9 Nonlinear control structures with direct decoupling for three-phase AC drive systems

9.1 Existing problems at linear controlled drive systems

It is clearly recognizable that the 3-phase drive system engineering has reached a relatively mature stage of development (cf. chapters 1-8). The principle of the field orientated control also has largely asserted to be the most used method in commercial systems. The spectrum of solved questions extends from the control and observer structures over the problems of parameter identification (on-line, off-line) and adaptation to the self-tuning and the self-commissioning.

The most implemented structure (chapter 5 or [Quang 1999]) contains a 2-dimensional current controller for decoupling between the magnetization and the generation of torque as well as for undelayed impression of torque. Because of the decoupling the flux and speed control loops could be designed rather liberally. In these structures the current controller and the flux observer are always based on motor models linearized within a sampling period (cf. section 3.2.2).

The linearization is made under the assumption that the sampling time T is small enough for the stator frequency ω_s to be regarded constant within T. Because of this assumption the frequency ω_s is now a parameter in the system matrix, and *the bilinear model becomes a linear time-variant system* for which the known design methods of linear systems (cf. chapter 5) can be used.

Although the present concept was very successful, it is recognizable that:

- because of the nonlinear process model (the input quantity ω_s appears in the system matrix) in high-speed drives with synchronous modulation (cf. section 2.5.2) or in sensorless controlled systems (cf. section 4.3), or

- because of the nonlinear parameters (the main inductance is strongly dependent on the state variable i_m or ψ_r) with respect to the system stability in systems with parameter identification and adaptation,

some problems often appear, particularly if the system must work at the voltage limit (i.e. in the nonlinear mode) and therewith the condition ω_s = const is no longer fulfilled. If these problems remain unsolved, the drive quality will be affected considerably. In such cases at least a nonlinear design would be able to deliver better results.

Within the last approx. 15 years different new ways to design nonlinear controllers were shown ([Isidori 1995], [Krstíc 1995], [Wey 2001]) or even experimented in motor control ([Ortega 1998], [Bodson 1998], [Khorrami 2003], [Dawson 2004]), but they were mostly theoretical works. The practical developments were completely missing. Recently, some more thorough investigations ([Cuong 2003], [Ha 2003], [Duc 2004], [Nam 2004]) concerned with practical implementation of the methods had been forthcoming, particularly on the exact linearization method discussed in this book.

9.2 Nonlinear control structure for drive systems with IM

In the section 3.6.2 the nonlinear process model of the IM was already derived as a starting point to the controller design:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{h}_1 u_1 + \mathbf{h}_2 u_2 + \mathbf{h}_3 u_3 \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \end{cases} \tag{9.1}$$

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -d\,x_1 + c\,\psi'_{rd} \\ -d\,x_2 - c\,T_r\,\omega\,\psi'_{rd} \\ 0 \end{bmatrix}; \mathbf{h}_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}; \mathbf{h}_1 = \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix}; \mathbf{h}_3 = \begin{bmatrix} x_2 \\ -x_1 \\ 1 \end{bmatrix} \tag{9.2}$$

$$y_1 = g_1(\mathbf{x}) = x_1;\ y_2 = g_2(\mathbf{x}) = x_2;\ y_3 = g_3(\mathbf{x}) = x_3$$

- Parameters: $a = 1/\sigma L_s\,;\ b = 1/\sigma T_s\,;\ c = (1-\sigma)/\sigma T_r\,;\ d = b + c$
- State variables: $x_1 = i_{sd}\,;\ x_2 = i_{sq}\,;\ x_3 = \vartheta_s$
- Input variables: $u_1 = u_{sd}\,;\ u_2 = u_{sq}\,;\ u_3 = \omega_s$
- Output variables: $y_1 = i_{sd}\,;\ y_2 = i_{sq}\,;\ y_3 = \vartheta_s$

9.2.1 Nonlinear controller design based on "exact linearization"

Using the idea in the section 3.6.1 the design can be realized in the following steps:

- Step 1: Calculation of the vector $\mathbf{r} = [r_1, r_2, \cdots, r_m]$ of the relative difference orders.
- Step 2: Calculation of the matrix $\mathbf{L}(\mathbf{x})$ using the formula (3.99) and check of its invertibility.

$$\mathbf{L}(\mathbf{x}) = \begin{pmatrix} L_{h_1} L_f^{r_1-1} g_1(\mathbf{x}) & \cdots & L_{h_m} L_f^{r_1-1} g_1(\mathbf{x}) \\ \vdots & \ddots & \vdots \\ L_{h_1} L_f^{r_m-1} g_m(\mathbf{x}) & \cdots & L_{h_m} L_f^{r_m-1} g_m(\mathbf{x}) \end{pmatrix}; \det[\mathbf{L}(\mathbf{x})] \neq 0 \quad (9.3)$$

After that the fulfillment of the condition (3.95) is checked.

- Step 3: Realization of the coordinate transformation using (3.96).

$$\mathbf{z} = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix} = \mathbf{m}(\mathbf{x}) = \begin{pmatrix} m_1^1(\mathbf{x}) \\ \vdots \\ m_{r_1}^1(\mathbf{x}) \\ \vdots \\ m_1^m(\mathbf{x}) \\ \vdots \\ m_{r_m}^m(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} g_1(\mathbf{x}) \\ \vdots \\ L_f^{r_1-1} g_1(\mathbf{x}) \\ \vdots \\ g_m(\mathbf{x}) \\ \vdots \\ L_f^{r_m-1} g_m(\mathbf{x}) \end{pmatrix} \quad (9.4)$$

- Step 4: Calculation of the state-feedback control law (3.98):

$$\mathbf{u} = \mathbf{a}(\mathbf{x}) + \mathbf{L}^{-1}(\mathbf{x})\mathbf{w} = -\mathbf{L}^{-1}(\mathbf{x}) \begin{pmatrix} L_f^{r_1} g_1(\mathbf{x}) \\ \vdots \\ L_f^{r_m} g_m(\mathbf{x}) \end{pmatrix} + \mathbf{L}^{-1}(\mathbf{x})\mathbf{w} \quad (9.5)$$

Using the represented steps or the formulae (9.3) - (9.5) the design for drive systems with IM can now be proceeded as follows:

- Step 1: Calculation of the vector **r**.

  a) Case $j = 1$:

$$L_{h_1} g_1(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_1 = [1 \quad 0 \quad 0] \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = a \neq 0 \quad (9.6)\text{a}$$

$$L_{h_2} g_1(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} = 0 \tag{9.6)b}$$

$$L_{h_3} g_1(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_3 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 \\ 1 \end{bmatrix} = x_2 \neq 0 \tag{9.6)c}$$

Therewith $r_1 = 1$ follows from the equation (9.6).
b) Case $j = 2$:

$$L_{h_1} g_2(\mathbf{x}) = \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = 0 \tag{9.7)a}$$

$$L_{h_2} g_2(\mathbf{x}) = \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} = a \neq 0 \tag{9.7)b}$$

$$L_{h_3} g_2(\mathbf{x}) = \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_3 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 \\ 1 \end{bmatrix} = -x_1 \neq 0 \tag{9.7)c}$$

From the equation (9.7), $r_2 = 1$ similarly follows.
c) Case $j = 3$:

$$L_{h_1} g_3(\mathbf{x}) = \frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = 0 \tag{9.8)a}$$

$$L_{h_2} g_3(\mathbf{x}) = \frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} = 0 \tag{9.8)b}$$

$$L_{h_3} g_3(\mathbf{x}) = \frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 \\ 1 \end{bmatrix} = 1 \neq 0 \tag{9.8)c}$$

From the equation (9.8), $r_3 = 1$ follows then.

