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Math Foundations:

1. $\sum_{i=1}^{n-1} 1 = (n-1-1+1) = n-1$

upperbound − lowerbound + 1

2. $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1-i-1+1) = \sum_{i=0}^{n-2} n-1-i$

$= \sum_{i=0}^{n-2} n-1 - \sum_{i=0}^{n-2} i$

$= n-1 \sum_{i=0}^{n-2} 1 - \sum_{i=0}^{n-2} i$ → sum of n natural numbers $= \frac{n(n+1)}{2}$

$= (n-1)[n-2-0+1] - \frac{(n-2)(n-1)}{2}$

$= (n-1)(n-1) - \frac{(n-1)(n-2)}{2}$

$= \frac{2(n-1)(n-1) - (n-1)(n-2)}{2}$

$= \frac{(n-1)}{2}[2n-2-n+2] = \frac{(n-1)}{2}[n]$

$= \frac{n^2}{2} - \frac{n}{2} \approx \frac{n^2}{2} \approx n^2$

3. $\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n-1-0+1 \quad = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n$$

$$= \sum_{i=0}^{n-1} n \sum_{j=0}^{n-1} \quad = n^2 \sum_{i=0}^{n-1} 1 \quad = n^2 [n-1-0+1]$$

$$= n^3$$

4. Important Summation Formulas:

a) $\sum_{i=1}^{n} i = 1+2+\ldots+n = \frac{n(n+1)}{2} \approx \frac{n^2}{2}$

b) $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{n^3}{3}$

c) $\sum_{i=1}^{n} i^k = 1^k + 2^k + \ldots n^k \approx \frac{1}{k+1} n^{k+1}$

d) $\sum_{i=0}^{n} a^i = 1 + a + \ldots + a^n = \frac{a^{n+1}-1}{a-1} \quad (a \neq 1)$

e) $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

f) $\sum_{i=1}^{n} i \cdot 2^i = 1 \times 2 + 2 \times 2^2 + \ldots n \times 2^n$

$$= (n-1)2^{n+1} + 2$$

g) $\sum_{i=1}^{n} \lg i = n \lg n$

h) $\sum_{i=1}^{n} \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n} = \ln n + \gamma$

$\gamma = 0.5772$, Euler's constant

nth Harmonic number.

5. Solve for T(n)

$$T(n) = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n=1 \\ 2T(n-1)+1 & \text{otherwise} \end{cases}$$

$T(n) = 2T(n-1)+1 \rightarrow (1)$

put $n=n-1$ in (1)

$T(n-1) = 2T(n-2)+1 \rightarrow (2)$

put (2) in (1)

$T(n) = 2[2T(n-2)+1]+1$

$= 2^2 T(n-2) + 2 + 1$

$= 2^3 T(n-3) + 2^2 + 2 + 1$

$= 2^4 T(n-4) + 2^3 + 2^2 + 2 + 1$

$= \dots$

$\dots$

$= 2^n T(n-n) + 2^{n-1} + 2^{n-2} + \dots 2^3 + 2^2 + 1$

$= 2^n T(0) + 2^{n-1} + \dots 2^3 + 2^2 + 1$

$= 2^{n-1} + 2^{n-2} + \dots + 2^3 + 2^2 + 1$

This is a GP Series.

$S = \frac{a(r^n - 1)}{r-1}$

a = first term

r = common ratio

n = no of terms

$a = 1$

$r = 2$

$n = n$

$S = \frac{1(2^n - 1)}{2-1} = \frac{2^n - 1}{1} = 2^n - 1$

$= 2^n - 1$

The result can also be obtained from 4.e formula

6. Solve for

$$C(n) = \begin{cases} 0 & n = 0 \\ 1 + C(n-1) & \text{otherwise} \end{cases}$$

$C(n) = 1 + C(n-1) \rightarrow (1)$

put $n = n-1$ in (1)

$C(n-1) = 1 + C(n-2) \rightarrow (2)$

put (2) in (1)

$C(n) = 1 + 1 + C(n-2)$

$= 2 + C(n-2)$

$= 3 + C(n-3)$

$= \ldots$

$= n + C(n-n)$

$= n$

$C(n) = n$

7. Solve,

$$T(n) = \begin{cases} 0 & \text{if } n=1 \\ T(n/2)+1 & \text{otherwise} \end{cases}$$

$T(n) = T(n/2) + 1$

Assume $n = 2^k$

$T(2^k) = T(2^{k-1}) + 1 \rightarrow (1)$

put $k = k-1$ in (1)

$T(2^{k-1}) = T(2^{k-2}) + 1 \longrightarrow (2)$

put (2) in (1)

$T(2^k) = T(2^{k-2}) + 1 + 1$

$= T(2^{k-2}) + 2$

$= T(2^{k-3}) + 3$

$= \ldots\ldots$

$= T(2^{k-k}) + k$

$= T(1) + k$

$= k.$

$n = 2^k$

$\log n = \log 2^k$

$\log n = k \log 2$

Assume base 2.

$\log_2 n = k \log_2 2$

$k = \log_2 n$

$\therefore T(n) = \log_2 n.$

From the last 3 examples, what are your insights?

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8. Solve:

$x(n) = x(n-1) + 5$ for $n > 1$ & $x(1) = 0$

$x(n) = x(n-1) + 5 \rightarrow (1)$

Solving the Same way,

$x(n) = x(n-2) + 5 + 5$

$= x(n-2) + 2*5$

$= x(n-3) + 3*5$

$= x(n-4) + 4*5$

$= \ldots$

$= x(n-(n-1)) + (n-1)*5$

$= x(1) + (n-1)*5$

$= 5(n-1)$

$x(n) = 5(n-1)$

9. Solve:

$x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

$x(n) = 3x(n-1) \rightarrow (1)$

put $n = n-1$ in (1)

$x(n-1) = 3x(n-2) \rightarrow (2)$

put (2) in (1)

$x(n) = 3[3x(n-2)]$

$= 3^2 x(n-2)$

$= 3^3 x(n-3)$

$= \ldots$

$= 3^{n-1} x[n-(n-1)]$

$= 3^{n-1} x(1)$

$= 3^{n-1} . 4$

$= \frac{4}{3} . 3^n$