Obstacle Avoidance Trajectory Planning for Gaussian Motion of Robot Based on Probability Theory

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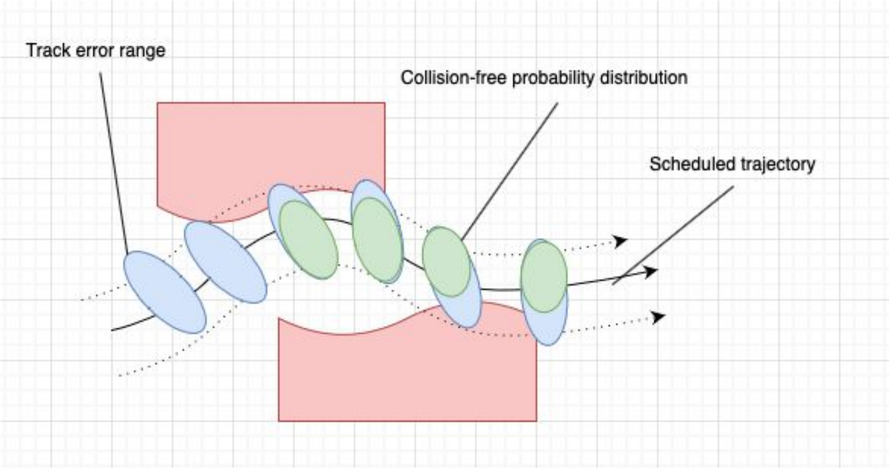
Date:	11/10/2021

Introduction

The linear control method is used in combination with Kalman filter to establish error model of Gaussian motion system. Then, all feasible trajectories are assessed by the Gaussian motion model by calculating the probability of avoiding obstacles and arriving at the target.



Introduction

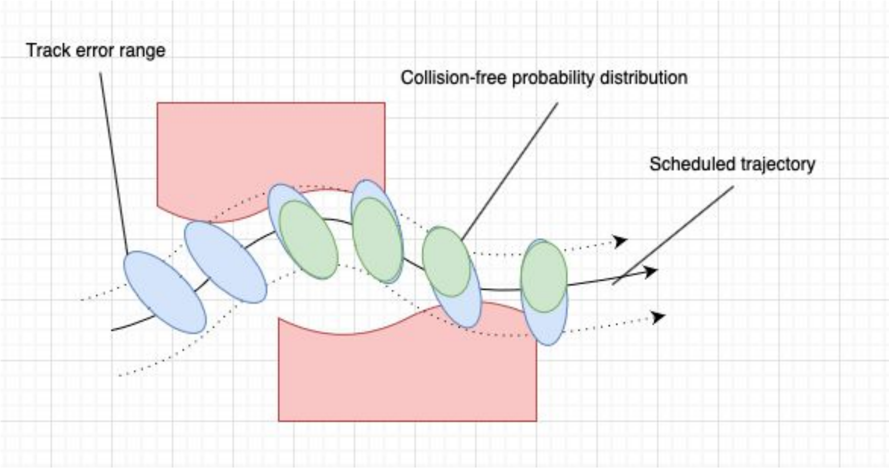
The total project will be executed in two stages:

Stage 1

Using the Gaussian distribution in probability theory is to analyze the motion of the robot on the predetermined trajectory, and the motion error distribution in each cycle is analyzed to calculate the collision probability between the robot and the obstacle at different moments.

Stage 2

On the basis of the stage 1 experiment, the robot is assigned multiple trajections, and the robot searches for the optimal trajectory according to the collision probability of different trajections.



Priori Estimation of Trajectory Error Distribution

Due to the errors in the process of robot movement, the position deviation between the actual output of the system and the system instruction is subject to Gaussian distribution in each control cycle. And because the observation system also has observation noise subject to Gaussian distribution, the deviation state of robot trajectory cannot be accurately reflected. Therefore, moving Kalman filter can effectively estimate the position of the robot.

