3 Machine models as a prerequisite to design the controllers and observers

3.1 General issues of state space representation

The mathematical modelling of the physical relations in 3-phase machines generally leads to differential equations of higher order and to state models with mutual coupling of the state variables respectively. For such systems the state space representation provides a very clear notation and a suitable starting point for the design of controllers, process models or observers.

Consistently, the equations to be derived in the following chapters will be predominantly based on the state space representation, making it sensible to introduce this chapter with some basic ideas. There the main focus will be on some important topics of the modelling of 3-phase machines such as time variance of the parameters and nonlinearity of the system equations, and their consequences for the discretization of the state equations.

3.1.1 Continuous state space representation

A time-continuous dynamic system can generally be represented in the following form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}\left(\mathbf{x}(t), \mathbf{u}(t)\right); \quad \mathbf{x} \in \mathrm{R}^n;\ \mathbf{u} \in \mathrm{R}^m;\ \mathbf{x}_0 = \mathbf{x}(t_0) \\ \mathbf{y}(t) &= \mathbf{h}\left(\mathbf{x}(t), \mathbf{u}(t)\right); \quad \mathbf{y} \in \mathrm{R}^p \end{aligned} \tag{3.1}$$

In equation (3.1) **f** and **h** are general analytical vector functions of the state vector **x** and the input vector **u**. The equation (3.1) describes a system of differential equations of first order, in which the system order n is equal to the number of contained independent energy storages. The system has n state, m input and p output quantities.

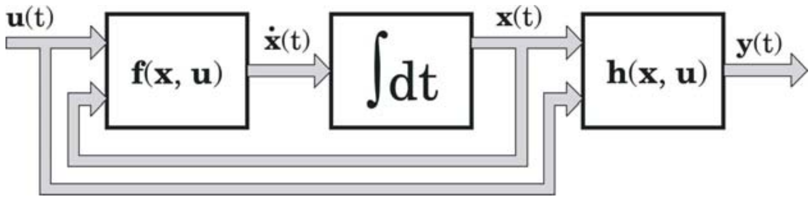

Fig. 3.1 Block circuit diagram of a system in state space representation

In many cases, a system description will not be required in the general form of equation (3.1), or for analysis and controller design a model must be found, which represents an adequately exact approximation of the physical conditions and is more accessible to the further processing. The usual way to achieve this is *the linearization of* (3.1) *along a (quasi-stationary) trajectory* ($\mathbf{X}(t)$, $\mathbf{U}(t)$) *or around a stationary operating point* ($\mathbf{X}_0$, $\mathbf{U}_0$). After TAYLOR series expansion and truncation after the linear term, the following system is obtained:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}_{\mathbf{x}}\big(\mathbf{X}(t),\mathbf{U}(t)\big)\,\mathbf{x}(t) + \mathbf{f}_{\mathbf{u}}\big(\mathbf{X}(t),\mathbf{U}(t)\big)\,\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{h}_{\mathbf{x}}\big(\mathbf{X}(t),\mathbf{U}(t)\big)\,\mathbf{x}(t) \end{aligned} \tag{3.2}$$

Depending on the choice of the trajectory ($\mathbf{X}(t)$, $\mathbf{U}(t)$), the operating point ($\mathbf{X}_0$, $\mathbf{U}_0$) or the degree of the linearization respectively the following special cases can be distinguished.

1. *Linear system with time-variant parameters*

The linearization is performed along the trajectory of a slowly variable quantity. Nonlinear combinations of state quantities are interpreted as products of a state quantity and a time variable parameter. Such a representation proffers itself primarily if products of state quantities with appropriately big differences of their eigendynamics appear. The equation system takes on the following form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(t)\,\mathbf{x}(t) + \mathbf{B}(t)\,\mathbf{u}(t); \quad \mathbf{x}_0 = \mathbf{x}(t_0);\, t \ge t_0 \\ \mathbf{y}(t) &= \mathbf{C}(t)\,\mathbf{x}(t) \end{aligned} \tag{3.3}$$

In (3.3) **A** is the system matrix, **B** the input matrix and **C** the output matrix. Because no direct feed-through of the input to the output vector **y** exists in electrical drive control systems (*no step-change capability*), we will abstain from explicitly including this dependency in the following.

2. *Bilinear system*

If the transfer matrices are constant in time, and if a nonlinearity only exists regarding the control input, and not regarding the state vector, we speak of a bilinear system.

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{A}\,\mathbf{x}(t)+\sum_{i=1}^{m}\mathbf{N}_i\,u_i(t)\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t);\quad \mathbf{x}_0=\mathbf{x}(t_0)\\&\mathbf{y}(t)=\mathbf{C}\,\mathbf{x}(t)\end{aligned}\tag{3.4}$$

The multiplicative couplings between input and state quantities are summarized in the matrices $\mathbf{N}_i$.

3. *Linear system with constant parameters*

The class of the linear time-invariant systems finally represents the most simple case. The system equations are:

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\,\mathbf{u}(t);\quad \mathbf{x}_0=\mathbf{x}(t_0)=\mathbf{x}(0)\\&\mathbf{y}(t)=\mathbf{C}\,\mathbf{x}(t)\end{aligned}\tag{3.5}$$

3.1.2 Discontinuous state space representation

Control algorithms and models are processed in micro computers, and therefore in discrete time. The computer receives the system output quantity $\mathbf{y}(t)$ at definite equidistant points of time – e.g. after sampling and A/D conversion or U/f conversion and integration – as discrete quantity $\mathbf{y}(k)$. The calculated control variables are realized discontinuously as voltages by a PWM inverter. The complete control system represents a sampling system (fig. 3.2).

Because of the sampling operation of the computer, as a rule, a discrete design of the control system will be preferred. This is motivated firstly, because special phenomena caused by the sampling can specifically be considered in the design. Secondly, the application of special design methods particularly adopted to sampling operation, such as the dead beat design, will be possible. It is prerequisite that an equivalent time-discrete description can be found for the continuous system, which exactly reflects the dynamic behaviour of the continuous system at the sampling instants.

Unfortunately, for time-variant or nonlinear systems it will only in some rare cases be possible to find such an equivalent time-discrete system representation. The reasons will become clear at the derivation of the discontinuous state equations in later sections. Therefore, to design a discontinuous control system we can principally choose between the following two alternatives:

1. Controller design for the continuous system and then discrete implementation (*quasi-continuous design)*.
2. Derivation of an approximated time-discrete process model and then *discrete controller design*.

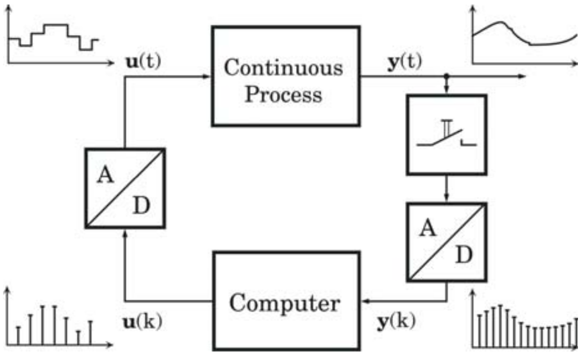

Fig. 3.2 Overview of a sampling system

In many cases it will not be required to take into account all or single nonlinearities of the system because certain approximations for the time-variant parameters and nonlinearities are possible and acceptable depending on the concrete application. In addition, because also in the linear case the discretization of the process model raises some important problems, the second way will be discussed in more detail in the following. The description of the continuous system shall be idealized as far as possible, to enable its discretization like for the linear time-invariant case.

For this purpose it is assumed that *the time-variant and state dependent parameters in equation* (3.3) are *constant within a sampling period,* therefore the sampling period has to be chosen sufficiently small. Thus equation (3.3) can be regarded piecewise linear and time-invariant for each sampling period, and the discretization of the continuous model is possible in a conventional way like for linear time-invariant systems. The discretization starts from the system equation (3.5) with the sampling period T presumed constant.

The discrete-time state model arises from the solution of the continuous state equation, yielding for the time-variant system (3.3) with continuous matrices $\mathbf{A}(t)$ and $\mathbf{B}(t)$:

$$\varphi(t,\mathbf{x}_0,t_0)=\mathbf{\Phi}(t,t_0)\mathbf{x_0}+\int_{t_0}^{t}\mathbf{\Phi}(t,\tau)\mathbf{B}(\tau)\mathbf{u}(\tau)\,d\tau\,, \quad t\geq t_0 \tag{3.6}$$

The matrix $\mathbf{\Phi}(t, t_0)$ describes the transition of the system from the state $\mathbf{x}(t_0)=\mathbf{x}_0$ to the state $\mathbf{x}(t)$ on the trajectory φ, and is therefore called the fundamental matrix or transition matrix. The matrix $\mathbf{\Phi}$ fulfils the following matrix differential equation with the system matrix $\mathbf{A}(t)$:

$$d\boldsymbol{\Phi}(t,t_0)/dt = \mathbf{A}(t)\,\boldsymbol{\Phi}(t,t_0);\quad \boldsymbol{\Phi}(t,t_0) = \mathbf{I} \tag{3.7}$$

For a constant system matrix **A** the fundamental matrix from equation (3.7) can be calculated analytically and represented as a matrix exponential function:

$$\boldsymbol{\Phi}(t,t_0) = \mathrm{e}^{\mathbf{A}(t-t_0)} \tag{3.8}$$

For the derivation of the discrete state equation the transient response between two sampling instants is of interest. That means equation (3.6) must be integrated over a sampling period T. With (3.8) and $t_0 = 0$ the following result is obtained:

$$\mathbf{x}\big((k+1)T\big) = \mathrm{e}^{\mathbf{A}T}\,\mathbf{x}(kT) + \int_0^T \mathrm{e}^{\mathbf{A}(T-\tau)}\,\mathbf{B}\mathbf{u}(kT+\tau)\,d\tau \tag{3.9}$$

To comply with (3.8) and (3.9), for the discretization of a time-variant system, the system matrix **A** must be presumed constant over one sampling period, as already indicated above. The transition matrix $\boldsymbol{\Phi}((k+1)T,\ kT)$ becomes the discrete system matrix $\boldsymbol{\Phi}(k)$ and has to be recalculated online for every sampling period. Thus a time-variant discrete system is obtained. If one assumes further that the input vector $\mathbf{u}(t)$ is sampled by a zero-order hold function and therefore is also constant over one sampling period, $\mathbf{u}(t)$ may be extracted from the integral, and the complete system of state equations can be rewritten into the following matrix form:

$$\begin{aligned}&\mathbf{x}(k+1) = \boldsymbol{\Phi}(k)\,\mathbf{x}(k) + \mathbf{H}(k)\,\mathbf{u}(k);\quad \mathbf{x}_0 = \mathbf{x}(0);\quad k \geq 0\\ &\mathbf{y}(k) = \mathbf{C}(k)\,\mathbf{x}(k)\end{aligned} \tag{3.10}$$

The system matrix $\boldsymbol{\Phi}(k)$ is defined by:

$$\boldsymbol{\Phi}(k) = \mathrm{e}^{\mathbf{A}(kT)T} \tag{3.11}$$

Because the input matrix **B** is constant, **B** can also be written outside the integral in (3.9). After substitution of the integration variable τ, the discrete input matrix **H** can be written as follows:

$$\mathbf{H} = \int_0^T \mathrm{e}^{\mathbf{A}(kT)\tau}\,d\tau\,\mathbf{B} = \int_0^T \boldsymbol{\Phi}(k)\big|_{T=\tau}\,d\tau\,\mathbf{B} \tag{3.12}$$

With regular **A**, (3.12) can be solved further to:

$$\mathbf{H} = \mathbf{A}(kT)^{-1}\left[\mathrm{e}^{\mathbf{A}(kT)T} - \mathbf{I}\right]\mathbf{B} \tag{3.13}$$

The output matrix **C** is identical to the continuous system. The system matrix $\boldsymbol{\Phi}(k)$ is the decisive component of the discretization procedure. It determines the dynamics and stability of the discrete system. For its evaluation different methods are known, characterized by more or less calculation effort and higher or lower degree of approximation. Some of them, which are suitable for real time applications, shall be discussed in the following in more detail.

1. *Series expansion*

With this method, (3.11) is expanded directly into a power series.

$$\mathbf{\Phi} = \mathrm{e}^{\mathbf{A}T} = \mathbf{I} + \mathbf{A}T + \frac{(\mathbf{A}T)^2}{2} + ... = \sum_{\nu=0}^{\infty} \frac{(\mathbf{A}T)^{\nu}}{\nu!} \tag{3.14}$$

After truncating the series expansion after the linear term, we obtain the solution for the Euler or RK1 procedure. This quite simple and easily comprehensible solution already suffices for many electrical drive applications at usual sampling times in the range 0.1 ... 1ms with respect to stability and precision. Because of possible numerical stability problems of the Euler procedure a more exact analysis is, however, appropriate.

The stability range of the Euler procedure in the continuous state plane is a circle with the radius $1/T$ and center at $-1/T$ on the real axis. Therefore all eigenvalues λ_i of the continuous system must hold to the following inequality:

$$\left|\lambda_i + \frac{1}{T}\right| < \frac{1}{T} \tag{3.15}$$

Particularly for complex frequency dependent eigenvalues of the system matrix **A** an exact check of this stability condition is required. Discretization-induced instabilities may be avoided by:

- Increasing the order of the series expansion of (3.14).
- Eluding to an integration method of higher order, e.g. RK4, which however, probably will be less feasible for real time applications.
- Avoiding complex eigenvalues of the system matrix **A** or its partial matrices.

For the latter variant the discretization of the state equations has to be first carried out in a coordinate system in which no frequency dependent eigenvalues of **A** or partial matrices $\mathbf{A}_{ii}$ appear. After that the discrete state equations are transformed into the final coordinate system (refer to example in the section 12.2). This procedure already yields decisive improvements for the Euler method. The use of suitable coordinate systems for the discretization can at the same time help to avoid errors, which result from the necessarily idealizing assumption of constant parameters of the system matrix **A** within a sampling period.

A similar approach would consist in transforming the input quantities of the partial system of interest into the respective natural coordinate system (without frequency dependent eigenvalues for the $\mathbf{A}_{ii}$). In these coordinates all required calculations (model, controller and observer) would be processed, and then the output quantities would be transformed back into the original reference system.

