Relay Feedback Auto Tuning of PID Controllers

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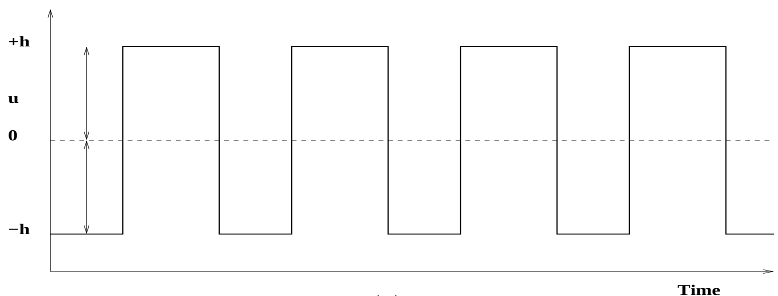
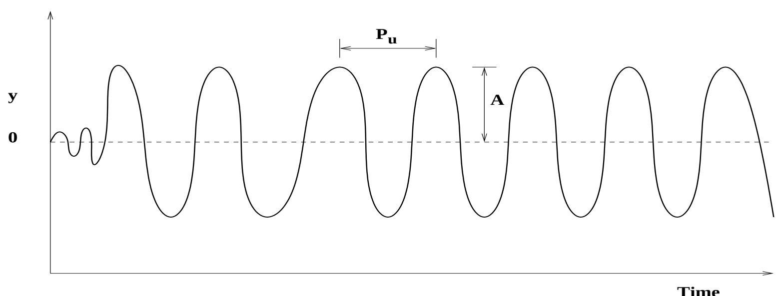
1 Introduction

For a certain class of process plants, the so-called "auto tuning" procedure for the automatic tuning of PID controllers can be used. Such a procedure is based on the idea of using an on/off controller (called a relay controller) whose dynamic behaviour resembles to that shown in Figure 1(a). Starting from its nominal bias value (denoted as **0** in the Figure) the control action is increased by an amount denoted by **h** and later on decreased until a value denoted by $-\mathbf{h}$.



(a)



(b)

Figure 1:

The closed-loop response of the plant, subject to the above described actions of the relay controller, will be similar to that depicted in Figure 1(b). Initially, the plant oscillates without a definite pattern around the nominal output value (denoted as **0** in the Figure) until a definite and repeated output response can be easily identified. When we reach this closed-loop plant response pattern the oscillation period (P_u) and the amplitude (A) of the plant response can be measured and used for PID controller tuning. In fact, the ultimate gain can be computed as:

$$K_{cu} = \frac{4h}{\pi A} \tag{1}$$

Having determined the ultimate gain K_{cu} and the oscillation period P_u the PID controller tuning parameters can be obtained from the following table:

	K_c	τ_I	τ_D
P	$0.5K_{cu}$		
PI	$0.45K_{cu}$	$P_u/1.2$	
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$

Example

Let us consider a process system given by the following transfer function:

$$G_p = \frac{6}{48s^3 + 44s^2 + 12s + 1}$$

and assume that the plant will be controlled by a PI feedback control system. Design the control system using the relay auto tuning method.

There are some decisions that ought to be taken before testing the relay auto tuning procedure:

- Pure gain controller value (K_c).
- Size of the manipulated variable deviation from the bias value (h).

By the time being let us pick up a small deviation of the manipulated variable from the bias value: $h = \pm 0.1$. The value of the controller gain should be large enough so that the value of the manipulated variable will lie between the bounds as represented by h. Therefore after some trials $K_c = 100$. The implementation of the relay auto tuning procedure is depicted in Figure 2.

In Figure 3 the dynamic behaviour of both the manipulated (u) and controlled (y) variables is shown. From both plots the following values can be easily read:

$$\begin{aligned} P_u &= 12.5 \\ A &= 0.07785 \end{aligned}$$

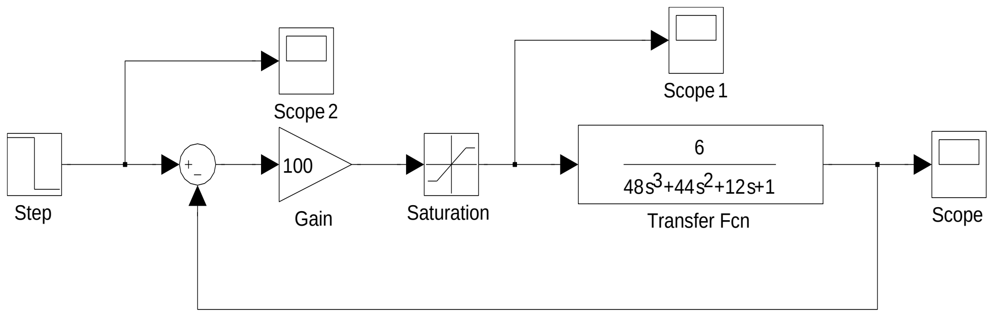


Figure 2:

hence,

$$K_{cu} = 1.6355$$

and the tuning parameters of the PI controller will given by:

$$\begin{aligned} K_c &= 0.7360 \\ \tau_i &= 10.4167 \end{aligned}$$

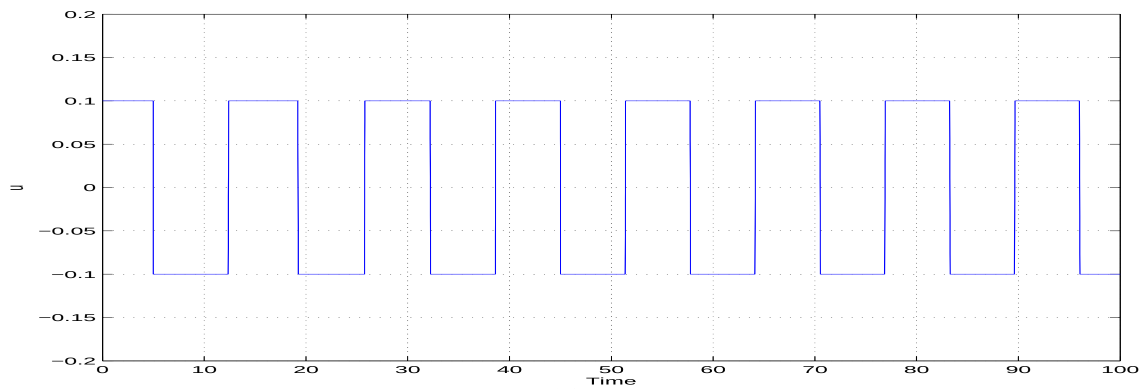
Similar results are also obtained by using the `relay` function available in `Simulink` whose closed-loop implementation is shown in Figure 4.

Example

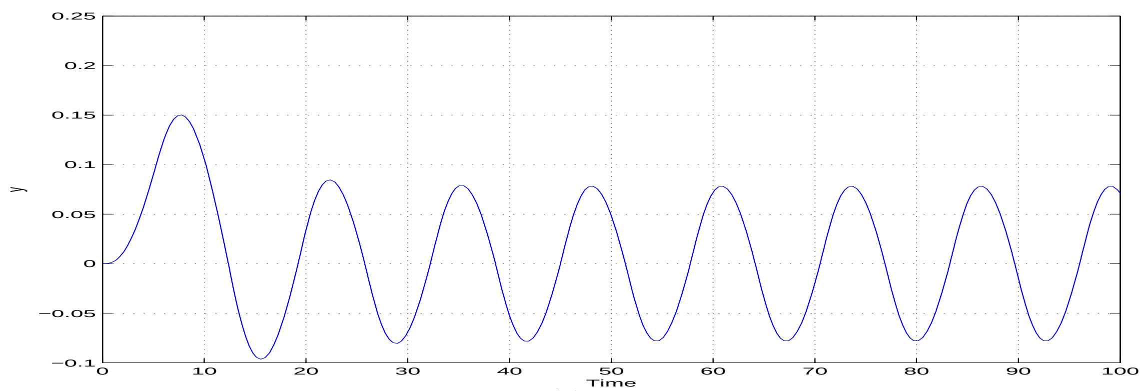
The dynamic mathematical model of a reaction train of 3 series connected CSTRs (see Figure 5) where the reaction $2A \xrightarrow{k} B$ takes place reads as follows:

$$\begin{aligned} \frac{dC_{A1}}{dt} &= \frac{Q}{V}(C_{Ao} - C_{A1}) - kC_{A1}^2 \\ \frac{dC_{A2}}{dt} &= \frac{Q}{V}(C_{A1} - C_{A2}) - kC_{A2}^2 \\ \frac{dC_{A3}}{dt} &= \frac{Q}{V}(C_{A2} - C_{A3}) - kC_{A3}^2 \end{aligned}$$

using the following values of the design parameters:



(a)



(b)

Figure 3:

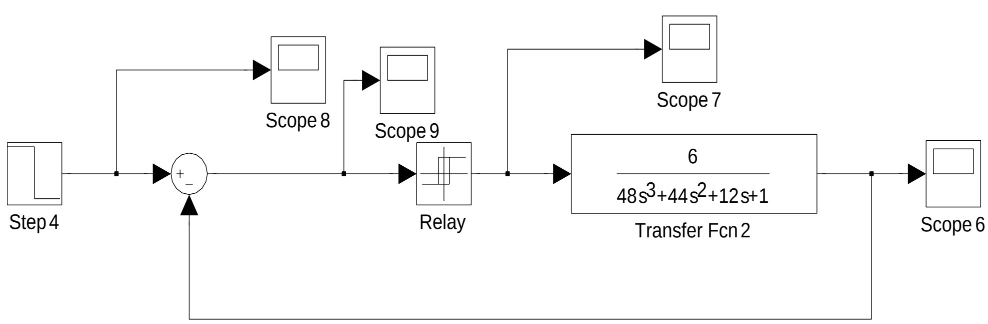


Figure 4:

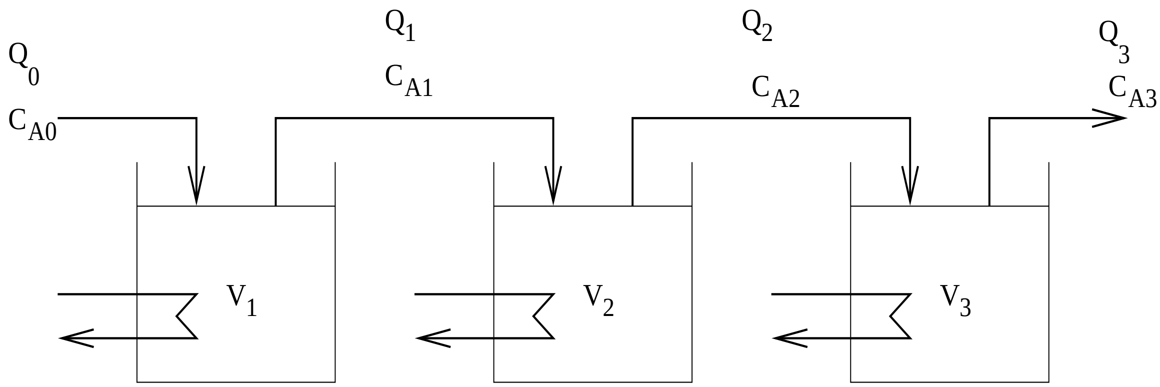


Figure 5: Flowsheet of the 3 CSTRs isothermal reaction train.

Parameter	Value	Units
k	5x10^{-4}	L/(mol-min)
C_{Ao}	100	mol/L
Q_i	50	L/min
V_j	1000	L

the conversion degree of reactant A coming out from the third reactor is around 67% which corresponds to $C_{A3} = 32.5641$ mol/L. Using the relay auto tuning procedure design and implement a closed-loop PI control system capable of raising the reaction train conversion from 67% up to 80% (which is equivalent to $C_{A3} = 20$ mol/L). The manipulated variable is the reactant A feed stream volumetric flow rate sent to the first reactor (Q), which happens to be the same along the reaction train, whereas the controlled variable is the reactant A concentration leaving the third reactor C_{A3}.

The transfer function between the controlled and manipulated variables reads as follows:

$$G_p(s) = \frac{C_{A3}(s)}{Q(s)} = \frac{0.0106s^2 + 0.003105s + 0.0003101}{s^3 + 0.2875s^2 + 0.02734s + 0.00086}$$

which can be approximated by the following first order plus time delay transfer function:

$$G(s) = \frac{0.3603}{18.389s + 1} e^{-4.1845}$$

using a Padé first order approximation for the representation of the delay:

$$G(s) = \frac{-0.3603s + 0.1722}{18.39s^2 + 9.789s + 0.478}$$

The `Simulink` implementation of the relay auto tuning procedure is shown in Figure 6, whereas the closed-loop response obtained from the application of this automatic tuning procedure is depicted in Figure 7.