The KF process has two steps, namely:

* **Prediction step:** the next step state of the system is predicted given the previous measurements
* **Update step:** the current state of the system is estimated given the measurement at that time step

Prediction

$$X_k = A_k X_k + B_k u_{k-1} \quad (1)$$

$$P_k = A_k P_{k-1} A_k^T + V_k M_k V_k^T \quad (2)$$

Adjusting

$$K_k = P_k H_k^T (H_k P_k H_k^T + W_k N_k W_k^T)^{-1} \quad (3)$$

$$X_k = X_k + K_k (Z_k - H_k X_k) \quad (4)$$

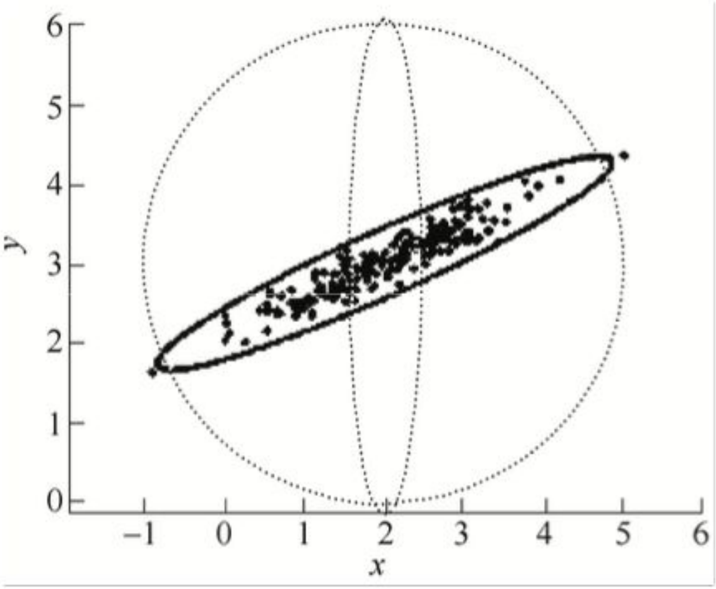
$$Pk = (I - K_k H_k) P_k \quad (5)$$

Xk is the optimal estimation of the system state within the k cycle, and Pk is the covariance estimation of the system state. Kk is Kalman gain matrix

Collision probability calculation

I. Gaussian motion probability ellipse representation

Obtained the normal distribution variance of the robot trajectory points in each control cycle



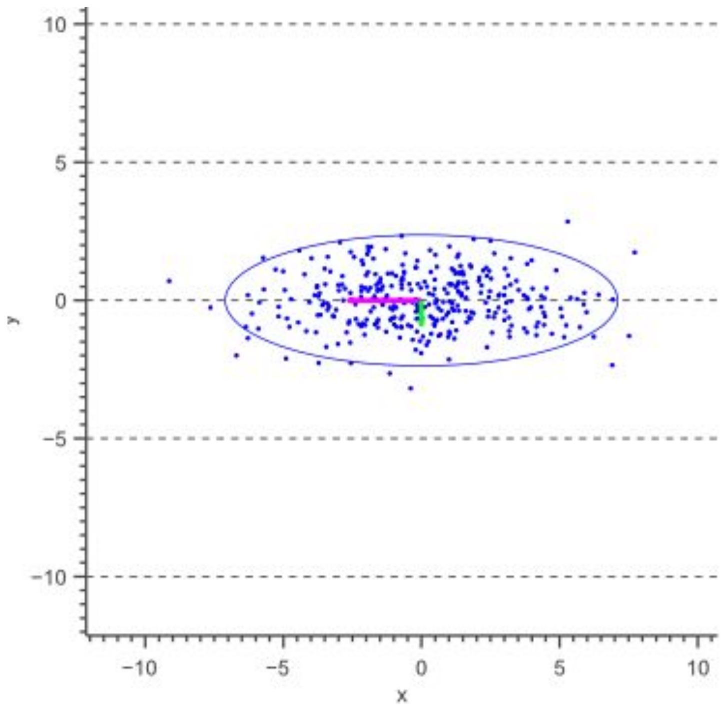
Collision probability calculation

I. Gaussian motion probability ellipse representation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

In the motion probability ellipse, the length of the axes is defined by the standard deviations and of the data. Then choose s and let the scale of the resulting ellipse represents a chosen confidence level.

$$\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 = s$$



Collision probability calculation

I. Gaussian motion probability ellipse representation

Sum of squares of independent normally distributed data samples.