2. *Equivalent function*

The matrix function $\mathbf{F}(\mathbf{A}) = \mathrm{e}^{\mathbf{A}T}$ is recreated by an equivalent polynomial function $\mathbf{R}(\mathbf{A})$ with:

$$\mathbf{R}(\mathbf{A}) = \sum_{i=0}^{n-1} r_i\,\mathbf{A}^i = \mathrm{e}^{\mathbf{A}T} \tag{3.16}$$

In this function, n is the order of the continuous system. This substitution is based on the Cayley Hamilton theorem, which states that every square matrix satisfies its own characteristic equation. As a consequence, it can be derived that every *(n×n)* matrix function of order $p \geq n$, therefore also $p \to \infty$ like in (3.14), may be represented by a function of not more than (n-1)$^{\text{th}}$ order. The equivalent function (3.16) corresponds exactly to this statement.

With known factors r_i the system matrix $\mathbf{\Phi}$ can be calculated from (3.16) whereby completely avoiding discretization errors, as in the case of truncated series expansion. For the calculation of the factors r_i the already mentioned property of (3.16) is used, that it is satisfied not only by the matrix $\mathbf{A}$ but also by the eigenvalues λ_j. This leads to the following linear system of equations:

$$\sum_{i=0}^{n-1} r_i\,\lambda_j^i = \mathrm{e}^{\lambda_j T} \qquad \left(j = 1, 2 \ldots n\right), \tag{3.17}$$

which holds at first for single eigenvalues. For p-fold eigenvalues (p>1) equation (3.17) is differentiated (p-1) times with respect to λ_j:

$$\begin{aligned} T\,\mathrm{e}^{\lambda_j T} &= \sum_{i=1}^{n-1} r_i\, i\,\lambda_j^{i-1} \\ &\;\vdots \\ T^{p-1}\,\mathrm{e}^{\lambda_j T} &= \sum_{i=p-1}^{n-1} r_i\, i(i-1)\ldots(i-p+2)\,\lambda_j^{i-p+1} \end{aligned} \tag{3.18}$$

A second possibility for the calculation of r_i is offered by the *Sylvester-Lagrange equivalent polynomial method.* A minimal polynomial $M(\lambda)$ is defined, which is equal to the characteristic polynomial for the case of exclusively single eigenvalues of **A**:

$$M(\lambda) = |\lambda \mathbf{I} - \mathbf{A}| = \prod_{i=1}^{n} (\lambda - \lambda_i) \tag{3.19}$$

In the case of multiple eigenvalues, $M(\lambda)$ contains only the eigenvalues different from each other with number $m<n$. Furthermore the following auxiliary functions are defined.

$$M_i(\lambda) = \frac{M(\lambda)}{\lambda - \lambda_i}; \quad i = 1, 2, ..., n \tag{3.20}$$

$$m_i = M_i(\lambda)\big|_{\lambda=\lambda_i}; \quad i = 1, 2, ..., n \tag{3.21}$$

With these, the substitute function $R(\lambda) = \mathbf{R}(\mathbf{A})\big|_{\mathbf{A}=\lambda}$

$$R(\lambda) = \sum_{i=1}^{n} \frac{e^{\lambda_i T}}{m_i} M_i(\lambda) \tag{3.22}$$

is finally calculated, from which the factors r_i are obtained by organizing after powers of λ. In the case of multiple eigenvalues, n is to be replaced by m in equations (3.19) to (3.22).

The previous explanations for the state space representation shall promote the understanding of the procedure for the later controller and observer design. The example in the section 12.2 (appendices) shall clarify the theoretical explanations.

As opposed to linear systems, no representation of an equivalent time-discrete system can be given for general nonlinear and time-variant systems. The bilinear systems (3.4) are an exception up to a certain point. The system

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \left[\mathbf{A} + \sum_{i=1}^{m} \mathbf{N}_i u_i(t)\right] \mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t); \quad \mathbf{x}_0 = \mathbf{x}(t_0) \\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t) \end{aligned} \tag{3.23}$$

can be integrated over T like an ordinary linear system under the prerequisite of the constancy of the control vector **u** during a sampling period. For the system matrices of the equivalent discrete system the following results are obtained:

$$\mathbf{\Phi}(k) = e^{\left(\mathbf{A} + \sum_{i=1}^{m} \mathbf{N}_i u_i(k)\right) T}; \quad \mathbf{H}(k) = \int_0^T e^{\left(\mathbf{A} + \sum_{i=1}^{m} \mathbf{N}_i u_i(k)\right) \tau} d\tau\, \mathbf{B} \tag{3.24}$$

However, this derivation also will have practical meaning only in special cases.

3.2 Induction machine with squirrel-cage rotor (IM)

As indicated in the previous section, the 3-phase AC machine can be described by a complicated system of higher order differential equations. To derive a machine or a system model, which allows a convenient handling from the control point of view, a series of simplifying assumptions must be met regarding the reproduction accuracy of constructive and electrical details (refer to chapter 6).

The reference axis for the field angle is the axis of the phase winding u and therefore the α axis of the stator-fixed coordinate system. The coordinate transformations (vector rotations for voltage output and current measurement) are assumed as well-known methods. The same applies to the inverter control by means of space vector modulation. These transfer blocks are regarded as error-free with respect to phase and amplitude, and will be considered negligible for the benefit of a clear control structure representation.

In this book, the three-phase machines will be represented using their state space models. In the classical, computer-based control structures the controller designs almost always were based on continuous state models. This approach does not suffice any more today. Therefore, in the first step the continuous state space models of the 3-phase AC machines shall be worked out in this section. Then the equivalent discrete state models will be derived to support the design of the discrete controllers.

The electrical quantities are represented as vectors with real components. As a reminder the important indices to be used shall be listed here.

a) *Superscript*:		
	f	field synchronous (or field orientated, rotor flux / pole flux orientated) quantities
	s	stator-fixed quantities
	r	rotor-fixed (or rotor orientated) quantities
b) *Subscript*:		
1st letter:	s	stator quantities
	r	rotor quantities
2nd letter:	d, q	field synchronous components
	α, β	stator-fixed components
c) *Letters in bold*:		vectors, matrices

3.2.1 Continuous state space models of the IM in stator-fixed and field-synchronous coordinate systems

Starting-point for all derivations are the stator and rotor voltage equations in their natural and easily comprehensible winding systems: The stator-fixed coordinate system, and the rotor-fixed coordinate system.

- *Stator voltage in the stator winding system*:

$$\mathbf{u}_s^s = R_s\,\mathbf{i}_s^s + \frac{d\psi_s^s}{dt} \tag{3.25}$$

R_s: Stator resistance; ψ_s^s: Stator flux vector

- *Rotor voltage in the short-circuited rotor winding system*:

$$\mathbf{u}_r^r = R_r\,\mathbf{i}_r^r + \frac{d\psi_r^r}{dt} = \mathbf{0} \tag{3.26}$$

R_r: Rotor resistance; ψ_r^r: Rotor flux vector, $\mathbf{0}$: Zero vector

- *Stator and rotor flux*:

$$\begin{cases} \psi_s = L_s\,\mathbf{i}_s + L_m\,\mathbf{i}_r \\ \psi_r = L_m\,\mathbf{i}_s + L_r\,\mathbf{i}_r \end{cases} \text{with} \begin{cases} L_s = L_m + L_{\sigma s} \\ L_r = L_m + L_{\sigma r} \end{cases} \tag{3.27}$$

L_m: Mutual inductance; L_s, L_r: Stator and rotor inductances

$L_{\sigma s}$, $L_{\sigma r}$: Leakage inductances on the side of the stator and rotor

Due to the mechanically symmetrical construction the inductances are equal in all Cartesian coordinate systems. Therefore the superscripts are dropped in equation (3.27). The mechanical equations also are part of the machine description.

- *Torque equation*:

$$m_M = \frac{3}{2} z_p \left(\psi_s \times \mathbf{i}_s\right) = -\frac{3}{2} z_p \left(\psi_r \times \mathbf{i}_r\right)^{1} \tag{3.28}$$

$$m_M = \frac{3}{2} z_p \operatorname{Im}\left\{\psi_s^* \,\mathbf{i}_s\right\} = -\frac{3}{2} z_p \operatorname{Im}\left\{\psi_s \,\mathbf{i}_s^*\right\}^{2} \tag{3.29}$$

- *Equation of motion*:

$$m_M = m_W + \frac{J}{z_p}\frac{d\omega}{dt} \tag{3.30}$$

m_M, m_W: Motor and load torque, z_p: Number of pole pair

J: Torque of inertia, ω: Mechanical angular velocity

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[1] × cross product of vectors

[2] Im{ } Imaginary part of the term in brackets; * conjugated complex value

Now a coordinate system is introduced which rotates with angular frequency ω_k, as shown in section 1.1, and all quantities are transformed from the winding-coupled systems into the rotating one:

1. *Stator voltage equation*

After applying the transformation rules the following results are obtained:

$$\mathbf{u}_s^s = \mathbf{u}_s^k e^{j\vartheta_k}, \mathbf{i}_s^s = \mathbf{i}_s^k e^{j\vartheta_k}, \psi_s^s = \psi_s^k e^{j\vartheta_k}, \frac{d\psi_s^s}{dt} = \frac{d\psi_s^k}{dt} e^{j\vartheta_k} + j\omega_k \psi_s^k e^{j\vartheta_k}$$

Inserting the transformed quantities into equation (3.25), the equation (3.31) of the stator voltage in the new rotating system is obtained:

$$\mathbf{u}_s^k = R_s \mathbf{i}_s^k + \frac{d\psi_s^k}{dt} + j\omega_k \psi_s^k \tag{3.31}$$

However, the voltage equation is not to be represented in an arbitrary system, but for special practically relevant cases: in the stator-fixed or in the field synchronous (field-orientated) systems. These representations are obtained by setting:

- $\omega_k = \omega_s$: Here ω_s is the angular velocity of the stator-side space vectors or the rotating rotor flux vector.

$$\mathbf{u}_s^f = R_s \mathbf{i}_s^f + \frac{d\psi_s^f}{dt} + j\omega_s \psi_s^f \tag{3.32}$$

This coordinate system is chosen to lock the real or the *d*-axis of the system to the rotor flux (refer to section 1.2). Thus the cross component of the rotor flux becomes equal to zero. The axes of the system are denoted by *dq coordinates*.

- $\omega_k = 0$: This means, that the system is fixed in space, whereat the real axis or the α-axis of the coordinate system coincides with the axis of the phase winding *u*.

$$\mathbf{u}_s^s = R_s \mathbf{i}_s^s + \frac{d\psi_s^s}{dt} \tag{3.33}$$

The axes of this stator-fixed coordinate system are denoted as $\alpha\beta$-coordinates. For the case $\omega_k = \omega$ (mechanical angular velocity or respectively motor speed) a rotor-orientated equation of the stator voltage can also be derived. However, since there is hardly any advantage to be obtained from this representation we will not follow it further.

2. *Rotor voltage equation*

The transformation rules are applied in similar way to the stator voltage equation.

$$\mathbf{i}_r^r = \mathbf{i}_r^k\, e^{j\vartheta_k}\,,\ \psi_r^r = \psi_r^k\, e^{j\vartheta_k}\,,\ \frac{d\psi_r^r}{dt} = \frac{d\psi_r^k}{dt} e^{j\vartheta_k} + j\omega_k\, \psi_r^k\, e^{j\vartheta_k}$$

After inserting the transformed quantities into equation (3.26) the following result is obtained:

$$\mathbf{0} = R_r\,\mathbf{i}_r^k + \frac{d\psi_r^k}{dt} + j\omega_k\,\psi_r^k \tag{3.34}$$

The equation (3.34) can also be written for the field-orientated and stator-fixed coordinate systems.

- $\omega_k = \omega_s - \omega = \omega_r$: This coordinate system is rotating ahead of the rotor with angular velocity ω_r and coincides with the field synchronous coordinate system. Inserting ω_r into equation (3.34) yields:

$$\mathbf{0} = R_r\,\mathbf{i}_r^f + \frac{d\psi_r^f}{dt} + j\omega_r\,\psi_r^f \tag{3.35}$$

Equation (3.35) represents the rotor voltage in dq-coordinates.

- $\omega_k = -\omega$: Assuming the rotor to rotate with the mechanical angular velocity ω, this coordinate system turns with the same angular velocity in the opposite direction. Therefore, the coordinate system is fixed to the stator and can be chosen to coincide with the $\alpha\beta$-coordinates mentioned above.

$$\mathbf{0} = R_r\,\mathbf{i}_r^s + \frac{d\psi_r^s}{dt} - j\omega\,\psi_r^s \tag{3.36}$$

The equation (3.36) represents the rotor voltage equation in the stator-fixed, $\alpha\beta$-coordinates.

So far the transformation of all voltage equations from their original winding systems into the required dq- or $\alpha\beta$-coordinates is complete. With the equations (3.32), (3.33), (3.35) and (3.36) the starting point to derive the continuous state space models of the IM is reached.

