UC DAVIS

STA 135

HW 3 SOLUTIONS

5.1 a) $\bar{\underset{\sim}{x}} = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$; $S = \begin{bmatrix} 8 & -10/3 \\ -10/3 & 2 \end{bmatrix}$

$T^2 = 150/11 = 13.64$

b) T^2 is $3F_{2,2}$ (see (5-5))

c) $H_0: \underset{\sim}{\mu}' = [7, 11]$

$\alpha = .05$ so $F_{2,2}(.05) = 19.00$

Since $T^2 = 13.64 < 3F_{2,2}(.05) = 3(19) = 57$; do not reject H_0 at the $\alpha = .05$ level

5.3 a)
$$T^2 = \frac{(n-1)\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)'\right|}{\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\bar{\underset{\sim}{x}})(\underset{\sim}{x}_j-\bar{\underset{\sim}{x}})'\right|} - (n-1) = \frac{3(244)}{44} - 3 = 13.64$$

b)
$$\Lambda = \left(\frac{\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\bar{\underset{\sim}{x}})(\underset{\sim}{x}_j-\bar{\underset{\sim}{x}})'\right|}{\left|\sum_{j=1}^{n}(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)(\underset{\sim}{x}_j-\underset{\sim}{\mu}_0)'\right|}\right)^{n/2} = \left(\frac{44}{244}\right)^2 = .0325$$

Wilks' lambda $= \Lambda^{2/n} = \Lambda^{1/2} = \sqrt{.0325} = .1803$

5.5 $H_0: \underset{\sim}{\mu}' = [.55, .60]$; $T^2 = 1.17$

$\alpha = .05$; $F_{2,40}(.05) = 3.23$

Since $T^2 = 1.17 < \frac{2(41)}{40} F_{2,40}(.05) = 2.05(3.23) = 6.62$,

we do not reject H_0 at the $\alpha = .05$ level. The result is consistent with the 95% confidence ellipse for $\underset{\sim}{\mu}$ pictured in Figure 5.1 since $\underset{\sim}{\mu}' = [.55, .60]$ is <u>inside</u> the ellipse.

5.10 a) 95% T^2 simultaneous confidence intervals:

Lngth2: (130.65, 155.93)	Lngth4: (160.33, 185.95)
Lngth3: (127.00, 191.58)	Lngth5: (155.37, 198.91)

b) 95% T^2 simultaneous intervals for change in length (ΔLngth):

ΔLngth2-3: (−21.24, 53.24)
ΔLngth3-4: (−22.70, 50.42)
ΔLngth4-5: (−20.69, 28.69)

c) 95% confidence region determined by all $\mu_{2\text{-}3}, \mu_{4\text{-}5}$ such that

$$\left(16-\mu_{2\text{-}3}, 4-\mu_{4\text{-}5}\right)\begin{bmatrix} .011024 & .009386 \\ .009386 & .025135 \end{bmatrix}\begin{pmatrix} 16-\mu_{2\text{-}3} \\ 4-\mu_{4\text{-}5} \end{pmatrix} \le 72.96/7 = 10.42$$

where $\mu_{2\text{-}3}$ is the mean increase in length from year 2 to 3, and $\mu_{4\text{-}5}$ is the mean increase in length from year 4 to 5.

Beginning at the center $\underset{\sim}{\bar{x}}' = (16,4)$, the axes of the 95% confidence ellipsoid are:

$$\text{major axis} \quad \pm\sqrt{157.8}\sqrt{72.96}\begin{pmatrix} .895 \\ -.447 \end{pmatrix}$$

$$\text{minor axis} \quad \pm\sqrt{33.53}\sqrt{72.96}\begin{pmatrix} .447 \\ .895 \end{pmatrix}$$

(See confidence ellipsoid in part e.)

d) Bonferroni 95% simultaneous confidence intervals ($m = 7$):

Lngth2: (137.37, 149.21)
Lngth3: (144.18, 174.40)