The sum of squared Gaussian data points is known to be distributed according to a so called Chi-Square distribution

$$\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 = s$$

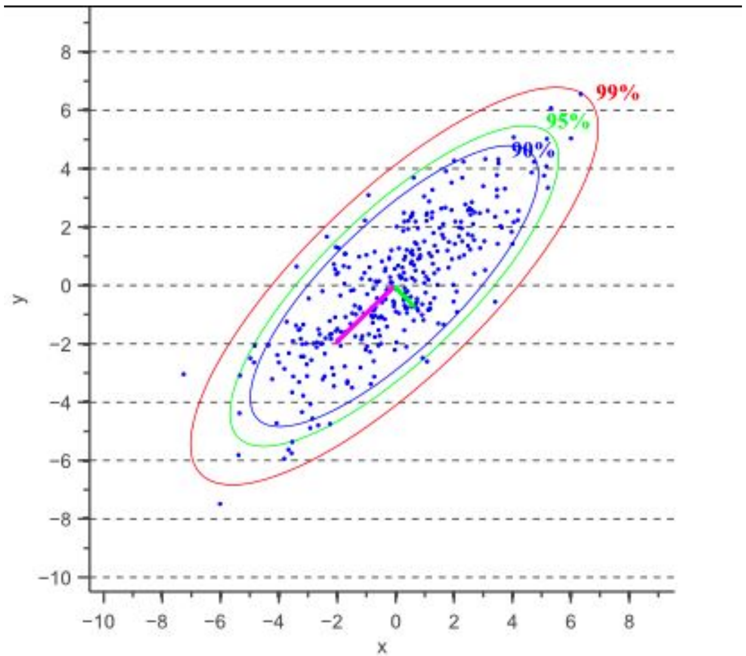
df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Collision probability calculation

I. Gaussian motion probability ellipse representation

$$P(s < 5.991) = 1 - 0.05 = 0.95$$

$$\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2 = 5.991$$

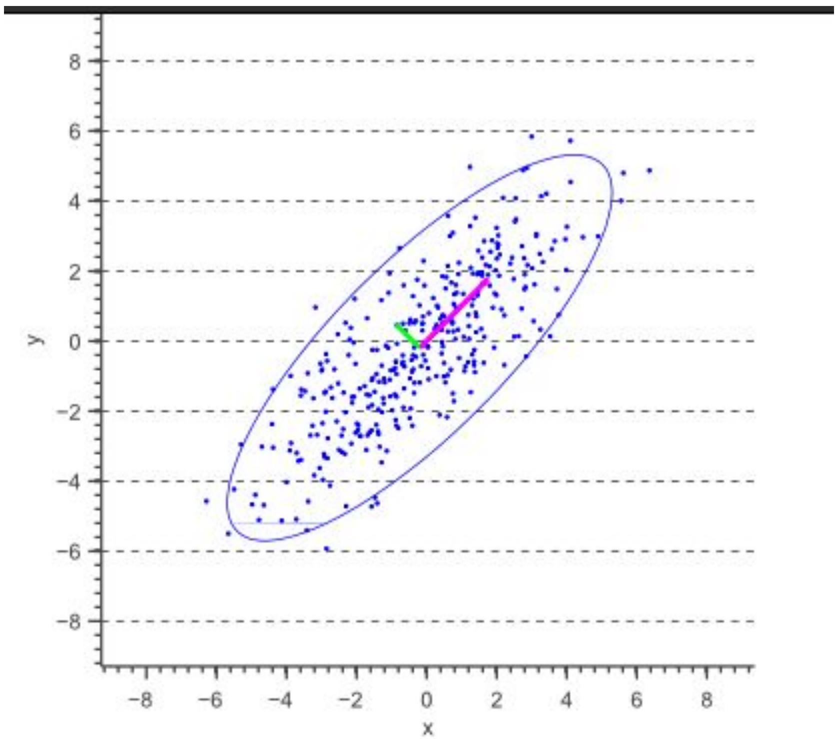


Collision probability calculation

I. Gaussian motion probability ellipse representation

- calculate the covariance matrix of random variables x and y, calculate the eigenvector of the matrix according to the covariance matrix
- Define the ellipse The eigenvector in the major axis direction is v1, and the eigenvector in the minor axis direction of the ellipse is v2.