3. *Continuous state space model of the IM in the stator-fixed coordinate system ($\alpha\beta$-coordinates)*

The equations (3.33) and (3.36) are combined into the following equation system:

$$\begin{cases} \mathbf{u}_s^s = R_s\,\mathbf{i}_s^s + \dfrac{d\boldsymbol{\psi}_s^s}{dt} \\ \mathbf{u}_r^s = R_r\,\mathbf{i}_r^s + \dfrac{d\boldsymbol{\psi}_r^s}{dt} - j\omega\,\boldsymbol{\psi}_r^s = \mathbf{0} \\ \boldsymbol{\psi}_s^s = L_s\,\mathbf{i}_s^s + L_m\,\mathbf{i}_r^s \\ \boldsymbol{\psi}_r^s = L_m\,\mathbf{i}_s^s + L_r\,\mathbf{i}_r^s \end{cases} \tag{3.37}$$

Not all electrical quantities in the system (3.37) are actually of interest. These are e.g. the not measurable rotor current $\mathbf{i}_r^s$, or, depending on the viewpoint of the observer, also the stator flux $\boldsymbol{\psi}_s^s$. Therefore these quantities shall be eliminated from the equation system. From the two flux equations it follows:

$$\mathbf{i}_r^s = \frac{1}{L_r}\left(\boldsymbol{\psi}_r^s - L_m\,\mathbf{i}_s^s\right);\;\; \boldsymbol{\psi}_s^s = L_s\,\mathbf{i}_s^s + \frac{L_m}{L_r}\left(\boldsymbol{\psi}_r^s - L_m\,\mathbf{i}_s^s\right)$$

Now $\mathbf{i}_r^s$ and $\boldsymbol{\psi}_s^s$ are substituted into the voltage equations (3.37) to yield:

$$\begin{cases} \mathbf{u}_s^s = R_s\,\mathbf{i}_s^s + \sigma L_s\,\dfrac{d\mathbf{i}_s^s}{dt} + \dfrac{L_m}{L_r}\dfrac{d\boldsymbol{\psi}_r^s}{dt} \\ \mathbf{0} = -\dfrac{L_m}{T_r}\mathbf{i}_s^s + \left(\dfrac{1}{T_r} - j\omega\right)\boldsymbol{\psi}_r^s + \dfrac{d\boldsymbol{\psi}_r^s}{dt} \end{cases} \tag{3.38}$$

With: $\sigma = 1 - L_m^2 \big/ \left(L_s\,L_r\right)$ Total leakage factor

$T_s = L_s / R_s\,;\, T_r = L_r / R_r$ Stator, rotor time constants

After separating the real and imaginary components from (3.38) we finally obtain:

$$\begin{cases} \dfrac{di_{s\alpha}}{dt} = -\left(\dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}\right) i_{s\alpha} + \dfrac{1-\sigma}{\sigma T_r}\psi'_{r\alpha} + \dfrac{1-\sigma}{\sigma}\omega\psi'_{r\beta} + \dfrac{1}{\sigma L_s}u_{s\alpha} \\ \dfrac{di_{s\beta}}{dt} = -\left(\dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}\right) i_{s\beta} - \dfrac{1-\sigma}{\sigma}\omega\psi'_{r\alpha} + \dfrac{1-\sigma}{\sigma T_r}\psi'_{r\beta} + \dfrac{1}{\sigma L_s}u_{s\beta} \\ \dfrac{d\psi'_{r\alpha}}{dt} = \dfrac{1}{T_r}i_{s\alpha} - \dfrac{1}{T_r}\psi'_{r\alpha} - \omega\psi'_{r\beta} \\ \dfrac{d\psi'_{r\beta}}{dt} = \dfrac{1}{T_r}i_{s\beta} + \omega\psi'_{r\alpha} - \dfrac{1}{T_r}\psi'_{r\beta} \end{cases} \tag{3.39}$$

With: $\psi_r^{s/} = \psi_r^s / L_m$ and $\psi_{r\alpha}^/ = \psi_{r\alpha} / L_m ; \psi_{r\beta}^/ = \psi_{r\beta} / L_m$

To get the complete model of the IM the $\alpha\beta$-components of flux and current have to be inserted into the torque equation. The vector $\mathbf{i}_r^s$ is extracted from the last equation of the system (3.37) and substituted into equation (3.28).

$$m_M = \frac{3}{2} z_p \frac{L_m^2}{L_r} \left(\psi_{r\alpha}^/ \, i_{s\beta} - \psi_{r\beta}^/ \, i_{s\alpha} \right) \tag{3.40}$$

The equations (3.39), (3.40) can now be summarized to a complete continuous model of the IM. The figure 3.3 illustrates the block structure of this model.

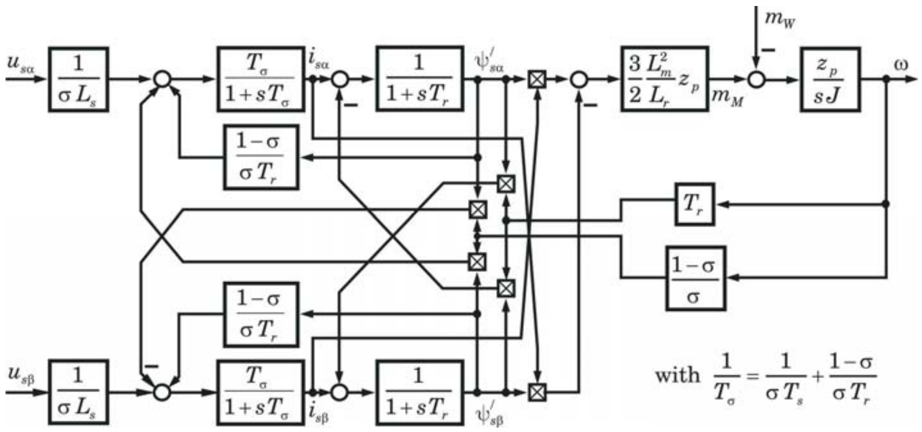

Fig. 3.3 Model of the IM with squirrel-cage rotor in stator-fixed coordinate system

The α- and β-components of stator voltage, stator current and rotor flux may be comprised in the following vectors with real components.

$$\mathbf{x}^{sT} = \left[i_{s\alpha}, i_{s\beta}, \psi_{r\alpha}^/, \psi_{r\beta}^/ \right]; \mathbf{u}_s^{sT} = \left[u_{s\alpha}, u_{s\beta} \right]$$

Superscript index T: Transposed vector

With the newly defined state vector $\mathbf{x}$ the continuous state space model of the IM with squirrel-cage rotor is finally obtained from the equations (3.39).

$$\frac{d\mathbf{x}^s}{dt} = \mathbf{A}^s \, \mathbf{x}^s + \mathbf{B}^s \, \mathbf{u}_s^s \tag{3.41}$$

$\mathbf{A}^s$, $\mathbf{B}^s$: System and input matrix
$\mathbf{x}^s$: State vector in stator-fixed coordinate system
$\mathbf{u}_s^s$: Input vector in stator-fixed coordinate system

The equations (3.42) show in detail the matrices $\mathbf{A}^s$ and $\mathbf{B}^s$ with the machine parameters.

$$\mathbf{A}^s = \left[\begin{array}{cc|cc} -\left(\frac{1}{\sigma T_s}+\frac{1-\sigma}{\sigma T_r}\right) & 0 & \frac{1-\sigma}{\sigma T_r} & \frac{1-\sigma}{\sigma}\omega \\ 0 & -\left(\frac{1}{\sigma T_s}+\frac{1-\sigma}{\sigma T_r}\right) & -\frac{1-\sigma}{\sigma}\omega & \frac{1-\sigma}{\sigma T_r} \\ \hline \frac{1}{T_r} & 0 & -\frac{1}{T_r} & -\omega \\ 0 & \frac{1}{T_r} & \omega & -\frac{1}{T_r} \end{array}\right]; \; \mathbf{B}^s = \left[\begin{array}{cc} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ \hline 0 & 0 \\ 0 & 0 \end{array}\right] \tag{3.42}$$

The equation (3.41) introduces a time-variant state system with the rotor speed ω as a measurable time-variant parameter in the system matrix $\mathbf{A}^s$. This continuous state model of the IM (figure 3.4) forms the basis for the design of discrete controllers in the stator-fixed coordinate system in which the components of the state vector $\mathbf{x}^s$ appear as sinusoidal quantities.

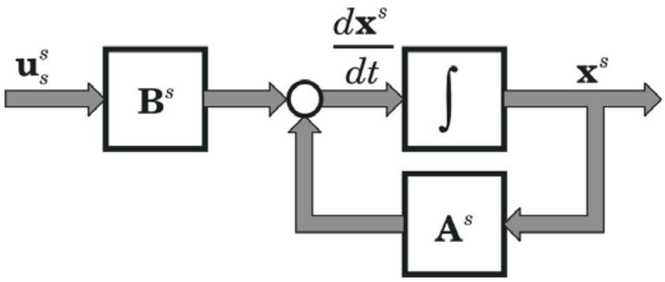

Fig. 3.4 Continuous state model of the IM in stator-fixed $\alpha\beta$-coordinates

4. *Continuous state space model of the IM in the field synchronous or field-orientated coordinate system (dq-coordinates)*:

The equations (3.32), (3.35) are summarized in the following system.

$$\begin{cases} \mathbf{u}_s^f = R_s\,\mathbf{i}_s^f + \dfrac{d\boldsymbol{\psi}_s^f}{dt} + j\omega_s\,\boldsymbol{\psi}_s^f \\ \mathbf{0} = R_r\,\mathbf{i}_r^f + \dfrac{d\boldsymbol{\psi}_r^f}{dt} + j\omega_r\,\boldsymbol{\psi}_r^f \\ \boldsymbol{\psi}_s^f = L_s\,\mathbf{i}_s^f + L_m\,\mathbf{i}_r^f \\ \boldsymbol{\psi}_r^f = L_m\,\mathbf{i}_s^f + L_r\,\mathbf{i}_r^f \end{cases} \tag{3.43}$$

As in the case of the stator-fixed coordinate system the not measurable rotor current as well as the stator flux are eliminated.

$$
\left\{
\begin{aligned}
\frac{di_{sd}}{dt} &= -\left(\frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r}\right) i_{sd} + \omega_s\, i_{sq} + \frac{1-\sigma}{\sigma T_r}\psi'_{rd} + \frac{1-\sigma}{\sigma}\omega\psi'_{rq} + \frac{1}{\sigma L_s}u_{sd} \\
\frac{di_{sq}}{dt} &= -\omega_s\, i_{sd} - \left(\frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r}\right) i_{sq} - \frac{1-\sigma}{\sigma}\omega\psi'_{rd} + \frac{1-\sigma}{\sigma T_r}\psi'_{rq} + \frac{1}{\sigma L_s}u_{sq} \\
\frac{d\psi'_{rd}}{dt} &= \frac{1}{T_r} i_{sd} - \frac{1}{T_r}\psi'_{rd} + \left(\omega_s - \omega\right)\psi'_{rq} \\
\frac{d\psi'_{rq}}{dt} &= \frac{1}{T_r} i_{sq} - \left(\omega_s - \omega\right)\psi'_{rd} - \frac{1}{T_r}\psi'_{rq}
\end{aligned}
\right.
\tag{3.44}
$$

Here are: $\psi'_{rd} = \psi_{rd}/L_m$; $\psi'_{rq} = \psi_{rq}/L_m$; $\omega_s - \omega = \omega_r$

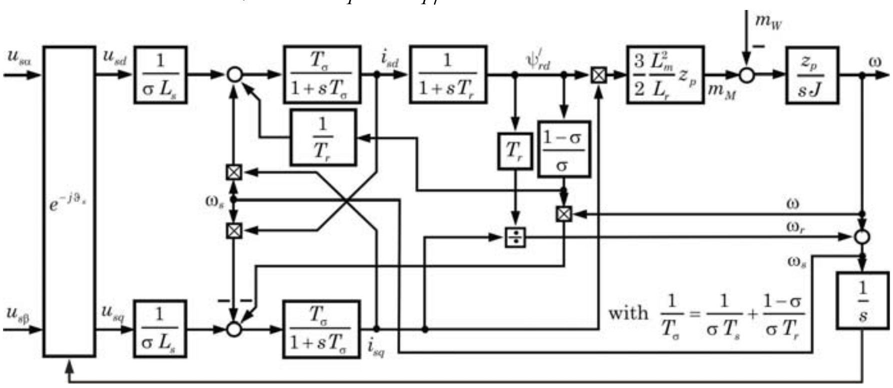

Fig. 3.5 Model of the IM with squirrel-cage rotor in field synchronous coordinate system

After extraction of $\mathbf{i}_r^f$ from equation (3.27), substituting into (3.28) or (3.29) and setting ψ_{rq} to zero due to fixing of the rotor flux vector to the real axis of the coordinate system, the equation (3.45) for the torque is arrived at:

$$
m_M = \frac{3}{2} z_p \frac{L_m^2}{L_r} \psi'_{rd}\, i_{sq} = \frac{3}{2} z_p \left(1-\sigma\right) L_s\, \psi'_{rd}\, i_{sq} \tag{3.45}
$$

The equations (3.44) and (3.45) together form the complete, continuous model of the IM like shown in figure 3.5. The equation system (3.44) can be condensed into the following state space model:

$$
\frac{d\mathbf{x}^f}{dt} = \mathbf{A}^f\, \mathbf{x}^f + \mathbf{B}^f\, \mathbf{u}_s^f + \mathbf{N}\,\mathbf{x}^f\, \omega_s \tag{3.46}
$$

with the state vector $\mathbf{x}_f$, the input vector $\mathbf{u}_s^f$:

$$\mathbf{x}^{fT} = \left[i_{sd}, i_{sq}, \psi'_{rd}, \psi'_{rq} \right]; \mathbf{u}_s^{fT} = \left[u_{sd}, u_{sq} \right]$$

the system matrix $\mathbf{A}^f$, the input matrix $\mathbf{B}^f$ and the nonlinear coupling matrix $\mathbf{N}$:

$$\mathbf{A}^f = \left[\begin{array}{cc|cc} -\left(\frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r} \right) & 0 & \frac{1-\sigma}{\sigma T_r} & \frac{1-\sigma}{\sigma}\omega \\ 0 & -\left(\frac{1}{\sigma T_s} + \frac{1-\sigma}{\sigma T_r} \right) & -\frac{1-\sigma}{\sigma}\omega & \frac{1-\sigma}{\sigma T_r} \\ \hline \frac{1}{T_r} & 0 & -\frac{1}{T_r} & -\omega \\ 0 & \frac{1}{T_r} & \omega & -\frac{1}{T_r} \end{array} \right] \tag{3.47}$$

$$\mathbf{B}^f = \left[\begin{array}{cc} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ \hline 0 & 0 \\ 0 & 0 \end{array} \right]; \ \mathbf{N} = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{array} \right]$$

The state equation (3.46) with the matrices (3.47) obviously points to *a bilinear characteristic* (refer to section 3.1.1, equation (3.4)). Here the field synchronous components u_{sd}, u_{sq} of the stator voltage and the angular velocity ω_s of the stator circuit represent the input quantities. The mechanical angular velocity ω in the system matrix $\mathbf{A}^f$ is regarded as a measurable variable system parameter. The only formal difference between the two continuous state models (3.41) and (3.46) is the nonlinear term with the matrix $\mathbf{N}$. The other matrices of both models are identical. The figure 3.6 illustrates the derived state model.

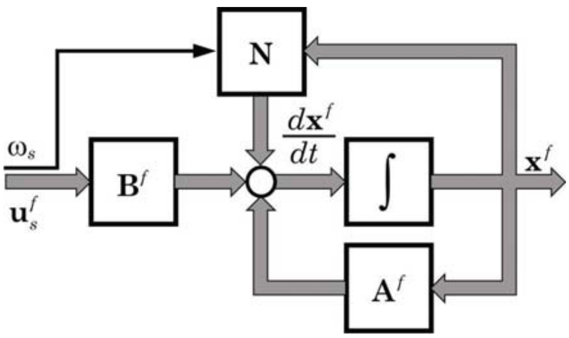

Fig. 3.6 Continuous state model of the IM in field synchronous *dq*-coordinates

So far the basic prerequisites for the further work are completed. However, for the controller design the continuous models are not particularly suitable. The microcomputer works discretely and processes only the motor quantities measured at discrete instants. A discrete model of the motor corresponding to this reality is therefore necessary for the controller design. The development of the discrete models is subject of the following section. It is useful to derive the models in the field synchronous as well as in the stator-fixed coordinate system because in practice control methods are developed in both coordinate systems.