- Step 2: Calculation of the matrix **L**.

$$\mathbf{L}(\mathbf{x})=\begin{bmatrix} L_{h_1}g_1(\mathbf{x}) & L_{h_2}g_1(\mathbf{x}) & L_{h_3}g_1(\mathbf{x}) \\ L_{h_1}g_2(\mathbf{x}) & L_{h_2}g_2(\mathbf{x}) & L_{h_3}g_2(\mathbf{x}) \\ L_{h_1}g_3(\mathbf{x}) & L_{h_2}g_3(\mathbf{x}) & L_{h_3}g_3(\mathbf{x}) \end{bmatrix}=\begin{bmatrix} a & 0 & x_2 \\ 0 & a & -x_1 \\ 0 & 0 & 1 \end{bmatrix} \quad (9.9)$$

It is easy to see that $\det[\mathbf{L}(\mathbf{x})]=a^2\neq 0$, and therefore the matrix $\mathbf{L}(\mathbf{x})$ can be inverted. The necessary and sufficient conditions are summarized:

$$\begin{cases} \det[\mathbf{L}(\mathbf{x})]=a^2\neq 0 \\ r_1+r_2+r_3=3=n \end{cases} \quad (9.10)$$

The fulfilled condition (9.11) indicates that the system (9.1) can be linearized exactly, or that the coordinate transformation can now be made.

- Step 3: Realization of the coordinate transformation.

a) The state space **x** is transformed into a new state space **z** using (9.4).

$$\mathbf{z}=\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}=\begin{bmatrix} m_1^1(\mathbf{x}) \\ m_1^2(\mathbf{x}) \\ m_1^3(\mathbf{x}) \end{bmatrix}=\begin{bmatrix} g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ g_3(\mathbf{x}) \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (9.11)$$

b) The new state space model is calculated as follows.

$$\begin{cases} \dfrac{dz_1}{dt}=\dfrac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}}\dfrac{d\mathbf{x}}{dt} \\ \qquad =L_f g_1(\mathbf{x})+L_{h_1}g_1(\mathbf{x})u_1+L_{h_2}g_1(\mathbf{x})u_2+L_{h_3}g_1(\mathbf{x})u_3 \\ \dfrac{dz_2}{dt}=\dfrac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}}\dfrac{d\mathbf{x}}{dt} \\ \qquad =L_f g_2(\mathbf{x})+L_{h_1}g_2(\mathbf{x})u_1+L_{h_2}g_2(\mathbf{x})u_2+L_{h_3}g_2(\mathbf{x})u_3 \\ \dfrac{dz_3}{dt}=\dfrac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}}\dfrac{d\mathbf{x}}{dt} \\ \qquad =L_f g_3(\mathbf{x})+L_{h_1}g_3(\mathbf{x})u_1+L_{h_2}g_3(\mathbf{x})u_2+L_{h_3}g_3(\mathbf{x})u_3 \end{cases} \quad (9.12)$$

The unknown terms in the equation (9.12) have to be calculated now.

$$L_f g_1(\mathbf{x})=\frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}}f=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix} -d\,x_1+c\,\psi_{rd}' \\ -d\,x_2-c\,T_r\,\omega\,\psi_{rd}' \\ 0 \end{bmatrix}=-d\,x_1+c\psi_{rd}'$$

$$L_f g_2(\mathbf{x}) = \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} f = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -d\,x_1 + c\,\psi_{rd}' \\ -d\,x_2 - c\,T_r\,\omega\,\psi_{rd}' \\ 0 \end{bmatrix} = -d\,x_2 + c\,T_r\,\omega\,\psi_{rd}'$$

$$L_f g_3(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} f = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -d\,x_1 + c\,\psi_{rd}' \\ -d\,x_2 - c\,T_r\,\omega\,\psi_{rd}' \\ 0 \end{bmatrix} = 0$$

