UC DAVIS

STA 135

HW 4 SOLUTIONS

6.2 Using a critical value $t_{n-1}(\alpha/2p) = t_{10}(0.0125) = 2.6338$,

	LOWER	UPPER
Bonferroni C. I.:	-20.57	1.85
	-2.97	29.52
Simultaneous C. I.:	-22.45	3.73
	-5.70	32.25

6.6 a) Treatment 2: Sample mean vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$; sample covariance matrix

$$\begin{bmatrix} 1 & -3/2 \\ -3/2 & 3 \end{bmatrix}$$

Treatment 3: Sample mean vector $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$; sample covariance matrix

$$\begin{bmatrix} 2 & -4/3 \\ -4/3 & 4/3 \end{bmatrix}$$

$$S_{pooled} = \begin{bmatrix} 1.6 & \\ -1.4 & \end{bmatrix}$$

b) $T^2 = [2-3,\ 4-2] \left[\left(\frac{1}{3} + \frac{1}{4}\right) \begin{bmatrix} 1.6 & -1.4 \\ -1.4 & 2 \end{bmatrix} \right]^{-1} \begin{bmatrix} 2-3 \\ 4-2 \end{bmatrix} = 3.88$

$$\frac{(n_1+n_2-2)p}{(n_1+n_2-p-1)} F_{p,n_1+n_2-p-1}(.01) = \frac{(5)2}{4}(18) = 45$$

Since $T^2 = 3.88 < 45$ do not reject $H_0: \underset{\sim}{\mu}_2 - \underset{\sim}{\mu}_3 = \underset{\sim}{0}$ at the $\alpha = .01$ level.

c) 99% simultaneous confidence intervals:

$$\mu_{21} - \mu_{31}: (2-3) \pm \sqrt{45} \sqrt{\left(\frac{1}{3}+\frac{1}{4}\right)1.6} = -1 \pm 6.5$$

$$\mu_{22} - \mu_{32}: 2 \pm 7.2$$

6.9 For <u>any</u> matrix C

$$\bar{\underset{\sim}{d}} = \frac{1}{n} \Sigma \underset{\sim}{d}_j = C\left(\frac{1}{n} \Sigma \underset{\sim}{x}_j\right) = C\bar{\underset{\sim}{x}}$$

and $\underset{\sim}{d}_j - \bar{\underset{\sim}{d}} = C(\underset{\sim}{x}_j - \bar{\underset{\sim}{x}})$

so $S_d = \frac{1}{n-1} \Sigma(\underset{\sim}{d}_j - \bar{\underset{\sim}{d}})(\underset{\sim}{d}_j - \bar{\underset{\sim}{d}})' = C\left(\frac{1}{n-1} \Sigma(\underset{\sim}{x}_j - \bar{\underset{\sim}{x}})(\underset{\sim}{x}_j - \bar{\underset{\sim}{x}})'\right)C' = CSC'$

Note: For 6.16, this is one solution for the specific contrast matrix, C, specified. There are many contrast matrices that could have been specified.

6.16 $H_0: C\underline{\mu} = \underline{0}$; $H_1: C\underline{\mu} \neq \underline{0}$ where $C = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$.

Summary statistics:

$$\bar{\mathbf{x}} = \begin{bmatrix} 1906.1 \\ 1749.5 \\ 1509.1 \\ 1725.0 \end{bmatrix}; \qquad S = \begin{bmatrix} 105625 & 94759 & 87249 & 94268 \\ & 101761 & 76166 & 81193 \\ & & 91809 & 90333 \\ & & & 104329 \end{bmatrix}$$

$$T^2 = n(C\bar{\mathbf{x}})'(CSC')^{-1}(C\bar{\mathbf{x}}) = 254.7$$

$$\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(\alpha) = \frac{(30-1)(4-1)}{(30-4+1)} F_{3,27}(.05) = 9.54$$

Since $T^2 = 254.7 > 9.54$ we reject H_0 at $\alpha = .05$ level.

