An Online Truthful Mechanism for Software Defined Collaborative Offloading in Mobile Edge Computing

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I. Theoretical Analysis

In this section, we prove the desired properties of our online auction, including computational tractability, individual rationality, and truthfulness. Moreover, a detailed competitive analysis is also conducted.

A. Properties of the Auction

***Theorem** 1:* The proposed auction mechanism is computationally tractable, and the time complexity is $O(MRTJ)$.

Proof:

The for-loop (line 2-9) takes $O(MRT)$ steps and the running time of line 10-19 is $O(M)$. Therefore, the time complexity of ONTA-SWM-Alloc is $O(MRT)$. In the ONTA-SWM, for every bid $\theta_i^j \in \Theta$, line 8 which calls function ONTA-SWM-Alloc consumes $O(MRT)$ time. In summary, the time complexity of ONTA-SWM is $O(MRT\Theta)$. ■

***Theorem** 2:* The proposed auction mechanism is individually rational.

Proof: Recall that μ_{ij} definition. If $\mu_{ij} = 0$, the auctioneer rejects the bid θ_i^j, and the user's utility is $u_{ij} = 0$. If $\mu_{ij} > 0$, the bid θ_i^j is accepted. Hence, the demander's utility $u_{ij} = \mu_{ij} > 0$. In conclusion, the user's utility u_j is always non-negative to guarantee the individual rationality. ■

***Lemma** 1:* The allocation rule of ONTA-SWM is monotonic.

Proof: We assume $\theta_i^{\prime j} \succeq \theta_i^j$ where $\theta_i^j = \{t_{ij}^-, \eta_{ij}, D_{ij}, b_i^j\}$ and $\theta_i^j = \{t_{ij}^{\prime -}, \eta_{ij}^\prime, D_{ij}^\prime, b_i^{\prime j}\}$. Then we have to show that the fact that the bid θ_i^j is accepted implies that the bid $\theta_i^{\prime j}$ is too accepted. Recall the definition, there are four cases : (1) the smaller execution time, i.e, $\eta_{ij} \geq \eta_{ij}^\prime$. Recall the payment rule is non-decreasing with the task exciton time. Therefore, $\pi_{ij} \geq \pi_{ij}^\prime$. According to the allocation rule, bid $\theta_i^{\prime j}$ must wins if θ_i^j wins. (2) the less resource demand, i.e, $D_{ij} \succeq D_{ij}^\prime$. Likewise, the payment rule is non-decreasing with less resource demand and a user will pay more if it occupies more resource. Hence, $\pi_{ij} \geq \pi_{ij}^\prime$ and bid $\theta_i^{\prime j}$ must wins if θ_i^j wins. (3) the less bid value, i.e, $b_i^j \geq b_i^{\prime j}$. Recall the allocation rule is increasing with the bid value and bid $\theta_i^{\prime j}$ must wins if θ_i^j wins. In conclusion, the allocation rule of ONTA-SWM is monotonic ■

***Lemma** 2:* The ONTA-SWM charges each winning bid by its critical value.

Proof: According allocation rule and payment rule, the bid θ_i^j is accepted if bid value $b_i^j \geq \pi_{ij}$ and is rejectd otherwise. By the definition , π_{ij} is exactly the critical value c_i^j for bid θ_i^j. ■

***Theorem** 3:* The proposed auction mechanism is truthful.

Proof: By lemma 1, 2 and the Myerson Theorem, we have proven the ONTA-SWM is: (1)truthful in execution time; (2)truthful in bid value; (3)truthful in resource demand. ■

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B. Competitive Analysis

***Lemma** 3:* If we can find out a constant $\sigma \geq 1$ such that the inequality $(S^{[\gamma]} - S^{[\gamma-1]}) \geq \frac{1}{\sigma}(D^{[\gamma]} - D^{[\gamma-1]})$ holds true for each iteration γ, the competitive ratio of our online algorithm is σ.

Proof: By summing up the inequalities for each iteration γ, we have:

$$\begin{aligned} S^{[\gamma]} - S^{[0]} &= \sum_{\gamma \in \Gamma} (S^{[\gamma]} - S^{[\gamma-1]}) \\ &\geq \sum_{\gamma \in \Gamma} \frac{1}{\sigma} (D^{[\gamma]} - D^{[\gamma-1]}). \end{aligned} \tag{1}$$

Namely, $(S^{[\Gamma]} - S^{[0]}) \geq \frac{1}{\sigma}(D^{[\Gamma]} - D(0))$. Specifically, the proposed algorithm guarantees that $S^{[0]} = 0$ and $D(0) = 0$ through initialization. Hence, it suffices to show that $S^{[\gamma]} \geq \frac{1}{\sigma} D^{[\Gamma]}$. By the weak duality, $D^{[\Gamma]} \geq OPT$ holds true. Then, we have the inequality $S^{[\gamma]} \geq \frac{1}{\sigma} OPT$, which means that our auction mechanism is σ-competitive. ■

***Lemma** 4:* If the inequality

$$\begin{aligned} &\frac{1}{\sigma_r} (\bar{O}_m^r(\rho_m^{r[\gamma]}(t)) - \bar{O}_m^r(\rho_m^{r[\gamma-1]}(t))) \leq \\ &\qquad \rho_m^{r[\gamma]}(t)(z_m^{r[\gamma]}(t) - z_m^{r[\gamma-1]}(t)) \\ &\qquad\quad - (O_m^r(d_m^{r[\gamma]}(t)) - O_m^r(d_m^{r[\gamma-1]}(t)) \end{aligned} \tag{2}$$

holds, then $(S^{[\gamma]} - S^{[\gamma-1]}) \geq \frac{1}{\sigma}(D^{[\gamma]} - D^{[\gamma-1]})$.