The result of the coordinate transformation is then:

$$\begin{cases} \dfrac{dz_1}{dt} = -d\,x_1 + c\,\psi_{rd}' + a\,u_1 + x_2\,u_3 = w_1 \\ \dfrac{dz_1}{dt} = -d\,x_2 - c\,T_r\,\omega\,\psi_{rd}' + a\,u_2 - x_1\,u_3 = w_2 \\ \dfrac{dz_3}{dt} = u_3 = w_3 \end{cases} \tag{9.13}$$

The following equation will be obtained from (9.13):

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -d\,x_1 + c\,\psi_{rd}' \\ -d\,x_2 - c\,T_r\,\omega\,\psi_{rd}' \\ 0 \end{bmatrix}}_{\mathbf{p}(\mathbf{x})} + \underbrace{\begin{bmatrix} a & 0 & x_2 \\ 0 & a & -x_1 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{L}(\mathbf{x})} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{\mathbf{u}} \tag{9.14}$$

$$\mathbf{w} = \mathbf{p}(\mathbf{x}) + \mathbf{L}(\mathbf{x})\mathbf{u}$$

- Step 4: The control law, thereat **w** represents the new input vector, can be calculated by equation (9.14).

$$\mathbf{u} = \underbrace{-\mathbf{L}^{-1}(\mathbf{x})\mathbf{p}(\mathbf{x})}_{\mathbf{a}(\mathbf{x})} + \mathbf{L}^{-1}(\mathbf{x})\mathbf{w} = \mathbf{a}(\mathbf{x}) + \mathbf{L}^{-1}(\mathbf{x})\mathbf{w} \tag{9.15}$$

Using the matrix **L** in equation (9.9), $\mathbf{L}^{-1}$ is then obtained to:

$$\mathbf{L}^{-1} = \begin{bmatrix} 1/a & 0 & -x_2/a \\ 0 & 1/a & x_1/a \\ 0 & 0 & 0 \end{bmatrix} \tag{9.16}$$

The state-feedback control law or the coordinate transformation law (9.15) can be written in detailed form:

$$\mathbf{u}=\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}=\begin{bmatrix}d\,x_1/a-c\,\psi'_{rd}/a\\d\,x_2/a+c\,T_r\,\omega\,\psi'_{rd}/a\\0\end{bmatrix}+\begin{bmatrix}1/a&0&-x_2/a\\0&1/a&x_1/a\\0&0&0\end{bmatrix}\begin{bmatrix}w_1\\w_2\\w_3\end{bmatrix}$$

$$=\underbrace{\begin{bmatrix}\left(\dfrac{1}{T_s}+\dfrac{1-\sigma}{T_r}\right)L_s x_1-\dfrac{1-\sigma}{T_r}L_s\psi'_{rd}\\\left(\dfrac{1}{T_s}+\dfrac{1-\sigma}{T_r}\right)L_s x_2+(1-\sigma)\omega L_s\psi'_{rd}\\0\end{bmatrix}}_{\mathbf{a}(\mathbf{x})}+\underbrace{\begin{bmatrix}\sigma L_s&0&-\sigma L_s x_2\\0&\sigma L_s&\sigma L_s x_1\\0&0&1\end{bmatrix}}_{\mathbf{L}^{-1}(\mathbf{x})}\underbrace{\begin{bmatrix}w_1\\w_2\\w_3\end{bmatrix}}_{\mathbf{w}}\tag{9.17}$$

9.2.2 Feedback control structure with direct decoupling for IM

Using the state feedback or the coordinate transformation (9.17) the exact linearized IM model can be represented as in the figure 9.1. The new state model will now be the starting point for the controller design.