$$\alpha = arctan\frac{v_1(y)}{v_2(x)}$$



Collision probability calculation

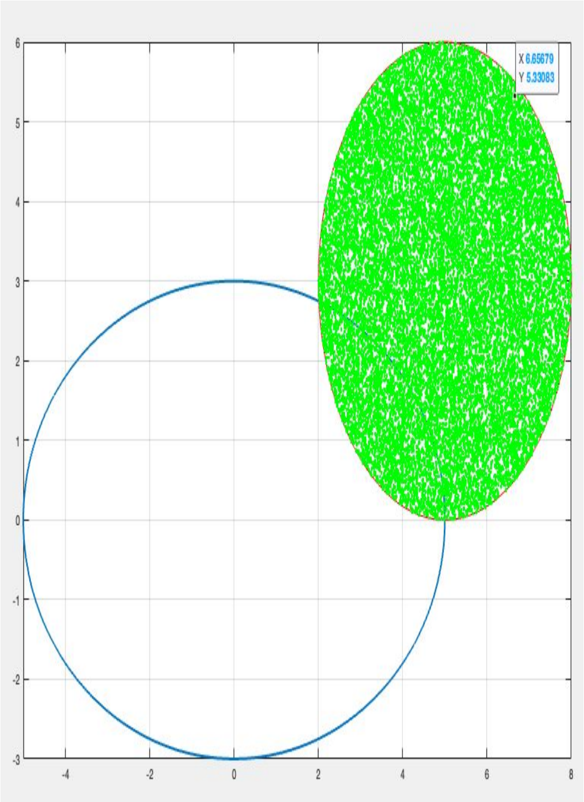
II. Obstacle avoidance probability calculation

Way1: **Monte Carlo**

Calculate the area ratio of the motion probability ellipse and the obstacle collision interference.

N: Number of total sample
n: Number of sample fall into specific area

$$area = \frac{n}{N} * (Area\ of\ ellipse)$$

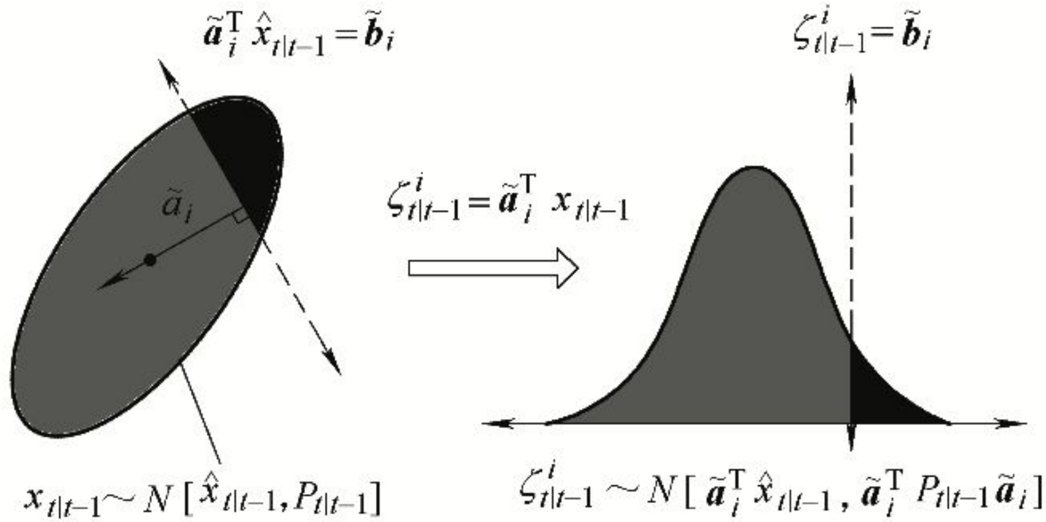


Collision probability calculation

II. Obstacle avoidance probability calculation

Way2: **Projection**

- The obstacle is expressed by the local linearization method
- project the two-dimensional probability ellipse into a one-dimensional space for normal distribution probability calculation

