ECS 170 Assignment 1 Part I

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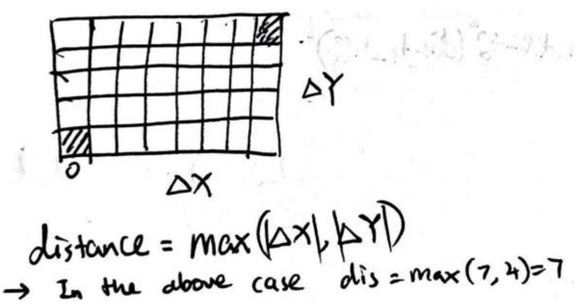
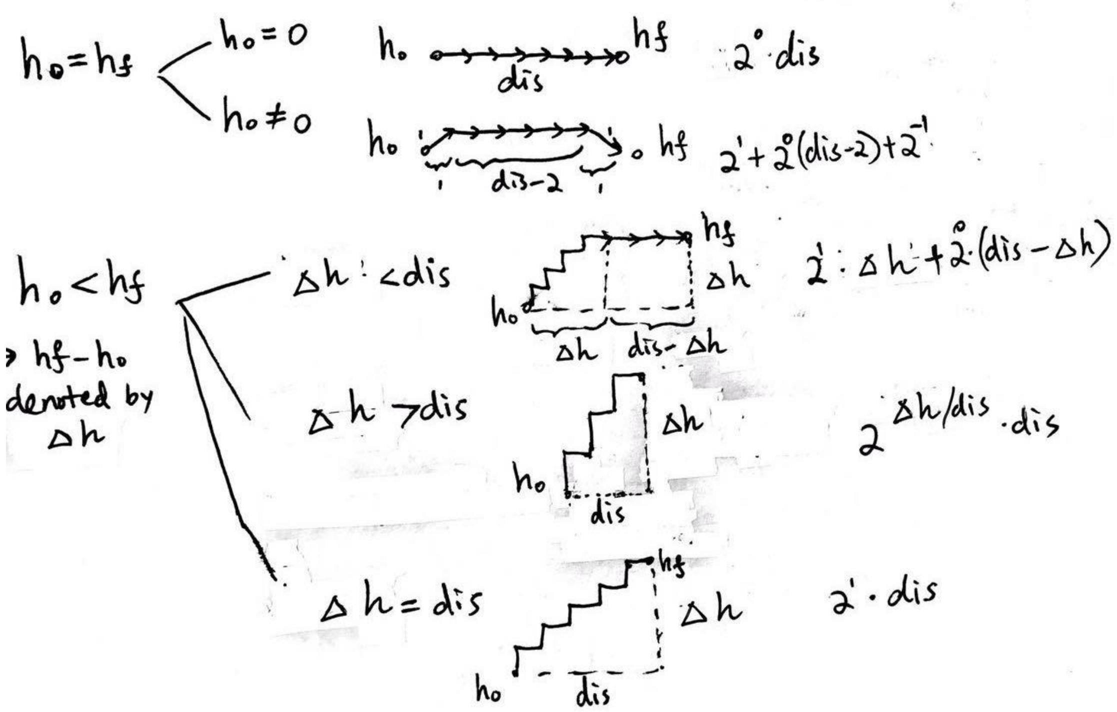
1. Exponential of Height Difference

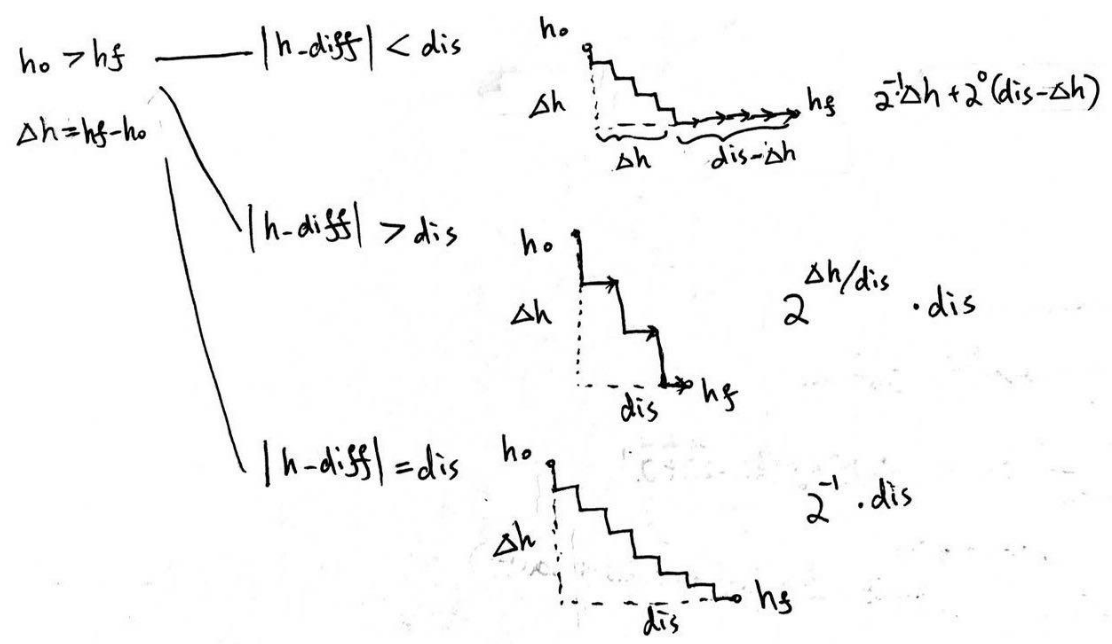
$$D = Height\ \ Difference(End - Current)$$
$$S = Minimum\ \ Steps\ \ to\ \ reach\ \ the\ \ EndPoint(chebyshev\ \ distance)$$
$$h(n) = \begin{cases} 2^{D}, & S = 1 \quad (1.1) \\ S, & D = 0 \quad (1.2) \\ S * 2^{\frac{D}{S}}, & D \neq 0, |D| > S \quad (1.3) \\ (S - |D|) + 2 * |D|, & |D| < S, D > 0 \quad (1.4) \\ (S - |D|) + 0.5 * |D|, & |D| < S, D \leq 0 \quad (1.5) \end{cases}$$

