ECS 170 Assignment 1 Part 4

Members: Xie Zhou 912143385, Haozhe Gu 999200555

For the part 4, climbing Mount Saint Helens During the Eruption, the biggest challenge is the efficiency to calculate the optimal path. I used modified Dijkstra's method in part 3 using arrays like Visited, Distance etc.… This approach has poor efficiency. First, the way to calculate the minimum node(minimum f(x)) require O(n) because we are searching in the array storing all point's f(x). We used a priority queue with customized compare function and Hash maps to make the cost of searching and traversing cheaper. The searching in hash map only takes O(1) and the priority queue will be sorted when we add Point to the queue but not O(n) approach to find the minimum f(x). Also, the array approach I used calls Point to index and index to Point calculation extensively, we fixed this problem in the new version using hash key.

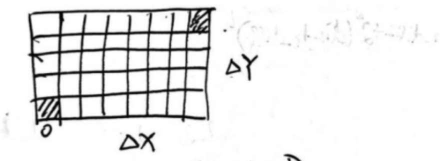
1. Exponential of Height Difference

$$D = Height\ \ Difference(End - Current)$$

$$S = Minimum\ \ Steps\ \ to\ \ reach\ \ the\ \ EndPoint(chebyshev\ \ distance)$$

$$h(n) = \begin{cases} 2^{D}, & S = 1 \quad (1.1) \\ S, & D = 0 \quad (1.2) \\ S * 2^{\frac{D}{S}}, & D \neq 0, |D| > S \quad (1.3) \\ (S - |D|) + 2 * |D|, & |D| < S, D > 0 \quad (1.4) \\ (S - |D|) + 0.5 * |D|, & |D| < \mathrm{S}, \mathrm{D} \leq 0 \quad (1.5) \end{cases}$$

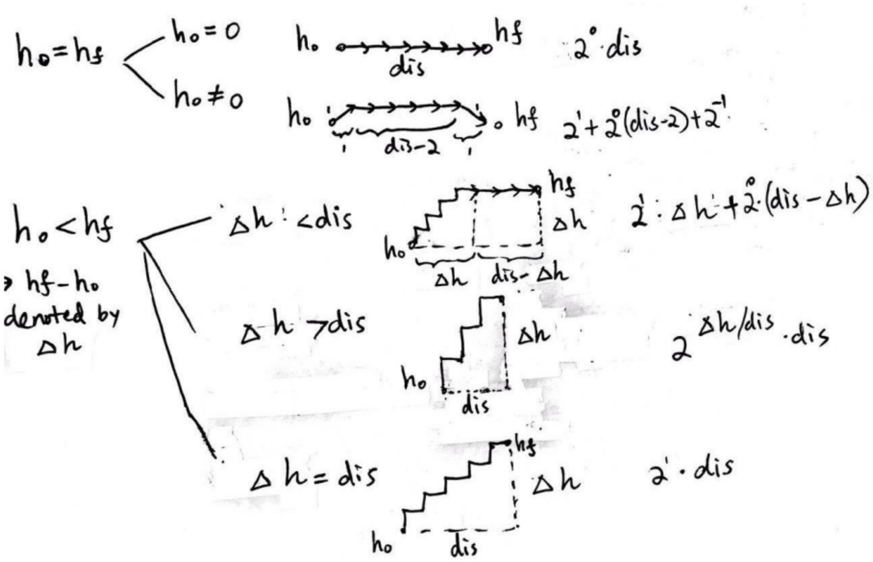
Graph illustration of D:

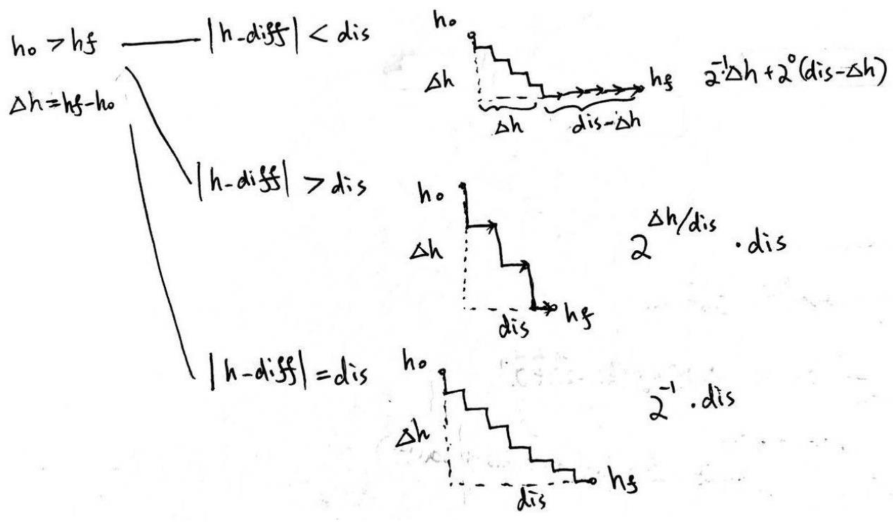


distance = max(|ΔX|, |ΔY|)

→ In the above case dis = max(7, 4) = 7

Graph illustration of Heuristic:





Prove of Consistency:

(1.1) It is the exact form of the true cost to the goal state when we are one move away from EndPoint.

