Network Performance Analysis: Solutions to Tutorial 1

1. (a) The curve $P(k|\mu)$ against k has one maximum. As k increases, the value of k at which the maximum value of $P(k|\mu)$ occurs, increases, but the value of the maximum decreases.

   (b) The curve achieves its maximum value at $k \approx \mu$. Note that k is an integer and μ is a real number.

   (c) From the data, $k = 12$ and $\mu = \lambda t = 1 \times 10 = 10$. Thus

$$P(k|t,\lambda) = P(k|\mu) = \frac{10^{12}\exp(-10)}{12!} = \frac{4.54 \times 10^{7}}{12!} = 0.09478$$

2. (a) The mean value of X is

$$E\{X\} = 1(0.05) + 2(0.2) + 3(0.3) + 4(0.45) = 3.15$$

   (b) The variance of X is

$$\mathrm{var}\{X\} = E\left\{(X - E\{X\})^2\right\} = E\left\{X^2\right\} - (E\{X\})^2$$

   Evaluate the first term.

$$E\left\{X^2\right\} = 1(0.05) + 4(0.2) + 9(0.3) + 16(0.45) = 10.75$$

   Thus

$$\mathrm{var}\{X\} = 10.75 - (3.15)^2 = 0.8275$$

   (c) The following table is required:

X	1	2	3	4
$P(X)$	0.05	0.2	0.3	0.45
$\ln X$	0	0.693	1.099	1.386

   Thus

$$E\{\ln X\} = 0(0.05) + 0.693(0.2) + 1.099(0.3) + 1.386(0.45) = 1.092$$

3. (a)

$$\frac{\sigma}{\mu} = \frac{\sqrt{\mathrm{var}\{k\}}}{\mu} = \frac{\sqrt{\mu}}{\mu} = \frac{1}{\sqrt{\mu}} = \frac{1}{\sqrt{\lambda t}}$$

   and the result follows.

(b) This result shows that the distribution is closely packed about the mean μ for large values of μ. Note that the mean and standard deviation increase as μ increases, but their ratio decreases, as shown in part (a).

4. (a) The mean of the exponential distribution is $\frac{1}{\lambda}$ and

$$\int_0^\infty \lambda x^2 \exp(-\lambda x)\,\mathrm{dx} = \frac{2}{\lambda^2}$$

It follows that the variance is equal to

$$\frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

(b) The probability is

$$\int_1^2 0.5 \exp\left(-\frac{x}{2}\right)\,\mathrm{dx} = -\exp\left(-\frac{x}{2}\right)\Big|_1^2 = \exp\left(-\frac{1}{2}\right) - \exp\left(-1\right) = 0.239$$

5. The arrival rate is $\lambda = \frac{1000}{3600}$ calls per second.

Since

P(1 or more calls arrive in an interval of 1 second) $=$ 1-P(there are no calls in an interval of 1 second)

the answer is

$$1 - P(0|1,\lambda) = 1 - \exp\left(-\frac{1000}{3600}\right) = 0.24$$

6. (a) Since the Poisson process has no memory, the fact that no packets arrived within a 5 millisecond interval has no effect. Thus historical information about a Poisson process does improve predictions about the future.

(b) Since

P(no arrivals in first 3 milliseconds) $+$ $P(k \geq 1$ arrivals in first 3 milliseconds)$=$1

it follows that

$P(k \geq 1$ arrivals in first 3 milliseconds)$=$1-$P(0|3,1) = 1 - \exp(-(3 \times 1)) = 0.95$