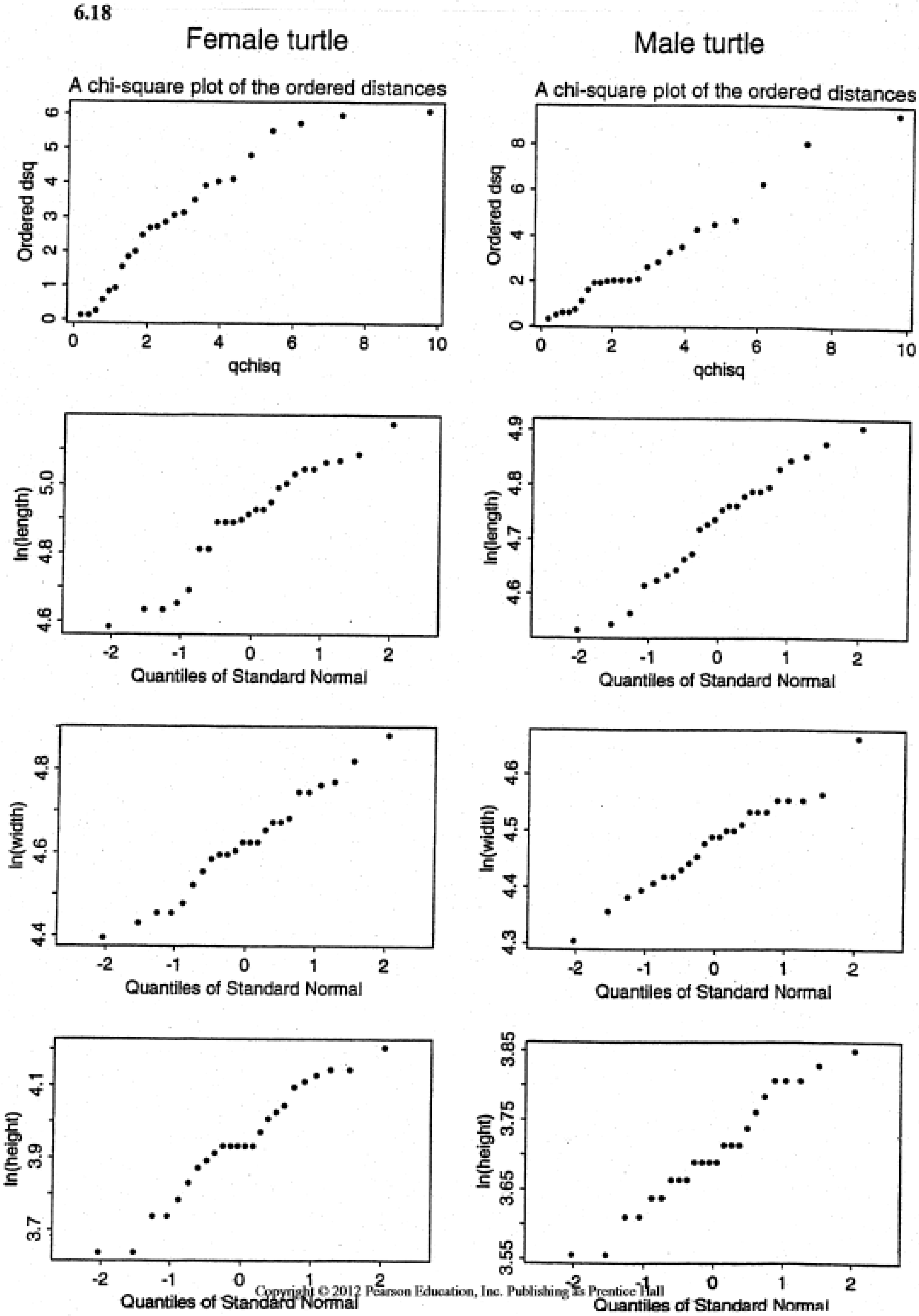
95% simultaneous confidence interval for "dynamic" versus "static" means $(\mu_1 + \mu_2) - (\mu_3 + \mu_4)$ is, with $\mathbf{c}' = [1 \quad 1 \quad -1 \quad -1]$,

$$\mathbf{c}'\bar{\mathbf{x}} \pm \sqrt{\frac{(n-1)(q-1)}{(n-q+1)} F_{q-1,n-q+1}(\alpha)} \sqrt{\frac{\mathbf{c}'S\mathbf{c}}{n}}$$

$$= 421.5 \pm 174.5 \longrightarrow (247, 596)$$

Note: the QQ diagnostic plots on 6.18 are for reference and were not required to answer the question.