3.2.2 Discrete state space models of the IM

Depending on the choice of the control coordinate system, the starting point for the derivation of a discrete state model for the IM is given by one of the two continuous state models (3.41) or (3.46).

In principle the discretization of the continuous model is relatively simple for linear and time-invariant systems. This presumption is fulfilled to a large degree if the IM model in the stator-fixed coordinate system is used and one assumes that the electrical transient processes settle essentially faster than the mechanical ones. Thus the system matrix $\mathbf{A}^s$ or the stator-fixed system (3.41) can be considered as virtually time-invariant within one sampling period of the current control. The mechanical angular velocity ω of the rotor can be regarded as a slowly variable parameter and is measured by a resolver or an incremental encoder.

This condition however is no longer fulfilled if the system is processed in the field synchronous coordinate system. The system model (3.46) indicates a *bilinear characteristic* additionally, the stator frequency ω_s consisting of the mechanical speed ω and the load ω_r leads to a time-variant system, further complicating the derivation of the required model. But under the prerequisite that the input quantities u_{sd}, u_{sq} and ω_s are constant within one sampling period T the discretization of this *bilinear and time-variant system* becomes feasible. The result is *a time-variant however linear system* which allows the application of a similar design methodology as for linear systems, like in stator-fixed coordinates. The demanded prerequisite is largely fulfilled for modern drive systems with sampling periods below 500µs. The pulsed stator voltage is processed as mean average over one period, and therefore also regarded constant in T.

1. *Discrete state model in the stator-fixed coordinate system*

After integrating the equation (3.41) (refer to equations (3.10) to (3.14)) the following equivalent discrete state model of the IM is obtained.

$$\mathbf{x}^s(k+1) = \mathbf{\Phi}^s\,\mathbf{x}^s(k) + \mathbf{H}^s\,\mathbf{u}_s^s(k) \tag{3.48}$$

$$\mathbf{\Phi}^s = \mathrm{e}^{\mathbf{A}^s T} = \sum_{\nu=0}^{\infty}\left(\mathbf{A}^s\right)^{\nu}\frac{T^{\nu}}{\nu!};\ \mathbf{H}^s = \int_{kT}^{(k+1)T}\mathrm{e}^{\mathbf{A}^s\tau}\,d\tau\,\mathbf{B}^s = \sum_{\nu=1}^{\infty}\left(\mathbf{A}^s\right)^{\nu-1}\frac{T^{\nu}}{\nu!}\mathbf{B}^s \tag{3.49}$$

The input vector $\mathbf{u}_s^s$ (k) is given by the microcontroller and therefore has step-shaped components. The transition matrix $\mathbf{\Phi}_s$ and the input matrix $\mathbf{H}_s$ depend on the sampling period T and the mechanical angular velocity ω. The two matrices can be derived from the matrix exponential function $\mathrm{e}^{\mathbf{A}T}$, which may be developed into a series expansion like in (3.49). But for the practical application a further simplification would be very helpful and wished for. Here the consideration may help that the discrete model to be developed is not intended for mathematical simulation of the IM, but to serve the design of the discrete controller. For this purpose the series expansion may be truncated at an early stage if the inaccuracy hereby produced is compensated by appropriate control means, e.g. by an implicit integral part in the control algorithms.

The practical experience shows that an approximation of first order for $\mathbf{\Phi}_s$ and $\mathbf{H}_s$ suffices completely for small sampling times (under 500µs). An approximation of higher order would increase the needed computation power unnecessarily. A special issue is the investigation of the stability of such discrete systems. It shall only be mentioned at this place that the stability very strongly depends on the sampling time T. The smaller the sampling time T, the larger becomes the stability area and thus also the utilizable speed range. Therefore a compromise must be found between decreasing the sampling time and increasing the stability area as well as the speed range, and the acceptable computation power or the computing time. The following formulae (3.50) show the approximation of first order for transition and input matrix.

The representation of the discrete state models with partial matrices (figure 3.7) gives a good insight into the inner physical structure of the IM.

$$
\mathbf{\Phi}^s = \left[\begin{array}{cc|cc}
1-\dfrac{T}{\sigma}\left(\dfrac{1}{T_s}+\dfrac{1-\sigma}{T_r}\right) & 0 & \dfrac{1-\sigma}{\sigma}\dfrac{T}{T_r} & \dfrac{1-\sigma}{\sigma}\omega T \\
0 & 1-\dfrac{T}{\sigma}\left(\dfrac{1}{T_s}+\dfrac{1-\sigma}{T_r}\right) & -\dfrac{1-\sigma}{\sigma}\omega T & \dfrac{1-\sigma}{\sigma}\dfrac{T}{T_r} \\
\hline
\dfrac{T}{T_r} & 0 & 1-\dfrac{T}{T_r} & -\omega T \\
0 & \dfrac{T}{T_r} & \omega T & 1-\dfrac{T}{T_r}
\end{array}\right] = \left[\begin{array}{c|c} \mathbf{\Phi}_{11}^s & \mathbf{\Phi}_{12}^s \\ \hline \mathbf{\Phi}_{21}^s & \mathbf{\Phi}_{22}^s \end{array}\right]
$$

$$
\mathbf{H}^s = \left[\begin{array}{cc}
\dfrac{T}{\sigma L_s} & 0 \\
0 & \dfrac{T}{\sigma L_s} \\
\hline
0 & 0 \\
0 & 0
\end{array}\right] = \left[\begin{array}{c} \mathbf{H}_1^s \\ \hline \mathbf{H}_2^s \end{array}\right] \tag{3.50}
$$

The equation (3.48) can be written in detail as follows:

$$
\begin{cases}
\mathbf{i}_s^s(k+1) = \mathbf{\Phi}_{11}^s \mathbf{i}_s^s(k) + \mathbf{\Phi}_{12}^s \psi_r^{s/}(k) + \mathbf{H}_1^s \mathbf{u}_s^s(k) \\
\psi_r^{s/}(k+1) = \mathbf{\Phi}_{21}^s \mathbf{i}_s^s(k) + \mathbf{\Phi}_{22}^s \psi_r^{s/}(k)
\end{cases} \tag{3.51}
$$

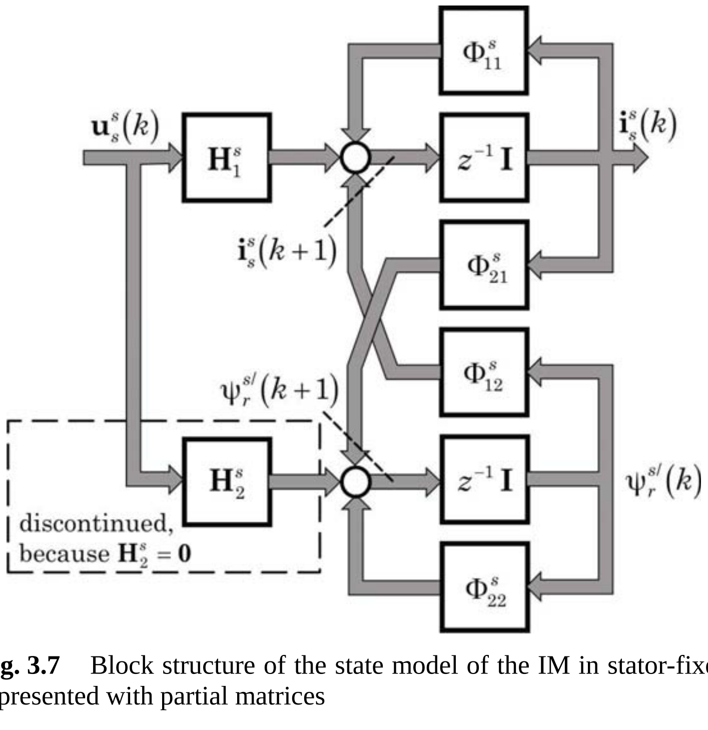

Fig. 3.7 Block structure of the state model of the IM in stator-fixed coordinates represented with partial matrices

With the separation of the complete model (3.48) into two submodels (3.51) a favourable starting point arises for the practical controller design. The first equation of (3.51) represents the current process model of the IM. The system has two input vectors: The stator voltage $\mathbf{u}_s^s(k)$ and the slowly variable rotor flux $\psi_r^{s/}(k)$ (figure 3.8a).

In the chapter 5 it will be worked out, that the slowly variable rotor flux can be understood as a disturbance variable and therefore can be eliminated separately by a disturbance feed-forward compensation. The rotor flux is not measurable, it must be estimated. For this purpose the second equation of (3.51) may be used and is for this reason designated as i-ω flux model (figure 3.8b). From the measured currents and speed the rotor flux can be calculated using this model.

The special issue of the flux estimation has been treated in some detail in earlier works. Besides this simple flux model, different flux observers have been proposed for flux estimation (refer to section 4.4). The rotor flux estimates are used:

- to calculate the slip frequency ω_r or the field angle ϑ_s for the field orientation, and
- as actual flux values for the flux controller.

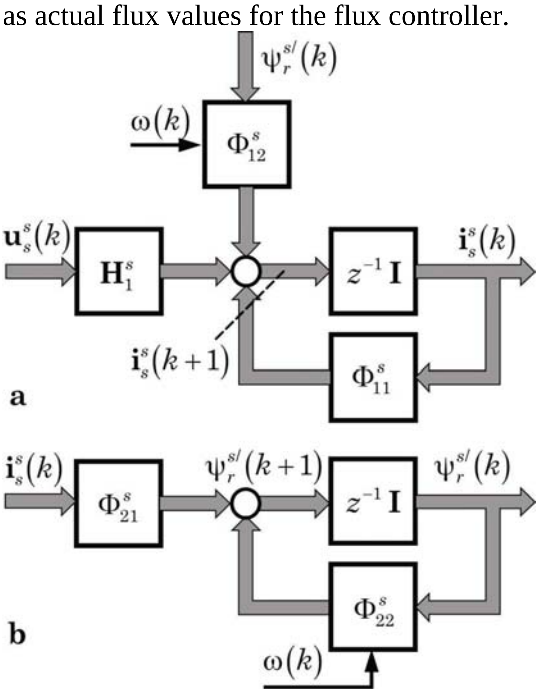

Fig. 3.8 Structure of the current process model (**a**) and the i-ω flux model (**b**) of the IM in stator-fixed coordinates

The structures in figure 3.8 have been derived by splitting-up the structure in the figure 3.7 with $\mathbf{H}_2^s = \mathbf{0}$.

2. *Discrete state model in the field synchronous coordinate system*

The derivation of a discrete state model or the discretization of the continuous bilinear state model (3.46) is carried out under the prerequisite that the input components u_{sd}, u_{sq} and ω_s are constant within a sampling period T. It was already indicated in the introduction of this section that this demand can be looked-at as largely fulfilled for modern three-phase AC drives with PWM inverters due to their high sampling and pulse frequencies.

After iterative integration of the equation (3.46) the following equivalent discrete state model of the IM is obtained.

$$\mathbf{x}^f(k+1) = \mathbf{\Phi}^f \mathbf{x}^f(k) + \mathbf{H}^f \mathbf{u}_s^f(k) \tag{3.52}$$

$$\begin{aligned} \mathbf{\Phi}^f &= \mathrm{e}^{\left[\mathbf{A}^f + \mathbf{N}\omega_s(k)\right]T} = \sum_{\nu=0}^{\infty}\left[\mathbf{A}^f + \mathbf{N}\omega_s(k)\right]^{\nu}\frac{T^{\nu}}{\nu!} \\ \mathbf{H}^f &= \int_{kT}^{(k+1)T} \mathrm{e}^{\left[\mathbf{A}^f + \mathbf{N}\omega_s(k)\right]\tau} d\tau\, \mathbf{B}^f = \sum_{\nu=1}^{\infty}\left[\mathbf{A}^f + \mathbf{N}\omega_s(k)\right]^{\nu-1}\frac{T^{\nu}}{\nu!}\mathbf{B}^f \end{aligned} \tag{3.53}$$

The discrete model (3.52) is a time-variant, however linear model, unlike the continuous one. The elements of the transition matrix $\mathbf{\Phi}^f(k)$ and the input matrix $\mathbf{H}^f(k)$ are calculated on-line. Like for the discrete state model in the stator-fixed system, useable formulae are obtained by first-order approximation of the series expansions of the exponential functions in (3.53).

$$\mathbf{\Phi}^f = \left[\begin{array}{cc|cc} 1-\dfrac{T}{\sigma}\left(\dfrac{1}{T_s}+\dfrac{1-\sigma}{T_r}\right) & \omega_s T & \dfrac{1-\sigma}{\sigma}\dfrac{T}{T_r} & \dfrac{1-\sigma}{\sigma}\omega T \\ -\omega_s T & 1-\dfrac{T}{\sigma}\left(\dfrac{1}{T_s}+\dfrac{1-\sigma}{T_r}\right) & -\dfrac{1-\sigma}{\sigma}\omega T & \dfrac{1-\sigma}{\sigma}\dfrac{T}{T_r} \\ \hline \dfrac{T}{T_r} & 0 & 1-\dfrac{T}{T_r} & (\omega_s-\omega)T \\ 0 & \dfrac{T}{T_r} & -(\omega_s-\omega)T & 1-\dfrac{T}{T_r} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{\Phi}_{11}^f & \mathbf{\Phi}_{12}^f \\ \hline \mathbf{\Phi}_{21}^f & \mathbf{\Phi}_{22}^f \end{array}\right]$$

$$\mathbf{H}^f = \left[\begin{array}{cc} \dfrac{T}{\sigma L_s} & 0 \\ 0 & \dfrac{T}{\sigma L_s} \\ \hline 0 & 0 \\ 0 & 0 \end{array}\right] = \left[\begin{array}{c} \mathbf{H}_1^f \\ \hline \mathbf{H}_2^f \end{array}\right] \tag{3.54}$$

After rewriting the discrete state model (3.52) in the form with partial matrices:

$$\begin{cases} \mathbf{i}_s^f(k+1) = \mathbf{\Phi}_{11}^f \mathbf{i}_s^f(k) + \mathbf{\Phi}_{12}^f \psi_r^{f\prime}(k) + \mathbf{H}_1^f \mathbf{u}_s^f(k) \\ \psi_r^{f\prime}(k+1) = \mathbf{\Phi}_{21}^f \mathbf{i}_s^f(k) + \mathbf{\Phi}_{22}^f \psi_r^{f\prime}(k) \end{cases} \tag{3.55}$$

and considering, that $\mathbf{H}_2^f$ is a zero matrix, the current process model of the IM and the *i*-ω flux model for the field synchronous coordinate system are obtained like in equations (3.55) and in figure 3.10. The formal similarity of the two discrete state models of the IM, which is recognizable from the equations and from the pictures, can surely be noticed in the stator-fixed as well as in the field synchronous coordinate system. This formal similarity permits a generalization of both cases and their later summarizing into a common controller design.