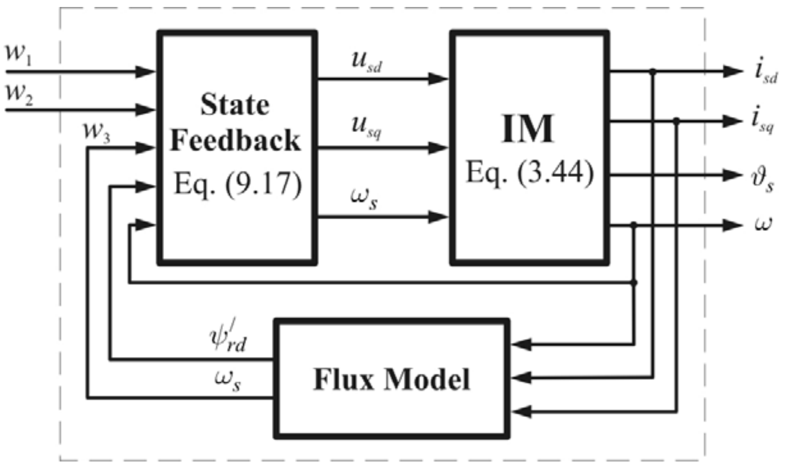

Fig. 9.1 Substitute linear process model of the IM as starting point for controller design (cf. figure 3.17)

It is not difficult for the new linear model to derive the input-output relation. After some transformations the following transfer function will be obtained:

$$\mathbf{y}(s)=\begin{bmatrix}1/s^{r_1} & 0 & 0\\ 0 & 1/s^{r_2} & 0\\ 0 & 0 & 1/s^{r_3}\end{bmatrix}\mathbf{w}(s)=\begin{bmatrix}1/s & 0 & 0\\ 0 & 1/s & 0\\ 0 & 0 & 1/s\end{bmatrix}\mathbf{w}(s) \qquad (9.18)$$

At more exact analysis of the equation (9.18) the following essential knowledge can be learned:

- Besides the exact linearization achieved in the complete new state space **z**, *the input-output decoupling relations are totally guaranteed*.
- The three transfer functions respectively contain only one element of integration.

Based on both these new results it seems to be possible to replace the two-dimensional current controller (figure 1.6) by a coordinate transformation and two separate current controllers for both axes *dq* (figure 9.2).

The *direct decoupling* concept in the figure 9.2 is dynamically effective for the complete state space. The two current controllers R_{isd} and R_{isq} need not have the PI characteristic, and can be designed with modern algorithms such as dead-beat control. A dynamical and nearly undelayed impression of the motor torque can be guaranteed without breaking any linearization condition.

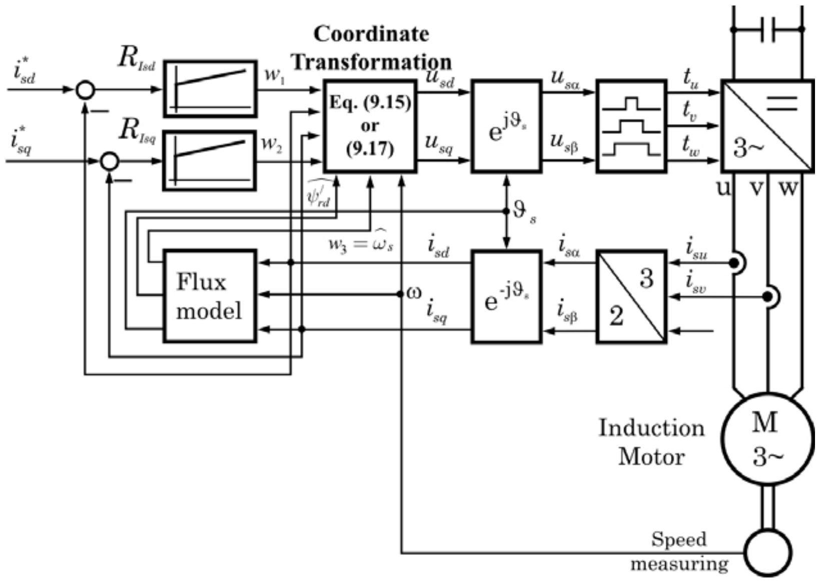

Fig. 9.2 The new control structure of the inner loop (impression of the stator current components) with direct decoupling designed using the method of exact linearization