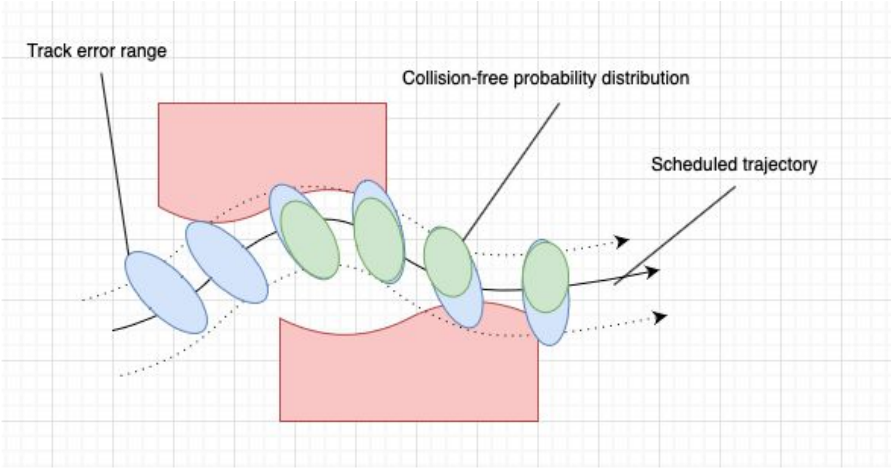


$$\tilde{a}_l^T \tilde{x}_{k|k-1} = \tilde{b}_i$$

Collision probability calculation

II. Obstacle avoidance probability calculation

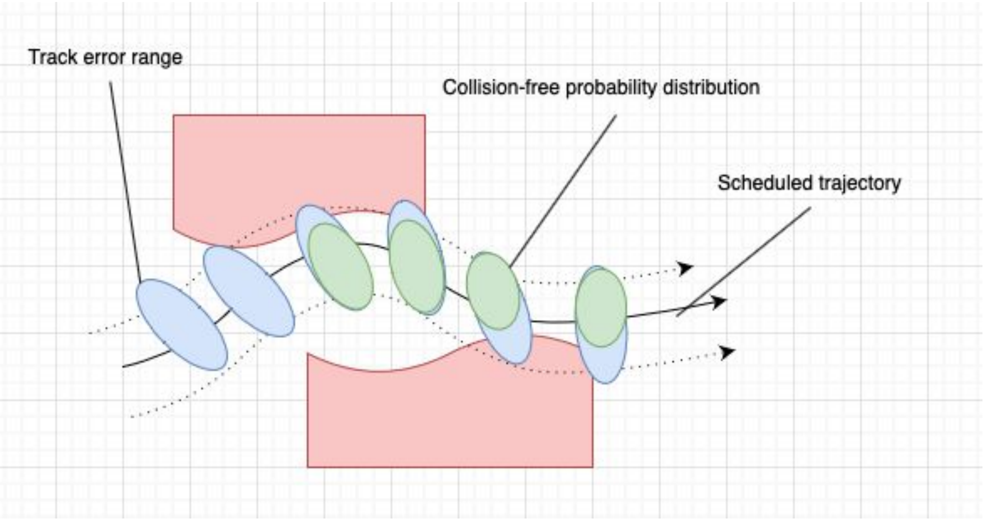
As long as the probability ellipse intersects the obstacle, the robot may collide with the obstacle.



Collision probability calculation

II. Obstacle avoidance probability calculation

Total success probability



$$p_{suc} = p(\Lambda_{k=0}^{l} x_k \in X_s) = \Pi_{k=0}^{l} p(x_{k|k-1} \in X_s)$$