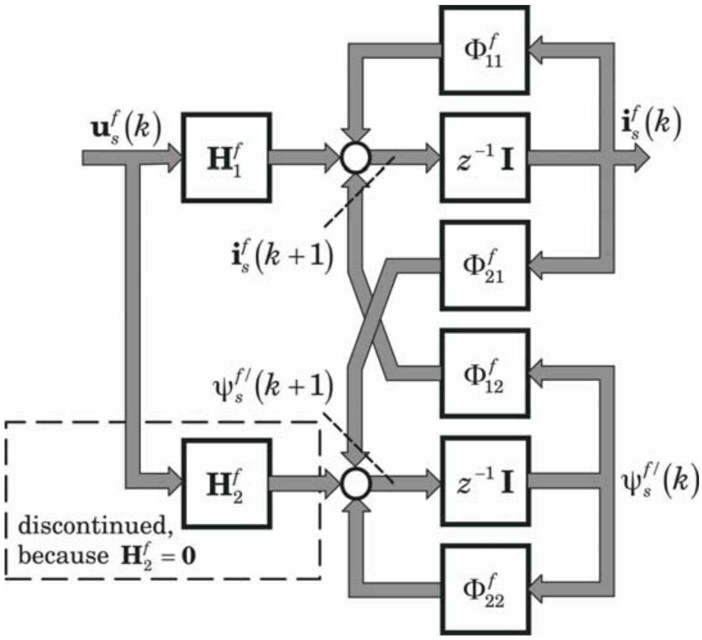

Fig. 3.9 Block structure of the state model of the IM in field synchronous coordinates represented with partial matrices

A decisive difference between the two models can be found in the appearance of ω_s in the transition matrix $\mathbf{\Phi}^f$, expressing the coupling between the two current components i_{sd} and i_{sq}. This coupling, as already mentioned in chapter 1, cannot be removed effectively, e.g. by using a decoupling network like indicated in the classical control structure in figure 1.4. This becomes particularly evident if the system is operated constantly with strong field weakening.

The discrete state model (3.52) of the IM in the *dq*-coordinate system was derived by discretization of the continuous model (3.46), which in turn was obtained by transformation from the original $\alpha\beta$-coordinate system into the *dq*-coordinate system, i.e. *the transformation took place before the discretization*.

Another order also may be chosen alternatively: *Discretization before transformation*; i.e. the discrete *dq*-model results from the coordinate transformation of the discrete $\alpha\beta$-model (3.48) (refer to sections 3.1.2 and 12.2). *This way complex eigenvalues of the system matrix or instabilities caused by discretization can be avoided*. In the result a discrete state model is obtained, which provides a larger stable working range for the controller. The parameters of the transition matrix $\mathbf{\Phi}$ will contain sin/cos functions of $\omega_s T$ (e.g. $\omega_s T \rightarrow \sin(\omega_s T)$). Especially for high-speed drives this procedure may yield significant advantages.

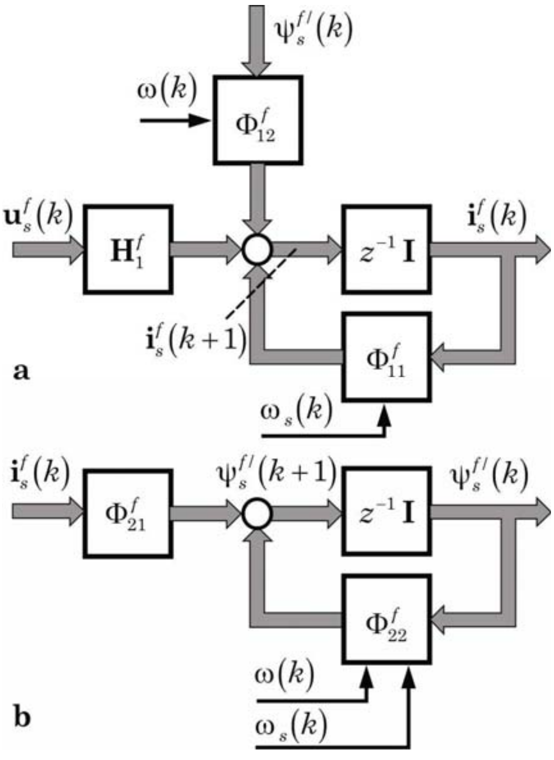

Fig. 3.10 Structure of the current process model (a) and of the i-ω flux model (b) of the IM in field synchronous coordinates

3.3 Permanent magnet excited synchronous machine (PMSM)

Unlike the IM the PMSM has a permanent and constant rotor flux (also pole flux) with a certain preferred axis. With a simultaneous use of a position sensor (resolver, incremental encoder with zero pulse) the pole position can always be clearly identified, and field orientation (also pole flux orientation) is always ensured. For this reason the system design in the stator-fixed coordinate system will be abstained from, and the field synchronous coordinate system will be immediately chosen for the treatment of the machine.

3.3.1 Continuous state space model of the PMSM in the field synchronous coordinate system

The equation (3.25) is the general stator voltage equation of three-phase AC machines, and valid also for the PMSM. A coordinate system rotating with ω or ω_s is conceivable whose axes are the d and q axis. For the PMSM ω and ω_s are identical which means, that the coordinate system rotates not only field synchronously, but is also fixed to the rotor. If the coordinate system is chosen to match the real d-axis with the preferred axis of the pole flux, this coordinate system represents the desired field or pole flux orientation. If the equation (3.25) of the PMSM (in a similar way as in the case of the IM) is transformed from the three winding system of the stator into the field synchronous system, we obtain:

$$\mathbf{u}_s^f = R_s \, \mathbf{i}_s^f + \frac{d\boldsymbol{\psi}_s^f}{dt} + j\omega_s \, \boldsymbol{\psi}_s^f \tag{3.56}$$

For the flux the following equation holds:

$$\boldsymbol{\psi}_s^f = L_s \, \mathbf{i}_s^f + \boldsymbol{\psi}_p^f \tag{3.57}$$

Here $\boldsymbol{\psi}_p^f$ is the vector of the pole flux. Because the real axis of the coordinate system is directly orientated to the preferred axis of the pole flux, the quadrature component of $\boldsymbol{\psi}_p^f$ is zero. Therefore the pole flux vector has only the real direct component ψ_p. From that follows:

$$\boldsymbol{\psi}_p^f = \psi_{pd} + j\psi_{pq} = \psi_p \text{ with } \psi_{pq} = 0 \tag{3.58}$$

In addition it has to be taken into account that due to the construction dependent pole gaps on the rotor surface, the stator inductance assumes different values L_{sd}, L_{sq} in the real and quadrature axis, respectively. For

PMSM with cylindrical (non-salient) rotor both inductances are nearly identical and therefore usually equalized in classical control structures. The difference is not pronounced unlike in the case of salient-pole machines and to amounts approx. 3...12%. To obtain an effective decoupling between the current components i_{sd} and i_{sq}, this difference should be and will be taken into account in the following. Application to the stator flux equations thus yields:

$$\begin{cases} \psi_{sd} = L_{sd}\, i_{sd} + \psi_p \\ \psi_{sq} = L_{sq}\, i_{sq} \end{cases} \tag{3.59}$$

Substituting equations (3.57), (3.59) into the equation (3.56) then yields:

$$\begin{cases} u_{sd} = R_s\, i_{sd} + L_{sd} \dfrac{di_{sd}}{dt} - \omega_s L_{sq}\, i_{sq} \\ u_{sq} = R_s\, i_{sq} + L_{sq} \dfrac{di_{sq}}{dt} + \omega_s L_{sd}\, i_{sd} + \omega_s\, \psi_p \end{cases} \tag{3.60}$$

From the general torque equation (3.28) or (3.29) of three-phase AC machines we obtain:

$$m_M = \frac{3}{2} z_p \left(\psi_{sd}\, i_{sq} - \psi_{sq}\, i_{sd} \right) \tag{3.61}$$

After inserting (3.59) into (3.61) the following torque equation results:

$$m_M = \frac{3}{2} z_p \left[\psi_p\, i_{sq} + i_{sd}\, i_{sq} \left(L_{sd} - L_{sq} \right) \right] \tag{3.62}$$

The torque of the PMSM consists of two parts: the main and the reaction torque. With pole flux orientated control of the PMSM the stator current usually will be controlled to obtain a right angle between stator current and pole flux (i_{sd} = 0) and therefore not to contribute to magnetization, but only to torque production. Therefore a similar equation as (3.45) for the IM can be obtained:

$$m_M = \frac{3}{2} z_p\, \psi_p\, i_{sq} \tag{3.63}$$

Now equation (3.60) will be rewritten as follows:

$$\begin{cases} \dfrac{di_{sd}}{dt} = -\dfrac{1}{T_{sd}} i_{sd} + \omega_s \dfrac{L_{sq}}{L_{sd}} i_{sq} + \dfrac{1}{L_{sd}} u_{sd} \\ \dfrac{di_{sq}}{dt} = -\omega_s \dfrac{L_{sd}}{L_{sq}} i_{sd} - \dfrac{1}{T_{sq}} i_{sq} + \dfrac{1}{L_{sq}} u_{sq} - \omega_s \dfrac{\psi_p}{L_{sq}} \end{cases} \tag{3.64}$$

The PMSM is completely described by equations (3.62) and (3.64) in field synchronous coordinates (figure 3.11). The equations (3.62), (3.64) are summarized to the following state space model.

$$\frac{d\mathbf{i}_s^f}{dt} = \mathbf{A}_{SM}^f\,\mathbf{i}_s^f + \mathbf{B}_{SM}^f\,\mathbf{u}_s^f + \mathbf{N}_{SM}\,\mathbf{i}_s^f\,\omega_s + \mathbf{S}\,\psi_p\,\omega_s \tag{3.65}$$

$$\mathbf{A}_{SM}^f = \begin{bmatrix} -\frac{1}{T_{sd}} & 0 \\ 0 & -\frac{1}{T_{sq}} \end{bmatrix}; \mathbf{B}_{SM}^f = \begin{bmatrix} \frac{1}{L_{sd}} & 0 \\ 0 & \frac{1}{L_{sq}} \end{bmatrix}; \mathbf{N}_{SM} = \begin{bmatrix} 0 & \frac{L_{sq}}{L_{sd}} \\ -\frac{L_{sd}}{L_{sq}} & 0 \end{bmatrix}; \mathbf{S} = \begin{bmatrix} 0 \\ -\frac{1}{L_{sq}} \end{bmatrix} \tag{3.66}$$

$\mathbf{A}_{SM}^f =$	System matrix;	$\mathbf{S}$ =	Disturbance vector
$\mathbf{B}_{SM}^f =$	Input matrix;	$T_{sd} = L_{sd}/R_s =$	Time constant of d axis
$\mathbf{N}_{SM} =$	Nonlinear coupling matrix;	$T_{sq} = L_{sq}/R_s =$	Time constant of q axis

The figure 3.12 illustrates the model (3.65) of the PMSM.

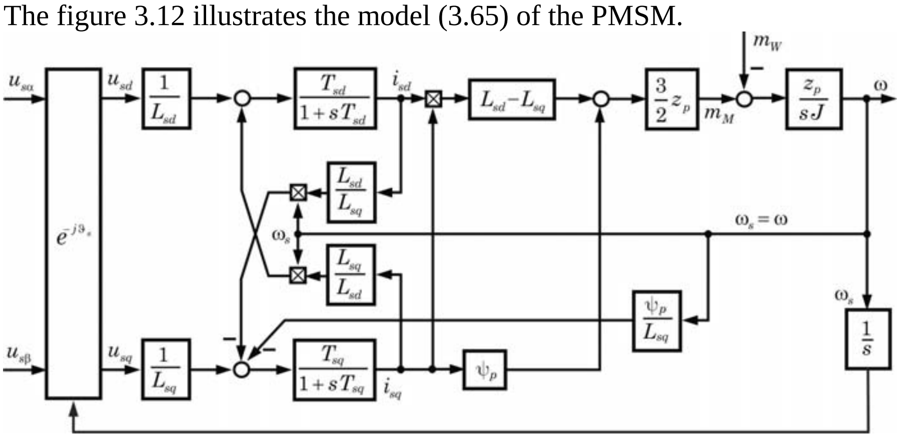

Fig. 3.11 Model of the PMSM in field synchronous or pole flux orientated coordinate system

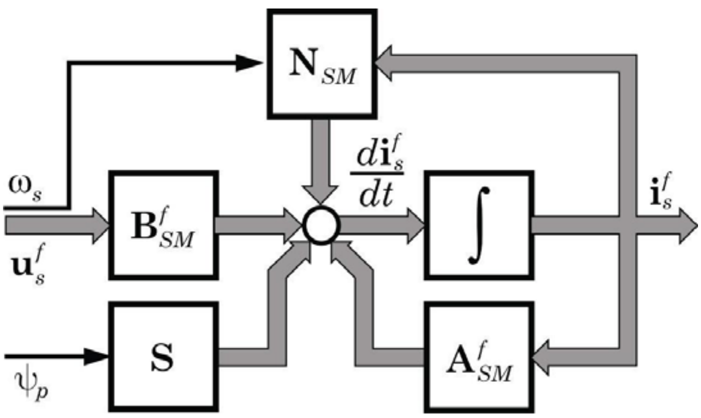

Fig. 3.12 Continuous state model of the PMSM in field synchronous coordinates

The bilinear characteristic of the model is recognizable like in the case of the IM by the matrix $\mathbf{N}_{SM}$. The disturbance, acting on the system through the pole flux ψ_p, does not depend on the stator current but is constant unlike for the IM. The constant excitation shows some advantages for the further treatment:

- The system model is a model of 2nd order (i_{sd}, i_{sq}) – the IM has a model of 4th order (i_{sd}, i_{sq}, ψ_{rd}, ψ_{rq} or $i_{s\alpha}$, $i_{s\beta}$, $\psi_{r\alpha}$, $\psi_{r\beta}$) – and immediately yields the current control process model. For the IM the system of 4th order must be split into partial models to obtain the current process model and the flux model.
- The constant flux ψ_p may be regarded as a system parameter.
- The constant disturbance ψ_p is documented by the machine manufacturer and can, similarly as for the IM, later be separately compensated by a disturbance feed-forward.

