Network Performance Analysis: Tutorial 1

1. Leaves fall from a tree to the ground as a Poisson process at a rate of one every minute.

   (a) Sketch the form of the Poisson distribution for several values of $\mu = \lambda t$. What happens as μ increases?

   (b) At what value of k does the curve achieve its maximum value?

   (c) You arrive at 4:00 pm. During the next ten minutes, what is the probability that you see twelve leaves fall?

2. X takes on four different values with differing probabilities, as shown in the following table.

X	1	2	3	4
$P(X)$	0.05	0.2	0.3	0.45

   (a) What is the mean of X?

   (b) What is the standard deviation of X?

   (c) What is the expectation of $\ln X$?

3. (a) Show that the standard deviation of the Poisson distribution, normalised by its mean value, tends to zero as $\mu = \lambda t$ increases.

   (b) What does this imply about the form of the distribution for large values of μ?

4. Compute the mean and variance of the exponential distribution,

$$p(x) = \lambda \exp(-\lambda x), \quad \text{where}\, 0 < x < \infty.$$

   What is the probability that x will be between 1 and 2 if $\lambda = 0.5$?

5. Calls arrive at a telephone exchange at a rate of 1000 calls per hour. What is the probability that one or more calls arrive in an interval of one second?

6. (a) Suppose one observes a Poisson process for which a packet arrives, on average, every millisecond. No packets arrive within a 5 millisecond interval. How does this affect the probability of observing an arrival during the next millisecond?

   (b) Packets arrive according to a Poisson process at an average rate of one packet every millisecond. What is the probability that the first arrival occurs within 3 milliseconds?