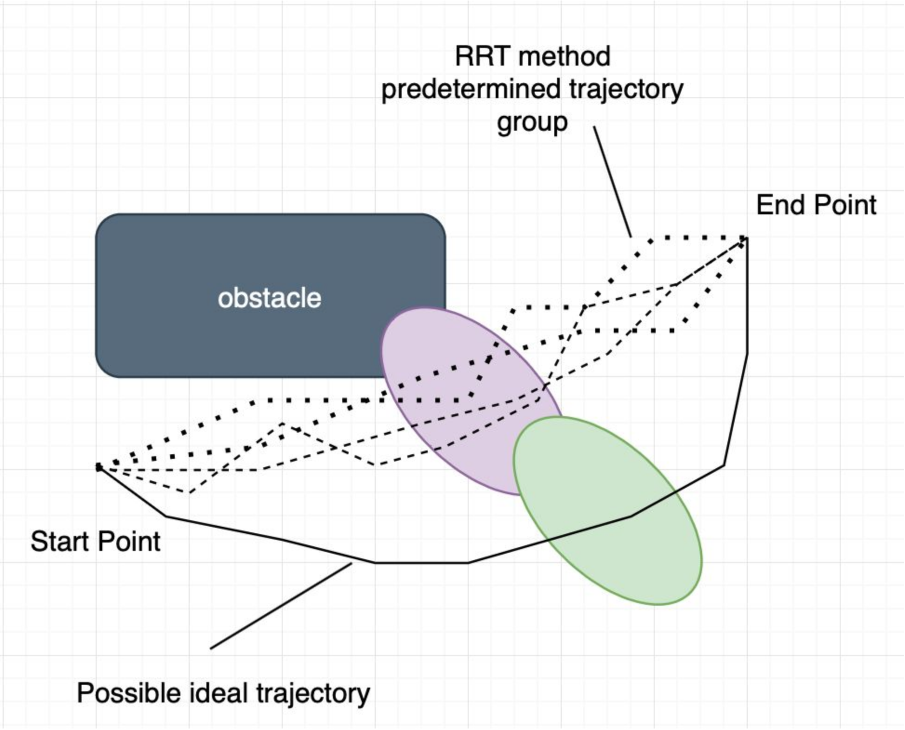
Collision probability calculation

II. Trajectory optimization method

- ★ PRM
- ★ RRT
- ★ A*
- ★ D*
- ★ GA
- ★ Dijkstra

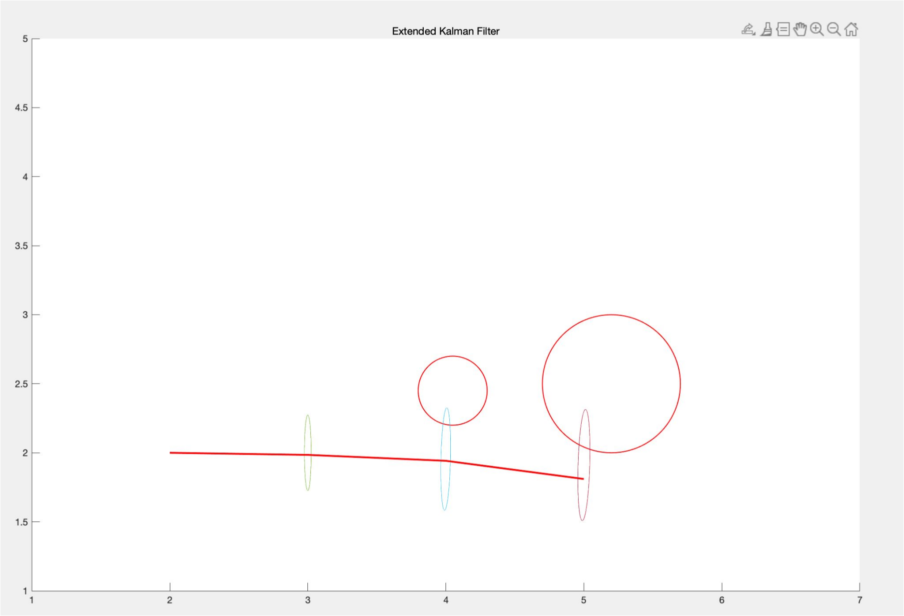
Remind:

- ❏ these trajectories cannot uniformly cover the robot state space due to optimization screening
- ❏ the optimization strategy cannot guarantee consistency with the safety of the robot's Gaussian motion.



Simulation test

As for the experimental setup, two obstacle areas are set up within the 6*6 area. The starting position of the robot is (2,2) and the ending position is (5,1.8).



Simulation test

While simulating the trajectory motion, the geometric information of the probability ellipse for each cycle and the collision with environmental obstacles are calculated.

No.	P(collision)
1	0
2	0.0432
3	0.0815
Total	0.00352

Conclusion

- Motion errors need to be calculated through experiments before the imputation planning, and its accuracy directly affects the accuracy of the probability estimation of the imputation results in this paper.

- It can be seen from the policy test that the greater the trajectory probability ellipse, the greater the robot motion error. When the probability ellipse intersects the obstacle, the robot has the probability of colliding with the obstacle.