The University Of Sheffield.

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NONE

DEPARTMENT OF COMPUTER SCIENCE **Spring Semester 2019-2020**

NETWORK PERFORMANCE ANALYSIS **2 hours**

ANSWER ALL QUESTIONS.

All questions carry equal weight. Figures in square brackets indicate the percentage of available marks allocated to each part of a question.

Registration number from U-Card (9 digits) — to be completed by student

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1. a) Consider a steady state queue in which the number of servers is unbounded.

   (i) Write down the equations that define the arrival rate and service rate in terms of the state k. Explain the differences between them. [15%]

   (ii) Use the steady state balance equations to derive the steady state probability that the system is in state k. [30%]

   (iii) What is the average number of people in the system and what is the average delay? [10%]

   b) The system now changes to a finite number of servers m, and a customer is not allowed to wait. Thus if a customer arrives and finds all the service tills busy, the customer leaves the system.

   (i) What are the service and arrival rates for this system? [20%]

   (ii) Calculate the blocking probability. [25%]

2. a) Consider an $M/M/m$ queue in which there is one queue and m servers. The steady state probabilities for this system are

$$P_k = \begin{cases} P_0\left(\frac{(m\rho)^k}{k!}\right) & k < m \\ P_0\left(\frac{m^m\rho^k}{m!}\right) & k \geq m \end{cases}, \qquad \rho = \frac{\lambda}{m\mu}.$$

(i) What do λ and μ represent? Also, state the restriction on the value of ρ. [15%]

(ii) Derive an expression for P_0 in terms of m and ρ. Simplify the expression as much as possible. [20%]

b) Consider a switch in a computer network that has one input port and three output ports, and is modelled as an $M/M/3$ queue.

(i) Write down the expressions for the probabilities P_k for $k < 3$ and $k \geq 3$, and the restriction on the value of ρ. [10%]

(ii) Show that [15%]

$$P_0 = \left[1 + 3\rho + \frac{9}{2}\rho^2 + \frac{9\rho^3}{2(1-\rho)}\right]^{-1}.$$

(iii) Show that the average number of packets in the system is

$$P_0 \sum_{k=0}^{2} \frac{k(3\rho)^k}{k!} + \frac{9P_0}{2}\sum_{k=3}^{\infty} k\rho^k$$

and that this expression simplifies to

$$P_0\left[\frac{9\rho}{2(1-\rho)^2} - \frac{3\rho}{2}\right].$$

[40%]

3. a) The Poisson process is the arrival process that is most frequently used to model the behaviour of queues.

   (i) Derive expressions for the mean and variance of the Poisson distribution at a specific time in terms of the rate λ. [25%]

   (ii) What is the probability that there are no arrivals in the time interval T? [5%]

   (iii) What is the probability that there is at least one arrival in the time interval T? [5%]

b) Consider an M/M/1 queue for which the arrival and service rates at state k are

$$\begin{aligned} \lambda_k &= \lambda\alpha^k, \qquad && k \geq 0,\ 0 \leq \alpha < 1 \\ \mu_k &= \mu, && k \geq 1 \end{aligned}$$

   (i) Calculate the probability P_k that there are k customers in the system. Express your answer in terms of P_0. [30%]

   (ii) Deduce an expression for P_0 and calculate the probability that there are two or more people in the system. [15%]

   (iii) Show that if $\frac{\lambda}{\mu} < 1$, then

$$P_0 > 1 - \frac{\lambda}{\mu}$$

[10%]

   (iv) Is the condition $\frac{\lambda}{\mu} < 1$ necessary for a steady state solution to exist? Can this solution exist for $\frac{\lambda}{\mu} \geq 1$? Explain your answer. [10%]

END OF QUESTION PAPER