9.3 Nonlinear control structure for drive systems with PMSM

The nonlinear process model of the PMSM was derived in the section 3.6.4 as follows:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{h}_1(\mathbf{x})u_1 + \mathbf{h}_2(\mathbf{x})u_2 + \mathbf{h}_3(\mathbf{x})u_3 \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \end{cases} \tag{9.19}$$

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -cx_1 \\ -dx_2 \\ 0 \end{bmatrix}; \mathbf{h}_1(\mathbf{x}) = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}; \mathbf{h}_2(\mathbf{x}) = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}; \mathbf{h}_3(\mathbf{x}) = \begin{bmatrix} ax_2/b \\ -bx_1/a - b\psi_p \\ 1 \end{bmatrix} \tag{9.20}$$

$$y_1 = g_1(\mathbf{x}) = x_1;\ y_2 = g_2(\mathbf{x}) = x_2;\ y_3 = g_3(\mathbf{x}) = x_3$$

- Parameters: $a = 1/L_{sd};\ b = 1/L_{sq};\ c = 1/T_{sd};\ d = 1/T_{sq}$
- State variables: $x_1 = i_{sd};\ x_2 = i_{sq};\ x_3 = \vartheta_s$
- Input variables: $u_1 = u_{sd};\ u_2 = u_{sq};\ u_3 = \omega_s$
- Output variables: $y_1 = i_{sd};\ y_2 = i_{sq};\ y_3 = \vartheta_s$

9.3.1 Nonlinear controller design based on "exact linearization"

The design is carried out similarly as in the case of the IM.

- Step 1: Calculation of the vector **r**.

a) Case $j = 1$:

$$L_{h_1} g_1(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = a \neq 0 \tag{9.21a}$$

$$L_{h_2} g_1(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = 0 \tag{9.21b}$$

$$L_{h_3} g_1(\mathbf{x}) = \frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_3 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} ax_2/b \\ -bx_1/a - b\psi_p \\ 1 \end{bmatrix} = \frac{ax_2}{b} \neq 0 \tag{9.21c}$$

From the equation (9.21) $r_1 = 1$ follows.

b) Case j = 2:

$$L_{h_1} g_2(\mathbf{x}) = \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = 0 \tag{9.22a}$$

$$L_{h_2} g_2(\mathbf{x}) = \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = b \neq 0 \tag{9.22b}$$

$$\begin{aligned} L_{h_3} g_2(\mathbf{x}) &= \frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_3 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} ax_2/b \\ -bx_1/a - b\psi_p \\ 1 \end{bmatrix} \\ &= -\frac{bx_1}{a} - b\psi_p \neq 0 \end{aligned} \tag{9.22c}$$

With the equation (9.22) r_2 = 1 is obtained.

c) Case j = 3:

$$L_{h_1} g_3(\mathbf{x}) = \frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} = 0 \tag{9.23a}$$

$$L_{h_2} g_3(\mathbf{x}) = \frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = 0 \tag{9.23b}$$

$$L_{h_3} g_3(\mathbf{x}) = \frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}} \mathbf{h}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ax_2/b \\ -bx_1/a - b\psi_p \\ 1 \end{bmatrix} = 1 \neq 0 \tag{9.23c}$$

With the equation (9.23) r_3 = 1 is obtained.

- Step 2: Calculation of the matrix **L**.

$$\begin{aligned} \mathbf{L}(\mathbf{x}) &= \begin{bmatrix} L_{h_1} g_1(\mathbf{x}) & L_{h_2} g_1(\mathbf{x}) & L_{h_3} g_1(\mathbf{x}) \\ L_{h_1} g_2(\mathbf{x}) & L_{h_2} g_2(\mathbf{x}) & L_{h_3} g_2(\mathbf{x}) \\ L_{h_1} g_3(\mathbf{x}) & L_{h_2} g_3(\mathbf{x}) & L_{h_3} g_3(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} a & 0 & ax_2/b \\ 0 & b & -bx_1/a - b\psi_p \\ 0 & 0 & 1 \end{bmatrix} \\ \Rightarrow \quad & \det\left[\mathbf{L}(\mathbf{x})\right] = ab \neq 0 \end{aligned} \tag{9.24}$$

- Step 3: Realization of the coordinate transformation.

  a) The state space $\mathbf{x}$ is transformed into a new state space $\mathbf{z}$ using (9.4).