Proof: Assume that the bid θ_i^j is processed in the γ-th iteration. Consider two kinds of cases, i.e., the bid θ_i^j is rejected, and the opposite where the bid θ_i^j is accepted. In the former case, we easily obtain $S^{[\gamma]} - S^{[\gamma-1]} = D^{[\gamma]} - D^{[\gamma-1]} = 0$. In the following, we elaborate the latter case. If the bid θ_i^j is accepted and scheduled on MD m,

$$\begin{aligned} &S^{[\gamma]} - S^{[\gamma-1]} \\ &= b_i^j - \sum_{t \in [t_{ij}^-, t_{ij}^+]} \sum_{r \in \mathcal{R}} (O_m^r(z_m^{r[\gamma]}(t)) - O_m^r(z_m^{r[\gamma-1]}(t))) \\ &\overset{(a)}{=} \mu_{ij} + \sum_{t \in [t_{ij}^-, t_{ij}^+]} \sum_{r \in \mathcal{R}} \rho_m^{r[\gamma-1]}(t)(z_m^{r[\gamma]}(t) - z_m^{r[\gamma-1]}(t)) \\ &\quad - \sum_{t \in [t_{ij}^-, t_{ij}^+]} \sum_{r \in \mathcal{R}} (O_m^r(z_m^{r[\gamma]}(t)) - O_m^r(z_m^{r[\gamma-1]}(t))) \end{aligned} \tag{3}$$

The equality (a) follows from the fact in the Algorithm. Additionally, we can obtain the difference of the dual obejective function.

$$\begin{aligned} &D^{[\gamma]} - D^{[\gamma-1]} \\ &= \mu_{ij} + \sum_{t \in [t_{ij}^-, t_{ij}^+]} \sum_{r \in \mathcal{R}} (\bar{O}_m^r(\rho_m^{r[\gamma]}(t)) - \bar{O}_m^r(\rho_m^{r[\gamma-1]}(t))) \end{aligned}$$

By summing up inequality (2) over $r \in \mathcal{R}, t \in [t_{ij}^-, t_{ij}^+]$, we can have

$$\begin{aligned} &S^{[\gamma]} - S^{[\gamma-1]} \\ &\geq \mu_{ij} + \frac{1}{\sigma} \sum_{t \in [t_{ij}^-, t_{ij}^+]} \sum_{r \in \mathcal{R}} (\bar{g}_m^r(\rho_m^{r[\gamma]}(t)) - \bar{g}_m^r(\rho_m^{r[\gamma-1]}(t))) \\ &= \mu_{ij} + \frac{1}{\sigma}(D^{[\gamma]} - D^{[\gamma-1]} - \mu_{ij}) \\ &= \frac{1}{\sigma}(D^{[\gamma]} - D^{[\gamma-1]}) + (1 - \frac{1}{\sigma})\mu_{ij} \end{aligned} \tag{4}$$

Since $\mu_{ij} \geq 0$ and $\sigma > 1$, it is obvious that $S^{[\gamma]} - S^{[\gamma-1]} \geq \frac{1}{\sigma}(D^{[\gamma]} - D^{[\gamma-1]})$. ■

***Theorem** 4:* Our auction is σ-competitive, where $\sigma = \max_{r \in \mathcal{R}, m \in \mathcal{M}} \ln(\dfrac{2R(H_r - h_m^r)}{L_r - h_m^r})$.

Proof:

Note that $\mathrm{d}P_m^r(z_m^r(t)) = \frac{1}{C_m^r}(P_m^r - h_m^r)\sigma_r \mathrm{d}z_m^r(t)$, where $\sigma_r = \ln(\dfrac{2R(H_r - h_m^r)}{L_r - h_m^r})$. If we take $\sigma = \max_{r\in R}\sigma_r$, we obtain the following inequality for $\forall m \in \mathcal{M}$, $\forall t \in \mathcal{T}$, $\forall r \in R$:

$$\frac{1}{\sigma}C_m^r \mathrm{d}\rho_m^r(t) \leq (\rho_m^r(t) - h_m^r)\mathrm{d}z_m^r(t), \tag{5}$$

In this case, $\mathrm{d}O_m^r(z_m^r(t)) = h_m^r \mathrm{d}z_m^r(t)$ and $\mathrm{d}\bar{O}_m^r(\rho_m^r(t)) = C_m^r \mathrm{d}\rho_m^r(t)$. By putting them into (5), we have the inequality:

$$\frac{1}{\sigma}\mathrm{d}\bar{O}_m^r(\rho_m^r(t)) \leq \rho_m^r(t)\mathrm{d}z_m^r(t) - \mathrm{d}O_m^r(z_m^r(t)), \tag{6}$$

which has proven inequality (2).

■

In conclusion, we have $S^{[\gamma]} - S^{[\gamma-1]} \geq \frac{1}{\sigma}(D^{[\gamma]} - D^{[\gamma-1]})$ for every iteration γ. By Lemma 3, we have proven that our auction is σ-competitive.