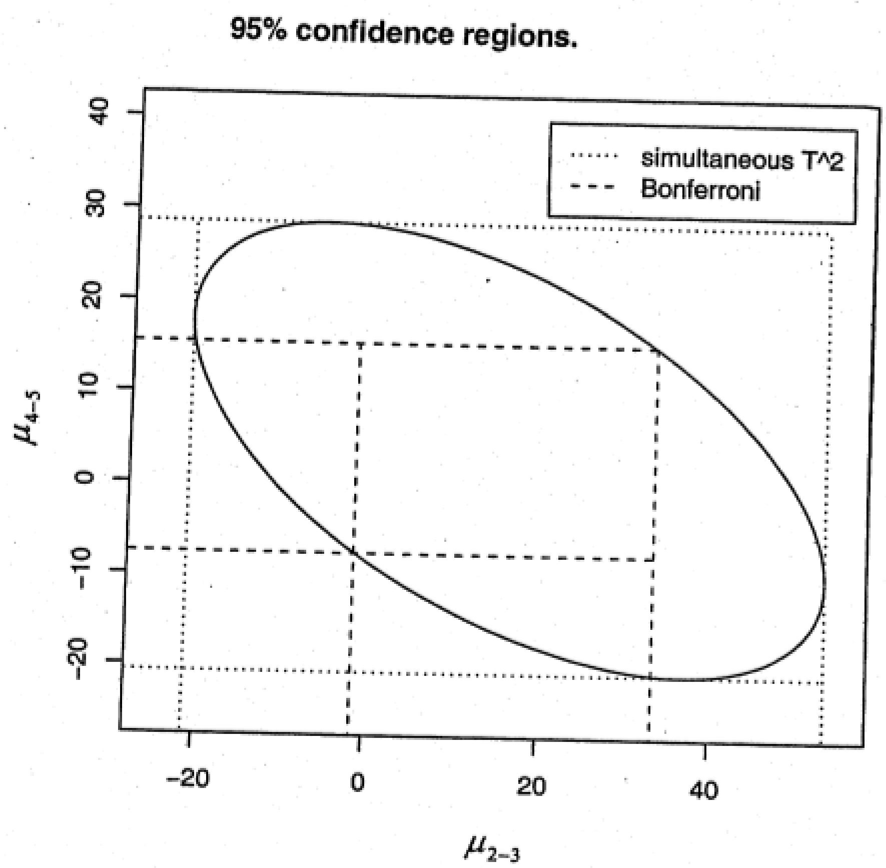
Lngth4: (167.14, 179.14)
Lngth5: (166.95, 187.33)

ΔLngth2-3: (−1.43, 33.43)
ΔLngth3-4: (−3.25, 30.97)

ΔLngth4-5: (−7.55, 15.55)

5.10 (Continued)

e) The Bonferroni 95% confidence rectangle is much smaller and more informative than the 95% confidence ellipse.



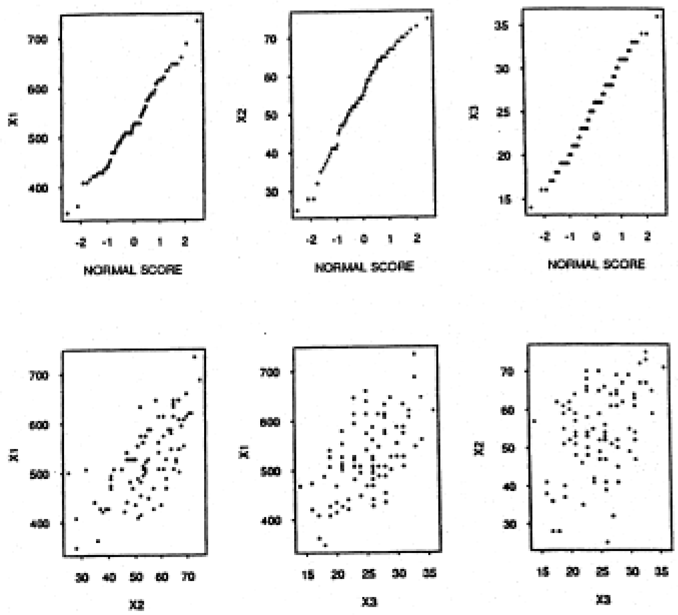
5.18

(a). Hotelling's $T^2 = 223.31$. The critical point for the statistic ($\alpha = 0.05$) is 8.33. We reject $H_0 : \mu = (500, 50, 30)'$. That is, The group of students represented by scores are significantly different from average college students.

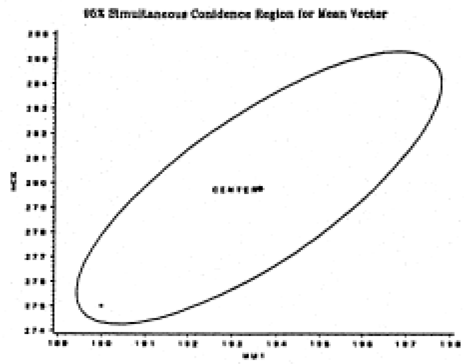
(b). The lengths of three axes are 23.730, 2.473, 1.183. And directions of corresponding axes are

$$\begin{pmatrix} 0.994 \\ 0.103 \\ 0.038 \end{pmatrix}, \quad \begin{pmatrix} -0.104 \\ 0.995 \\ 0.006 \end{pmatrix}, \quad \begin{pmatrix} -0.037 \\ -0.010 \\ 0.999 \end{pmatrix}.$$

(c). Data look fairly normal.



5.20 (a). Yes, they are plausible since the hypothesized vector $\underset{\sim}{\mu}_0$ (denoted as $*$ in the plot) is inside the 95% confidence region.



(b).

	LOWER	UPPER
Bonferroni C. I.:	189.822	197.423
	274.782	284.774
Simultaneous C. I.:	189.422	197.823
	274.256	285.299

Simultaneous confidence intervals are larger than Bonferroni's confidence intervals. Simultaneous confidence intervals will touch the simultaneous confidence region from outside.
(c). Q-Q plots suggests non-normality of (X_1, X_2). Could try transforming X_1.