  After replacing $r_1 = r_2 = r_3 = 1$ into (9.4), the same result like (9.11) is obtained:

$$\mathbf{z}=\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}=\begin{bmatrix} m_1^1(\mathbf{x}) \\ m_1^2(\mathbf{x}) \\ m_1^3(\mathbf{x}) \end{bmatrix}=\begin{bmatrix} g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ g_3(\mathbf{x}) \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{9.25}$$

  b) The new state model is similar to (9.12).

$$\begin{cases} \dfrac{dz_1}{dt}=\dfrac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}}\dfrac{d\mathbf{x}}{dt} \\ \qquad =L_f g_1(\mathbf{x})+L_{h_1} g_1(\mathbf{x})u_1+L_{h_2} g_1(\mathbf{x})u_2+L_{h_3} g_1(\mathbf{x})u_3 \\ \dfrac{dz_2}{dt}=\dfrac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}}\dfrac{d\mathbf{x}}{dt} \\ \qquad =L_f g_2(\mathbf{x})+L_{h_1} g_2(\mathbf{x})u_1+L_{h_2} g_2(\mathbf{x})u_2+L_{h_3} g_2(\mathbf{x})u_3 \\ \dfrac{dz_3}{dt}=\dfrac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}}\dfrac{d\mathbf{x}}{dt} \\ \qquad =L_f g_3(\mathbf{x})+L_{h_1} g_3(\mathbf{x})u_1+L_{h_2} g_3(\mathbf{x})u_2+L_{h_3} g_3(\mathbf{x})u_3 \end{cases} \tag{9.26}$$

  with:

$$L_f g_1(\mathbf{x})=\frac{\partial g_1(\mathbf{x})}{\partial \mathbf{x}}f=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix} -c\,x_1 \\ -d\,x_2 \\ 0 \end{bmatrix}=-c\,x_1$$

$$L_f g_2(\mathbf{x})=\frac{\partial g_2(\mathbf{x})}{\partial \mathbf{x}}f=\begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix} -c\,x_1 \\ -d\,x_2 \\ 0 \end{bmatrix}=-d\,x_2$$

$$L_f g_3(\mathbf{x})=\frac{\partial g_3(\mathbf{x})}{\partial \mathbf{x}}f=\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix} -c\,x_1 \\ -d\,x_2 \\ 0 \end{bmatrix}=0$$

  After inserting of the above calculated terms into the equation (9.26), the result of the coordinate transformation is given as follows:

$$\begin{aligned}
\frac{dz_1}{dt} &= -c\,x_1 + a\,u_1 + \frac{a}{b}x_2 u_3 = w_1 \\
\frac{dz_2}{dt} &= -d\,x_2 + b\,u_2 - \left(\frac{b}{a}x_1 + b\,\psi_p\right)u_3 = w_2 \\
\frac{dz_3}{dt} &= u_3 = w_3
\end{aligned} \tag{9.27}$$

From the equation (9.27) the new input vector **w** is derived as:

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -cx_1 \\ -dx_2 \\ 0 \end{bmatrix}}_{\mathbf{p}(\mathbf{x})} + \underbrace{\begin{bmatrix} a & 0 & a\,x_2/b \\ 0 & b & -b\,x_1/a - b\,\psi_p \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{L}(\mathbf{x})} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{\mathbf{u}} \tag{9.28}$$

- Step 4: The control law or the transformation law results from the equation (9.28).