(1.2) When the current point and end point are at the same height, we claim the Minimum moves underestimate the true cost from current to EndPoint.

$$\text{N: Current Node on Optimal Path} \quad \text{N}'\text{: Node succssor of N}$$

$$h(N) \le h(N') + c(N, N')$$

$$\textit{In order to return to same height, there must be}$$

$$N''\textit{that counteract the change in height}$$

$$S \le (S-2) + 2^{\Delta D} + 2^{-\Delta D}$$

$$\text{Because } 2^{\Delta D} + 2^{-\Delta D} \ge 2, \textit{So it is consistent and admissible}$$

(1.3) Let $\frac{\mathrm{D}}{\mathrm{S}} = a, |D| > S, \textit{then we need to prove } 2^{aS} \ge S * 2^a$

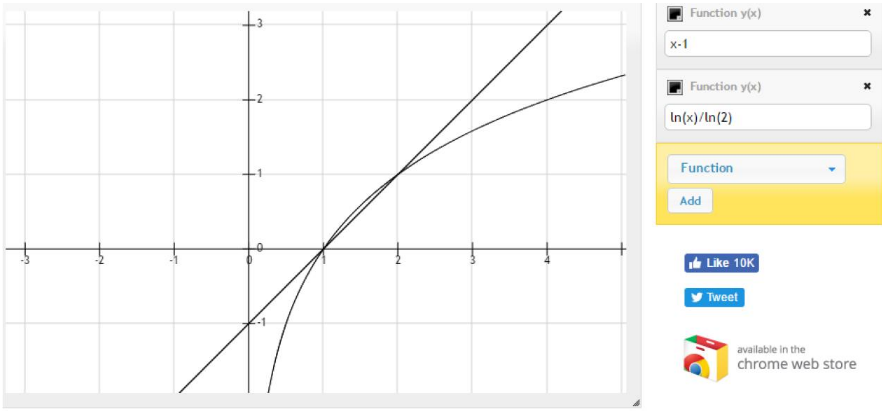
$$\ln(2^{aS}) \ge a\ln(2S)$$

$$aS\ln(2) \ge a\ln(2) + a\ln(S)$$

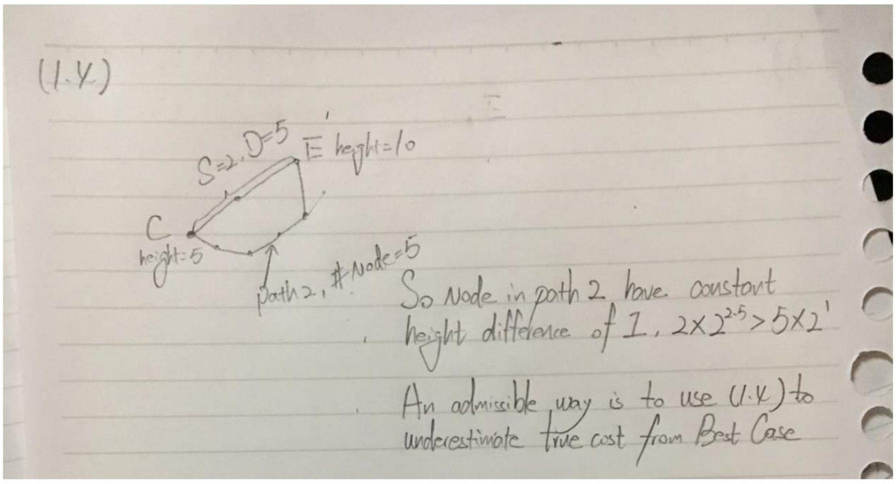
$$S \ge \frac{\ln(2) + \ln(S)}{\ln(2)}$$

$$S - 1 \ge \frac{\ln(S)}{\ln(2)}$$

From the plot below, S-1 is always larger or equal to RHS when S-1 is Integer value.
So we proved that the (1.3) is admissible to divide height into different steps as much as possible(When |D| is larger than S)



(1.4) When |D|<S, so we cannot move by divide difference equally into integer steps. We give an example of underestimate the true cost.



(1.5) Similar to (1.4) except we multiply by $2^{-1} = 0.5$