Graph illustration of D:



Graph illustration of Heuristic:





Prove of Consistency:

(1.1) It is the exact form of the true cost to the goal state when we are one move away from EndPoint.

(1.2) When the current point and end point are at the same height, we claim the Minimum moves underestimate the true cost from current to EndPoint.

$$\text{N: Current Node on Optimal Path} \quad \text{N}'\text{: Node succssor of N}$$

$$h(N) \leq h(N') + c(N, N')$$

$$\textit{In order to return to same height, there must be}$$

$$N'' \textit{that counteract the change in height}$$

$$S \leq (S-2) + 2^{\Delta D} + 2^{-\Delta D}$$

$$\text{Because } 2^{\Delta D} + 2^{-\Delta D} \geq 2, \textit{So it is consistent and admissible}$$

(1.3) Let $\frac{\mathrm{D}}{\mathrm{S}} = a, |D| > S, \textit{then we need to prove } 2^{aS} \geq S * 2^{a}$

$$\ln(2^{aS}) \geq aln(2S)$$

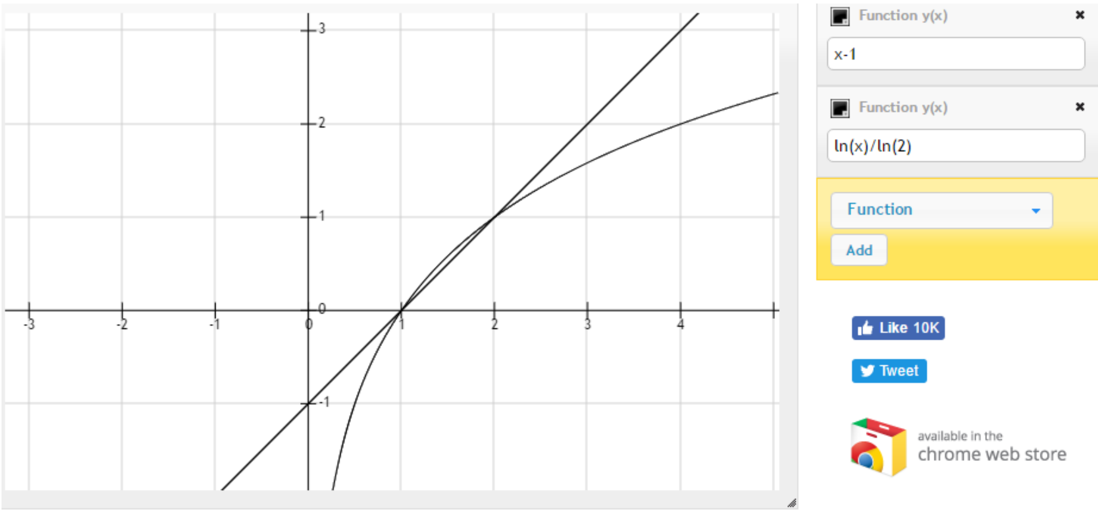
$$aSln(2) \geq aln(2) + aln(S)$$

$$S \geq \frac{\ln(2) + \ln(S)}{\ln(2)}$$

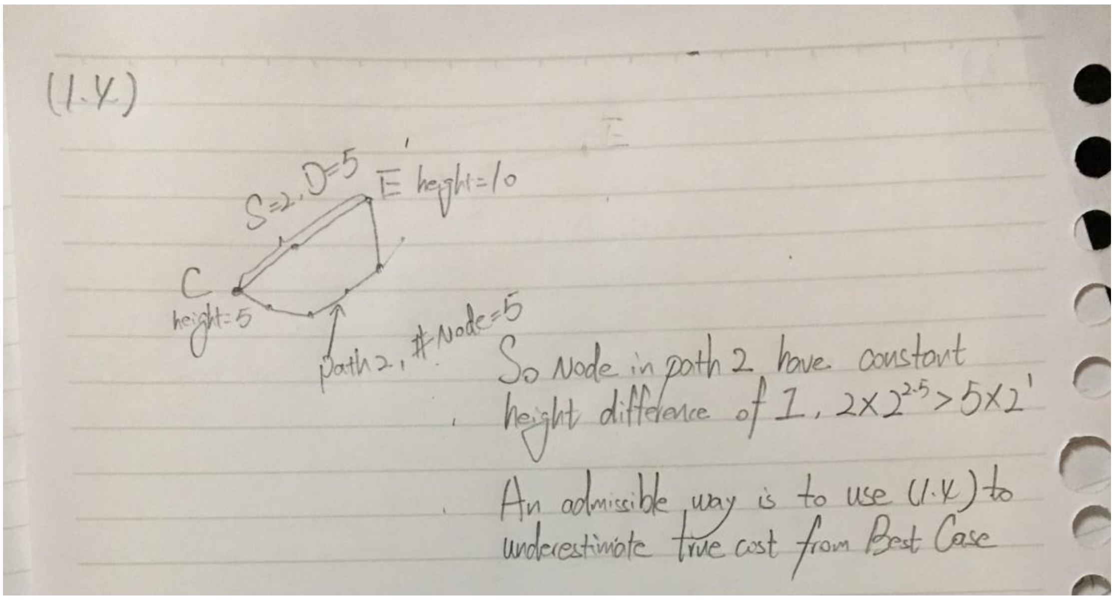
$$S - 1 \geq \frac{\ln(S)}{\ln(2)}$$

From the plot below, S-1 is always larger or equal to RHS when S-1 is Integer value.

So we proved that the (1.3) is admissible to divide height into different steps as much as possible(When |D| is larger than S)



(1.4) When |D|<S, so we cannot move by divide difference equally into integer steps. We give an example of underestimate the true cost.



(1.5) Similar to (1.4) except we multiply by $2^{-1} = 0.5$

2. Bizzaro World Cost Function

$$D = Height\ \ Difference(End - Current)$$

$$S = Minimum\ \ Steps\ \ to\ \ reach\ \ the\ \ EndPoint(chebyshev\ \ distance)$$

$$\mathrm{c} = \begin{cases} \sqrt[S]{D}, & c > 2 \\ 2, & c \leq 2 \end{cases}$$