3.3.2 Discrete state model of the PMSM

To show clearly that the pole flux ψ_p represents only a constant disturbance variable, ψ_p was introduced into the system by a separate term through the disturbance vector in equation (3.65) and in figure 3.12. However, a discretization of the model is hardly possible in this form. To advance the situation, ψ_p will be viewed as a constant system parameter. The equation (3.65) must be rewritten as follows:

$$\frac{d\mathbf{i}_s^f}{dt} = \mathbf{A}_{SM}^f \, \mathbf{i}_s^f + \mathbf{B}_{SM}^{f*} \, \mathbf{v}^f + \mathbf{N}_{SM} \, \mathbf{i}_s^f \omega_s \tag{3.67}$$

with:

$$\mathbf{v}^{fT} = \left[u_{sd}, u_{sq}, \omega_s\right]; \; \mathbf{B}_{SM}^{f*} = \left[\mathbf{B}_{SM}^f, \mathbf{S}\psi_p\right] \tag{3.68}$$

With (3.67) the formal, complete identity with (3.46) has been achieved, allowing to treat the PMSM in the same way as the IM to derive its discrete state space description.

Also in this case the state space description (3.67) is characterized by a *bilinear characteristic* because of the multiplicative combination between the state vector $\mathbf{i}_s$ and the element ω_s of the input vector $\mathbf{v}^f$. Under the same assumption regarding the input quantities as in the case of the IM, and after an iterative integration of (3.67) the following equivalent, discrete state model of the permanent magnet excited synchronous machine is obtained.

$$\mathbf{i}_s^f\left(k+1\right)=\mathbf{\Phi}_{SM}^f\,\mathbf{i}_s^f\left(k\right)+\mathbf{H}_{SM}^{f*}\,\mathbf{v}^f\left(k\right) \tag{3.69}$$

There are:

$$\mathbf{\Phi}_{SM}^f=\mathrm{e}^{\left[\mathbf{A}_{SM}^f+\mathbf{N}_{SM}\,\omega_s(k)\right]T}=\sum_{\nu=0}^{\infty}\left[\mathbf{A}_{SM}^f+\mathbf{N}_{SM}\,\omega_s\left(k\right)\right]^{\nu}\frac{T^{\nu}}{\nu!}$$

$$\mathbf{H}_{SM}^{f*}=\int_{kT}^{(k+1)T}\mathrm{e}^{\left[\mathbf{A}_{SM}^f+\mathbf{N}_{SM}\,\omega_s(k)\right]\tau}\,d\tau\,\mathbf{B}_{SM}^{f*}=\sum_{\nu=1}^{\infty}\left[\mathbf{A}_{SM}^f+\mathbf{N}_{SM}\,\omega_s\left(k\right)\right]^{\nu-1}\frac{T^{\nu}}{\nu!}\mathbf{B}_{SM}^{f*} \tag{3.70}$$

The approximation of first order for the transition matrix $\mathbf{\Phi}_{SM}^f$ and the input matrix $\mathbf{H}_{SM}^{f*}$ arise from the series expansion (3.70):

$$\mathbf{\Phi}_{SM}^f=\begin{bmatrix}1-\dfrac{T}{T_{sd}} & \omega_s T\dfrac{L_{sq}}{L_{sd}}\\ -\omega_s T\dfrac{L_{sd}}{L_{sq}} & 1-\dfrac{T}{T_{sq}}\end{bmatrix};\ \mathbf{H}_{SM}^{f*}=\begin{bmatrix}\dfrac{T}{L_{sd}} & 0 & 0\\ 0 & \dfrac{T}{L_{sq}} & -\dfrac{\psi_p T}{L_{sq}}\end{bmatrix} \tag{3.71}$$

The discrete state model (3.69) simultaneously represents the expected current control system of the PMSM. The input matrix $\mathbf{H}_{SM}^{f*}$ can be split up as follows:

$$\mathbf{H}_{SM}^{f*}=\left[\mathbf{H}_{SM}^f\,,\mathbf{h}\right]\text{ with }\mathbf{H}_{SM}^f=\begin{bmatrix}\dfrac{T}{L_{sd}} & 0\\ 0 & \dfrac{T}{L_{sq}}\end{bmatrix};\mathbf{h}=\begin{bmatrix}0\\ -\dfrac{\omega_s T}{L_{sq}}\end{bmatrix} \tag{3.72}$$

The figure 3.13 shows the discrete state model or the current process model of the PMSM arrived at so far. The splitting of $\mathbf{H}_{SM}^{f*}$ in equation (3.72) into two partial matrices is necessary, because:

1. Identical structures of the current process model are obtained in both cases. This commonality later allows the summarized treatment of the current control problem for both machine types and spares a repeated representation of similar designs.
2. It is necessary to later invert the input matrix for the compensation of the disturbance quantity ψ_p. This would not be possible if $\mathbf{H}_{SM}^{f*}$ keeps the form of a 3×2 matrix.

With that the final equation of the discrete state model or the current process model of the PMSM is obtained as:

$$\mathbf{i}_s^f(k+1) = \mathbf{\Phi}_{SM}^f\,\mathbf{i}_s^f(k) + \mathbf{H}_{SM}^f\,\mathbf{u}_s^f(k) + \mathbf{h}\,\psi_p \tag{3.73}$$

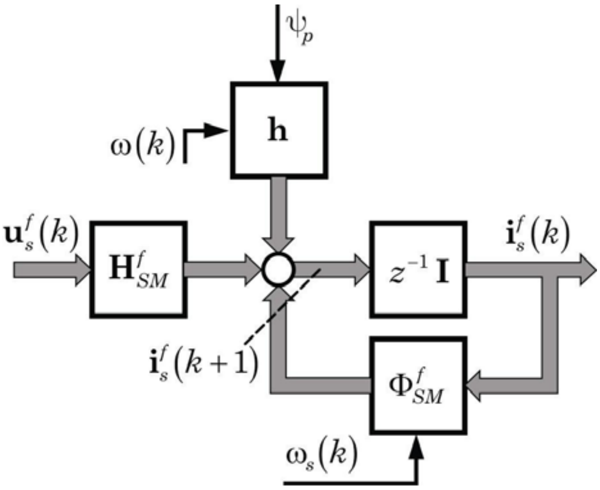

Fig. 3.13 Discrete state model and current process model of the PMSM

3.4 Doubly-fed induction machine (DFIM)

3.4.1 Continuous state space model of the DFIM in the grid synchronous coordinate system

Starting-point for the derivation of the state space model of the DFIM are the voltage equations for stator and rotor winding respectively:

- *Stator voltage in the stator winding system*:

$$\mathbf{u}_s^s = R_s\,\mathbf{i}_s^s + \frac{d\boldsymbol{\psi}_s^s}{dt} \tag{3.74}$$

- *Rotor voltage in the rotor winding system*:

$$\mathbf{u}_r^r = R_r\,\mathbf{i}_r^r + \frac{d\boldsymbol{\psi}_r^r}{dt} \tag{3.75}$$

- *Stator and rotor flux*:

$$\begin{cases} \boldsymbol{\psi}_s = L_s\,\mathbf{i}_s + L_m\,\mathbf{i}_r \\ \boldsymbol{\psi}_r = L_m\,\mathbf{i}_s + L_r\,\mathbf{i}_r \end{cases} \text{with} \begin{cases} L_s = L_m + L_{\sigma s} \\ L_r = L_m + L_{\sigma r} \end{cases} \tag{3.76}$$

All symbols in the formulae (3.74), (3.75) and (3.76) have the same meaning as in the section 3.2.1.

After transforming equations (3.74) and (3.75) to a reference frame rotating with the stator frequency ω_s the following equation is obtained:

$$\begin{cases} \mathbf{u}_s = R_s\mathbf{i}_s + \dfrac{d\boldsymbol{\psi}_s}{dt} + j\omega_s\boldsymbol{\psi}_s \\ \mathbf{u}_r = R_r\mathbf{i}_r + \dfrac{d\boldsymbol{\psi}_r}{dt} + j\omega_r\boldsymbol{\psi}_r \end{cases} \tag{3.77}$$

Eliminating of stator current $\mathbf{i}_s$ and rotor flux $\boldsymbol{\psi}_r$ from equation (3.77) gives:

$$\begin{cases} \dfrac{d\mathbf{i}_r}{dt} = -\dfrac{1}{\sigma}\left(\dfrac{1}{T_r} + \dfrac{1-\sigma}{T_s}\right)\mathbf{i}_r - j\omega_r\mathbf{i}_r + \dfrac{1-\sigma}{\sigma}\left(\dfrac{1}{T_s} + j\omega\right)\boldsymbol{\psi}_s' \\ \qquad + \dfrac{1}{\sigma L_r}\mathbf{u}_r - \dfrac{1-\sigma}{\sigma L_m}\mathbf{u}_s \\ \dfrac{d\boldsymbol{\psi}_s'}{dt} = \dfrac{1}{T_s}\mathbf{i}_r - \left(\dfrac{1}{T_s} + j\omega_s\right)\boldsymbol{\psi}_s' + \dfrac{1}{L_m}\mathbf{u}_s \end{cases} \tag{3.78}$$

with: $\boldsymbol{\psi}_s' = \boldsymbol{\psi}_s / L_m$

After separating both equations into real and imaginary components, we obtain the complete electrical equation system of the DFIM.

$$\begin{cases} \dfrac{di_{rd}}{dt} = -\dfrac{1}{\sigma}\left(\dfrac{1}{T_r} + \dfrac{1-\sigma}{T_s}\right)i_{rd} + \omega_r i_{rq} + \dfrac{1-\sigma}{\sigma}\left(\dfrac{1}{T_s}\psi_{sd}' - \omega\psi_{sq}'\right) \\ \qquad + \dfrac{1}{\sigma L_r}u_{rd} - \dfrac{1-\sigma}{\sigma L_m}u_{sd} \\ \dfrac{di_{rq}}{dt} = -\omega_r i_{rd} - \dfrac{1}{\sigma}\left(\dfrac{1}{T_r} + \dfrac{1-\sigma}{T_s}\right)i_{rq} + \dfrac{1-\sigma}{\sigma}\left(\dfrac{1}{T_s}\psi_{sq}' + \omega\psi_{sd}'\right) \\ \qquad + \dfrac{1}{\sigma L_r}u_{rq} - \dfrac{1-\sigma}{\sigma L_m}u_{sq} \\ \dfrac{d\psi_{sd}'}{dt} = \dfrac{1}{T_s}i_{rd} - \dfrac{1}{T_s}\psi_{sd}' + \omega_s\psi_{sq}' + \dfrac{1}{L_m}u_{sd} \\ \dfrac{d\psi_{sq}'}{dt} = \dfrac{1}{T_s}i_{rq} - \omega_s\psi_{sd}' - \dfrac{1}{T_s}\psi_{sq}' + \dfrac{1}{L_m}u_{sq} \end{cases} \tag{3.79}$$

The main control objectives stated above is always the decoupled control of active and reactive current components. This suggests to choose the stator voltage – and respectively grid voltage – orientated reference frame for the further control design.

The realization of the grid voltage orientation requires the accurate and robust acquisition of the phase angle of the grid voltage fundamental wave, considering strong distortions due to converter mains pollution or background grid harmonics. Usually this is accomplished by means of a phase locked loop (PLL).

Summarizing the equation system (3.79) yields the following state space model for the DFIM in the grid voltage orientated reference frame:

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\,\mathbf{x} + \mathbf{B}_s\mathbf{u}_s + \mathbf{B}_r\mathbf{u}_r \tag{3.80}$$

with:

- State vector $\mathbf{x}^T = \left[i_{rd}, i_{rq}, \psi'_{sd}, \psi'_{sq}\right]$
- Stator voltage vector $\mathbf{u}_s^T = \left[u_{sd}, u_{sq}\right]$ as input vector on stator side
- Rotor voltage vector $\mathbf{u}_r^T = \left[u_{rd}, u_{rq}\right]$ as input vector on rotor side

The system matrix **A**, the rotor input matrix $\mathbf{B}_r$ and the stator input matrix $\mathbf{B}_s$ may be written as follows:

$$\mathbf{A} = \left[\begin{array}{cc|cc} -\frac{1}{\sigma}\left(\frac{1}{T_r}+\frac{1-\sigma}{T_s}\right) & \omega_r & \frac{1-\sigma}{\sigma T_s} & -\frac{1-\sigma}{\sigma}\omega \\ -\omega_r & -\frac{1}{\sigma}\left(\frac{1}{T_r}+\frac{1-\sigma}{T_s}\right) & \frac{1-\sigma}{\sigma}\omega & \frac{1-\sigma}{\sigma T_s} \\ \hline \frac{1}{T_s} & 0 & -\frac{1}{T_s} & \omega_s \\ 0 & \frac{1}{T_s} & -\omega_s & -\frac{1}{T_s} \end{array}\right]$$

$$\mathbf{B}_s = \left[\begin{array}{cc} -\frac{1-\sigma}{\sigma L_m} & 0 \\ 0 & -\frac{1-\sigma}{\sigma L_m} \\ \hline \frac{1}{L_m} & 0 \\ 0 & \frac{1}{L_m} \end{array}\right];\ \mathbf{B}_r = \left[\begin{array}{cc} \frac{1}{\sigma L_r} & 0 \\ 0 & \frac{1}{\sigma L_r} \\ \hline 0 & 0 \\ 0 & 0 \end{array}\right] \tag{3.81}$$

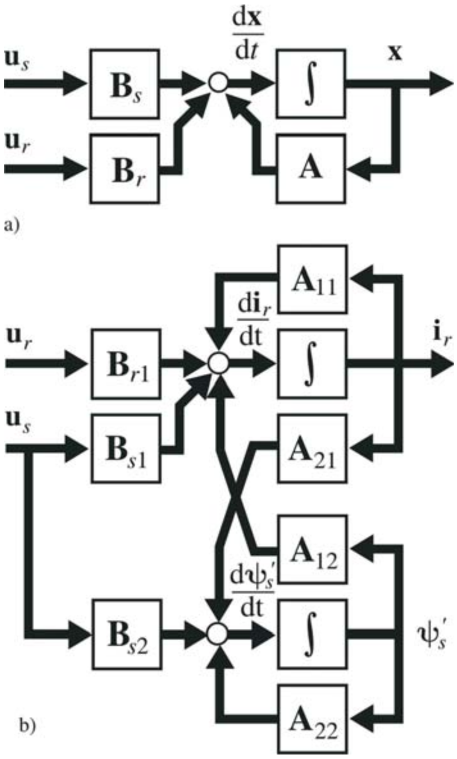

Fig. 3.14 Continuous state space model of the DFIM with stator flux and rotor current as state variables: (**a**) Common representation; (**b**) Split in partial matrices

The state space model of the DFIM is shown in figure 3.14a. The matrices of (3.81) may be split into partial matrices as follows, refer also to figure 3.14b.

$$\mathbf{A} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{A}_{21} & \mathbf{A}_{22} \end{array}\right]; \ \mathbf{B}_s = \left[\begin{array}{c} \mathbf{B}_{s1} \\ \hline \mathbf{B}_{s2} \end{array}\right]; \ \mathbf{B}_r = \left[\begin{array}{c} \mathbf{B}_{r1} \\ \hline \mathbf{0} \end{array}\right] \tag{3.82}$$

The state space model in partial matrices according to figure 3.14b shows that the rotor voltage $\mathbf{u}_r$ does not influence the stator flux ψ_s directly, but only in an indirect way through the rotor current $\mathbf{i}_r$. The stator flux is determined mainly by the stator voltage. The influence of $\mathbf{u}_s$ to $\mathbf{i}_r$ is like a constant disturbance, and therefore may be compensated by simple feedforward compensation.

