to go to work by one of three modes of transportation, car, bus, or train.
- Because of high traffic, if he decides to go by car, there is a 50% chance he will be late.
- If he goes by bus, which has special reserved lanes but is sometimes overcrowded, the probability of being late is only 20%.
- The train is almost never late, with a probability of only 1%, but is more expensive than the bus.



Example 1



- Suppose that Bob is late one day, and his boss wishes to estimate the probability that he drove to work that day by car.
- Since he does not know which mode of transportation Bob usually uses, he gives a prior probability of 1 of 3 to each of the three possibilities.
- What is the boss' estimate of the probability that Bob drove to work?



Example 1

$$P(\text{car}) = \frac{1}{3}$$

$$P(\text{late}|\text{car}) = \frac{1}{2}$$

$$P(\text{bus}) = \frac{1}{3}$$

$$P(\text{late}|\text{bus}) = \frac{1}{5}$$

$$P(\text{train}) = \frac{1}{3}$$

$$P(\text{late}|\text{train}) = \frac{1}{100}$$

Example 1

$$\begin{aligned} P(\text{car}|\text{late}) &= \frac{P(\text{late}|\text{car})P(\text{car})}{P(\text{late})} \\ &= \frac{P(\text{late}|\text{car})P(\text{car})}{P(\text{late}|\text{car})P(\text{car}) + P(\text{late}|\text{bus})P(\text{bus}) + P(\text{late}|\text{train})P(\text{train})} \\ &= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{100} \times \frac{1}{3}} \\ &\approx 0.704 \end{aligned}$$

Example 1 (cont)



- Bob's co-worker Alice knows Bob's travel preferences much better than his boss.
 - She knows that Bob never takes the bus, and 9 out of 10 times he will take the train to work.
- What is Alice's estimate of the probability that Bob drove to work?



Example 1 (cont)

$$P(\text{car}) = \frac{1}{10}$$

$$P(\text{bus}) = 0$$

$$P(\text{train}) = \frac{9}{10}$$

$$P(\text{late}|\text{car}) = \frac{1}{2}$$

$$P(\text{late}|\text{bus}) = \frac{1}{5}$$

$$P(\text{late}|\text{train}) = \frac{1}{100}$$

Example 1

$$\begin{aligned} P(\text{car}|\text{late}) &= \frac{P(\text{late}|\text{car})P(\text{car})}{P(\text{late})} \\ &= \frac{P(\text{late}|\text{car})P(\text{car})}{P(\text{late}|\text{car})P(\text{car}) + P(\text{late}|\text{train})P(\text{train})} \\ &= \frac{\frac{1}{2} \times \frac{1}{10}}{\frac{1}{2} \times \frac{1}{10} + \frac{1}{100} \times \frac{9}{10}} \\ &\approx 0.848 \end{aligned}$$

Example 2

- A clinical test, designed to diagnose a specific illness, comes out positive for a certain patient.
- We are told that
 - The test is 79 percent reliable (that is, it misses 21 percent of actual cases)
 - On average, this illness affects 1 percent of the population in the same age group as the patient
 - The test has a false positive rate of 10 percent.
- Taking this into account and assuming you know nothing about the patient's symptoms or signs, what is the probability that this patient actually has the illness?

Example 2

$$P(D) = 0.01$$

$$P(T|D) = 0.79$$

$$P(T|\neg D) = 0.1$$

Example 2

$$\begin{aligned} P(D|T) &= \frac{P(T|D)P(D)}{P(T)} \\ &= \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|\neg D)P(\neg D)} \\ &= \frac{0.79 \times 0.01}{0.79 \times 0.01 + 0.1 \times 0.99} \\ &\approx 0.074 \end{aligned}$$

Next Time

Probability Distributions and
Probabilistic Programming