

University of California, Davis  
Department of Statistics

Name .....  
P R I N T P L E A S E

STA 135

Sample  
Midterm I

Instructions: 1. **WORK ALL PROBLEMS.** Please, give details and explanations and  
**SHOW ALL YOUR WORK** so that partial credits can be given.  
2. You may use **two** sheets of **notes** and a **calculator** but **no** other reference materials.

---

**Points**

1. Let  $A$  be an  $n \times n$  matrix and  $Q$  an  $n \times n$  orthogonal matrix.
    - (a) Show that  $\text{tr}(Q' A Q) = \text{tr}(A)$ .
    - (b) Find the determinant of  $Q$ .
    - (c) Show that  $Q' A Q$  and  $A$  have the same eigenvalues.
- (25)

2.  $A$  is a  $2 \times 2$  symmetric matrix with  $a_{11} = 1$ ,  $a_{12} = 2$  and  $a_{22} = -2$ .

- (a) Find  $A^{-1}$ .
- (b) Compute eigenvalues and normalized eigenvectors of  $A^{-1}$ .
- (c) Write down the spectral decomposition of  $A^{-1}$ .

(25)

3. The following data matrix is observed for a three-dimensional random vector  $\underline{X}$ .

$$\mathbf{X} = \begin{bmatrix} 1 & 4 & 3 \\ 6 & 2 & 6 \\ 8 & 3 & 3 \end{bmatrix}$$

- (a) Compute the sample mean vector and sample covariance matrix of  $\underline{X}$ .  
(b) Let  $\underline{b}' = [1 \quad 2 \quad -3]$  and compute sample mean vector and sample covariance matrix of  $\underline{b}' \underline{X}$ .

(20)

4. The three-dimensional random vector  $\underline{X}$  has a multivariate normal distribution. The means of  $X_1$ , and  $X_2$  are 1, and the mean of  $X_3$  is 0. Variance of  $X_1$  is 6, variance of  $X_2$  is 4 and variance of  $X_3$  is 2.  $X_2$  and  $X_3$  are uncorrelated, but the correlation between  $X_1$  and  $X_2$  is  $1/\sqrt{24}$ , and the correlation between  $X_1$  and  $X_3$  is  $-1/\sqrt{12}$ .
- (a) Find the distribution of  $X_2$  given  $X_3$ .
  - (b) Find the distribution of  $X_1 + 2X_2 + 3X_3$ .
  - (c) Partition  $\underline{X}$  into  $\underline{X}^{(1)}$  and  $\underline{X}^{(2)}$ , where  $\underline{X}^{(2)}$  is a 2x1 vector. Find the distribution of  $\underline{X}^{(2)}$ .
  - (d) Find the conditional distribution of  $\underline{X}^{(2)}$  given  $X_1 = 2$ .
- (30)