3.4.2 Discrete state model of the DFIM

Like in sections 3.2.2 and 3.3.2 the time discrete state model of the DFIM may be obtained by iterative integration of (3.80), yielding the following matrix equation system as base model for the controller design:

$$\mathbf{x}(k+1)=\mathbf{\Phi}\mathbf{x}(k)+\mathbf{H}_s\mathbf{u}_s(k)+\mathbf{H}_r\mathbf{u}_r(k) \tag{3.83}$$

Transition matrix $\mathbf{\Phi}$, stator input matrix $\mathbf{H}_s$ and rotor input matrix $\mathbf{H}_r$ are given by:

$$\mathbf{\Phi}=\left[\begin{array}{cc|cc} 1-\frac{T}{\sigma}\left(\frac{1}{T_r}+\frac{1-\sigma}{T_s}\right) & \omega_r T & \frac{1-\sigma}{\sigma}\frac{T}{T_s} & -\frac{1-\sigma}{\sigma}\omega T \\ -\omega_r T & 1-\frac{T}{\sigma}\left(\frac{1}{T_r}+\frac{1-\sigma}{T_s}\right) & \frac{1-\sigma}{\sigma}\omega T & \frac{1-\sigma}{\sigma}\frac{T}{T_s} \\ \hline \frac{T}{T_s} & 0 & 1-\frac{T}{T_s} & \omega_s T \\ 0 & \frac{T}{T_s} & -\omega_s T & 1-\frac{T}{T_s} \end{array}\right]=\left[\begin{array}{c|c} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} \\ \hline \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} \end{array}\right]$$

$$\mathbf{H}_s=\left[\begin{array}{cc} -\frac{1-\sigma}{\sigma}\frac{T}{L_m} & 0 \\ 0 & -\frac{1-\sigma}{\sigma}\frac{T}{L_m} \\ \hline \frac{T}{L_m} & 0 \\ 0 & \frac{T}{L_m} \end{array}\right]=\left[\begin{array}{c} \mathbf{H}_{s1} \\ \hline \mathbf{H}_{s2} \end{array}\right];\ \mathbf{H}_r=\left[\begin{array}{cc} \frac{T}{\sigma L_r} & 0 \\ 0 & \frac{T}{\sigma L_r} \\ \hline 0 & 0 \\ 0 & 0 \end{array}\right]=\left[\begin{array}{c} \mathbf{H}_{r1} \\ \hline \mathbf{0} \end{array}\right] \tag{3.84}$$

The discrete state space model is shown in partial matrix form in figure 3.15a. Figure 3.15b shows the rotor current system, being the starting-point for the rotor current controller design. Due to the stiff mains system stator voltage $\mathbf{u}_s$ and stator flux ψ_s can be recognized as almost constant disturbances.

The figure 3.15a was produced by splitting of the equation (3.83) as follows:

$$\begin{cases} \mathbf{i}_r(k+1)=\mathbf{\Phi}_{11}\mathbf{i}_r(k)+\mathbf{\Phi}_{12}\psi_s'(k)+\mathbf{H}_{s1}\mathbf{u}_s(k)+\mathbf{H}_{r1}\mathbf{u}_r(k) \\ \psi_s'(k+1)=\mathbf{\Phi}_{21}\mathbf{i}_r(k)+\mathbf{\Phi}_{22}\psi_s'(k)+\mathbf{H}_{s2}\mathbf{u}_s(k) \end{cases} \tag{3.85}$$

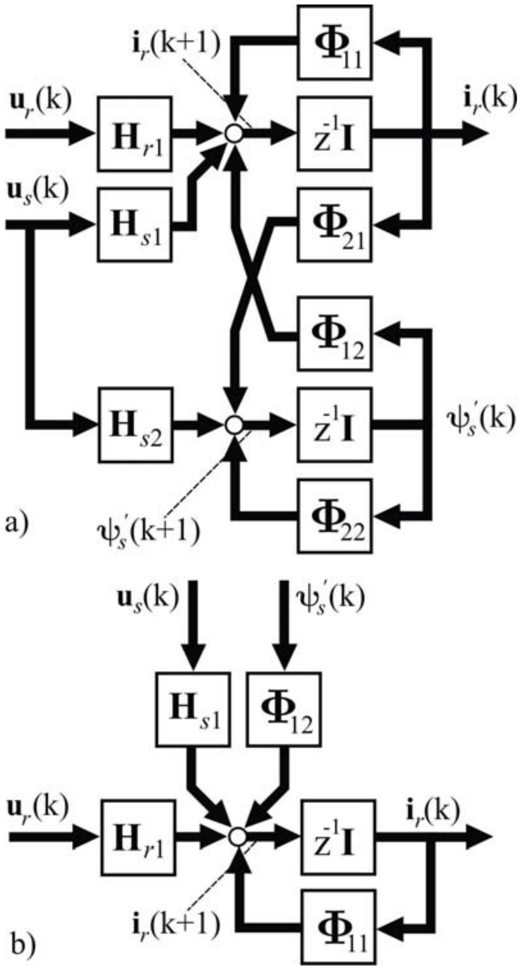

Fig. 3.15 Discrete state model of the DFIM: (**a**) in grid voltage orientated reference frame; (**b**) rotor current process model

3.5 Generalized current process model for the two machine types IM and PMSM

In evaluation of the equations (3.51), (3.55) and (3.73) as well as the figures 3.8, 3.10 and 3.13 the formal identity of the two machine types IM and PMSM regarding system structure and system order becomes clearly visible. Therefore it can be regarded theoretically proven, that with respect to hardware and software an identical concept for the stator current impression may be applied in the stator-fixed as well as in the field synchronous coordinate system. In this section a uniform description for all system elements will be derived to support a parallel treatment of both current process models under investigation. The following symbols are defined:

- $\mathbf{\Phi}$: Transition matrices $\mathbf{\Phi}_{11}^{s}$ or $\mathbf{\Phi}_{11}^{f}$ or $\mathbf{\Phi}_{SM}^{f}$
- $\mathbf{H}$: Input matrices $\mathbf{H}_{1}^{s}$ or $\mathbf{H}_{1}^{f}$ or $\mathbf{H}_{SM}^{f}$
- $\mathbf{h}$: Disturbance matrices or vector $\mathbf{\Phi}_{12}^{s}$ or $\mathbf{\Phi}_{12}^{f}$ or $\mathbf{h}$, which represent the intervention of the flux dependent disturbance quantity.

$\mathbf{h}$ is a 2×2 matrix in the case of the IM and only a simple vector in the case of the PMSM. The input vector $\mathbf{u}_s$ and the state vector $\mathbf{i}_s$ will be written without the subscripts „*s*“ (for stator-fixed) or „*f*“ (for field synchronous coordinate system). This index can be attached later in the concrete choice of the coordinate system to be used. For the rotor and pole flux the symbol ψ is used instead of ψ_r^s or ψ_r^f or ψ_p. With these arrangements the following common equation results for the current process models:

$$\mathbf{i}_s(k+1) = \mathbf{\Phi}\,\mathbf{i}_s(k) + \mathbf{H}\,\mathbf{u}_s(k) + \mathbf{h}\,\psi(k) \tag{3.86}$$

and in the z domain:

$$z\,\mathbf{i}_s(z) = \mathbf{\Phi}\,\mathbf{i}_s(z) + \mathbf{H}\,\mathbf{u}_s(z) + \mathbf{h}\,\psi(z) \tag{3.87}$$

with the characteristic equation:

$$\det\left[z\,\mathbf{I} - \mathbf{\Phi}\right] = 0 \text{ with } \mathbf{I} = \text{ unity matrix} \tag{3.88}$$

The figure 3.16 shows the current process models for the following three cases in the overview:

1. IM in the stator-fixed,
2. IM in the field synchronous and
3. PMSM in the field synchronous coordinate system.

The equation (3.87) as well as the characteristic equation (3.88) are given in the z domain which is advisable for the treatment of discrete systems.

Here the similarity between the model in the figure 3.16 and the current process model of the DFIM in equation (3.85), shown in the figure 3.15b, can also be easily recognized. Because the three models represent linear and time-variant processes, linear current controllers using:

- output feedback or
- state feedback

can be designed. The method to derive these three linear and time-variant process models can be called the linearization within the sampling period. This is possible because the models have been derived under the conditions that:

- the stator-side angular velocity ω_s in the case of the IM or PMSM, and
- the rotor-side angular velocity ω_r in the case of the DFIM are constant within one sampling period.

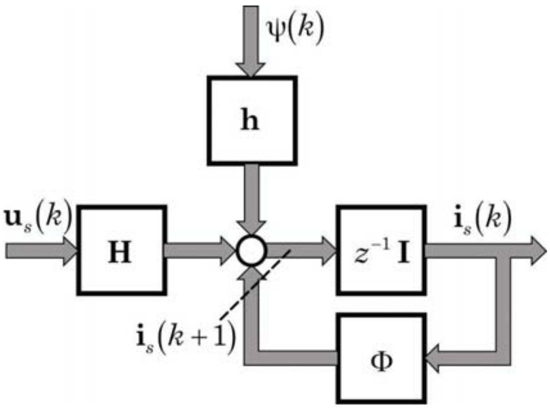

Fig. 3.16 General current process model of IM and PMSM

3.6 Nonlinear properties of the machine models and the way to nonlinear controllers

Electrical 3-phase AC machines exhibit different nonlinearities because of the mechanical construction of their magnetic paths with slots and air-gaps. But only two types of nonlinearities are relevant for the controller design:

- *The nonlinear structure of the process models*: This nonlinearity is caused by products between states variables like current components and input variables ω_s (in cases IM and PMSM), ω_r (in the case DFIM). This structural nonlinearity can only be mastered completely by nonlinear controllers designed – for example – using methods like exact linearization or backstepping based concepts.
- *The nonlinear parameters*: Some parameters like the mutual inductance depends on the rotor flux which is a state variable. The problem with parametric nonlinearities can be solved by identification and adaptation methods.

Because the backstepping based design still is not a mature method for using in the practice, the section 3.6 only deals with the idea of the exact linearization which can be used to master the structural nonlinearities of the process models of the IM, DFIM and PMSM, and to design nonlinear controllers for improving the control performance in difficult operation situations.

3.6.1 Idea of the exact linearization

For the understanding, at first the idea of the relative difference order of a linear system without dead time – the SISO process – shall be explained. If the linear SISO process is represented by the following transfer function:

$$G(s)=\frac{y(s)}{u(s)}=\frac{b_0+b_1s+\cdots+b_ps^p}{a_0+a_1s+\cdots+a_qs^q};\quad p<q \tag{3.89}$$

then the pole surplus r with:

$$r=q-p\geq 1 \tag{3.90}$$

can be called the *relative difference order* of the process model described by equation (3.89). If the linear process model is a MISO system with m inputs and only one output, i.e. a system with m transfer functions in a form similar to equation (3.89), then the integer number r:

$$r=\min_i r_i \quad \text{with} \quad 1\leq i\leq m \tag{3.91}$$

means the relative difference order of the MISO system, in which r_i is the pole surplus of the ith transfer function. If the definition according to the formula (3.91) is applied to a linear process with m inputs and m outputs, the following vector **r** of the relative difference orders is obtained:

$$\mathbf{r}=\left[r_1,r_2,\cdots,r_m\right] \tag{3.92}$$

with m natural numbers r_j (j = 1, 2, ... , m), and r_j the relative difference order of the j^{th} output. Because the process described by the model (3.89) can be represented in the state space, the relative difference order r and respectively the vector **r** of relative difference orders can also be calculated using state space models.

Some classes of nonlinear systems with m inputs and m outputs, the so called nonlinear MIMO systems, can be described by the following equations:

$$\begin{cases}\dfrac{d\mathbf{x}}{dt}=\mathbf{f}(\mathbf{x})+\mathbf{H}(\mathbf{x})\mathbf{u}\\ \mathbf{y}=\mathbf{g}(\mathbf{x})\end{cases} \tag{3.93}$$

with:

$$\mathbf{x}=\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix};\mathbf{u}=\begin{pmatrix}u_1\\ \vdots\\ u_m\end{pmatrix};\mathbf{g}(\mathbf{x})=\begin{pmatrix}g_1(\mathbf{x})\\ \vdots\\ g_m(\mathbf{x})\end{pmatrix} \tag{3.94}$$

$$\mathbf{H}(\mathbf{x})=\left(\mathbf{h}_1(\mathbf{x}),\quad \mathbf{h}_2(\mathbf{x}),\quad \cdots\quad ,\mathbf{h}_m(\mathbf{x})\right)$$

Similarly to the linear systems, for the system in the equation (3.93) a vector of relative difference orders like (3.92) can also be derived.