6.18



6.18 continued

mean vector for females:

X1BAR
4.9006593
4.6229089
3.9402858

mean vector for males:

X2BAR
4.7254436
4.4775738
3.7031858

SPOOLED			
	0.0187388	0.0140655	0.0165386
	0.0140655	0.0113036	0.0127148
	0.0165386	0.0127148	0.0158563

TSQ	CVTSQ	F	CVF	PVALUE
85.052001	8.833461	27.118029	2.8164658	4.355E-10

linear combination most responsible for rejection

of H0 has coefficient vector:

COEFFVEC
-43.72677
-8.710687
67.546415

95% simultaneous CI for the difference

in female and male means

LOWER	UPPER
0.0577676	0.2926638
0.0541167	0.2365537
0.1290622	0.3451377

Bonferroni CI

LOWER	UPPER
0.0768599	0.2735714
0.0689451	0.2217252
0.1466248	0.3275751

6.19

a) $\bar{\mathbf{x}}_1 = \begin{bmatrix} 12.219 \\ 8.113 \\ 9.590 \end{bmatrix}$; $\bar{\mathbf{x}}_2 = \begin{bmatrix} 10.106 \\ 10.762 \\ 18.168 \end{bmatrix}$;

$$S_1 = \begin{bmatrix} 223.0134 & 12.3664 & 2.9066 \\ & 17.5441 & 4.7731 \\ & & 13.9633 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 4.3623 & .7599 & 2.3621 \\ & 25.8512 & 7.6857 \\ & & 46.6543 \end{bmatrix};$$

$$S_{pooled} = \begin{bmatrix} 15.8112 & 7.8550 & 2.6959 \\ & 20.7458 & 5.8960 \\ & & 26.5750 \end{bmatrix}$$

$$[(\frac{1}{n_1} + \frac{1}{n_2})S_{pooled}]^{-1} = \begin{bmatrix} 1.0939 & -.4084 & -.0203 \\ & .8745 & -.1525 \\ & & .5640 \end{bmatrix}$$

$H_0: \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = \mathbf{0}$

Since $T^2 = (\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)'[(\frac{1}{n_1} + \frac{1}{n_2})S_{pooled}]^{-1}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) = 50.92$

$$> \frac{(n_1+n_2-2)p}{(n_1+n_2-p-1)} F_{p,n_1+n_2-p-1}(.01) = \frac{(57)(3)}{55} F_{3,55}(.01) = 13,$$

we reject H_0 at the $\alpha = .01$ level. There is a difference in the (mean) cost vectors between gasoline trucks and diesel trucks.

b) $\hat{\mathbf{a}} \propto S_{pooled}^{-1}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) = \begin{bmatrix} 3.58 \\ -1.88 \\ -4.48 \end{bmatrix}$

c) 99% simultaneous confidence intervals are:

$\mu_{11} - \mu_{21}$: 2.113 ± 3.790

$\mu_{12} - \mu_{22}$: -2.650 ± 4.341

$\mu_{13} - \mu_{23}$: -8.578 ± 4.913

d) Assumption $\Sigma_1 = \Sigma_2$.

Since S_1 and S_2 are quite different, it may not be reasonable to pool. However, using "large sample" theory ($n_1 = 36$, $n_2 = 23$) we have, by Result 6.4,

$$(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2 - (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2))'[\frac{1}{n_1} S_1 + \frac{1}{n_2} S_2]^{-1}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2 - (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)) \sim \chi_p^2$$

Since

$$(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)'[\frac{1}{n_1} S_1 + \frac{1}{n_2} S_2]^{-1}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) = 43.15 > \chi_3^2(.01) = 11.34$$

we reject $H_0: \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = \mathbf{0}$ at the $\alpha = .01$ level. This is consistent with the result in part (a).

Note: part (d) was somewhat open-ended and not well defined. If students did not invoke Result 6.4, but repeated analysis with outliers removed and commented on it then that is okay.