Homework 1

Computation Camp

Due August 9, 2023

Submit Link: https://classroom.github.com/a/PIR0phPT (https://classroom.github.com/a/PIR0phPT)

Question 1

Recall that the factorial function n ! returns $n! = n \times (n-1) \times \ldots 2 \times 1$. Write a function that executes this called factorial2 that uses a for loop.

Question 2

Consider the polynomial

$$p(x) = a_0 + a_1 x + a_2 x^2 + \ldots a_n x^n = \sum_{i=0}^{n} a_i x_i$$

Using enumerate() in your loop, write a function p such that $p(x, coeff)$ computes the value of the polynomial given a point x and an array of coefficients $coeff$.

Question 3

Compute an approximation for π using Monte Carlo. You may only use rand() for random number generation. Hint: If U is a bivariate uniform random variable on the unit square $(0,1)^2$, then the probability that U lies in a subset B of $(0,1)^2$ is equal to the area of B.

Question 4

Let the data generating process for y be

$$y = ax_1 + bx_1^2 + cx_2 + d + \sigma w$$

where y, x_1, x_2 are scalars, a, b, c, d are parameters to estimate, and $w \sim N(0,1)$ iid.

- First, draw $N = 50$ values for x_1, x_2 from iid normal distributions.
- Then, draw a w vector for the N values to generate y for the simulated data. Use $a = 0.1, b = 0.2, c = 0.5, d = 1.0$, and $\sigma = 0.1$. Repeat draws of w until you have 200 different simulations of the y values.
- Finally, calculate ordinary least squares manually for each of the 200 simulations. Plot histograms for each parameter a, b, c, d,.

Question 5

Take a random walk starting from $x_0 = 1$:

$$x_{t+1} = \alpha x_t + \sigma \epsilon_{t+1},$$

where $t = 0, \ldots t_{\max}$. Assume that $x_{t_{\max}} = 0$ with certainty and that $\{\epsilon_t\}$ is drawn from an iid standard normal. Start with $\sigma = 0.2$ and $\alpha = 1.0$. For a given path $\{x_t\}$ define a first-passage time as $T_a = \min\{t \mid x_t \leq a\}$.

- Calculate the first-passage time T_0 for 100 simulated random walks (up to $t_{\max} = 200$) and plot a histogram.
- Plot the sample mean of T_0 from the simulation for $\alpha \in \{0.8, 1.0, 1.2\}$.

Question 6

Recall that the root of a univariate function $f(\cdot)$ is an x such that $f(x) = 0$. One solution method to find local roots of smooth functions is called Newton's method. Starting with an x_0 guess, a function $f(\cdot)$ and a first derivative $f'(\cdot)$, the algorithm is to repeat

$$x^{n+1} = x^n - \frac{f(x^n)}{f'(x^n}$$

until $\left|x^{n+1} - x^n\right|$ is below some tolerance threshold. Code a function that implements Newton's method. The function should accept arguments a function f, its derivative f_prime, a starting guess x_0, a tolerance tol, and a maximum number of iterations maxiter. Test this function with $f(x) = (x-1)^3$ and another function of your choice where you can analytically find the derivative.

Question 7

We consider the capital investment problem of an infinitely lived household with log preferences over consumption. Production is given by $Y_t = Z_t K_t^{\theta}$, where K_t is current capital, $\theta = 0.36$, and $Z_t \in \{Z_g, Z_b\}$ is the period's current productivity level. Assume that capital depreciates at rate $\delta = 0.025$, and households discount at rate $\beta = 0.99$. Assume further that the states of productivity are $Z_g = 1.25$, $Z_b = 0.2$, and that transitions between the two states are given by a two-state Markov process:

$$\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}$$

so, for instance, $P(Z_{t+1} = Z_g \mid Z_t = Z_g) = 0.977$. The household's value function $V(K, Z)$ is given by

$$V(K, Z) = \max_{K'} \left\{ u(c) + \beta \sum_{Z'} \Pi(Z', Z) V(K', Z') \right\}$$
$$c = ZK^{\theta} + (1-\delta)K - K'.$$

Set the capital grid to be from 0.01 to 45 with 1000 grid points.

A poorly-written version of this model exists in the Homework 1 repository. Update my code to include the stochastic transitions and to run faster than it currently does.