The basic idea of the *exact linearization* can be summarized as follows: If the nonlinear MIMO system in the form (3.93) contains a vector of relative difference orders like equation (3.92), which fulfills the following condition:

$$r=r_1+r_2+\cdots+r_m=n \tag{3.95}$$

then the system (3.93) can be transformed using the coordinate transformation:

$$\mathbf{z}=\begin{pmatrix}z_1\\ \vdots\\ z_n\end{pmatrix}=\mathbf{m}(\mathbf{x})=\begin{pmatrix}m_1^1(\mathbf{x})\\ \vdots\\ m_{r_1}^1(\mathbf{x})\\ \vdots\\ m_1^m(\mathbf{x})\\ \vdots\\ m_{r_m}^m(\mathbf{x})\end{pmatrix}=\begin{pmatrix}g_1(\mathbf{x})\\ \vdots\\ L_f^{r_1-1}g_1(\mathbf{x})\\ \vdots\\ g_m(\mathbf{x})\\ \vdots\\ L_f^{r_m-1}g_m(\mathbf{x})\end{pmatrix} \tag{3.96}$$

into the following linear MIMO system:

$$\begin{cases}\dfrac{d\mathbf{z}}{dt}=\mathbf{Az}+\mathbf{Bw}\\ \mathbf{y}=\mathbf{Cz}\end{cases} \tag{3.97}$$

The original input **u** is then controlled by the coordinate transformation law:

$$\mathbf{u}=\mathbf{a}(\mathbf{x})+\mathbf{L}^{-1}(\mathbf{x})\mathbf{w} \tag{3.98}$$

The vector **a**(**x**) and the matrix $\mathbf{L}^{-1}(\mathbf{x})$ in (3.98) look as follows:

$$\mathbf{L}(\mathbf{x})=\begin{pmatrix}L_{h_1}L_f^{r_1-1}g_1(\mathbf{x}) & \cdots & L_{h_m}L_f^{r_1-1}g_1(\mathbf{x})\\ \vdots & \ddots & \vdots\\ L_{h_1}L_f^{r_m-1}g_m(\mathbf{x}) & \cdots & L_{h_m}L_f^{r_m-1}g_m(\mathbf{x})\end{pmatrix};\mathbf{a}(\mathbf{x})=-\mathbf{L}^{-1}(\mathbf{x})\begin{pmatrix}L_f^{r_1}g_1(\mathbf{x})\\ \vdots\\ L_f^{r_m}g_m(\mathbf{x})\end{pmatrix} \tag{3.99}$$

Formula (3.99) also requires the ability, with respect to the coordinate transformation or to the exact linearization, to invert the matrix **L**(**x**). In equations (3.96) and (3.99), the term

$$L_f g(\mathbf{x}) = \frac{\partial g(\mathbf{x})}{\partial \mathbf{x}} \mathbf{f}(\mathbf{x}) \tag{3.100}$$

notifies the Lie derivation of the function $g(\mathbf{x})$ along the trajectory $\mathbf{f}(\mathbf{x})$. The details of the complicated general expressions for the matrices **A**, **B** and **C** are abandoned here. The figure 3.17 illustrates the explained facts so far.

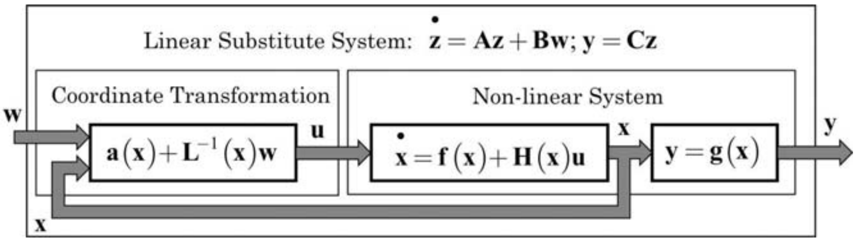

Fig. 3.17 Transformation of a nonlinear system into a linear substitute system

Here it must be highlighted that the coordinate transformation requires exact knowledge of the complete state vector **x**, which can not always be assumed for 3-phase AC machines.

3.6.2 Nonlinearities of the IM model

The nonlinearity of the IM is clearly represented by the equation (3.46). The two first equations of the system (3.44), which represent the current process model, will be separated and extended by the field angle ϑ_s.

$$\begin{cases} \dfrac{di_{sd}}{dt} = -\left(\dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}\right) i_{sd} + \omega_s\, i_{sq} + \dfrac{1-\sigma}{\sigma T_r} \psi'_{rd} + \dfrac{1}{\sigma L_s} u_{sd} \\ \dfrac{di_{sq}}{dt} = -\omega_s\, i_{sd} - \left(\dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}\right) i_{sq} - \dfrac{1-\sigma}{\sigma} \omega \psi'_{rd} + \dfrac{1}{\sigma L_s} u_{sq} \\ \dfrac{d\vartheta_s}{dt} = \omega_s \end{cases} \tag{3.101}$$

For better understanding temporary parameters and variables are introduced:

- Parameters: $a = 1/\sigma L_s\,;\, b = 1/\sigma T_s\,;\, c = (1-\sigma)/\sigma T_r\,;\, d = b + c$

- State variables: $x_1 = i_{sd}\,;\, x_2 = i_{sq}\,;\, x_3 = \vartheta_s$
- Input variables: $u_1 = u_{sd}\,;\, u_2 = u_{sq}\,;\, u_3 = \omega_s$
- Output variables: $y_1 = i_{sd}\,;\, y_2 = i_{sq}\,;\, y_3 = \vartheta_s$

Now the current process model looks as follows:

$$\begin{cases} \dfrac{dx_1}{dt} = -d\,x_1 + x_2 u_3 + a\,u_1 + c\psi'_{rd} \\ \dfrac{dx_2}{dt} = -x_1 u_3 - d\,x_2 + a\,u_{sq} - c\,T_r\,\omega\,\psi'_{rd} \\ \dfrac{dx_3}{dt} = u_3 \end{cases} \tag{3.102}$$

or:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -d\,x_1 + c\,\psi'_{rd} \\ -d\,x_2 - c\,T_r\,\omega\,\psi'_{rd} \\ 0 \end{bmatrix} + \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} x_2 \\ -x_1 \\ 1 \end{bmatrix} u_3 \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{cases} \tag{3.103}$$

The system (3.103) can now be transferred to the general form with the equation (3.93).

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{h}_1\,u_1 + \mathbf{h}_2\,u_2 + \mathbf{h}_3\,u_3 \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \end{cases} \tag{3.104}$$

with:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -d\,x_1 + c\,\psi'_{rd} \\ -d\,x_2 - c\,T_r\,\omega\,\psi'_{rd} \\ 0 \end{bmatrix};\, \mathbf{h}_1 = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix};\, \mathbf{h}_1 = \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix};\, \mathbf{h}_3 = \begin{bmatrix} x_2 \\ -x_1 \\ 1 \end{bmatrix} \tag{3.105}$$

$$y_1 = g_1(\mathbf{x}) = x_1;\; y_2 = g_2(\mathbf{x}) = x_2;\; y_3 = g_3(\mathbf{x}) = x_3$$

The equation (3.104) represents the new process model and will be used later to design the nonlinear current control loop using exact linearization.

3.6.3 Nonlinearities of the DFIM model

Similar to the case IM, the nonlinearity of the DFIM is represented by the following equation separated from the equation (3.79) and extended by the rotor angle ϑ_r.

$$\begin{cases} \dfrac{di_{rd}}{dt} = -\dfrac{1}{\sigma}\left(\dfrac{1}{T_r}+\dfrac{1-\sigma}{T_s}\right)i_{rd} + \omega_r i_{rq} + \dfrac{1-\sigma}{\sigma}\left(\dfrac{1}{T_s}\psi'_{sd} - \omega\psi'_{sq}\right) \\ \qquad + \dfrac{1}{\sigma L_r}u_{rd} - \dfrac{1-\sigma}{\sigma L_m}u_{sd} \\ \dfrac{di_{rq}}{dt} = -\omega_r i_{rd} - \dfrac{1}{\sigma}\left(\dfrac{1}{T_r}+\dfrac{1-\sigma}{T_s}\right)i_{rq} + \dfrac{1-\sigma}{\sigma}\left(\dfrac{1}{T_s}\psi'_{sq} + \omega\psi'_{sd}\right) \\ \qquad + \dfrac{1}{\sigma L_r}u_{rq} - \dfrac{1-\sigma}{\sigma L_m}u_{sq} \\ \dfrac{d\vartheta_r}{dt} = \omega_r \end{cases} \tag{3.106}$$

After substituting the newly defined temporary parameters:

$$a = \left(\frac{1}{\sigma T_r}+\frac{1-\sigma}{\sigma T_s}\right); b = \frac{1-\sigma}{\sigma}; c = \frac{1}{\sigma L_r}; d = \frac{1-\sigma}{\sigma L_m}; e = \frac{1-\sigma}{\sigma T_s}$$

in the partial model of the rotor current in the equation (3.106), the following model is obtained:

$$\begin{cases} \dfrac{di_{rd}}{dt} = -a i_{rd} + \omega_r i_{rq} + e\psi'_{sd} - b\omega\psi'_{sq} + c u_{rd} - d u_{sd} \\ \dfrac{di_{rq}}{dt} = -\omega_r i_{rd} - a i_{rq} + b\omega\psi'_{sd} + e\psi'_{sq} + c u_{rq} + d u_{sq} \\ \dfrac{d\vartheta_r}{dt} = \omega_r \end{cases} \tag{3.107}$$

New vectors will now be defined as follows:

- Vector of state variables:

$$\mathbf{x}^T = [x_1 \quad x_2 \quad x_3]; x_1 = i_{rd}; x_2 = i_{rq}; x_3 = \theta_r$$

- Vector of input variables:

$$\mathbf{u}^T = [u_1 \quad u_2 \quad u_3]; \quad u_1 = e\psi'_{sd} - b\omega\psi'_{sq} + c u_{rd} - d u_{sd}$$

$$u_2 = b\omega\psi'_{sd} + e\psi'_{sq} + c u_{rq} + d u_{sq}; \quad u_3 = \omega_r$$

- Vector of output variables:

$$\mathbf{y}^T=\begin{bmatrix}y_1 & y_2 & y_3\end{bmatrix};\ y_1=i_{rd};\ y_2=i_{rq};\ y_3=\theta_r$$

Finally, the following nonlinear DFIM model in the detailed:

$$\begin{cases}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-ax_1\\ -ax_2\\ 0\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}u_1+\begin{bmatrix}0\\1\\0\end{bmatrix}u_2+\begin{bmatrix}x_2\\ -x_1\\ 1\end{bmatrix}u_3\\ \begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\end{cases} \tag{3.108}$$

or in the generalized form is obtained:

$$\begin{cases}\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x})+\mathbf{h}_1(\mathbf{x})u_1+\mathbf{h}_2(\mathbf{x})u_2+\mathbf{h}_3(\mathbf{x})u_3\\ \mathbf{y}=\mathbf{g}(\mathbf{x})\end{cases} \tag{3.109}$$

with:

$$\mathbf{f}(\mathbf{x})=\begin{bmatrix}-ax_1\\ -ax_2\\ 0\end{bmatrix};\mathbf{h}_1(\mathbf{x})=\begin{bmatrix}1\\0\\0\end{bmatrix};\mathbf{h}_2(\mathbf{x})=\begin{bmatrix}0\\1\\0\end{bmatrix};\mathbf{h}_3(\mathbf{x})=\begin{bmatrix}x_2\\ -x_1\\ 1\end{bmatrix} \tag{3.110}$$

$$g_1(\mathbf{x})=x_1;\ g_2(\mathbf{x})=x_2;\ g_3(\mathbf{x})=x_3$$

The equations (3.108) and respectively (3.109) are starting points for the later design of the nonlinear controller for DFIM systems.

3.6.4 Nonlinearities of the PMSM model

The model (3.64) of the PMSM will now be extended by the field angle ϑ_s, similarly to the IM in equation (3.101).

$$\begin{cases} \dfrac{di_{sd}}{dt} = -\dfrac{1}{T_{sd}} i_{sd} + \omega_s \dfrac{L_{sq}}{L_{sd}} i_{sq} + \dfrac{1}{L_{sd}} u_{sd} \\ \dfrac{di_{sq}}{dt} = -\omega_s \dfrac{L_{sd}}{L_{sq}} i_{sd} - \dfrac{1}{T_{sq}} i_{sq} + \dfrac{1}{L_{sq}} u_{sq} - \omega_s \dfrac{\psi_p}{L_{sq}} \\ \dfrac{d\vartheta_s}{dt} = \omega_s \end{cases} \tag{3.111}$$

With newly introduced variables and temporary parameters:

- state variables: $x_1 = i_{sd}$; $x_2 = i_{sq}$; $x_3 = \vartheta_s$
- input variables: $u_1 = u_{sd}$; $u_2 = u_{sq}$; $u_3 = \omega_s$
- output variables: $y_1 = i_{sd}$; $y_2 = i_{sq}$; $y_3 = \vartheta_s$
- temporary parameters: $a = 1/L_{sd}$; $b = 1/L_{sq}$; $c = 1/T_{sd}$; $a = 1/T_{sq}$;

the equation (3.111) can be transferred to:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -cx_1 \\ -dx_2 \\ 0 \end{bmatrix} + \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} \dfrac{a}{b} x_2 \\ -\dfrac{b}{a} x_1 - b\psi_p \\ 1 \end{bmatrix} u_3 \\ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \end{cases} \tag{3.112}$$

or to the following generalized form:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{h}_1(\mathbf{x}) u_1 + \mathbf{h}_2(\mathbf{x}) u_2 + \mathbf{h}_3(\mathbf{x}) u_3 \\ \mathbf{y} = \mathbf{g}(\mathbf{x}) \end{cases} \tag{3.113}$$

with:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -cx_1 \\ -dx_2 \\ 0 \end{bmatrix}; \mathbf{h}_1(\mathbf{x}) = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}; \mathbf{h}_2(\mathbf{x}) = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}; \mathbf{h}_3(\mathbf{x}) = \begin{bmatrix} \dfrac{a}{b} x_2 \\ -\dfrac{b}{a} x_1 - b\psi_p \\ 1 \end{bmatrix} \tag{3.114}$$

$$y_1 = g_1(\mathbf{x}) = x_1;\ y_2 = g_2(\mathbf{x}) = x_2;\ y_3 = g_3(\mathbf{x}) = x_3$$

The equations (3.113) and (3.114) can be used to design nonlinear controllers for systems using 3-phase AC machines of type PMSM.

3.7 References to chapter 3

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