$$h(n) = \begin{cases} Cost\ \ function, S = 1 & (2.1) \\ \frac{1}{2} * S, \quad D = 0 & (2.2) \\ S * \frac{1}{c}, \quad D > 0 & (2.3) \\ S * c, \quad D < 0 & (2.4) \end{cases}$$

So for this cost function, climbing up will cost less while climbing down will cost more.
The greater height you climb up in one move, the less cost you will have.
The greater height you climb down in one move, the less cost you will have.
So, our initial thought is to split height into moves to make the cost of each move go down.

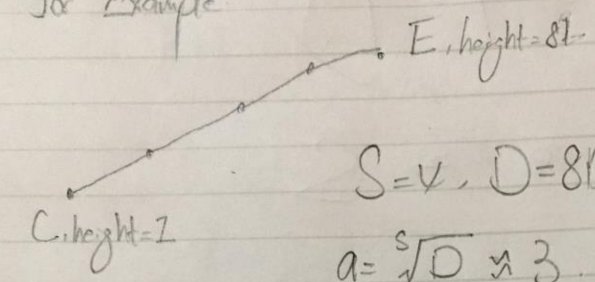
Prove of Consistency
(2.1) When the minimum distance is 1, then the cost is exactly the cost function

(2.2) If the height difference is 0, and 0.5 is the lower bound of the cost of moving horizontally.
Moving up or Moving down will need another move to return to the original height.

$$\text{N: Current Node on Optimal Path} \quad \text{N}': \text{Node succssor of N}$$
$$h(N) \leq h(N') + c(N, N')$$
$$In\ order\ to\ return\ to\ same\ height, there\ must\ be$$
$$N''that\ counteract\ the\ change\ in\ height$$
$$\frac{1}{2} * S \leq \left(\frac{1}{2} * S\right) - 1 + c(\frac{N}{N'}) + c(\frac{N'}{N''})$$
$$1 \leq c(\frac{N}{N'}) + c(\frac{N'}{N''})$$
$$\text{Because } at\ least\ one\ of\ the\ RHS\ cost\ will$$
$$be\ \geq 1(For\ going\ down), So\ it\ is\ consistent\ and\ admissible$$

(2.3)

2.3 For Example



$S=4,\ D=80$

$a = \sqrt[S]{D} \approx 3.$

$h(n) = \frac{1}{3+1} + \frac{3}{9+1} + \frac{9}{27+1} + \frac{27}{81+1} = 1.2007$

$\because \frac{1}{a} \leq \frac{1+c}{a+c} \leq \frac{1+c}{a+c+1}$

$\therefore$ Each Move $\frac{1}{\text{True}}$ cost $\geq \frac{1}{c}$

$\therefore S \cdot \frac{1}{c} \leq$ True Cost (C, E).

(2.4) Similar to (2.3)