$$\begin{aligned}
\mathbf{u} &= \underbrace{-\mathbf{L}^{-1}(\mathbf{x})\mathbf{p}(\mathbf{x})}_{\mathbf{a}(\mathbf{x})} + \mathbf{L}^{-1}(\mathbf{x})\mathbf{w} = \mathbf{a}(\mathbf{x}) + \mathbf{L}^{-1}(\mathbf{x})\mathbf{w} \\
&= \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} c\,x_1/a \\ d\,x_2/b \\ 0 \end{bmatrix} + \begin{bmatrix} 1/a & 0 & -x_2/b \\ 0 & 1/b & x_1/a + \psi_p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \\
&= \underbrace{\begin{bmatrix} L_{sd}x_1/T_{sd} \\ L_{sq}x_2/T_{sq} \\ 0 \end{bmatrix}}_{\mathbf{a}(\mathbf{x})} + \underbrace{\begin{bmatrix} L_{sd} & 0 & -L_{sq}x_2 \\ 0 & L_{sq} & L_{sd}x_1 + \psi_p \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{L}^{-1}(\mathbf{x})} \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}}_{\mathbf{w}}
\end{aligned} \tag{9.29}$$

9.3.2 Feedback control structure with direct decoupling for PMSM

Similarly to the IM and using the state feedback or the coordinate transformation (9.29) the exact linearized PMSM model can be represented as in the figure 9.3. The difference between IM and PMSM consists here in the fact that a flux model is not needed any more, because the pole flux is permanently available.

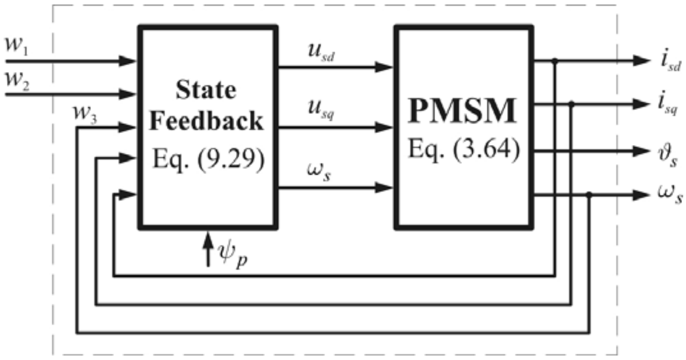

Fig. 9.3 Substitute linear process model of the PMSM

The new model in the figure 9.3 can be written in equation form as follows:

$$\begin{cases} \dfrac{d\mathbf{z}}{dt} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{w} \\ \mathbf{y} = \begin{bmatrix} z_1 \\ z_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{z} \end{cases} \tag{9.30}$$

Similarly to the case IM with the equation (9.18), the following transfer function for the PMSM is obtained:

$$\mathbf{y}(s) = \begin{bmatrix} 1/s^{r_1} & 0 & 0 \\ 0 & 1/s^{r_2} & 0 \\ 0 & 0 & 1/s^{r_3} \end{bmatrix} \mathbf{w}(s) = \begin{bmatrix} 1/s & 0 & 0 \\ 0 & 1/s & 0 \\ 0 & 0 & 1/s \end{bmatrix} \mathbf{w}(s) \tag{9.31}$$

It can be seen easily that the same conclusions about the direct decoupling between the *dq* axes and the transfer functions of the decoupled input-output couples are valid also here. The summary then is that the control structure with direct decoupling in the figure 9.2 (of course without the flux model) can be used for the PMSM.

The new control concept in the two cases IM and PMSM with direct decoupling has some features besides the mentioned advantages which should be mentioned here:

- The transformation laws or the control laws (9.17) and (9.29) contain only static feedbacks, no time dependent components like integration and differentiation so that this nonlinear concept is very well suitable for a digital implementation.

- Because of parameter faults caused by an inaccurate parameter setting or by parameter changes during operation, stationary faults of the coordinate transformation always exist. This can be avoided by a dynamic concept using an additional parameter identification and adaptation or by additional integral parts in the transformation law.

9.4 References to chapter 9

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