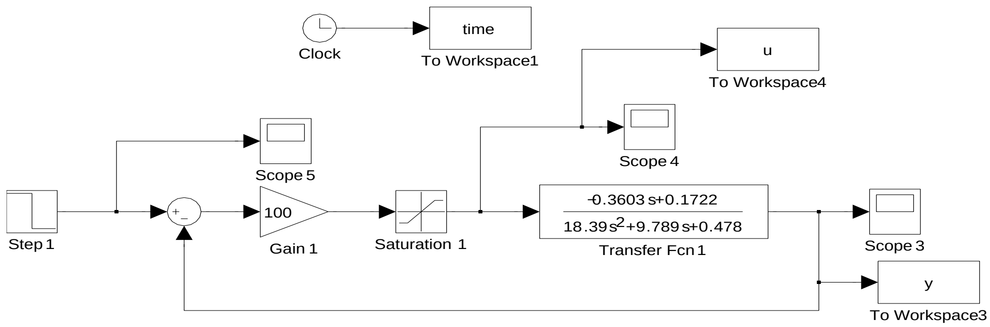
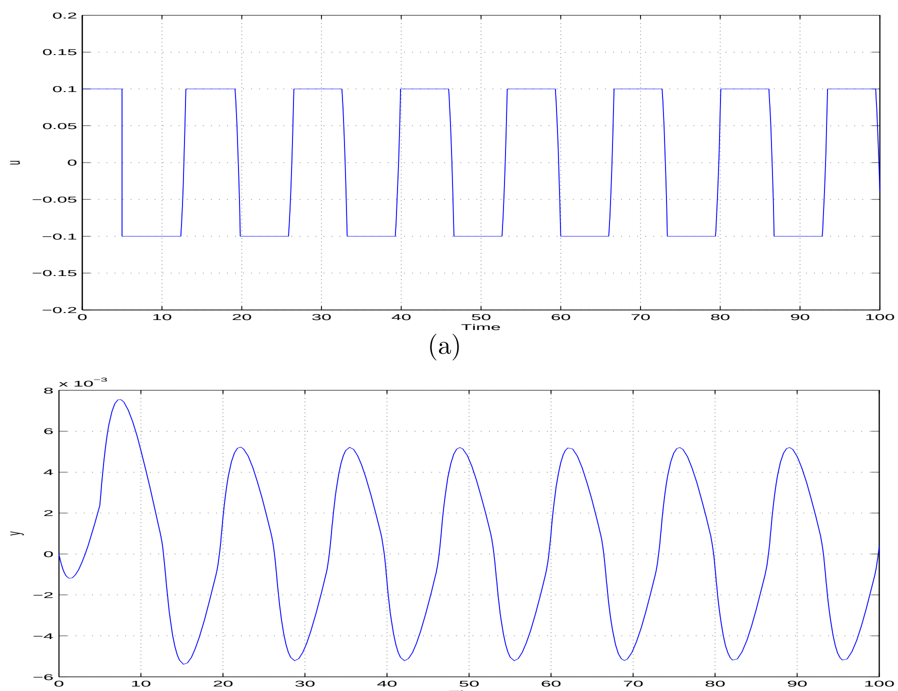


Figure 6:



(a)

(b)

Figure 7:

Hence,

$$
\begin{aligned}
h &= 0.1 \\
A &= 5.2\text{x}10^{-3} \\
P_u &= 13.4 \\
K_{cu} &= 24.4854
\end{aligned}
$$

therefore, the PI controller tuning parameters are given as follows:

$$
\begin{aligned}
K_c &= 11.0184 \\
\tau_i &= 11.1667
\end{aligned}
$$

The `Simulink` implementation of the closed-loop feedback control system is shown in Figure 8, whereas the closed-loop response obtained using a PI control system is depicted in Figure 9.

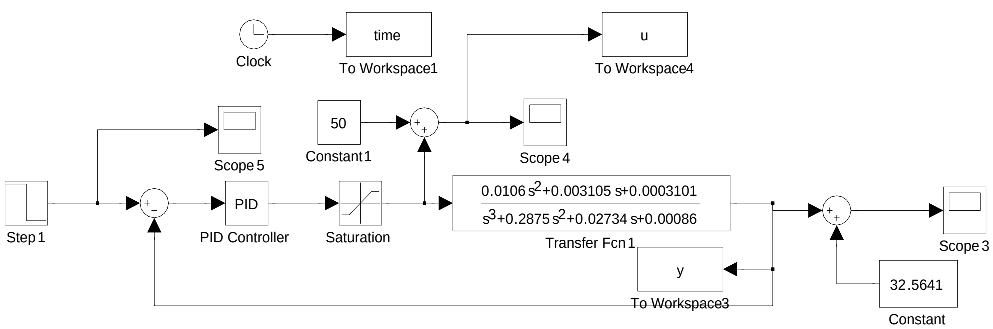
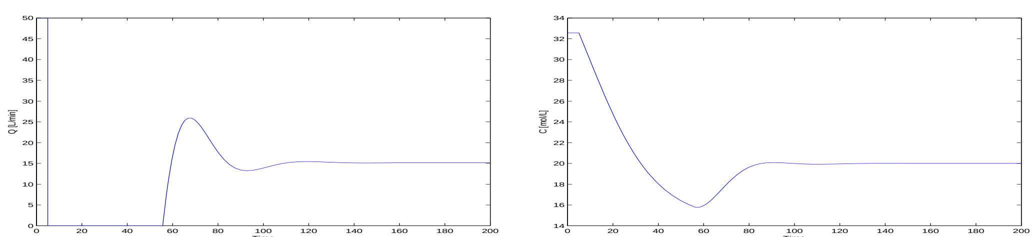


Figure 8:



(a) (b)

Figure 9: